Dark path holonomic qudit computation

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Non-adiabatic holonomic quantum computation (NHQC) is a method used to implement high-speed quantum gates with non-Abelian geometric phases associated with loops in state space. Due to their noise tolerance, these phases can be used to construct error resilient quantum gates. We extend the dark path qubit scheme in [Fundam. Res. (2022), doi:10.1016/j.fmre.2021.11.031] to qudits. Specifically, we demonstrate one-qudit universality by using the dark path technique. Explicit qutrit ($d = 3$) gates are demonstrated and the scaling of the number of loops with the dimension d is addressed. This scaling is linear and we show how any diagonal qudit gate can be implemented efficiently in any dimension.

I. INTRODUCTION

The emerging field of quantum technology has many promising applications, one of them is quantum computation, which currently is an active area of research. The most common form of quantum computation is the circuit model, which is analogous to the circuits used for classical computers. Gates are replaced by unitary transformations (quantum gates) and bits by qubits. To achieve the computational advantage it is important to construct robust, noise-resilient quantum gates. A candidate for this is holonomic quantum computation [1, 2], which is based on non-Abelian (matrix-valued) geometric phases in adiabatic [3] or non-adiabatic [4] evolution. Such holonomic gates are only dependent on the geometry of the system's state space and thus are resilient to local errors in the quantum evolution.

The idea that elements of computation should be limited to qubits is sort of an arbitrary choice that most likely rose out of convenience due to binary logic. So why binary logic? It is simply the easiest non-trivial example: in binary logic, things can be either 0 or 1, `True` or `False`, **on** or **off**, etc. Due to its simplicity, it is no wonder that this is how the first computer was designed. But are we limited to bits? As early as 1840, a mechanical trenary (three-valued logic) calculation device was built by Fowler [5], and in 1958 the first electronic trenary computer was developed by the Soviet Union [6]. Although the trenary computer had many advantages over the binary one, it never saw the same widespread success. There is nothing in theory that forbids a higher dimensional computational basis, even more so when it comes to quantum computers, where the implementation of the elements of computation already surpasses the simplicity of **on** and **off**. Thus, one may consider d dimensional 'qudits' as primitive units of quantum information, with promising results that show potential, some of them reviewed in Ref. [7].

Here, we develop a qudit generalization of the idea of dark paths proposed in [8] for implementing non-adiabatic holonomic qubit gates. By this, we combine the advantages of improved robustness associated with the dark path approach with the improved coding efficiency of higher dimensional quantum information units. We find explicit realizations of qutrit ($d = 3$) gates and consider extension to arbitrary d. We examine the robustness of the $d = 3$ gates to systematic errors in the Rabi frequencies of the laser induced transitions.

II. DARK PATH SETTING

In the dark path approach developed in Ref. [8], a qubit is encoded in two of the three lower levels of a tripod. The third auxiliary level is used to define a dark path, along which a certain time dependent linear combination of the bright, excited, and auxiliary states picks up zero dynamical phase. We now extend this idea to the qutrit ($d = 3$), leaving the case of general d to Sec. IV.

The key point of the qutrit dark path setting is to look for a level structure with three ground state levels $|k\rangle$, $k = 1, 2, 3$, encompassing a single fixed (time independent) dark eigenstate, as well as an extra fourth auxiliary lower level $|a\rangle$. As the number of dark eigenstates in the computational qutrit subspace $\text{Span}\{|1\rangle, |2\rangle, |3\rangle\}$ equals the difference between the number of excited states and the number of ground states [10], this amounts to coupling the ground state levels to two excited states $|e_1\rangle, |e_2\rangle$. The desired coupling structure is described by the Hamiltonian (see left panel of Fig. 1)

$$H^{(3)} = \sum_{k=1}^{3}\sum_{l=1}^{2} \omega_{k,l}|k\rangle\langle e_l| + \frac{\Omega_a(t)}{2}|a\rangle\langle e_2| + \text{H.c.} \tag{1}$$

with $\omega_{3,1} = 0$. The specific coupling structure allows for a Morris-Shore transformation [11] applied to the qutrit levels $|1\rangle, |2\rangle, |3\rangle$ only, yielding (see right panel of Fig. 1)

$$H^{(3)} = \sum_{l=1}^{2} \frac{\Omega_l(t)}{2} e^{-i\phi_l}|b_l\rangle\langle e_l| + \frac{\Omega_a(t)}{2}|a\rangle\langle e_2| + \text{H.c.} \tag{2}$$

