Student: Andrea Cavallo

Matricola: 245715

Course: Digital Communication Laboratory

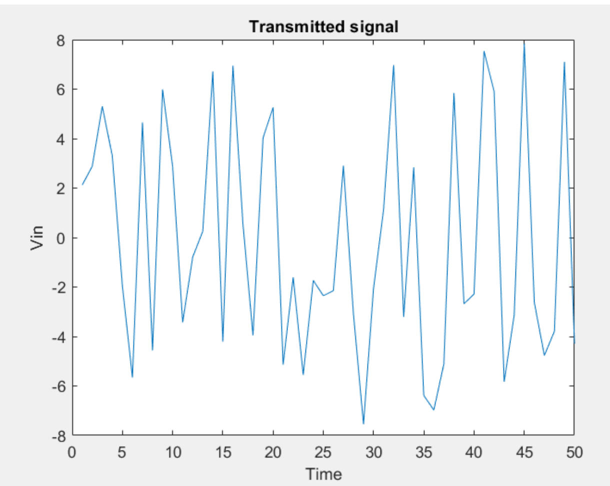
PROJECT #01

The project aims to build a numerical simulator to test a telecommunication system able to transmit an analog signal over a digital channel.

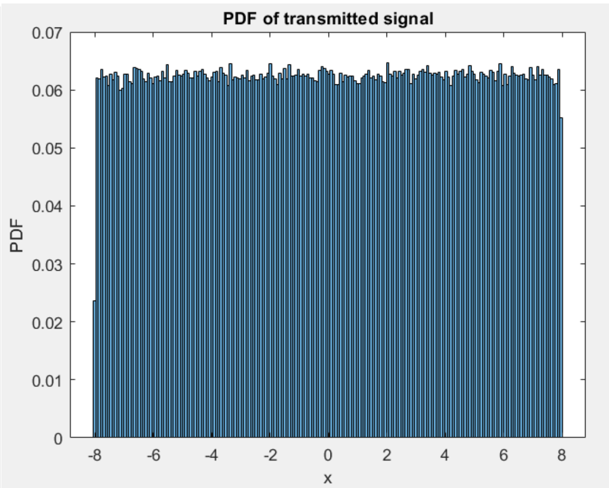
1. Uniform quantization and uniform pdf

The first part of the project consists in simulating a transmission of a signal with uniform probability density function (pdf) using uniform quantization.

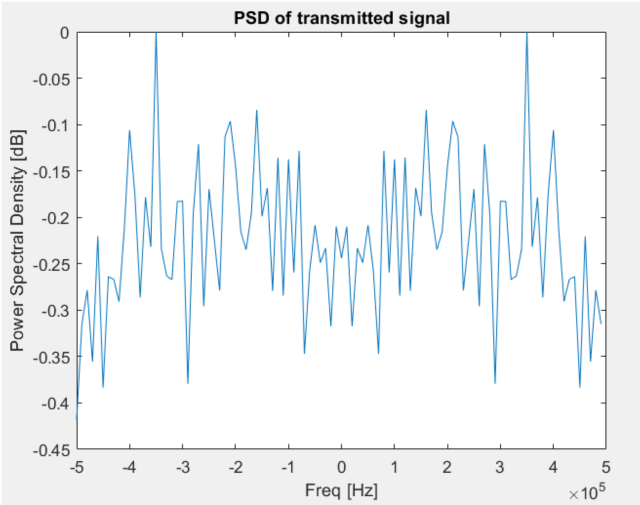
The signal is generated with the Matlab function rand, which generates a signal with uniform pdf. Even if Matlab generates a finite number of samples, the signal can still be considered analog since the samples are way denser than they are when the signal is quantized for the transmission.



