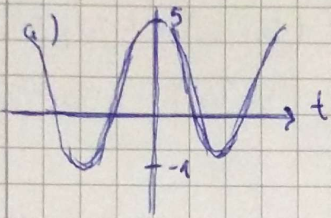
1º TESTE 07/04/2012

1. $x(t) = 2 + 3\cos(2\pi\, 10\, t - \pi/4)$
$y(t) = x(t - 1/20) - 4\sin(2\pi\, 20\, t)$

$T_0 = \frac{1}{f_0} = \frac{1}{10} = 0{,}1$

$f_{0x} = 10$ Hz

a) 

Um sinal contínuo/discreto diz-se periódico se se mantiver inalterado por um deslocamento de valor T.

$x(t) = x(t+T),\ T \in \mathbb{R}$ – contínuo
$x(n) = x(n+N),\ N \in \mathbb{Z}$ – discreto

Ao menor valor positivo de T/N dá-se o nome de período fundamental T_0 / N_0

$\text{mdc}(\{10, 20\}) = 10$ Hz

b) Cálculo do espectro pela f. Euler: $x(t) = X_0 + \sum_{k=1}^{N}\left[\frac{X_k}{2} e^{j2\pi f_k t} + \frac{\overline{X_k}}{2} e^{-j2\pi f_k t}\right]$

A_k amplitude k
ϕ_k fase k

$X_k = A_k e^{j\phi_k}$
$X_0 = 2e^{j0}$ ⟶ $2 + 0j \Rightarrow A = \sqrt{2^2 + 0^2},\ \phi = tg^{-1}\frac{0}{2} = 0 \Rightarrow 2e^{j0}$
$X_1 = 3e^{-j\pi/4}$
$f_0 = 0$
$f_1 = 10$

Então, $x(t) = X_0 + \frac{X_1}{2} e^{j2\pi f_1 t} + \frac{\overline{X_1}}{2} e^{-j2\pi f_1 t} =$

$= 2e^{j0} + \frac{3}{2} e^{-j\pi/4} e^{j2\pi 10 t} + \frac{3}{2} e^{j\pi/4} e^{-j2\pi 10 t}$

$\phi_0 = 0$ ϕ_1 f_1
$\phi_1 = -\pi/4$ X_1

$\{(f_k, |X_k|)\} = \{(0, 2); (10, 3/2); (-10, 3/2)\}$
$\{(f_k, \phi_k)\} = \{(0, 0); (10, -\pi/4); (-10, +\pi/4)\}$

(desenhar os pontos...)

$y(t) = x(t - 1/20) - 4\sin(2\pi 20 t = 2 - 3\sin(20\pi t + \pi/4) - 4\sin(2\pi 20 t) =$

$\sin(t) = \cos(t - \pi/2)$ $= 2 - 3\cos(20\pi t + \pi/4 - \pi/2) - 4\cos(2\pi 20 t - \pi/2) =$

$= 2 - 3\cos(2\pi 10 t - \pi/4) - 4\cos(2\pi 20 t - \pi/2)$

Igual a $x(t)$ mas com sinal negativo em A. "... y(t)"

$-1 + 0j \rightarrow \sqrt{(-1)^2 + 0^2} = 1$
$tg^{-1}\frac{0}{1} = 0 - \pi$

Transformar a A. em positivo

$-3 = 3 \times (-1) \rightarrow -1 = 1e^{j\pi} \rightarrow 3 \times 1e^{j\pi} = 3e^{j\pi/4}$

$3e^{j\pi}$ é no fundo igual ao que já estava por cima da fórmula e das trocas de sinais.

$y_0 = x_0$ $\quad\ldots - 4\cos(2\pi\, 20t - \pi/2)$

$y_1 = x_1$

Pela mesma maneira: $-4 = 4 \times (-1) = 4e^{j\pi}$

$\rightarrow \frac{4}{2} e^{j\pi/2} e^{j2\pi 20t} = 2 e^{j\pi/2} e^{j2\pi 20t} \rightarrow (20, 2) ; (-20, 2)$

