1 Introduction

1.1 Objectives

The purpose of this project was to get acquaintance of a serial manipulator robot and control it in order to perform simple tasks, *e.g.*, grasping an object and transporting it.

For such goal to be accomplished, we will define the robotic arm's direct and inverse kinematics in order to, respectively:

a) Calculate the end effector's position, given a set of angles;

b) Calculate the angles for each joint, given a specific position and orientation of the end-effector.

The report starts with a small introductive section (this section) to present the project objectives and introduce the reader to the equipment used in the project. Section 2 presents the rationale behind the choices of the coordinate. The choice of the frames will be performed in accordance to the D-H convention. This convention determines that each homogeneous transformation between frames shall be described thorough 4 parameters, three of which constant and one variable. The D-H parameters associated with the frame choice will be presented in section 3. Section 4 presents a description on the rationale chosen for representing the end-effector orientation. The two following sections (5 and 6) are dedicated to the direct and inverse kinematics simulation. A section (7) with experimental results will be presented in order to demonstrate that the kinematic models used are suitable for representing the ROB3/TR5 kinematics. This section shall also thoroughly describe the calibration procedure. Finally, section 8 concludes the report with a discussion of the project results and other relevant considerations.

1.2 The ROB3/TR5

The serial manipulator in this project is the ROB3/TR5 arm (see figure 1), which was connected to a standard PC platform through an RS232 interface and was ultimately controlled through MATLAB routines.

This manipulator has 5 joints and an end-effector that can be articulated in order to grasp objects. Each joint position is defined by a value in the interval between 0 and 255 and also has a theoretical dynamic range (Table 1) that needs to be respected.

Table 1: Dynamic range (in degrees) for the joints of the manipulator.

Joint	0	1	2	3	4
Range	160º	100º	100º	200º	200º

Figure 1 - ROB3/TR5 serial manipulator

2 Reference Frame Assignment

The kinematic model created for this project consists of five revolute joints (A1–A5) connected by six links (1–6), as it can be seen in Figure 2.

The frame position and orientation was chosen in accordance to the Denavit-Hartenberg (D-H) convention.

