LISTA 02

$\boxed{01-}$ $R_x = \begin{bmatrix} 1 & 0,5 \\ 0,5 & 1 \end{bmatrix}$ $\quad \underline{p}_{xd} = \begin{bmatrix} 0,5 \\ 0,25 \end{bmatrix}$

a) $\underline{w}_{OPT} = R_x^{-1} \cdot \underline{p}_{xd}$

$$R_x^{-1} = \frac{1}{|R_x|} \cdot \begin{bmatrix} 1 & -0,5 \\ -0,5 & 1 \end{bmatrix} = \frac{1}{0,75} \begin{bmatrix} 1 & -0,5 \\ -0,5 & 1 \end{bmatrix} = \begin{bmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{bmatrix}$$

$$\underline{w}_{OPT} = \begin{bmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/4 \end{bmatrix} \Rightarrow \boxed{\underline{w}_{OPT} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}}$$

b) $\varepsilon_{MIN} = \sigma_D^2 - \underline{p}_{xd}^T R_x^{-1} \underline{p}_{xd}$

$$= \sigma_D^2 - \begin{bmatrix} 0,5 & 0,25 \end{bmatrix} \cdot \begin{bmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{bmatrix} \cdot \begin{bmatrix} 0,5 \\ 0,25 \end{bmatrix}$$

$$= \sigma_D^2 - \begin{bmatrix} 0,5 \cdot 0,25 \end{bmatrix} \cdot \begin{bmatrix} 0,5 \\ 0 \end{bmatrix} \Rightarrow \boxed{\varepsilon_{MIN} = \sigma_D^2 - 0,25}$$

c) $\det(R_x - \lambda I) = 0 \Rightarrow$ AUTOVALORES

$$\begin{vmatrix} 1-\lambda & 0,5 \\ 0,5 & 1-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)^2 - 0,25 = 0$$

$$1 - 2\lambda + \lambda^2 - 0,25 = 0 \Rightarrow \lambda^2 - 2\lambda + 0,75 = 0$$

$$\Delta = 4 - 3 = 1$$

$$\lambda = \frac{2 \pm 1}{2} \quad \lambda_1 = 1,5 \quad \lambda_2 = 0,5$$

AUTOVETORES : $R_x \cdot v_1 = \lambda_1 v_1$

$$\begin{bmatrix} 1 & 0,5 \\ 0,5 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = 1,5 \begin{bmatrix} a \\ b \end{bmatrix}$$

$$a + 0,5b = 1,5a \Rightarrow 0,5b = 0,5a \Rightarrow b = a$$

$$0,5a + b = 1,5b \Rightarrow 0,5a = 0,5b \Rightarrow a = b$$

SE $a = b = 1$: $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0,5 \\ 0,5 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = 0,5 \begin{bmatrix} c \\ d \end{bmatrix}$$

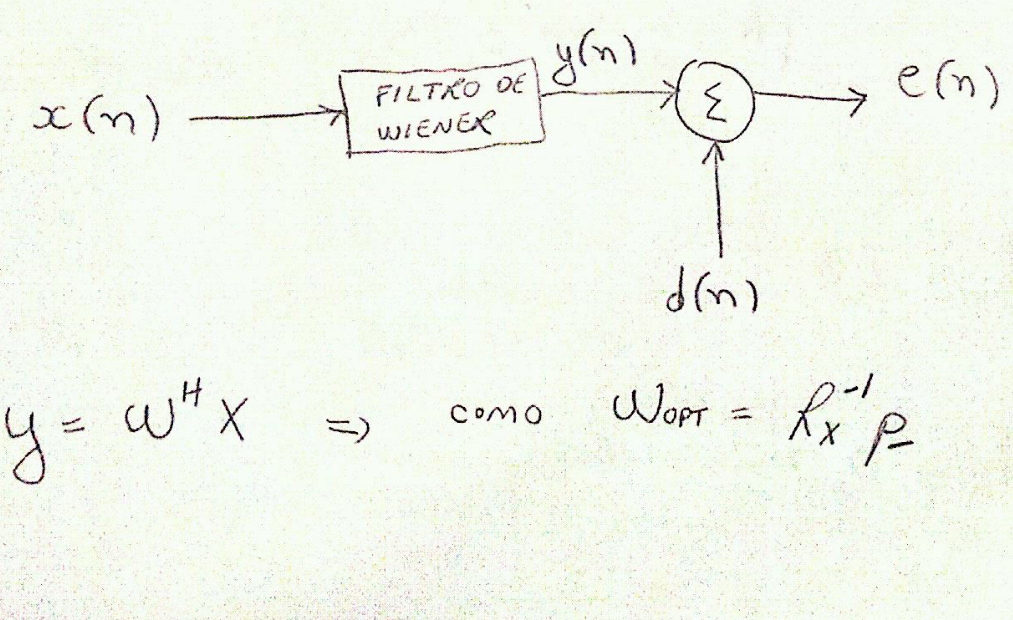
$$c + 0,5d = 0,5c \Rightarrow 0,5c = -0,5d \Rightarrow c = -d$$