First 50 samples of the signal

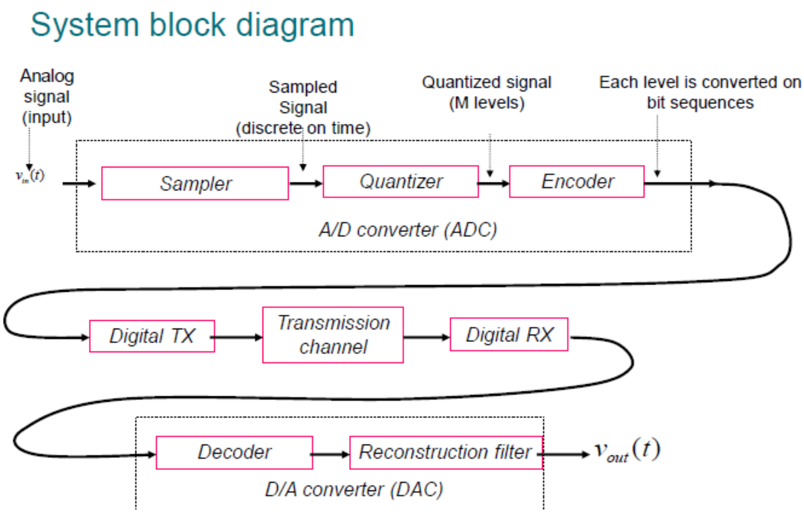


Pdf of the signal



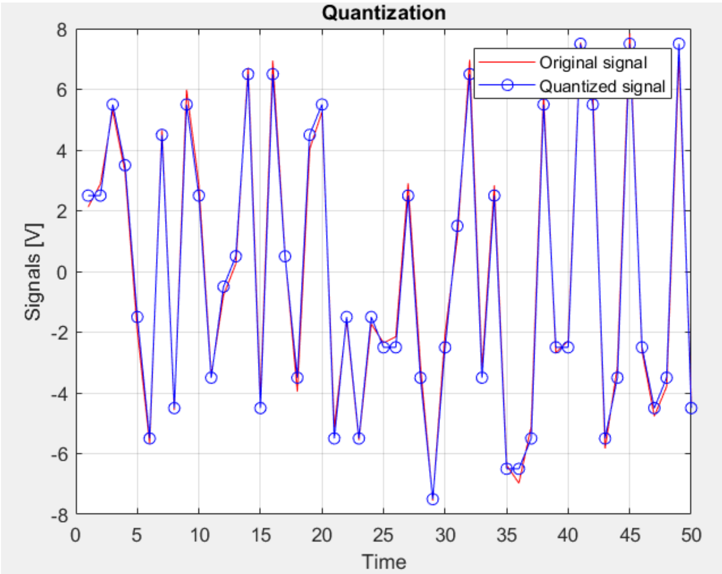
Power Spectral Density of the transmitted signal

The simulation is run according to the following scheme, where all possible sources of errors have been taken into consideration. In particular, the signal is sampled, quantized and encoded; then the transmission error is simulated according to a BSC model, and in conclusion the signal is decoded at the receiver and reconstructed.



Scheme of the blocks in the transmission system

The signal is quantized and the results of the quantization using 4 bits can be seen in the following picture.



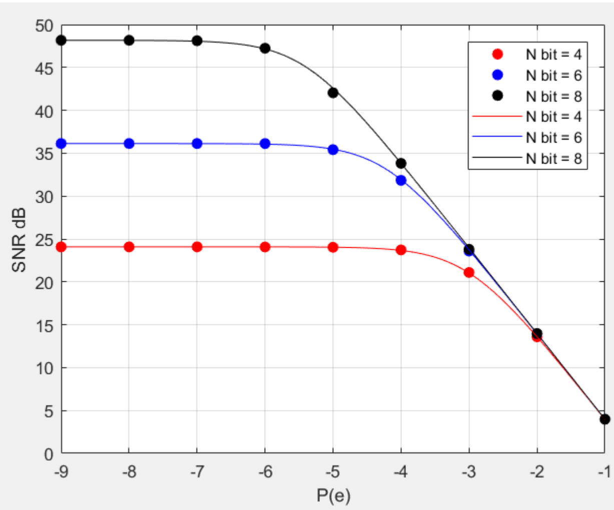
Results of signal quantization

In the figure, the red line represents the original signal and the blue line the quantized signal. In particular, the blue dots are the samples taken from the original signal. It can be seen that the quantization does not change the original signal too much.

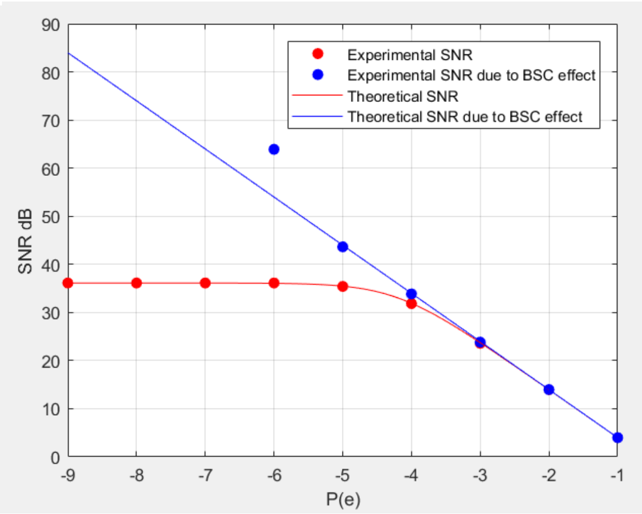
The following plot shows a comparison between the transmitted signal and the received signal, after the quantization and the transmission in the channel. Some errors can be clearly identified in this picture.