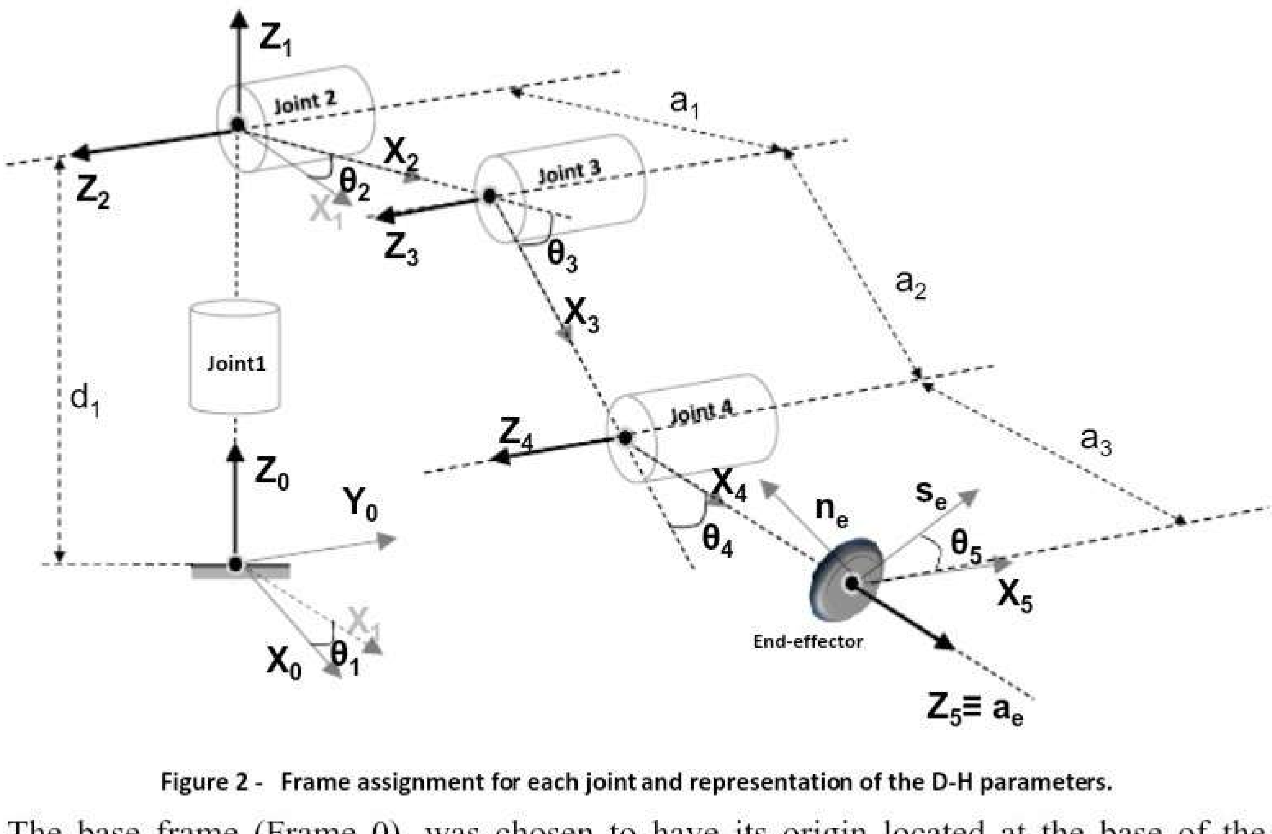

Figure 2 - Frame assignment for each joint and representation of the D-H parameters.

The base frame (Frame 0), was chosen to have its origin located at the base of the manipulator. Frame 1 and Frame 2 were chosen to have coincident origins located over joint 2 and the **Z** axis of both frames form a 90 degree angle around the $\mathbf{X}_1$ axis. Frame 3 origin is located over joint 3 and its **Z** axis is aligned with $\mathbf{Z}_2$, although separated by a distance of 'a_2' along the $\mathbf{X}_2$ axis. Similarly, frame 4 origin was located over joint 4 with its **Z** axis aligned with $\mathbf{Z}_3$ and displaced by a distance of 'a_3' along the $\mathbf{X}_3$ axis. Frame 5, was chosen to have the same origin as the end-effector frame which origin is displaced by 'a_4' from the frame 4 origin. Frame 5 **Z** axis was chosen to be aligned with the 'a_e' axis of the end-effector frame. The parameters that describe the transformation between frames 0-5 and from 5 to the end-effector frame are depicted in the Table 2.

3 D-H parameters

The D-H convention represents the homogenous transformations between frames by means of 4 parameters α, a, d, θ, as specified in Table 2 for our particular case.

Table 2: D-H convention parameters considered for the present model.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	275	θ_1
2	$\pi/2$	0	0	θ_2
3	0	200	0	θ_3
4	0	130	0	θ_4
5	$\pi/2$	130	0	$\pi/2$
6	$\pi/2$	0	0	θ_5

Special attention should be given to link 5, which is characterized by a fixed value of $\pi/2$ for θ_i. This constant transformation is necessary in order to align the approach direction of the end-effector with the rotation axis of the robot's claw.

Using the D-H convention, the homogeneous transformation from frame i-1 to frame i is given by the following matrix.

$$
{}^{i-1}_{i}T = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} & 0 & a_{i-1} \\ S_{\theta_i}C_{\alpha_{i-1}} & C_{\theta_i}C_{\alpha_{i-1}} & -S_{\alpha_{i-1}} & -S_{\alpha_{i-1}}d_i \\ S_{\theta_{i-1}}S_{\alpha_{i-1}} & C_{\theta_i}S_{\alpha_{i-1}} & C_{\alpha_{i-1}} & C_{\alpha_{i-1}}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

4. Orientation convention used for the end-effector

The end effector orientation frame was chosen in accordance with the following convention (see Figure 2):

- Axis '$\mathbf{a_e}$' is chosen to be aligned with the approach direction of the end-effector;
- Axis '$\mathbf{s_e}$' is chosen normal to $\mathbf{a_e}$ along the sliding direction of the end-effector jaws;
- Axis '$\mathbf{n_e}$' is chosen perpendicular to the plain defined by $\mathbf{s_e}$ and $\mathbf{a_e}$, following the right-hand frame rule;