SE $d = 1 \Rightarrow c = -1$: $v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$R_x = \Phi \Lambda \Phi^T$$

Φ → SUAS COLUNAS SÃO OS AUTOVETORES

$$R_x = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1,5 & 0 \\ 0 & 0,5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$



$y = w^H x \Rightarrow$ COMO $w_{OPT} = R_x^{-1} p$

LISTA 02

$\boxed{01-}$ CONTINUAÇÃO

c) $y = (R_x^{-1} p)^T \cdot x = p^T \cdot (R_x^{-1})^T \cdot x = p^T \left[(\Phi \Lambda \Phi^T)^{-1} \right]^T \cdot x =$

$= p^T \cdot \left[(\Phi \Lambda \Phi^T)^T \right]^{-1} \cdot x$

COMO A MATRIZ R_x É HERMITIANA:

$y = p^T \cdot (\Phi \Lambda \Phi^T)^{-1} \cdot x = p^T \cdot (\Phi^T)^{-1} \cdot \Lambda^{-1} \cdot \Phi^{-1} \cdot x$

$\Rightarrow y = (\Phi^{-1} p)^T \cdot \Lambda^{-1} \cdot (\Phi^{-1} x)$

DIMENSÕES:

$\Phi \Rightarrow N \times N$

$p \Rightarrow N \times 1$

$\Lambda \Rightarrow N \times N$

$x \Rightarrow N \times 1$

SEJA $A = \Phi^{-1} \cdot p$ DE ORDEM $N \times 1$ E $B = \Phi^{-1} x$ DE ORDEM $N \times 1$

PORTANTO: $y = A^T \cdot \Lambda^{-1} \cdot B$

EM QUE $\Lambda^{-1} = \begin{bmatrix} 1/\lambda_1 & 0 \\ 0 & 1/\lambda_N \end{bmatrix}$

NESTE CASO: $\Lambda^{-1} = \begin{bmatrix} 0{,}67 & 0 \\ 0 & 2 \end{bmatrix}$

LISTA 02

02 - NO FILTRO DE WIENER : $J_{min} = \sigma_d^2 - p_{xd}^T R_x^{-1} p_{xd}$

$$R_x w_{opt} = p_{xd}$$

SEJA $v(n)$ O VETOR AUMENTADO : $v(n) = \begin{bmatrix} d(n) \\ x(n) \end{bmatrix}$

PORTANTO : $A = \mathbb{E}\{v(n)\, v^T(n)\}$

$$A = \mathbb{E}\left\{ \begin{bmatrix} d(n) \\ x(n) \end{bmatrix} \begin{bmatrix} d(n) & x(n) \end{bmatrix} \right\} = \mathbb{E}\left\{ \begin{bmatrix} d(n)d^T(n) & d(n)x^T(n) \\ x(n)d^T(n) & x(n)x^T(n) \end{bmatrix} \right\}$$

$$\Rightarrow A = \begin{bmatrix} \sigma_d^2 & p_{xd}^T \\ p_{xd} & R_x \end{bmatrix}$$

ASSIM :

$$\begin{bmatrix} \sigma_d^2 & p_{xd}^T \\ p_{xd} & R_x \end{bmatrix} \begin{bmatrix} 1 \\ -w \end{bmatrix} = \begin{bmatrix} J_{min} \\ 0 \end{bmatrix}$$

E FINALMENTE :

$$J_{min} = \sigma_d^2 - p_{xd}^T w$$

$$p_{xd} - R_x w = 0 \Rightarrow w = w_{opt} = R_x^{-1} p_{xd}$$

$$\Rightarrow J_{min} = \sigma_d^2 - p_{xd}^T \cdot R_x^{-1} p_{xd}$$

LISTA 02

03-

$$e(n) = x(n) - \hat{v}_1(n)$$

$$x(n) = d(n) + v_1(n)$$

$$e(n) = d(n) + v_1(n) - \hat{v}_1(n)$$

$$\hat{v}_1(n) = w^H v_2(n)$$

NESTE CASO : SINAL DE ENTRADA $\longrightarrow v_2(n)$

SAÍDA DESEJADA $\longrightarrow v_1(n)$

FILTRO DE WIENER : $R_{v_2} \cdot w_{OPT} = P_{v_1 v_2}$

SABEMOS QUE $d(n)$ É DESCORRELACIONADO COM $v_1(n)$ E $v_2(n)$

SABEMOS AINDA QUE $v_1(n)$ E $v_2(n)$ SÃO CORRELACIONADOS.

