Holonomic Optimal Control for Qudits

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Outline

Motivation

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."
- Richard Feynman

History (1/2)

1980 Paul Benioff proposes a quantum mechanical model of the turing machine. [1]

1980s Both Richard Feynman and Yuri Manin suggests that a quantum based computer could perform calculations not possible on a conventional computer. [2] [3]

1994 Peter Shor publishes a quantum algorithm used for finding the prime-factors of an integer. [4] (This is big!)

[1] Benioff, Paul (1980). Journal of Statistical Physics.22(5):563.

[2] Feynman, Richard (June 1982). International Journal of Theoretical Physics. 21(6/7):467.

[3] Manin, Yu. I. (1980). Vychislimoe i nevychislimoe [Computable and Noncomputable]

[4] Shor, Peter (1994). IEEE Comput Soc Press:p.124

History (2/2)

1998 The first two-qubit quantum computer capable of running calculations is built by Isaac Chaung, Neil Gershenfeld and Mark Kubinec. [5]

2019 Google claims to have achieved computations on a quantum machine that would be infeasible on a classical computer. [6]

Today Both IBM and Google have active QC research and IBM has a quantum computer with 127 qubits. Despite much focus fault tolerant quantum computers are still a far way off.

[5] Chuang, Isaac L.; Gershenfeld, Neil; Kubinec, Markdoi(1998). Phys. Rev. Lett. American Physical Society. 80(15):3408.

[6] Gibney, Elizabeth (October 2019). Nature. 574(7779):461.

Outline

Quantum Computing

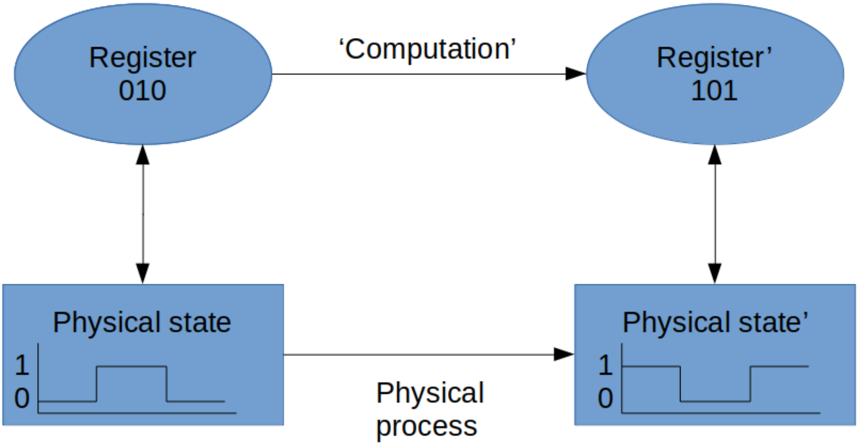
What is Quantum Computing?

- Qubits
- Quantum Gates
- superposition
- entanglement
- it do be whack yo

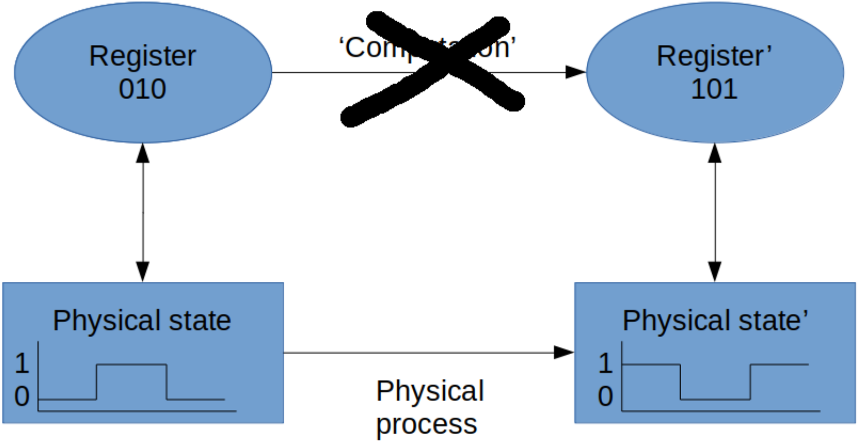
Outline

What is computation?