It is therefore possible to express the position and orientation of the end effector in the base frame using the homogeneous transformation, thus obtaining the following result:

$$
{}^{0}P = {}^{0}_{e}T \ {}^{e}P = \begin{bmatrix} \boldsymbol{n}_e^b & \boldsymbol{s}_e^b & \boldsymbol{a}_e^b & \boldsymbol{p}_e^b \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot {}^{e}P
$$

Where $\mathbf{n_e^b}$, $\mathbf{s_e^b}$, $\mathbf{a_e^b}$ designate the base frame representation of the $\mathbf{n_e}$, $\mathbf{s_e}$ and $\mathbf{a_e}$ vectors respectively and $\mathrm{p_e^b}$ designates the base frame representation of the end effector position. Therefore if we align the a_e axis with the rotation axis, the previous matrix may be interpreted as a XYZ Euler angle rotation. This was the orientation convention used in this project.

5. Direct Kinematics

The direct kinematics concerns the determination of the end-effector position and orientation, given the joint angles.

The application of the D-H homogeneous transformation matrix over a vector expressed in the frame i, will return the same vector expressed in the frame i-1 coordinates, as represented by the following expression:

$$^{i-1}P = {}^{i-1}_{i}T \ {}^{i}P$$

The homogeneous transformation from the base frame vector to the end effector frame is obtained by the successive application of the previous transformations thus giving the following result:

$$^{0}P = {}^{0}_{1}T \ {}^{1}_{2}T {}^{2}_{3}T {}^{3}_{4}T {}^{5}_{6}T {}^{6}_{e}T \ {}^{e}P$$

By direct substitution of the D-H parameters (expressed in Table 2) we obtain the homogeneous transformation matrixes for each link of the serial manipulator.

The direct kinematics approach was implemented in the MATLAB script that accompanies this report.

By exercising the model we see that whenever all joint angles are set to zero, the end-effector has no component in the $\mathbf{Y_0}$ axis, the component in the $\mathbf{X_0}$ axis corresponds to a1+a2+a3 and finally, the $\mathbf{Z_0}$ component is set to d1. In case of a rotation θ_1, the $\mathbf{Z_0}$ component remains constant, the $\mathbf{X_0}$ component changes its signal while maintain its length and Y_0 component remains 0, whenever θ_1 is equal to 180º. Similarly, whenever θ_1 is equal to 90º, the $\mathbf{X_0}$ component is set to 0, the $\mathbf{Y_0}$ component is set to the positive a1+a2+a3 quantity and $\mathbf{Z_0}$ remains d1. For θ_1= 270º a similar behavior is observed except for the $\mathbf{Y_0}$ component which is negative.

A rotation of the θ_2 by 90º sets the arm to a full vertical position ($\mathbf{Z_0}$ component set to d1+a1+a2+a3 not the remaining components set to 0). Setting the θ_3 angle to 90º, while maintain all the other joint angles set to zero, sets the $\mathbf{Z_0}$ component of the end effector's position to d1+a2+a3, the $\mathbf{X_0}$ component is set to a1 and the $\mathbf{Y_0}$ component is zero. Similarly, a rotation around the $\mathbf{Z_4}$ axis (θ_4) of 90º sets the $\mathbf{Z_0}$ component to d1+a3 and the $\mathbf{X_0}$ to a1+a2. A rotation of θ_5 produces a rotation of the end effector around the $\mathbf{a_e}$ axis.

Such results provide good indication that the Direct Kinematics model is adequate to the manipulator under study as its results are in accordance to the expected.

6. Inverse Kinematics

The Inverse Kinematics problem addresses the determination of the set of joint angles that match the user specified end-effector position and orientation. It is therefore the opposite problem of the Direct Kinematics.

The approach used was to consider the manipulator composed by the superposition of a two link planar manipulator in the X_1-Z_1 plain, with a rotation along the Z_0 axis (see figure 2). The following figure depicts the two-link planar arm approach as adjusted to our manipulator. Please note that in the real case, the angles increase in the opposite direction as indicated in the following figure. However, that is irrelevant for the overall results.