Comparison between transmitted and received signal

The results can be summarized in the following pictures.



Results for different number of bits



Results for 6 bits

For different number of bits in the channel, the continuous lines show the theoretical value of the Signal to Noise Ratio (SNR, expressed in dB), which represents the main parameter we consider to evaluate the performance of the system, with respect to the probability of error in the channel P(e). The dots, on the other side, represent the experimental values obtained during the simulation with 9 different values of P(e). By looking at the plot, it can be clearly noticed that the experimental results conform to the theoretical expectations. For a signal with uniform pdf, the simulator works as expected.

The experimental values (dots) are evaluated using the following formula:

$$(\frac{S}{N}) = \frac{\sigma_{Vin}^2}{\sigma_e^2}$$

where e = $V_{in} - V_{out}$. The expected theoretical results (continuous lines), on the other side, are evaluated with the formula below.

$$(\frac{S}{N}) = \frac{M^2}{1 + 4 * (M^2 - 1) * P_e}$$

From this formula it is possible to find a simple expression for the asymptotes as P(e) goes to zero, which is simply SNR = 6 * Nbits. It is possible to observe on the plot that these are the asymptotes of the curves plotted. For small values of P(e), the effect of the channel can be neglected and the main contribution to the SNR is given by the quantization error, whose effect on the SNR is M^2, and since M = 2^{Nbits} it is possible to derive the expression above for the asymptote.

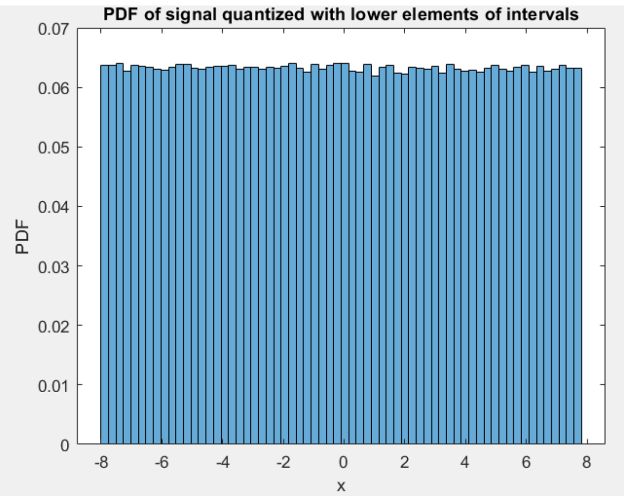
For big values of P(e), on the other side, the effect of the quantization error becomes negligible and the channel provides the main contribution to the SNR. This contribution is shown in the picture on the right (where 6 bits were used) and it can be expressed with the following formula.

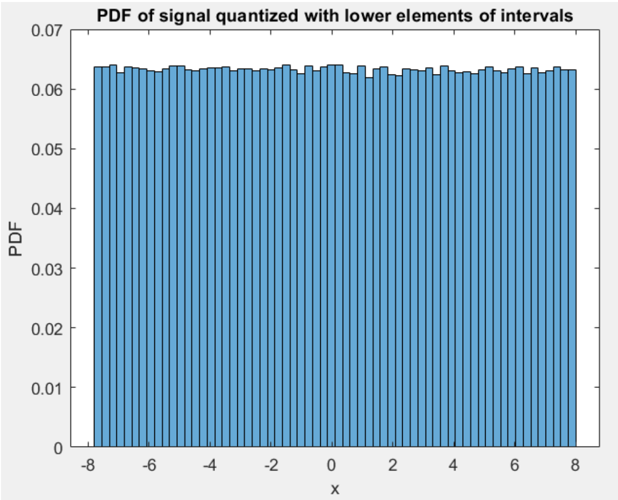
$$(\frac{S}{N}) = \frac{1}{4 * P_e}$$

2. Lower and upper limits of intervals as interval values

The second part of the project consists in simulating a transmission of a signal with uniform probability density function (pdf) using uniform quantization, as in the previous part, but now the values picked for each interval of the quantization are not the centre value, but the upper or the lower limit of the interval itself.