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by employing the dark-bright basis

$$|\mathcal{D}\rangle = \cos\theta|1\rangle + e^{i\chi}\sin\theta\cos\varphi|2\rangle + e^{i\xi}\sin\theta\sin\varphi|3\rangle,$$
$$|b_1\rangle = \frac{1}{\sqrt{1-\sin^2\theta\sin^2\varphi}}\left(-e^{-i\chi}\sin\theta\cos\varphi|1\rangle + \cos\theta|2\rangle\right),$$
$$|b_2\rangle = \frac{1}{\sqrt{1-\sin^2\theta\sin^2\varphi}}\Big(\sin\theta\cos\theta\sin\varphi|1\rangle + e^{i\chi}\sin^2\theta\sin\varphi\cos\varphi|2\rangle + e^{i\xi}(\sin^2\theta\sin^2\varphi-1)|3\rangle\Big) \tag{3}$$

with $|\mathcal{D}\rangle$ being a dark energy eigenstate satisfying $H_d|\mathcal{D}\rangle = 0$. Note that $|\mathcal{D}\rangle, |b_1\rangle, |b_2\rangle$ span the computational subspace, i.e., $\mathrm{Span}\{|\mathcal{D}\rangle, |b_1\rangle, |b_2\rangle\} = \mathrm{Span}\{|1\rangle, |2\rangle, |3\rangle\}$. The original $\omega_{k,l}$ can be expressed in the parameters $\theta, \varphi, \chi, \xi$ by expanding the bright states on the right-hand side of Eq. (2) in terms of the original qutrit levels $|k\rangle$ and by comparing with Eq. (1).

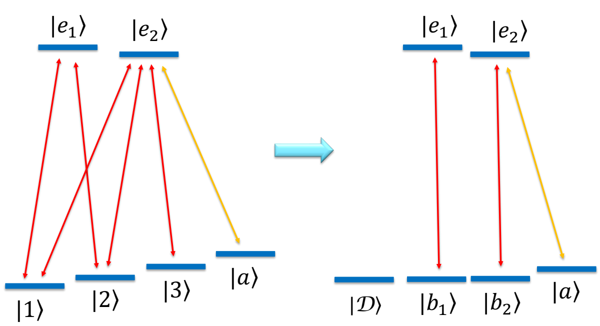

FIG. 1. Level setting for realizing a dark path holonomic qutrit gate (left panel). The specific coupling structure allows for a Morris-Shore transformation [11] applied to the qutrit levels $|1\rangle, |2\rangle, |3\rangle$, resulting in a single dark energy eigenstate $|\mathcal{D}\rangle$ and two bright state $|b_1\rangle, |b_2\rangle$, while leaving the auxiliary state $|a\rangle$ untouched.

A pair of dark path states $|D_1(t)\rangle, |D_2(t)\rangle$ can now be defined. These states should satisfy two conditions: (i) they should be orthogonal to $|\mathcal{D}\rangle$, and (ii) their average energy $\langle D_i(t)|H_d|D_i(t)\rangle, i = 1, 2$, should vanish along their evolution paths. Explicitly, one may check that

$$|D_1(t)\rangle = \cos u(t) e^{-i\phi_1}|b_1\rangle + i\sin u(t)|e_1\rangle,$$
$$|D_2(t)\rangle = \cos u(t)\cos v(t) e^{-i\phi_2}|b_2\rangle - i\sin u(t)|e_2\rangle - \cos u(t)\sin v(t)|a\rangle \tag{4}$$

satisfy the dark path conditions. In fact, these states even satisfy the stronger condition [2] $\langle D_1(t)|H_d|D_2(t)\rangle = 0$, which opens up for holonomic gates provided the parameters $u(t), v(t)$ are chosen such that the qutrit subspace $\mathrm{Span}\{|\mathcal{D}\rangle, |D_1(t)\rangle, |D_2(t)\rangle\}$ evolves in a cyclic manner, i.e., that $\mathrm{Span}\{|\mathcal{D}\rangle, |D_1(\tau)\rangle, |D_2(\tau)\rangle\} = \mathrm{Span}\{|1\rangle, |2\rangle, |3\rangle\}$ for some run time τ of the gate. This is achieved provided $u(0) = u(\tau) = v(0) = v(\tau) = 0$ so that each dark path starts in the respective bright state and travels along a curve and returns to the same bright state at $t = \tau$. We follow Ref. [8] and choose

$$u(t) = \frac{\pi}{2}\sin^2\frac{\pi t}{\tau},$$
$$v(t) = \eta\left[1 - \cos u(t)\right]. \tag{5}$$