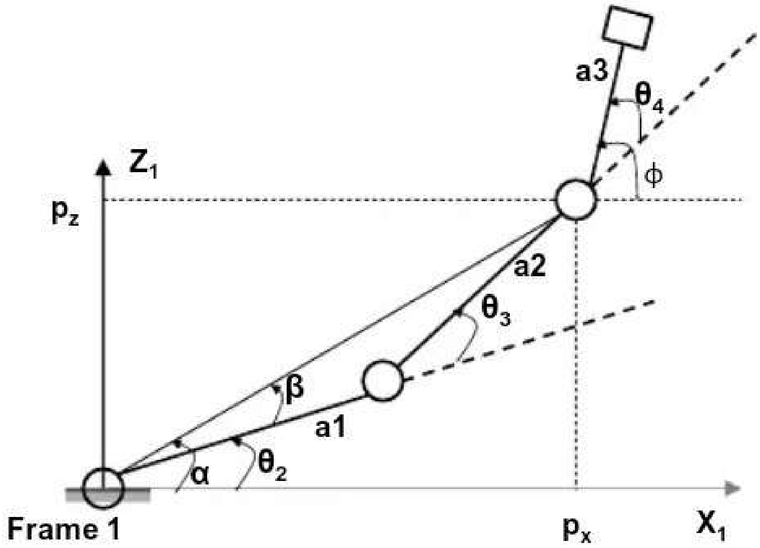

Figure 3 – Admissible postures for the two-link planar arm

The end-effector orientation will be defined by two different angles in order to uniquely define the end effector frame in relation to frame 1. One angle described the rotation about a_e as described in section 4 and the second angle is represented in the previous figure as Φ.

From the previous figure, using the cosine theorem and taking into consideration that $\cos(\pi\text{-}\theta)=\cos(\theta)$, the following expression may be derived:

$$c_3 = \cos(\theta_3) = \frac{p_z^2 + p_x^2 - a_1^2 - a_2^2}{2a_1a_2}$$

We thus obtain that the $\theta_3 = \pm\cos^{-1}(c_3)$.

From the figure we also conclude that

$$\propto = \text{Atan2}(p_z, p_x)$$

Applying the cosine theorem again we determine that β is given by the following expression:

$$\beta = \cos^{-1}\left(\frac{p_z^2 + p_x^2 + a_1^2 + a_2^2}{2a_1a_2\sqrt{p_z^2 + p_x^2}}\right)$$

We therefore conclude that $\theta_2 = \alpha\pm\beta$, where the positive sign applies to $\theta_3 > 0$ and the negative sign applies to $\theta_3 < 0$.

θ_4 is determined by noticing that the orientation angle Φ equals to $\theta_2 + \theta_3 + \theta_4$.

The θ_1 angle has two possibilities directly evaluated by Atan2(p_y,p_x) and Atan2($-p_y$,$-p_x$) respectively.

We therefore conclude that for this type of manipulator, for each value of θ_1 there are two possibilities for θ_2, thus pertaining 4 theoretical solutions possible for this type of manipulator. However, all solutions that do not fit into the ROB3/TR5 manipulator joint space will be discarded.

7 Experimental Results

This section describes the experimental trials using the ROB3-TR5 serial manipulator equipment. We present the approach for the calibration procedure, the techniques used in the laboratory to measure the end – effector position and orientation, and finally the results obtained from the applying the direct and inverse kinematics approach to actually move the manipulator arm to a specified position.

7.1 Calibration

The calibration process takes into consideration two important aspects:

- Alignment of the arm posture to be consistent with the base frame convention used by the kinematics model.
- To account for the transformation between the actual joint angle and the 8-bit value transmitted to the equipment controller.

Before transmitting data to the equipment, each joint specified joint angle is dully adjusted by the correspondent calibration curve. Whenever the values are not within the limits of the equipment (0 to 255), an error is reported and no data is transmitted.

This calibration is based on the assumption that the real joint angles depend linearly on the theoretical ones as presented in the following expression, where A and B are constants vectors:

$$\Theta_{Real} = A.\Theta_{Theor} + B$$

In the previous relation, Θ_{Real} is a vector where each component corresponds to the joint angle, measurable at the manipulator. On the other hand, the Θ_{Theor} vector components hold the expected joint angles.