The following pictures show the pdfs of the signal quantized using lower and upper elements of the interval. The simulation is run using 6 bits.





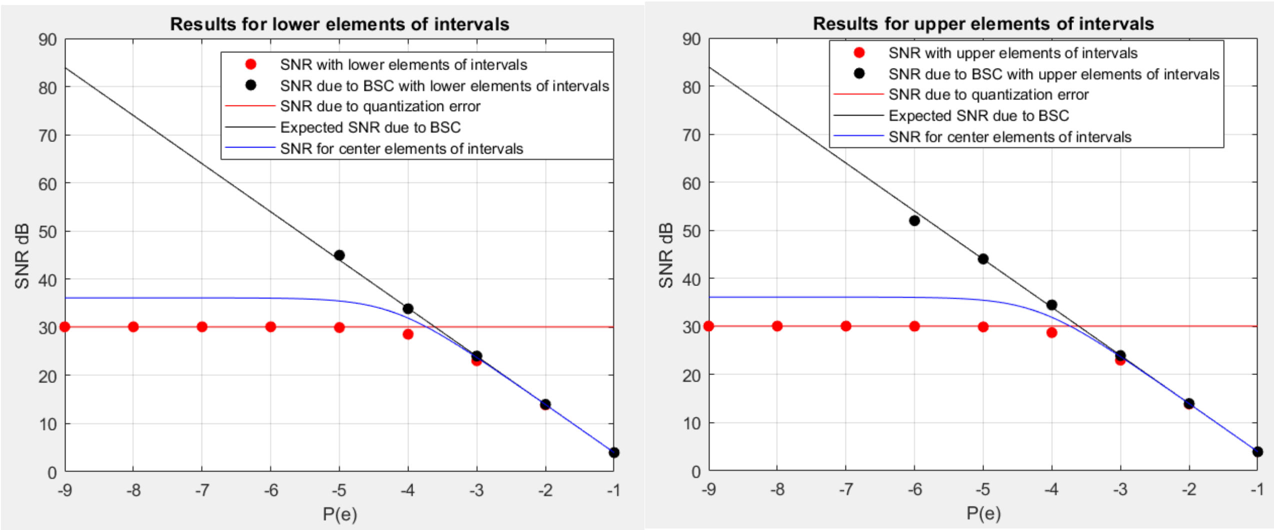
It can be noticed that the PDF plots are shifted with respect to the one obtained using centre elements of intervals for quantization.

Some differences occur with respect to the previous case. First of all, the formula for the SNR is now different due to the fact that the power of the error signal is not its variance anymore (the signal is not zero average). So, the experimental SNR is calculated as:

$$\left(\frac{S}{N}\right) = \frac{\sigma_{Vin}^2}{E[e^2]}$$

The contribution of the quantization error on the SNR is now $\frac{M^2}{4}$, which leads to a 6 dB down shift of the plot for small values of P(e), on a logarithmic scale. For large values of P(e), instead, no significant difference is detected with respect to the case of centre elements of intervals in quantization, since the contribution of the channel to the SNR (which prevails) does not change.

The results of the simulation are summarized in the following pictures.