The parameter η represents the coupling strength to the auxiliary state in the sense that $|D_2(t)\rangle$ becomes independent of $|a\rangle$ when $\eta = 0$.

One may now use the Schrödinger equation to reverse engineer the time dependent parameters $\Omega_l(t)$. A calculation yields

$$\Omega_1(t) = -2\dot{u}(t),$$
$$\Omega_2(t) = 2\left(\dot{v}(t)\cot u(t)\sin v(t) + \dot{u}(t)\cos v(t)\right),$$
$$\Omega_a(t) = 2\left(\dot{v}(t)\cot u(t)\cos v(t) - \dot{u}(t)\sin v(t)\right). \tag{6}$$

This completes the dark path construction in the qutrit case.

III. HOLONOMIC DARK PATH ONE-QUTRIT GATES

A. Gate construction

The multi-pulse single-loop setting [9] is used to dark path qutrit gates, thereby the loop is divided into two path segments by applying laser pulses. Explicitly, the pulses take the subspace $\mathrm{Span}\{|\mathcal{D}\rangle, |b_1\rangle, |b_2\rangle\}$ into $\mathrm{Span}\{|\mathcal{D}\rangle, |e_1\rangle, |e_2\rangle\}$ and back, by applying phase shifts γ_1, γ_2 to the second segment relative the first one. Note that $u\left(\frac{\tau}{2}\right) = \frac{\pi}{2}$, which implies that the duration is $\frac{\tau}{2}$ of both path segments This results in the unitaries

$$U\left(\frac{\tau}{2}, 0\right) = |\mathcal{D}\rangle\langle\mathcal{D}| - i(|e_1\rangle\langle b_1| + |b_1\rangle\langle e_1|) - i(|e_2\rangle\langle b_2| + |b_2\rangle\langle e_2|),$$
$$U\left(\tau, \frac{\tau}{2}\right) = |\mathcal{D}\rangle\langle\mathcal{D}| + ie^{i\gamma_1}(|b_1\rangle\langle e_1| + |e_1\rangle\langle b_1|) + ie^{i\gamma_2}(|b_2\rangle\langle e_2| + |e_2\rangle\langle b_2|) \tag{7}$$

restricted to the part with non-trvial action on the computational subspace. By combining these unitaries, we obtain the one-qutrit gate

$$\mathrm{U}_3^{(1)} = U\left(\tau, \frac{\tau}{2}\right)U\left(\frac{\tau}{2}, 0\right) = |\mathcal{D}\rangle\langle\mathcal{D}| + e^{i\gamma_1}|b_1\rangle\langle b_1| + e^{i\gamma_2}|b_2\rangle\langle b_2|. \tag{8}$$

The holonomy U_3 can be parametrized by $\chi, \xi, \theta, \varphi, \gamma_1, \gamma_2$; however, these parameters are not enough to construct all gates. For instance, X_3 requires two loops. The full gate is given by repeating U_3 with different set of parameters

$$\mathrm{U}_3^{(1)} = \mathrm{U}_3^{(1)}(\chi', \xi', \theta', \varphi', \gamma_1', \gamma_2')\mathrm{U}_3^{(1)}(\chi, \xi, \theta, \varphi, \gamma_1, \gamma_2). \tag{9}$$

