Expectation values

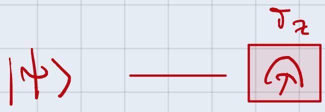
$$\langle \sigma_z \rangle = \langle \psi | \sigma_z | \psi \rangle = \langle \psi | 0 \rangle\langle 0 | \psi \rangle - \langle \psi | 1 \rangle\langle 1 | \psi \rangle$$

$$= |\langle 0 | \psi \rangle|^2 - |\langle 1 | \psi \rangle|^2$$

$$= p_0 - p_1$$

$$\langle \hat{\sigma}_z \rangle$$

Circuit:

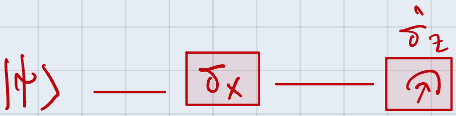


$$\langle \hat{\sigma}_x \rangle = \langle \psi | \sigma_x | \psi \rangle = \langle \psi | H . \sigma_z . H | \psi \rangle = \langle \psi H | \sigma_z | H \psi \rangle$$

$$= p_0^{\psi'} - p_1^{\psi'} \qquad \text{s.t. } |\psi'\rangle = H|\psi\rangle .$$

$$\langle \hat{\sigma}_x \rangle$$

Circuit :



$$\langle \hat{\sigma}_y \rangle = \langle \psi | \sigma_y | \psi \rangle = \langle \psi | S . H . \sigma_z . H . S^\dagger | \psi \rangle$$

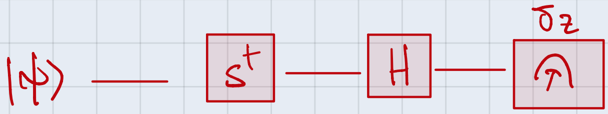
$$\hat{\sigma}_y = S . \sigma_x . S^\dagger \qquad = \langle \psi | (H S^\dagger)^\dagger . \sigma_z . H S^\dagger | \psi \rangle$$

$$= S . H . \sigma_z . H . S^\dagger$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}}_{S} . \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{X} . \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}}_{S^\dagger} = \begin{pmatrix} 0 & 1 \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Circuit :



$$\langle \hat{\sigma}_y \rangle$$

$$\langle \sigma_z \otimes \cdots \otimes \sigma_z \rangle = \langle \psi | \sigma_z \otimes \cdots \otimes \sigma_z | \psi \rangle$$

$$= P_{0\ldots0} \,\square\, P_{0\ldots1} \,\square\, P_{0\ldots10}$$

depend on the number of 1's in the binary string.

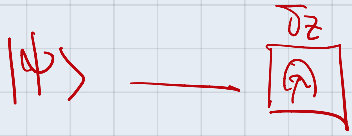
If # 1's is even then the sign is <u>plus</u> because the minus sign cancels.

otherwise if # 1's is odd then is <u>minus</u>

$$\Downarrow$$

$$\boxed{(-1)^{H(i) \bmod 2}}$$

<u>circuit</u>



Pauli string $\langle \sigma_z \otimes \sigma_z \otimes \cdots \otimes \sigma_z \rangle$