Results of the second simulation

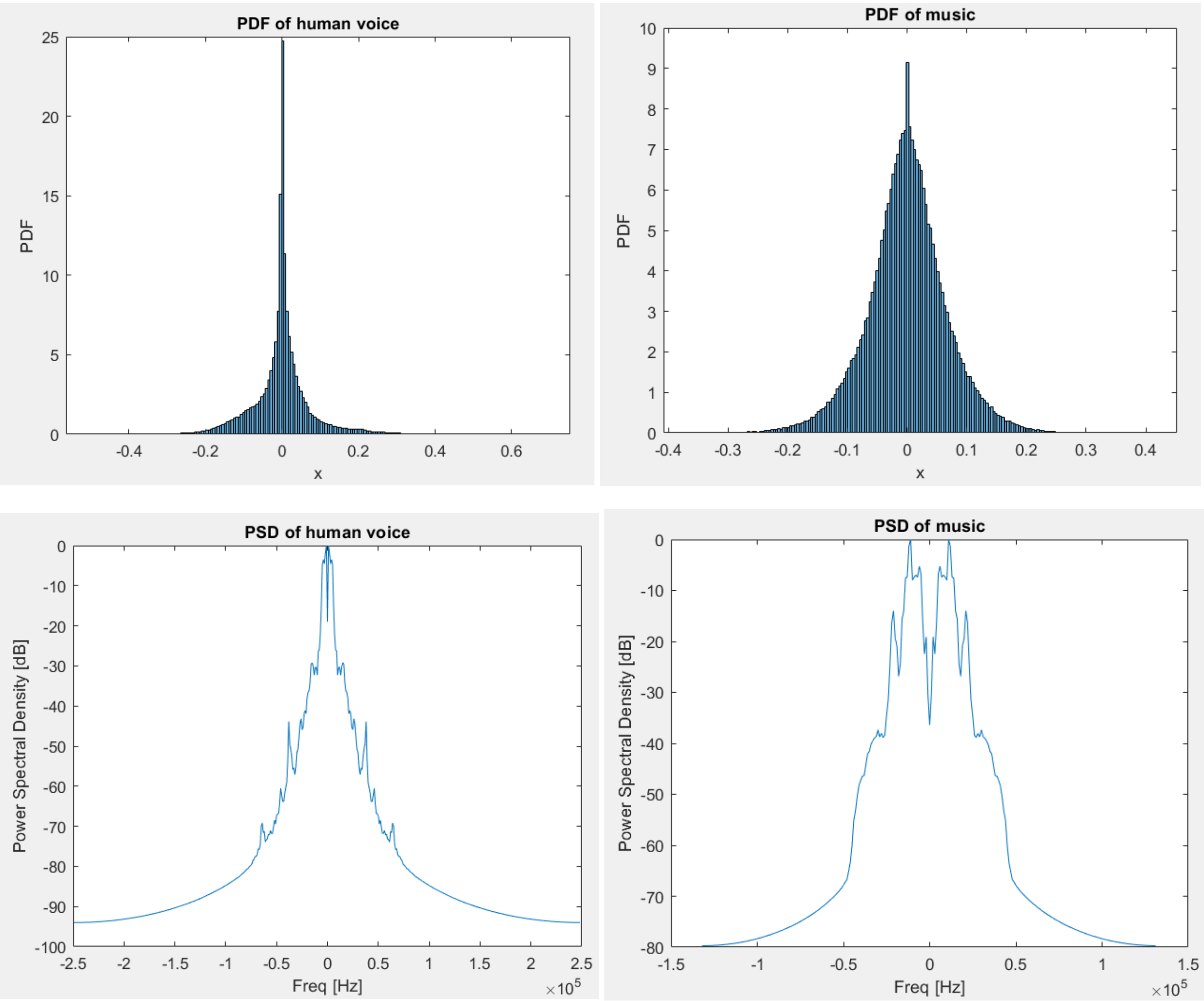
First of all, it is possible to observe that no difference occurs between the lower elements case and the upper elements case.

As expected, for small values of P(e) the quantization error prevails over the channel effects and the SNR values are smaller than the centre of intervals case by 6 dB. For high values of P(e), conversely, the values of the SNR are the same as the centre of intervals case because the BSC effects prevail on the quantization errors.

3. Audio signals

The third part of the project involves audio signals. Oppositely to the signals analysed before, audio signals do not have uniform pdf, so the transmission system used for the previous steps will show worse performances.

For the experience, two different audio signals were recorded: a sample of human voice, whose frequency spectrum is quite small, and a sample of a song, whose frequency spectrum is larger, as shown by the following pictures, which represent the PDFs and the PSDs (normalized) of the signals.