In this way, the following gates can be implemented:

$$\begin{aligned}
\mathrm{X}_3 &= U(0,0,\frac{\pi}{4},\frac{\pi}{2},0,\pi)\times U(0,0,\frac{\pi}{2},\frac{\pi}{4},0,\pi)\\
&= \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\\
\mathrm{Z}_3 &= U(0,0,0,0,\frac{2\pi}{3},\frac{4\pi}{3})\\
&= \begin{pmatrix}1&0&0\\0&e^{i\frac{2\pi}{3}}&0\\0&0&e^{i\frac{4\pi}{3}}\end{pmatrix},\\
\mathrm{T}_3 &= U(0,0,0,0,\frac{2\pi}{9},\frac{-2\pi}{9})\\
&= \begin{pmatrix}1&0&0\\0&e^{i\frac{2\pi}{9}}&0\\0&0&e^{-i\frac{2\pi}{9}}\end{pmatrix},\\
\mathrm{H}_3 &= U(6.41\cdot 10^{-4}, 6.56\cdot 10^{-4}, 0.48, 0.79, 1.58, 1.56)\\
&\quad\times U(9.81\cdot 10^{-3}, 0.00, 1.187, 2.15, 0.00, 1.57)\\
&\approx \frac{1}{\sqrt{3}}\begin{pmatrix}1&1&1\\1&e^{i\frac{2\pi}{3}}&e^{i\frac{4\pi}{3}}\\1&e^{i\frac{4\pi}{3}}&e^{i\frac{2\pi}{3}}\end{pmatrix}.
\end{aligned} \tag{10}$$

The set includes qutrit equivalents of the Hadamard and T-gate, which constitutes a universal set. Thus, no more than two loops for each gate are needed for qutrit universality. Figures 2 and 3 show the state population of the states during implementation of H_3 and X_3, respectively. As in Ref. [8], we have chosen $\eta = 4.0$ in the simulations.

All diagonal gates can be parametrized by a single loop by fixing $\theta = \varphi = \chi = \xi = 0$. The dark-bright basis states reduce to $|\mathcal{D}\rangle = |1\rangle, |b_1\rangle = |2\rangle, |b_2\rangle = |3\rangle$. By Eq. (8), it is possible to see that all diagonal unitaries can be specified by γ_1 and γ_2, up to a phase factor,

$$\mathrm{U}_3^{(1)}(0,0,0,0,\gamma_1,\gamma_2) = \begin{pmatrix}1&0&0\\0&e^{i\gamma_1}&0\\0&0&e^{i\gamma_2}\end{pmatrix}. \tag{11}$$

B. Robustness test

We quantify gate robustness by means of fidelity

$$F(\psi,\varphi) = \left|\langle\psi|\varphi\rangle\right| \tag{12}$$

with ψ and φ the ideal and non-ideal output states of the gate, given the same input. The fidelity is averaged by sampling initial states and letting them evolve with time by numerically solving the Schrödinger equation using the SciPy implementation of backwards differentiation [12].

We introduce systematic errors by shifting the Rabi frequencies $\Omega_p \mapsto \Omega_p(1+\delta)$ for $p = 1, 2, a$ in Eq. (2) and compare to the exact solution obtained by applying the ideal gate to the initial state. The calculated fidelities are shown in Fig. 4. In the figures, it can be seen that coupling to the auxiliary state (again with $\eta = 4.0$) improves the robustness to Rabi frequency errors compared to the NHQC scheme ($\eta = 0$). This result is similar to that found in [8] for the qubit case, and it is reasonable to expect that it applies for higher d as well. As qudits offer improved encoding efficiency, this result demonstrates potential for dark path holonomic qudit computation.

