Master thesis project in computational science specification

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Project: Holonomic optimal control for qudits

Background

A circuit-based quantum computer performs efficient computation by means of unitary transformations (gates) acting on quantum bits (two-level systems). These 'qubits' differ from their classical counterpart in that they can be superposed and entangled; features that can be used in order to design certain algorithms that are more efficient than what can be achieved by a classical computer [1, 2]. However, quantum computers are highly sensitive to decoherence and noise that may destroy the computational advantage. Therefore, it becomes pertinent to implement quantum gate operations that are resilient to errors.

Holonomic control is a promising tool for implementing robust quantum gates. They are based on Berry phases [3] and their non-Abelian and/or non-adiabatic generalizations [4, 5, 6]. These phases are only sensitive to the global geometry of quantum state spaces and are therefore conjectured to be resilient to local errors along the evolution of the quantum states. In particular, the non-Abelian versions of holonomic control are useful as the non-Abelian feature is essential as non-commuting gates are necessary for universal quantum computation [7, 8].

Description of task

In this project, we examine holonomic optimal control [10, 11], which combine standard non-adiabatic holonomic quantum computation [8] with inverse engineering and quantum control to optimize gate performance. In particular, the focus is on extending holonomic optimal control to 'qudits', which are $d \geq 3$ dimensional carriers of quantum information [9]. These qudits are implemented in quantum systems, typically a trapped atom or ion, exhibiting a bipartite structure, defined by two coupled sub-groups of mutually uncoupled energy levels. One such system is the d-pod, in which d uncoupled 'ground state' levels, forming the qudit, is coupled to a single excited state. The couplings are typically induced by external laser pulses with tailored shape and duration.

The key point with extending holonomic optimal control to qudits is twofold: first, qudits allow for an exponential increase of information capacity, when the number of primitive systems grows; secondly, the use of qudits increases the dimensionality of the parameter space and thus an increased flexibility in gate optimization. The purpose of the project is to optimize the gate performance to systematic errors combined with decoherence in such qudit systems. The aim is to contribute to the realization of efficient and error resilient quantum gates for robust quantum computation.

Method

The project involves a combination of (i) analytical derivation of Hamiltonian parameters via inverse engineering, and (ii) numerical simulation and optimisation of qudit gates in the presence of systematic and decoherence-induced errors.

The main focus is on the d-pod setting, which can be described by the Morris-Shore parametrization [12, 13] of the corresponding state space. Equations for these parameters are found by a combination of inverse engineering of the system Hamiltonian, and a cancellation of dynamical phases yielding purely holonomic quantum evolution. Resilience to systematic amplitude and phase errors, as well as decoherence simulated by means of Lindblad-type equations [14], are optimized over the restricted parameter space. Specifically, the optimization is carried out by maximizing the fidelity of the simulated gates with respect to the ideal gates over the Hamiltonian parameters.

Relevant courses

- Quantum Mechanics, 1FA352
- Quantum Information, 1FA592
- Scientific Computing 3, 1TD397
- Computational Physics, 1FA573
- High Performance Computing, 1TD062

Boundaries of task

The project is limited to:

- Finite dimensional Hilbert spaces with a d-pod structure;
- Classical treatment of the laser fields that induce transitions between the levels;
- Treatment of open system effects in the Markovian approximation, by means of the Lindblad equations.

These limitations are designed for efficient and accurate modelling of the qudit gates.

Time plan

- Week 35-36: Preparatory literature studies.
- Week 36-39: Derivation of parameter equations for the inverse engineering.
- Week 38-45: Code development.
- Week 43-49: Numerical optimisation of qudit gates in presence of errors.
- Week 47-51: Writing of report.
- Week 2 (2022): Presentation.

References

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