Single qubit measurement in a single qubit system

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$\langle \hat{\sigma}_z \rangle = \langle \psi | \sigma_z | \psi \rangle = \langle \psi | (|0\rangle\langle 0| - |1\rangle\langle 1|) | \psi \rangle =$$

$$= \langle \psi | 0 \rangle\langle 0 | \psi \rangle - \langle \psi | 1 \rangle\langle 1 | \psi \rangle \quad \circledast$$

$$= |\langle 0|\psi\rangle|^2 - |\langle 1|\psi\rangle|^2 = P_0 - P_1 \quad \circledast$$

Single qubit measurement in a 2-qubit system

$\circledast$ measuring the first qubit $\quad \hat{\sigma}_z \otimes \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= |00\rangle\langle 00| + |01\rangle\langle 01| - |10\rangle\langle 10| - |11\rangle\langle 11|$$

$$\langle \hat{\sigma}_z \otimes \hat{I} \rangle = \langle \psi | \hat{\sigma}_z \otimes \hat{I} | \psi \rangle = \langle \psi | (|00\rangle\langle 00| + |01\rangle\langle 01| - |10\rangle\langle 10| - |11\rangle\langle 11|) | \psi \rangle$$

following the same logic as in $\circledast$

$$= P_{00} + P_{01} - P_{10} - P_{11}$$