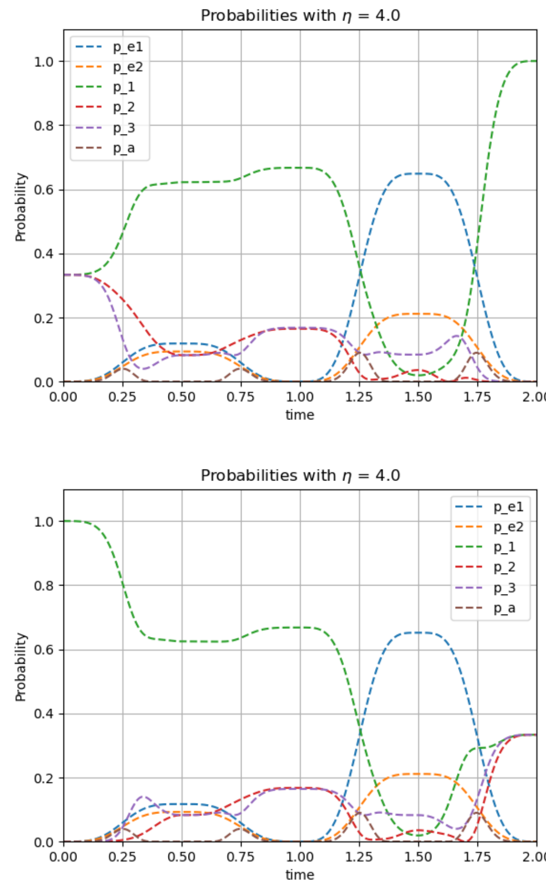

FIG. 2. The effect of the H_3-gate on the initial states $\frac{1}{\sqrt{3}}(|1\rangle+|2\rangle+|3\rangle)$ (upper) and $|1\rangle$ (lower). The coupling to the auxiliary state is $\eta = 4.0$. Note that since the plot shows the probabilities, phases cannot be seen in the plot.

IV. QUDIT GENERALIZATION

To generalize the dark path scheme to arbitrary qudit dimension d, we extend the above qutrit scheme by using d ground states $|k\rangle$, $d-1$ excited states $|e_l\rangle$, and an auxiliary state $|a\rangle$. The dark path Hamiltonian

$$H = \sum_{k=1}^{d}\sum_{l=1}^{d-1}\omega_{k,l}|k\rangle\langle e_l| + \frac{\Omega_a(t)}{2}|a\rangle\langle e_{d-1}| + \mathrm{H.c.} \tag{13}$$

is a direct extension of Eq. (1). As before, the relation between the number of excited states and qudit ground states is chosen so as to define a single fixed dark eigenstate $|\mathcal{D}\rangle$. The coupling structure has the following pattern: the lth excited state $|e_l\rangle$ is connected to ground states $|1\rangle, |2\rangle, \ldots, |l+1\rangle$, except the one

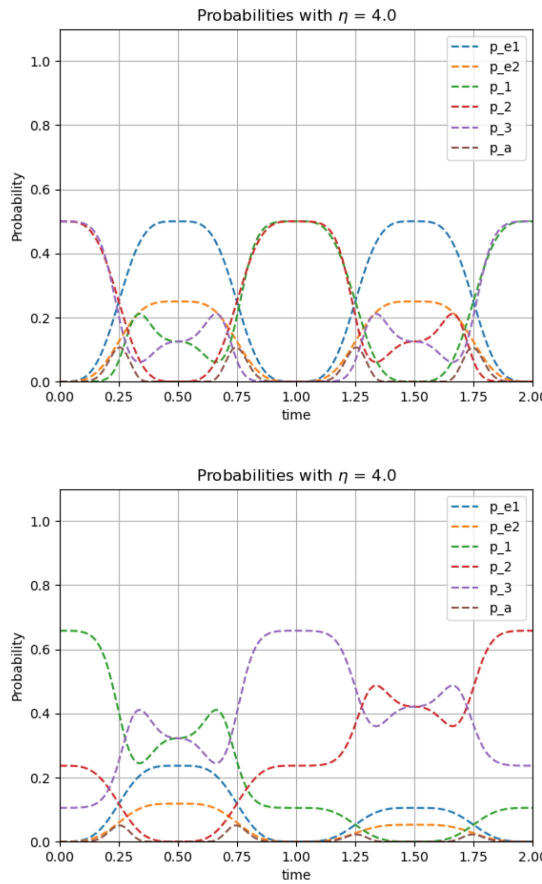

FIG. 3. The effect of the X_3-gate on the initial states $\frac{1}{\sqrt{2}}(|2\rangle + |3\rangle)$ (upper) and $\frac{1}{\sqrt{38}}(5|1\rangle + 3|2\rangle + 2|3\rangle)$ (lower). The coupling to the auxiliary state is $\eta = 4.0$. Note that since the plot shows the probabilities, phases cannot be seen in the plot.