What is computation?



What is computation?



Computation from Geometry

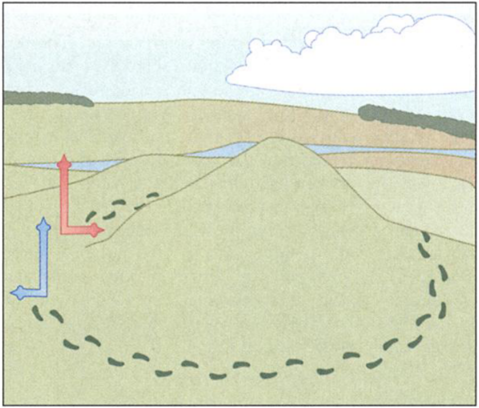


Image credit: Lloyd, Seth (june 2001)."Computation From Geometry". Science vol. 292

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Holonomies (1/2)

The change of a quantum state is called time-evolution, during one period of time T it is written $U(T, 0)$.
The Berry phase is an early example a 1D Holonomy in Quantum mechanics.[7]

$$|\psi(t)\rangle \xrightarrow{\text{time passes}} |\psi(t+T)\rangle = e^{i(\theta+\gamma)}\,|\psi(t)\rangle \tag{1}$$

in this case

$$U(0, T) = e^{i(\theta+\gamma)} \tag{2}$$

θ - dynamical phase, arises from the Hamiltonian of the system.
γ - 'berry phase', or geometric phase arises from the geometry of the 'landscape'.

[7] Berry, (19..)

Holonomies (2/2)

More generally the holonomy can be higher dimensional and non-abelian (matrix)

$$U(T,0) = \mathcal{T}\exp\left(i\int_0^T \underbrace{\mathbf{A}(t)}_{\text{Geometric}} - \underbrace{\mathbf{H}(t)}_{\text{Dynamic}} \, dt\right) \tag{3}$$

where $\mathbf{A}_{mn}(t) = i\langle m(t)|\frac{\partial}{\partial t}|n(t)\rangle$ and $\mathbf{H}_{mn}(t) = \langle m(t)|\mathbf{H}(t)|n(t)\rangle$. In the non-adiabatic case one wishes to remove dynamical phase contribution, then one is left with the Wilczek-Zee holonomy[8];

$$U(T,0) = \mathcal{T}\exp\left(i\int_0^T \mathbf{A}(t)\,dt\right) = \mathcal{P}\exp\left(i\oint_C \mathbf{A}(s)\,ds\right) = U(C) \tag{4}$$

[8] hej

Outline

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The qutrit

Higher dimensional computational elements?

Bit: 0 or 1 Qubit: $\frac{1}{\sqrt{2}}(\alpha\,|0\rangle + |1\rangle)$

The qutrit

Higher dimensional computational elements?

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trit: 0,1, or 2

qubit: $\frac{1}{\sqrt{2}}(\alpha\,|0\rangle + |1\rangle)$

qutrit: $\frac{1}{\sqrt{3}}(\alpha\,|0\rangle + |1\rangle + |2\rangle)$

The qutrit

Higher dimensional computational elements?

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trit: 0,1, or 2

dit 0,1,...,or d

qubit: $\frac{1}{\sqrt{2}}(\alpha\,|0\rangle + \beta\,|1\rangle)$

qutrit: $\frac{1}{\sqrt{3}}(\alpha\,|0\rangle + \beta\,|1\rangle + \gamma\,|2\rangle)$

$\vdots$

qudit: $\frac{1}{\sqrt{d}}(\alpha\,|0\rangle + \beta\,|1\rangle + \cdots + \Omega\,|d\rangle)$

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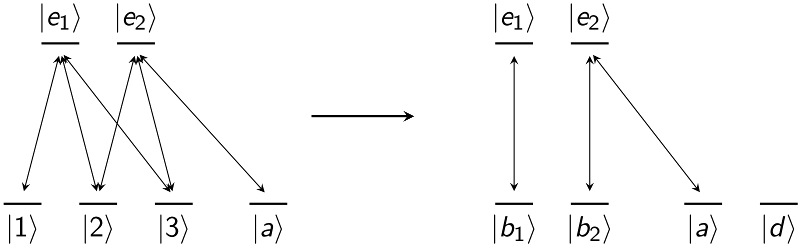
Pros and **Cons**

