Uppsala University

Degree Project D in Computational Science, 30c

Department of Physics and Astronomy
Division of Material stuff?

Holonomic optimal control for qudits

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Abstract

1 Engelskt abstrakt

Sammanfattning

Svenskt abstrakt

Contents

Test! Här är några relevant källor! [?],[?],[?]

1 Introduction

2 Background

2.1 Quantum mechanics, part I

This part offers a quick introduction to the necessary quantum mechanics for readers whom are not familiar with the subject. The section contains nothing relevant for later parts of the thesis and can be safely skipped. For a more complete introduction I suggest chapter 1 of Sakurai.

2.1.1 Dirac notation and quantum states

A **quantum state** is a complex vector in a **Hilbert space**, a complete space with inner product induced metric, with finite or infinite dimension depending on characteristics of the system it represents. A quantum state vector is represented by a so-called **ket**, which is written as $|\psi\rangle$. Kets follow the same rules as a standard vector, as in that they can be added to one another, multiplied by a scalars. The ket itself is just $|\rangle$, the content of the ket is just a label which (usually) give some sense of what the state represents.
An **operator** is a matrix that can act on a ket, often defined by a set of rules. Written as

$$\hat{O} \cdot (|\psi\rangle) = \hat{O} |\psi\rangle \tag{1}$$

Since operators are matrices and kets are vectors, the existence of **eigenkets** are analogous eigenvectors. A common example of this is the time-independent Schrödinger equation

$$\hat{H} |\psi\rangle = E |\psi\rangle \tag{2}$$

Where $\hat{H}$ is the Hamiltonian operator, and E is the energy of the quantum state $|\psi\rangle$. In this case $|\psi\rangle$ is an eigenstate to the Hamiltonian.

2.2 Lambda system

A traditional Λ-system is a quantum mechanical system with two decoupled 'ground states' and one 'excited state' coupled to both ground states. Such a state with two ground state could realize a qubit by letting the ground state act as $|0\rangle$ and $|1\rangle$.

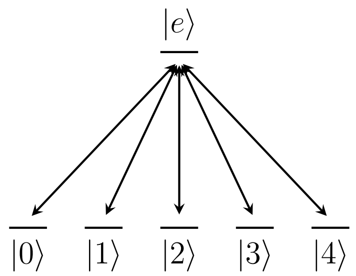

Figure 1: test figure

alfa $= \frac{\pi}{2}\sin^2(\frac{\pi t}{T})$
beta $= \eta(1-\cos(\frac{\pi}{2}\sin^2(\frac{\pi t}{T})))$
alfa' $= \frac{\pi^2}{2T}\sin(\frac{2\pi t}{T})$
beta' $= \frac{\eta\pi^2}{2T}\sin(\frac{2\pi t}{T})\sin(\frac{\pi}{2}\sin^2(\frac{\pi t}{T}))$

$$\Omega(t) = 2((\frac{\eta\pi^2}{2T}\sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{\pi}{2}\sin^2(\frac{\pi t}{T})\right))\cot\left(\frac{\pi}{2}\sin^2(\frac{\pi t}{T})\right)\sin\left(\eta(1-\cos\left(\frac{\pi}{2}\sin^2(\frac{\pi t}{T})\right))\right)$$
$$+\eta(1-\cos\left(\frac{\pi}{2}\sin^2(\frac{\pi t}{T})\right))\cos\left(\eta(1-\cos\left(\frac{\pi}{2}\sin^2(\frac{\pi t}{T})\right))\right))$$

2.3 Coupled excited states

Hamiltonian of a "double" tripod

$$H(t) = \sum_{i=1}^{2}\sum_{j=1}^{3}\omega_{ij}|j\rangle\langle e_i| + \text{h.c} \tag{3}$$

Find dark states $H(t)|d\rangle = 0|d\rangle$ on form $|d\rangle = \sum_{k=1}^{3} d_k |k\rangle$ Result

$$\begin{aligned} d_1 &= \frac{1}{\omega_{11}^*}(\omega_{12}^*\omega_{23}^* - \omega_{13}^*\omega_{22}^*) \\ d_2 &= \frac{\omega_{13}^*\omega_{21}^*}{\omega_{11}^*} - \omega_{23}^* \\ d_3 &= \omega_{22}^* - \frac{\omega_{12}^*\omega_{21}^*}{\omega_{11}^*} \end{aligned} \tag{4}$$

Bright state $\langle b|d\rangle = 0$

$$\begin{aligned} |b_1\rangle &= \omega_{11}|1\rangle + \omega_{12}|2\rangle + \omega_{13}|3\rangle \\ |b_2\rangle &= \omega_{21}|1\rangle + \omega_{22}|2\rangle + \omega_{23}|3\rangle \end{aligned} \tag{5}$$