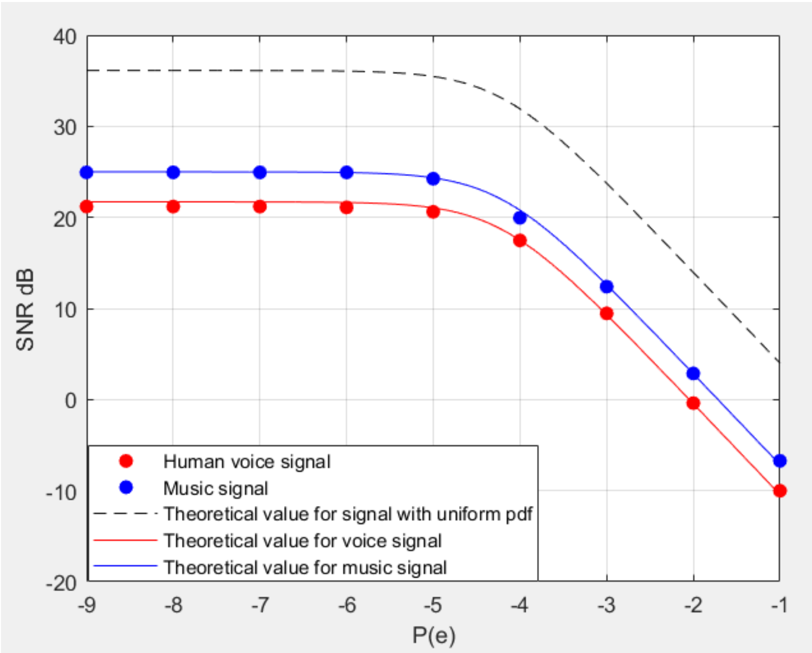
It is expected that the performances of a transmission system using uniform quantization are worse with signals with non-uniform pdf than with signals with uniform pdf. The loss of performance can be evaluated through a loss factor, which is calculated as follows:

$$a = \frac{\sigma^2_{Vuniform}}{\sigma^2_{Vsignal}}$$

where $\sigma^2_{Vuniform}$ is the variance of a generic signal with uniform pdf over the same range. The picture below shows the results of the simulation in terms of experimental SNR (dots), evaluated using the usual formula

$$\left(\frac{S}{N}\right) = \frac{\sigma^2_{Vin}}{\sigma^2_e}$$

and the expected values, evaluated by shifting down the theoretical SNR curve by the loss factor a. This factor is calculated for the two signals analysed and its values are 11.1306 (music signal) and 14.4233 (voice signal).



Results of the third simulation

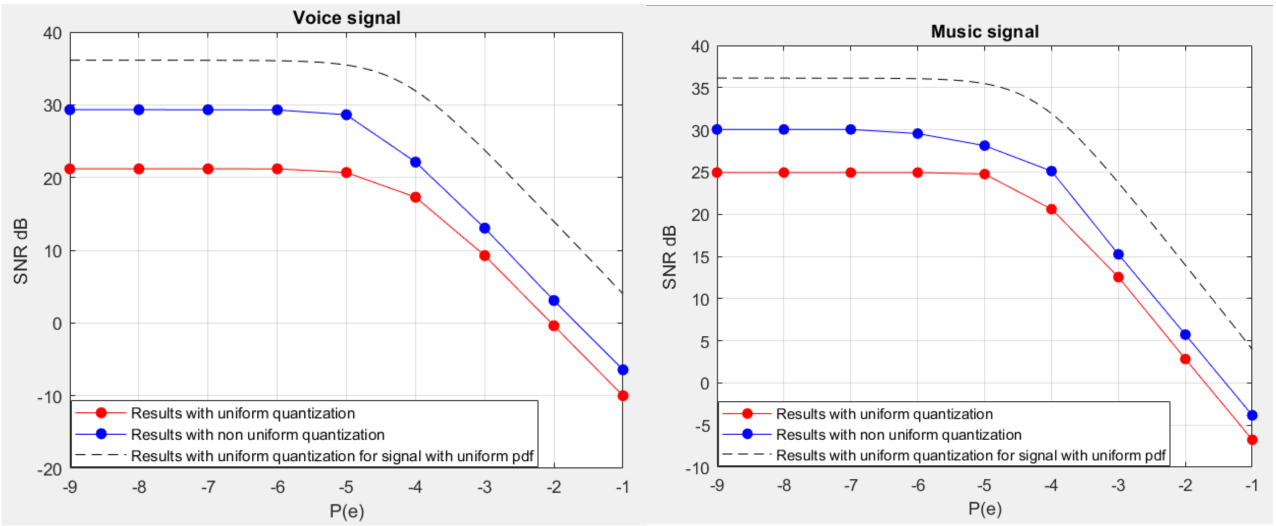
It can be noticed that the experimental values correspond to the expected results calculated through the loss factors.

It can also be observed that for the voice signal the performance of the transmission channel is worse than for the music signal. This can be explained by looking at the signals' PDFs. The music signal, in fact, has a wider PDF which is more similar to a uniform PDF than the voice signal, whose spectrum is smaller and shows a high peak for low frequencies.