$$R_{v_2} = \mathbb{E}\{ v_2(n)\, v_2^H(n) \}$$

$$P_{v_1 v_2} = \mathbb{E}\{ v_1(n)\, v_2(n) \} = \mathbb{E}\{ [x(n) - d(n)] \cdot v_2(n) \}$$

$$P_{v_1 v_2} = \mathbb{E}\{ x(n)\, v_2(n) \} - \underbrace{\mathbb{E}\{ d(n) \cdot v_2(n) \}}_{0}$$

$$\Rightarrow P_{v_1 v_2} = \mathbb{E}\{ x(n)\, v_2(n) \} = P_{x v_2}$$

E A EQUAÇÃO P/ O FILTRO DE WIENER TORNA-SE :

$$R_{v_2} \cdot w_{OPT} = P_{x v_2} \Rightarrow \boxed{w_{OPT} = R_{v_2}^{-1} \cdot P_{x v_2}}$$

LISTA 02

04-

$$x(n) = s(n+a) + s(n-4a)$$

$s(n)$ é WSS, a é CONSTANTE

$$d(n) = s(n-a)$$

$d(n)$ TEM MÉDIA ZERO E VARIÂNCIA UNITÁRIA

a)

$$\omega_{OPT} = R_x^{-1} p_{xd}$$

$$r_x(\tau) = \mathbb{E}\{x(n)x(n-\tau)\} = \mathbb{E}\{[s(n+a) + s(n-4a)] \cdot [s(n+a-\tau) + s(n-4a-\tau)]\}$$

$$= \mathbb{E}\{s(n+a) \cdot s(n+a-\tau) + s(n+a) \cdot s(n-4a-\tau) + s(n-4a)\, s(n+a-\tau) + s(n-4a) \cdot s(n-4a-\tau)\}$$

p/ $\tau = 0$: $r_x(0) = \mathbb{E}\{s^2(n+a)\} + \mathbb{E}\{s^2(n-4a)\} = 2$

p/ $\tau = 1$: $r_x(1) = 0$

p/ $\tau = -1$: $r_x(-1) = 0$

$$\Rightarrow R_x = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow R_x^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$p_{xd} = \mathbb{E}\{x(n)d(n)\} = \mathbb{E}\left\{\begin{bmatrix} x(n)\,d(n) \\ x(n-1)\,d(n) \end{bmatrix}\right\}$$

$$p_{xd} = \begin{bmatrix} \mathbb{E}\{[s(n+a) + s(n-4a)] \cdot s(n-a)\} \\ \mathbb{E}\{[s(n+a-1) + s(n-4a-1)]\, s(n-a)\} \end{bmatrix} = \begin{bmatrix} \mathbb{E}\{s(n+a)\,s(n-a)\} + \mathbb{E}\{s(n-4a)\,s(n-a)\} \\ \mathbb{E}\{s(n+a-1)\,s(n-a)\} + \mathbb{E}\{s(n-4a-1)\,s(n-a)\} \end{bmatrix}$$

$$p_{xd} = \begin{bmatrix} r_s(2a) + r_s(3a) \\ r_s(2a-1) + r_s(3a+1) \end{bmatrix}$$

PORTANTO: $W_{OPT} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} r_s(2a) + r_s(3a) \\ r_s(2a-1) + r_s(3a+1) \end{bmatrix}$

$$W_{OPT} = \frac{1}{2} \begin{bmatrix} r_s(2a) + r_s(3a) \\ r_s(2a-1) + r_s(3a+1) \end{bmatrix}$$

b) $w_{f,OPT} = R_x^{-1} \cdot r_{x,f}$

$$r_{x,f} = \mathbb{E}\left\{ \begin{bmatrix} x(n-1)\,x(n) \\ x(n-2)\,x(n) \end{bmatrix} \right\} = \begin{bmatrix} \mathbb{E}\{[s(n+a) + s(n-4a)][s(n+a-1) + s(n- \\ \mathbb{E}\{[s(n+a) + s(n-4a)][s(n+a-2) + s(n-4a- \end{bmatrix}$$

$$r_{x,f} = \begin{bmatrix} \mathbb{E}\{s(n+a)\cdot s(n+a-1)\} + \mathbb{E}\{s(n+a)\cdot s(n-4a-1)\} + \mathbb{E}\{s(n-4a)\,s(n+a-1)\} + \mathbb{E}\{s(n-4a)\,s(n-4a-1)\} \\ \mathbb{E}\{s(n+a)\cdot s(n+a-2)\} + \mathbb{E}\{s(n+a)\cdot s(n-4a-2)\} + \mathbb{E}\{s(n-4a)\cdot s(n+ + \mathbb{E}\{s(n-4a)\cdot s(n-4a-2)\} \end{bmatrix}$$

$$= \begin{bmatrix} r_s(1) + r_s(5a+1) + r_s(5a-1) + r_s(1) \\ r_s(2) + r_s(5a+2) + r_s(5a+2) + r_s(2) \end{bmatrix}$$

$$\Rightarrow \quad r_{x,f} = \begin{bmatrix} 2r_s(1) + r_s(5a+1) + r_s(5a-1) \\ 2r_s(2) + 2r_s(5a+2) \end{bmatrix}$$