Check if $\langle b_1|b_2\rangle = 0$? Holds if $\omega_{11} = -\frac{1}{\omega_{21}^*}(\omega_{12}^*\omega_{22} + \omega_{13}\omega_{23}^*)$

Change basis,

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \omega_{11} & \omega_{21} & d_1 \\ 0 & 0 & \omega_{12} & \omega_{22} & d_2 \\ 0 & 0 & \omega_{13} & \omega_{23} & d_3 \end{pmatrix} \tag{6}$$

New hamiltonian in $\{|e_1\rangle, |e_2\rangle, |b_1\rangle, |b_2\rangle, |d\rangle\}$ is

$$H_d(t) = T^\dagger H(t) T = \left(\sum_{j=1}^{2}\sum_{k=1}^{3}(\omega_{jk})^2|b_j\rangle\langle e_j|\right) + [\omega_{11}\omega_{12} + \omega_{12}\omega_{22} + \omega_{13}\omega_{23}](|b_2\rangle\langle e_1| + |b_1\rangle\langle e_2|) + \text{h.c} \tag{7}$$

Double check this calculation.

Subspace is $\{|e_1\rangle, |e_2\rangle, |b_1\rangle, |b_2\rangle\}$ since $|d\rangle$ is decoupled.

Simplify notation, $\Omega_j = \sum_{k=1}^{3}\omega_{jk}^2$ and $\Gamma = \omega_{11}\omega_{12} + \omega_{12}\omega_{22} + \omega_{13}\omega_{23}$ now Hamiltonian looks like

$$H_d = \sum_{j=1}^{2}\Omega_j|b_j\rangle\langle e_j| + \Gamma(|b_2\rangle\langle e_1| + |b_1\rangle\langle e_2|) + \text{ h.c} \tag{8}$$

Now find a "dark path" $|D_i(t)\rangle$ such that $\langle D_i(t)| H_d(t) |D_i(t)\rangle = 0$, for $i = 1, 2$
Set

$$\begin{aligned} |D_1\rangle &= a_1 |b_1\rangle + a_2 |3\rangle + a_3 |e_1\rangle \\ |D_2\rangle &= c_1 |b_2\rangle + c_2 |3\rangle + c_3 |e_2\rangle \end{aligned} \tag{9}$$

and

NEW TRY

$$H(t) = \sum_{i=1}^{2} \sum_{j=1}^{3} \omega_{ij} |j\rangle \langle e_i| + \text{h.c} \tag{10}$$

New dark state

$$|d\rangle = d_1 |1\rangle + d_2 |2\rangle \tag{11}$$

with $d_1 = -(\omega_{12}^* + \omega_{22}^*)$, $d_2 = (\omega_{11}^* + \omega_{21}^*)$ **There exists no dark state on this form ($d_1 = d_2 = 0$)**

bight state

$$|b\rangle = -d_2^* |1\rangle + d_1^* |2\rangle \tag{12}$$

CRITERA? $\omega_{11}\omega_{22} = \omega_{12}\omega_{21} \implies \frac{\omega_{11}}{\omega_{12}} = \frac{\omega_{21}}{\omega_{22}}$
change basis to $\{e_1, e_2, 3, b, d\}$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_2^* & d_1 \\ 0 & 0 & 0 & d_1^* & d_2 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{13}$$

New hamiltonian

$$H_d = \left(\sum_{j=1}^{2} \omega_{j3} |3\rangle \langle e_j| + \sigma_j |b\rangle \langle e_j| \right) + \text{h.c} \tag{14}$$

with

$$\begin{aligned} \sigma_1 &= \omega_{12} d_1^* - \omega_{11} d_2^* = -\omega_{12}(\omega_{12} + \omega_{22}) - \omega_{11}(\omega_{11} + \omega_{21}) \\ \sigma_2 &= \omega_{21} d_1^* - \omega_{12} d_2^* = -\omega_{23}(\omega_{12} + \omega_{22}) - \omega_{12}(\omega_{11} + \omega_{21}) \end{aligned}$$

Now find a "dark path" $|D_i(t)\rangle$ such that $\langle D_i(t)| H_d(t) |D_i(t)\rangle = 0$, for $i = 1, 2$
Set

$$\begin{aligned} |D_1\rangle &= a_1 |b\rangle + a_2 |3\rangle + a_3 |e_1\rangle \\ |D_2\rangle &= c_1 |b\rangle + c_2 |3\rangle + c_3 |e_2\rangle \end{aligned} \tag{15}$$