The following process was used to perform the geometrical calibration:

- Use the direct kinematics process to make the arm move to the end position defined by a set of theoretical angles;
- Measure the end-effector end position and orientation using the techniques described in section 7.2;
- Use the inverse kinematics simulation in order to determine the real joint angles associated with the measured position;
- Repeat the previous steps for several different sets of theoretical angles.

This results in populating the Θ_{Real} and Θ_{Theor} vectors with the real joint angles and the theoretical ones, respectively. To compute the A and B parameters we simply need to perform a linear regression of the Θ_{Real} data versus Θ_{Theor} thus obtaining the calibration curves for each joint angle.

7.2 Measurement techniques

The experimental results mostly concern measuring the end-effector position. It is therefore of utmost importance to describe the techniques used for such measurements.

The X Y Z position of the end effector in the world frame was measured using an off-the-shelf measuring tape. Although the error associated with the position measurement may be considerable, we believe that the accuracy level is sufficient for reaching to the conclusions presented in section 8. The gripper distance was also measured using the same measuring tape.

The end effector orientation was evaluated by measuring the X and Y position of the end-effector (in the end-effector frame) and evaluating the following expression:

$$\varphi = atan2(Y,X)$$

7.3 Direct Kinematics Results

The following table presents the expected positions (in the base frame) of the end effector after applying the direct kinematics procedure.

Table 3: Direct Kinematics expected positions.

JOINT ANGLES (Degrees)						Expected Position (mm)		
A1	A2	A3	A4	A5	G	X	Y	Z
0	0	0	0	0	1	450	5	257
45	45	-45	-45	45	0,2	251	243	319
-45	-45	45	45	90	0,5	Error (invalid A3)		
-45	45	45	45	120	0,6	Error (invalid A5)		
30	-30	45	60	180	0,8	281	160	12

The following table presents the measured positions (in the base frame) of the end effector after applying the direct kinematics procedure.

Table 4: Direct Kinematics measured positions.

JOINT ANGLES (Degrees)						Measured Position (mm)		
A1	A2	A3	A4	A5	G	X	Y	Z
0	0	0	0	0	1	460	0	275
45	45	-45	-45	45	0,2	256	257	324
-45	-45	45	45	90	0,5		-	
-45	45	45	45	120	0,6		-	
30	-30	45	60	180	0,8	288	166	16

7.4 Inverse Kinematics Results

The following table presents the manipulator's end-effector measured angles after application of the inverse kinematics procedure.

Table 5: Inverse Kinematics test results.

Expected End effector position & orientation (mm)					Measured angles (mm)				
X	Y	Z	φ_1	γ	A1	A2	A3	A4	A5
460	0	275	-100	0	No solutions				
455	0	273	?	0	0	0	0	0	0
200	0	200	-100	50	0	48	95	53	50
200	0	200	?	0	0	27	-60	99	0

8 Conclusions

The kinematic models described in this report have been implemented in a MATLAB script that allows us to retrieve the experimental results expressed in section 7. The MATLAB script permits to simulate and to position the end-effector in a specific position (referred to the base frame) by direct kinematics or by inverse kinematic processes. The user accesses this functionality through a very intuitive Graphical User Interface that among many features supports visualization of the expected results in both numerical and graphical formats.

The results obtained are quite satisfactory for validating the suitability of the theoretical kinematics models to the ROB3/TR5 serial manipulator. Although the techniques used for measuring the end-effector position (in relation to the base frame) were not very accurate, the results present a great proximity with the expected values simulated by our kinematics model.

In a real world situation it is important that the kinematic models do not depend on a specific equipment configuration. Otherwise we would need one model for each specific equipment. Even for the same equipment the same model may not work accurately throughout the manipulator lifetime because its geometrical configurations may change due to normal usage of the system. Therefore, the calibration process is of utmost importance. Without such process it would have been very hard to compare the theoretical results with the results obtained by simulation. For that reason, we highly recommend to run the calibration process before using any of the MATLAB functionality that actually sends commands to the equipment joints. Among many other reasons, this will also make sure that the validation routines will effectively prevent the user from damaging the equipment because the real limit angles are only available after performing a proper calibration process.