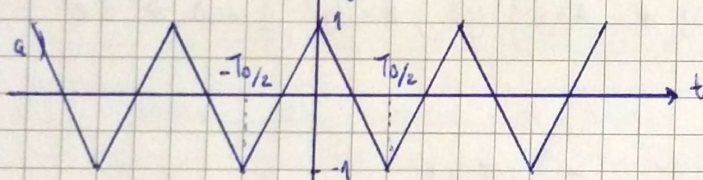
$(20, \pi/2) ; (-20, -\pi/2)$

c) $P_x = \sum_{k=-\infty}^{\infty} |X_k|^2 = 2^2 + (3/2)^2 + (3/2)^2$

$P_y = \sum_{k=-\infty}^{\infty} |Y_k|^2 = 2^2 + (3/2)^2 + (3/2)^2 + 2^2 + 2^2$

2. $A_k = \begin{cases} 0, & k \text{ par} \\ 4 \times \frac{2}{j\pi k}, & k \text{ ímpar} \end{cases}$ $\qquad T_0 = \frac{1}{f_0} = \frac{1}{10} = 0{,}1 \text{ s}$

$T_0/2 = 0{,}05$



Não tenho a certeza disto

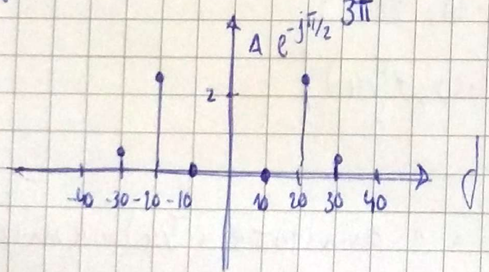
b) Domínio da frequência $\rightarrow (f_0, A_0) ; (f_1, A_1) ; (f_2, A_2) ; (f_3, A_3) \ldots$

$f_0 = 10$ Hz ; $A_0 = 0$

$f_1 = 20$ Hz ; $A_1 = \frac{8}{\pi} e^{-j\pi/2}$ (em polar / exponencial)

$f_2 = 30$ Hz ; $A_2 = 0$ ~~$\frac{4}{\pi} e^{-j\pi/2}$~~

$f_3 = 40$ Hz ; $A_3 = \frac{8}{3\pi} e^{-j\pi/2}$



e) Três primeiras harmónicas do sinal $(k < 4) \rightarrow k \in \{-3, -1, 1, 3\}$

Obter expressão analítica $a(t)$:

$$a(t) = \sum_{k=-\infty}^{\infty} A_k \, e^{j2\pi k f_0 t} = A_{-3} \, e^{j2\pi 10(-3)t} + A_{-1} e^{j2\pi 10(-1)t} +$$

$$+ A_1 e^{j2\pi 10 t} + A_3 e^{j2\pi 10 \times 3 t}$$

$$A_{-3} = 4 \times \frac{2}{-j\pi 3} \; ; \quad A_{-1} = 4 \times \frac{2}{-j\pi} \; ; \quad A_1 = 4 \times \frac{2}{j\pi} \; ; \quad A_3 = 4 \times \frac{2}{j\pi \times 3}$$

$$a(t) = \frac{8}{-j\pi 3} e^{-j2\pi 3 t \times 10} + \frac{8}{-j\pi} e^{-j2\pi 10 t} + \frac{8}{j\pi} e^{j2\pi 10 t} + \frac{8}{j\pi 3} e^{j2\pi 10 \times 3 t}$$

Pela f. de Euler $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$:

$$a(t) = -\frac{16}{3\pi} \times \frac{e^{-j2\pi 30 t}}{2j} + \frac{16}{3\pi} \times \frac{e^{+j2\pi 30 t}}{2j} + \quad \Bigg\| \; \frac{8}{-j\pi 3} = \frac{8}{\frac{3\pi}{2}} \times \frac{1}{j \times 2} = \frac{16}{3\pi} \times \frac{1}{2j}$$

$$-\frac{16}{\pi} \times \frac{e^{-j2\pi 10 t}}{2j} + \frac{16}{\pi} \times \frac{e^{+j2\pi 10 t}}{2j}$$

$$= \frac{16}{3\pi} \left[\frac{e^{+j2\pi 30 t}}{2j} - \frac{e^{-j2\pi 30 t}}{2j} \right] + \frac{16}{\pi} \left[\frac{e^{j2\pi 10 t}}{2j} - \frac{e^{-j2\pi 10 t}}{2j} \right]$$

$$= \frac{16}{3\pi} \times \sin(j\,2\pi 30 t) + \frac{16}{\pi} \times \sin(j\,2\pi 10 t)$$