LISTA 02

05-

$r_x(0) = 1$
$r_x(1) = 0$ $\Rightarrow$ $R_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\sigma_d^2 = 24{,}4$ $\qquad p_{xd} = \begin{bmatrix} 2 & 4{,}5 \end{bmatrix}^T$

a) $w_{opT} = R_x^{-1} \cdot p_{xd}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4{,}5 \end{bmatrix} = \begin{bmatrix} 2 \\ 4{,}5 \end{bmatrix}$$

b) $J(w) = \sigma_d^2 - 2w^T p_{xd} + w^T R_x w$

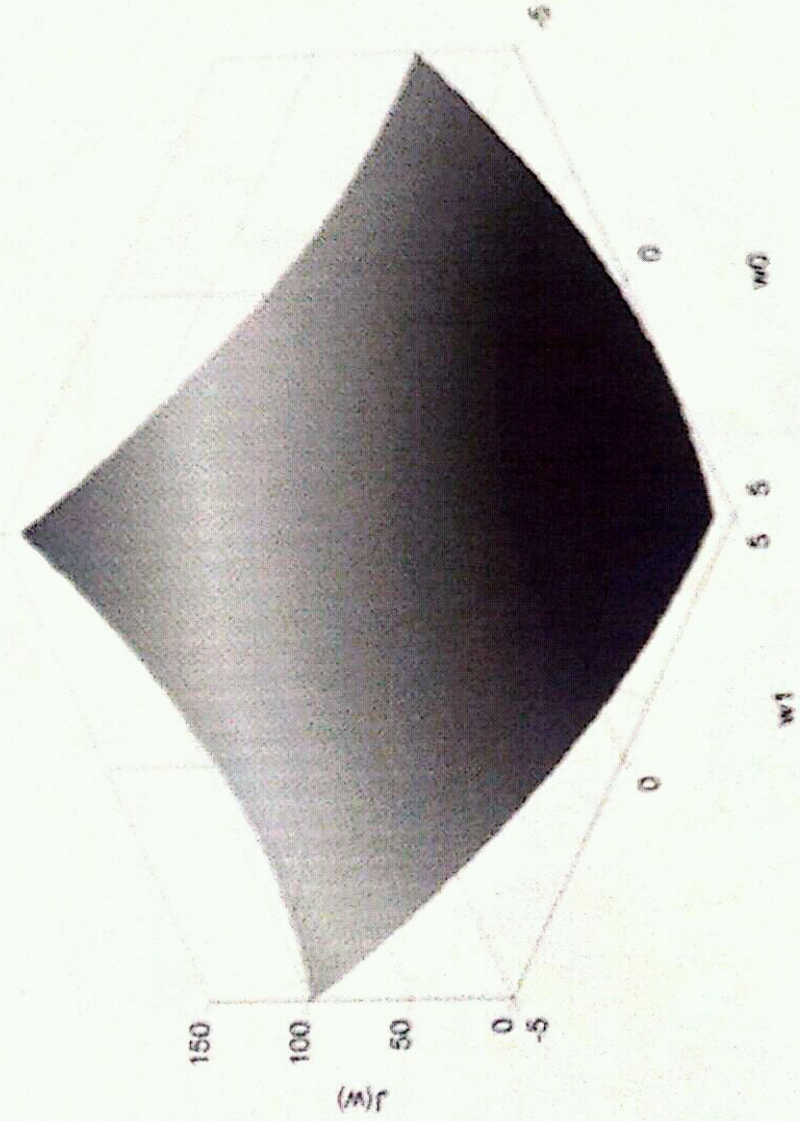
$$J(w) = 24{,}4 - 2\begin{bmatrix} w_0 & w_1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4{,}5 \end{bmatrix} + \begin{bmatrix} w_0 & w_1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot$$

$$= 24{,}4 - 2(2w_0 + 4{,}5w_1) + \begin{bmatrix} w_0 & w_1 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

$$= 24{,}4 - 4w_0 - 9w_1 + w_0^2 + w_1^2$$

$$\Rightarrow \boxed{J(w) = w_0^2 + w_1^2 - 4w_0 - 9w_1 + 24{,}4}$$

CONTINUA NA PRÓXIMA FOLHA


J(w)
150
100
50
0
-5
0
5
w1
5
0
-5
w0