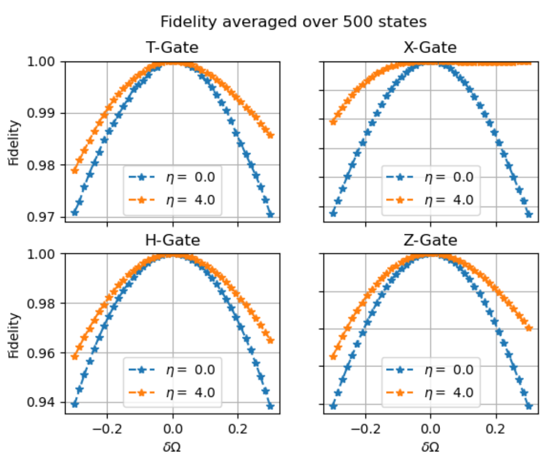

FIG. 4. Robustness test, average fidelity of the T_3, X_3, Z_3, and H_3 gates. The averages are calculated by sampling over 500 randomized initial states with a perturbation $\Omega \mapsto \Omega(1+\delta)$ of the Rabi frequency.

with largest index $l = d-1$, which is connected to all ground states and the auxiliary state $|a\rangle$. Thus, $\omega_{k>l+1,l} = 0$.

Assume now there exists a normalized dark state on the form

$$|\mathcal{D}\rangle = c_1|1\rangle + c_2|2\rangle + c_3|3\rangle \cdots + c_d|d\rangle \tag{14}$$

from which $d-1$ bright states $|b_k\rangle$, $k = 1, \ldots, d-1$ can be defined. Explicitly, we may take

$$\begin{aligned} |b_1\rangle &= \frac{1}{|c_1|^2 + |c_2|^2}\left(-c_2^*|1\rangle + c_1^*|2\rangle\right), \\ |b_2\rangle &= N_2(c_1|1\rangle + c_2|2\rangle + \Lambda_3|3\rangle), \\ &\vdots \\ |b_{d-1}\rangle &= N_{d-1}(c_1|1\rangle + \cdots + c_{d-1}|d-1\rangle + \Lambda_d|d\rangle). \end{aligned} \tag{15}$$

with $N_2, \ldots N_{d-1}$ being normalization factors. By construction, $|b_1\rangle$ is orthogonal to the dark state and all other bright states. For $k \geq 2$, $|b_k\rangle$ contains $k+1$ basis vectors, where the coefficient Λ_{k+1} is chosen such that $|b_k\rangle$ is orthogonal to the dark state $|\mathcal{D}\rangle$. This in turn makes any $|b_{l>k}\rangle$ orthogonal to $|b_k\rangle$ as they have the same states and coefficients as $|\mathcal{D}\rangle$ for all the states involved in the inner product, which implies that $\langle b_{l>k}|b_k\rangle \propto \langle \mathcal{D}|b_k\rangle$. Therefore, by choosing the Λ's such that these inner products are zero, the construction ensures an orthonormal dark-bright basis spanning the qudit subspace. Explicitly, for $k \geq 2$, one finds

$$\Lambda_{k+1} = -\frac{1}{c_{k+1}^*}\sum_{l=1}^{k}|c_l|^2 \tag{16}$$

and

$$\begin{aligned} N_k &= \left(\sum_{l=1}^{k}|c_l|^2 + |\Lambda_{k+1}|^2\right)^{-1/2} \\ &= \left(\sum_{l=1}^{k}|c_l|^2 + \frac{1}{|c_{k+1}|^2}\left|\sum_{l=1}^{k}|c_l|^2\right|^2\right)^{-1/2}. \end{aligned} \tag{17}$$

In the dark-bright basis, the Hamiltonian can be written as

$$H^{(d)} = \sum_{k=1}^{d-1}\frac{\Omega_k(t)}{2}e^{-i\phi_k}|b_k\rangle\langle e_k| + \frac{\Omega_a(t)}{2}|a\rangle\langle e_{d-1}| + \text{H.c.} \tag{18}$$

with Ω_k being real-valued time dependent parameters and ϕ_k time independent phases. This form of $H^{(d)}$ is depicted in Fig. 5 and can be used to define $d-1$ independent dark paths $|D_k(t)\rangle$; as above, these states must satisfy $\langle D_k(t)|H_d|D_k(t)\rangle = 0, k = 1, \ldots, d-1$, and $\langle D_k(t)|D_l(t)\rangle = \delta_{kl}$. The dark paths are traced out by the states

$$\begin{aligned} |D_k(t)\rangle &= \cos u(t)e^{-i\phi_k}|b_k\rangle + i\sin u(t)|e_k\rangle, \\ &\quad k = 1, \ldots, d-2, \\ |D_{d-1}(t)\rangle &= \cos u(t)\cos v(t)e^{-i\phi_n}|b_{d-1}\rangle - i\sin u(t)|e_{d-1}\rangle \\ &\quad - \cos u(t)\sin v|a\rangle, \end{aligned} \tag{19}$$

each of which starting and ending in one of the bright state provided $u(0) = u(\tau) = v(0) = v(\tau) = 0$. By using these states,