Assume that $|b_2\rangle = \Delta |b_1\rangle$

$$\begin{aligned} |D_1\rangle &= \cos\alpha\cos\beta(((e^{-i\varphi})))\,|b_1\rangle - \cos\alpha\sin\beta\,|3\rangle - i\sin\alpha\,|e_1\rangle \\ |D_2\rangle &= \cos\alpha\cos\beta(((e^{-i\varphi})))\,|b_2\rangle - \cos\alpha\sin\beta\,|3\rangle - i\sin\alpha\,|e_2\rangle \end{aligned} \tag{16}$$

then $\langle D_1|D_2\rangle = 0 \implies \langle b_1|b_2\rangle = -\tan^2\beta \implies \Delta = -i\tan\beta$

$$\begin{aligned} &\langle D_i|\, H_d\,|D_i\rangle = [\sigma_1^* + \sigma_2^* - \sigma_i]\,\Delta_i\cos\beta + \omega_{i3}\sin\beta = 0 \\ &\implies \omega_{i3} = \Delta_i(\sigma_i - \sigma_1^* - \sigma_2^*)\cot\beta \end{aligned} \tag{17}$$

NEW NEW TRY!

$$H(t) = \sum_{i=1}^{2}\sum_{j=1}^{3}\omega_{ij}\,|j\rangle\,\langle e_i| + \text{h.c} \tag{18}$$

Dark state

$$\begin{aligned} d_1 &= -\left(\omega_{22}^* + \frac{\omega_{12}^*\omega_{23}^*}{\omega_{13}^*}\right) \\ d_2 &= \left(\omega_{21}^* + \frac{\omega_{11}^*\omega_{23}^*}{\omega_{13}^*}\right) \\ d_3 &= \frac{1}{\omega_{13}^*}\left(\omega_{21}^*\omega_{12}^* - \omega_{11}^*\omega_{22}^*\right) \end{aligned} \tag{19}$$

bright state

$$|b\rangle = -d_2^*\,|1\rangle + d_1^*\,|2\rangle \tag{20}$$

$$H_d = \left(\sum_{j\ =1}^{2}\omega_{j3}\,|3\rangle\,\langle e_j| + \sigma_j\,|b\rangle\,\langle e_j|\right) + \text{h.c} \tag{21}$$

$$\sigma_1 = -\frac{\omega_{23}}{\omega_{13}}\left(\omega_{12}^2 + \omega_{11}^2\right) - \omega_{11}\omega_{21} - \omega_{12}\omega_{22}$$

$$\sigma_2 = -\frac{\omega_{23}}{\omega_{13}}\left(\omega_{12}\omega_{22} + \omega_{11}\omega_{21}\right) - \left(\omega_{12}^2 + \omega_{22}^2\right)$$

Then computations follow as before with new coeffcients.

$$H = \sum_{j=1}^{2} \alpha_j \left|\alpha_j\right\rangle \left\langle e_j\right| + \sum_{i=1}^{2} \omega_{ji} \left|i\right\rangle \left\langle e_j\right| + \text{ h.c} \tag{22}$$

in new basis

$$H_d = \sum_{j=1}^{2} -\alpha_j^2 - \sum_{i=1}^{2} \omega_{ji}^2 \left|b_j\right\rangle \left\langle e_j\right| - (\omega_{11}\omega_{21} + \omega_{12}\omega_{22})(\left|b_2\right\rangle \left\langle e_1\right| + \left|b_1\right\rangle \left\langle e_2\right|) + \text{ h.c} \tag{23}$$

simplify

$$H_d = \sum_{j=1}^{2} \sigma_j \left|b_j\right\rangle \left\langle e_j\right| + \Gamma(\left|b_2\right\rangle \left\langle e_1\right| + \left|b_1\right\rangle \left\langle e_2\right|) + \text{ h.c} \tag{24}$$

$$\begin{aligned} \left|D_1\right\rangle &= \cos\alpha \cos\beta e^{-i\varphi_1} \left|b_1\right\rangle - \cos\alpha \sin\beta \left|\alpha_1\right\rangle - i \sin\alpha \left|e_1\right\rangle \\ \left|D_2\right\rangle &= \cos\alpha \cos\beta e^{-i\varphi_2} \left|b_2\right\rangle - \cos\alpha \sin\beta \left|\alpha_2\right\rangle - i \sin\alpha \left|e_2\right\rangle \end{aligned} \tag{25}$$

$$\left\langle D_1 | D_2 \right\rangle = \cos^2\alpha \cos^2\beta e^{+i(\varphi_2 - \varphi_1)} \left\langle b_1 | b_2 \right\rangle == 0\text{?} \tag{26}$$

holds if

$$\left\langle b_1 | b_2 \right\rangle = \omega_{11}^* \omega_{21} + \omega_{12}^* \omega_{22} = 0 \tag{27}$$