4. Non-uniform quantization

Since the performances of the transmission system with uniform quantization turn out to be quite bad when dealing with signals with non-uniform pdf, in the fourth part of the project non-uniform quantization will be tested as a way to improve performances in these cases. In particular, the partition of the overall interval will be done according to Lloyd's algorithm.

The following pictures show the results of the experiment in terms of SNR on two different audio signals: human voice and music.



It can be clearly seen that the performances of the system with non-uniform quantization are better than the ones of the system with uniform quantization in both cases (blue lines vs red lines). This is due to the fact that a non-uniform quantization is more adaptable to the pdf of the signal and less information is lost during the quantization.

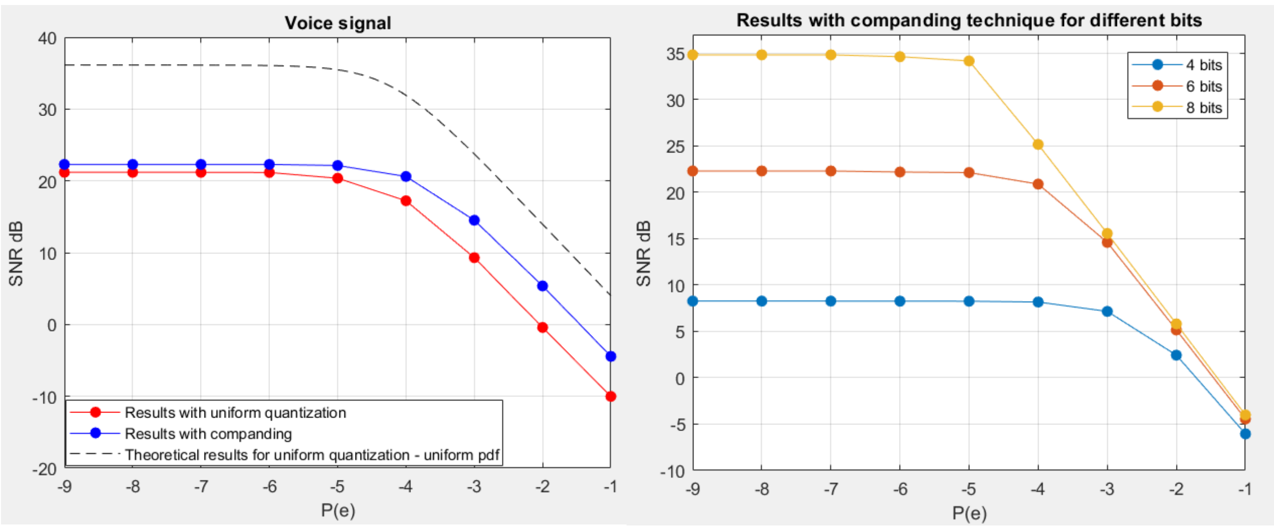
The improvement achieved by this method with respect to the uniform quantization method for small values of P(e) is better for voice signal (gain factor about 9 dB) than for the music signal (gain factor about 5 dB). However, the loss factor of the SNR for small values of P(e) with respect to the case of uniform quantization and uniform pdf is bigger for voice signal (more than 10 dB) than for music signal (around 6 dB). So, overall, the system analyzed performs better with music signal than with voice signal also with non-uniform quantization.

5. Companding technique

The fifth part of the project shows a way to improve the system's performances when dealing with signals with non-uniform pdf and at the same time use a uniform quantization. This is achieved with the companding technique, which consists in applying a non-linear component to compress the analog signal before converting it to digital, in order to make it look as uniform as possible. At the receiver, after the conversion from digital to analog, an equivalent component must be used to expand the signal and restore the original one. Several standards exist to design the non-linear component. In this example, the mu-law is used, which is the US standard. The transfer function of the component is the following.

$$|V_{out}| = \frac{\log(1+\mu\cdot|V_{in}|)}{\log(1+\mu)} \quad \text{dove:} \begin{cases} V_{in} \in [-1,+1] \\ \mu = 255 \end{cases}$$

This experiment is applied to an audio signal with human voice, and the results are shown and discussed below.



It can be observed from the plot that the companding technique provides some improvements with respect to a simple uniform quantization system, even though it is not as performing as a non-uniform quantization.

The picture on the right shows the results of the simulation with voice signal for different numbers of bits. It can be observed that the 6 dB rule is still valid with the companding technique. As a matter of fact, each bit added provides an improvement of around 6 dB in the SNR for small values of P(e).