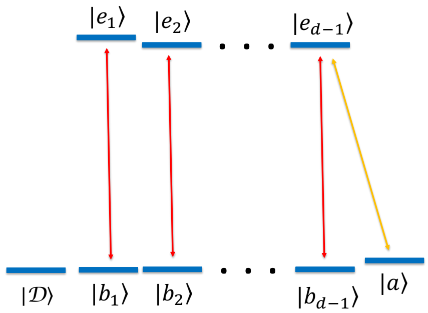

FIG. 5. Qudit setting in the Morris-Shore basis [11] applied to the qudit levels $|1\rangle, \ldots, |d\rangle$. As in the qutrit case, the auxiliary state $|a\rangle$ is untouched and a single dark state $|\mathcal{D}\rangle$ emerges in the computational qudit subspace Span$\{|1\rangle, \ldots, |d\rangle\}$.

one can reverse engineer the Hamiltonian to determine Ω_k and Ω_a, yielding

$$\begin{aligned}\Omega_1(t) = \Omega_2(t) = \ldots = \Omega_{d-2}(t) &= -2\dot{u},\\ \Omega_{d-1}(t) &= 2\left(\dot{v}\cot u \sin v + \dot{u}\cos v\right),\\ \Omega_a(t) &= 2\left(\dot{v}\cot u \cos v - \dot{u}\sin v\right).\end{aligned} \tag{20}$$

Holonomic one-qudit gates can be implemented by using the same $u(t), v(t)$ as in the qutrit setting and by using the single-loop multi-pulse technique [9]. This results in the gate

$$\mathrm{U}_d^{(1)} = |\mathcal{D}\rangle\langle\mathcal{D}| + \sum_{k=1}^{d-1} e^{i\gamma_k}|b_k\rangle\langle b_k| \tag{21}$$

acting on the qudit subspace Span$\{|\mathcal{D}\rangle, |b_1\rangle, \ldots, |b_{d-1}\rangle\}$. The unitary is parametrized by $3(d-1)$ parameters for $d \geq 2$, i.e.,

$$\mathrm{U}_d^{(1)} = \mathrm{U}_d^{(1)}(\varphi_1, \ldots, \varphi_{d-1}, \theta_1, \ldots, \theta_{d-1}, \gamma_1, \ldots, \gamma_{d-1}), \tag{22}$$

where we have assumed the c_k's are parametrized by the Euclidean components of the radius of the unit d-sphere with added phase factors:

$$\begin{aligned}c_1 &= \cos\varphi_1,\\ c_2 &= e^{i\theta_1}\sin\varphi_1\cos\varphi_2,\\ &\vdots\\ c_{d-1} &= e^{i\theta_{d-2}}\sin\varphi_1\ldots\sin\varphi_{d-2}\cos\varphi_{d-1},\\ c_d &= e^{i\theta_{d-1}}\sin\varphi_1\ldots\sin\varphi_{d-2}\sin\varphi_{d-1}.\end{aligned} \tag{23}$$

By applying the holonomy with different parameters in sequence up to k times is enough to create any desirable gate. To determine n, we use that the qudit state space is isomorphic to the special unitary group SU(d). By using dim(SU(d)) $= d^2 - 1$, we deduce that n must satisfy $3(d-1)n \geq d^2 - 1$, which implies $n \geq \frac{d+1}{3}$. Thus, the number of loops needed to create any unitary scales linearly since some gates can be created with fewer loops. In particular, $n = \frac{d+1}{3}$ when $d = 3j + 2$, $j \in \mathbb{N}$, which are optimal qudit dimensions in the sense that they require the smallest number of loops per dimension and these qudits could therefore be regarded as optimal carriers of information since the same number of loops must be carried out while higher dimension has higher information capacity.