- more information per unit
- operations require fewer gates
- harder to control

Designing the system

Designing the system



$$H = \sum_{j=1}^{2} \sum_{i=j}^{3} \omega_{ij} |i\rangle \langle e_j| + \frac{\Omega_a(t)}{2} |a\rangle \langle e_2| + \text{h.c} \tag{5}$$

$$H_d = \sum_{j=1}^{2} \frac{\Omega_j(t)}{2} e^{-i\phi_j} |b_j\rangle \langle e_j| + \frac{\Omega_a(t)}{2} |a\rangle \langle e_2| + \text{h.c} \tag{6}$$

Dark-Bright basis

The computational basis new computational basis $\{|1\rangle, |2\rangle, |3\rangle\} \to \{|d\rangle, |b_1\rangle, |b_2\rangle\}$ parametrized by angles $\theta, \varphi, \chi, \xi$.

$$\begin{aligned}
|d\rangle &= \cos\theta\,|1\rangle + e^{i\chi}\sin\theta\cos\varphi\,|2\rangle + e^{i\xi}\sin\theta\sin\varphi\,|3\rangle \\
|b_1\rangle &= \frac{1}{\sqrt{1-\sin^2\theta\sin^2\varphi}}\left(-e^{-i\chi}\sin\theta\cos\varphi\,|1\rangle + \cos\theta\,|2\rangle\right) \\
|b_2\rangle &= \frac{1}{\sqrt{1-\sin^2\theta\sin^2\varphi}}\Bigg(\frac{1}{2}\sin 2\theta\sin\varphi\,|1\rangle + \frac{e^{i\chi}}{2}\sin^2\theta\sin 2\varphi\,|2\rangle \\
&\quad + e^{i\xi}(\sin^2\theta\sin^2\varphi - 1)\,|3\rangle\Bigg)
\end{aligned} \tag{7}$$

Unitary gate

Constructing the unitary from the basis

$$\begin{aligned} U_1 &= |d\rangle\langle d| - i\,|e_1\rangle\langle b_1| - i\,|e_2\rangle\langle b_2|\,,\ \phi_1 = \phi_2 = 0 \\ U_2 &= |d\rangle\langle d| + ie^{i\gamma_1}\,|b_1\rangle\langle e_1| + ie^{i\gamma_2}\,|b_2\rangle\langle e_2|\,,\ \phi_1 = -\gamma_1,\ \phi_2 = -\gamma_2 \end{aligned} \tag{8}$$

$$U(\theta,\varphi,\chi,\xi,\gamma_1,\gamma_2) = U_2 U_1 = |d\rangle\langle d| + e^{i\gamma_1}\,|b_1\rangle\langle b_1| + e^{i\gamma_2}\,|b_2\rangle\langle b_2|\,. \tag{9}$$

Dark path

$$\begin{aligned} |D_1(t)\rangle &= \cos u e^{-i\phi_1} |b_1\rangle + i \sin u |e_1\rangle \\ |D_2(t)\rangle &= \cos u \cos v e^{-i\phi_2} |b_2\rangle - i \sin u |e_2\rangle - \cos u \sin v |a\rangle \end{aligned} \tag{10}$$

such that $u(0) = u(T) = v(0) = v(T) = 0$

$$|\psi(t)\rangle = f_0 |d\rangle + f_1 |D_1(t)\rangle + f_2 |D_2(t)\rangle, t \in [0, T]. \tag{11}$$

so for example

$$\begin{pmatrix}1\\0\\0\end{pmatrix} = f_0\,|d\rangle + f_1\,|D_1(0)\rangle + f_2\,|D_2(0)\rangle \xrightarrow{t\to T} \begin{pmatrix}0\\1\\0\end{pmatrix} = f_0\,|d\rangle + f_1\,|D_1(T)\rangle + f_2\,|D_2(T)\rangle \tag{12}$$

or equivalent

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \to U\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \tag{13}$$

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Generalization

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Conclusions