Furthermore, any diagonal gate only requires one loop. Explicitly, by setting $\varphi_1 = \cdots = \varphi_{d-1} = \theta_1 = \cdots = \theta_{d-1} = 0$ the unitary reduces to the form

$$\mathrm{U}_d^{(1)}(0, \ldots, 0, \gamma_1, \ldots, \gamma_{d-1}) = |1\rangle\langle 1| + \sum_{k=2}^{d} e^{i\gamma_k}|k\rangle\langle k|. \tag{24}$$

This corresponds to the choice $c_k = \delta_{k1}$.

V. TWO-QUDIT HOLONOMIC DARK PATH GATES

VI. CONCLUSIONS

We have shown how to explicitly create a quantum mechanical system, which could be used to emulate a qutrit and corresponding universal set of single-qutrit gates. This is done by expanding the dark path qubit scheme [8] into a higher dimension. We have shown how it generalizes in the qudit case and using auxiliary states to improve the robustness of the gates.

The qutrit gates have a high fidelity and their robustness is improved by the inclusion of the auxiliary state in a similar way as for the qubit, which suggests that the dark path method can be beneficial for higher dimensional qudits to improve robustness. In the general qudit case, we have shown how any single-qudit diagonal unitary could be created by a single multi-pulse loop in parameter space and that non-diagonal unitaries scale linearly in the number of loops required for control of each loop. The possibility that the scheme expands efficiently into certain dimension have been discussed.

ACKNOWLEDGMENT

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[1] P. Zanardi and M. Rasetti, Holonomic quantum computation, Phys. Lett. A **264**, 94 (1999).

[2] E. Sjöqvist, D. M. Tong, L. M. Andersson, B. Hessmo, M. Johansson, and K. Singh, Non-adiabatic holonomic quantum

computation, New J. Phys. **14**, 103035 (2012).

[3] F. Wilczek and A. Zee, Appearance of Gauge Structure in Simple Dynamical Systems, Phys. Rev. Lett. **52**, 2111 (1984).

[4] J. Anandan, Non-adiabatic non-Abelian geometric phase, Phys. Lett. A **133** 171 (1988).

[5] M. Glusker, D. M. Hogan and P. Vass, The ternary calculating machine of Thomas Fowler, IEEE Annals of the History of Computing, **27**, 4 (2005).

[6] N. P. Brusentsov and J. R. Alvarez, Ternary Computers: The Setun and the Setun 70, In J. Impagliazzo and E. Proydakov (Eds.), Perspectives on Soviet and Russian Computing **357**, 74 (2011).

[7] Y. Wang, Z. Hu, B. C. Sanders, and S. Kais, Qudits and High-Dimensional Quantum Computing, Front. Phys. **8**, 589504 (2020).

[8] M. Z. Ai, S. Li, R. He, Z. Y. Xue, J. M. Cui, Y. F. Huang, C. F. Li and G. C. Guo, Experimental Realization of Nonadiabatic Holonomic Single-Qubit Quantum Gates with Two Dark Paths in a Trapped Ion, Fundam. Res. (2022), doi:10.1016/j.fmre.2021.11.031.

[9] E. Herterich and E. Sjöqvist, Single-loop multiple-pulse nonadiabatic holonomic quantum gates, Phys. Rev. A **94**, 052310 (2016).

[10] V. O. Shkolnikov and G. Burkard, Effective Hamiltonian theory of the geometric evolution of quantum systems, Phys. Rev. A **101**, 042101 (2020).

[11] J. R. Morris and B. W. Shore, Reduction of degenerate two-level excitation to independent two-state system, Phys. Rev. A **27**, 906 (1983).

[12] L. F. Shampine and M. W. Reichelt, The MATLAB ODE Suite, SIAM J. Sci Comput. **18**, 1 (1997).

[13] G. F. Xu, P. Z. Zhao, E. Sjöqvist, and D. M. Tong, Realizing nonadiabatic holonomic quantum computation beyond the three-level setting, Phys. Rev. A **103**, 052605 (2021).

[14] D. F. V. James and J. Jerke, Effective Hamiltonian theory and its applications in quantum information, Can. J. Phys. **85**, 625 (2007).