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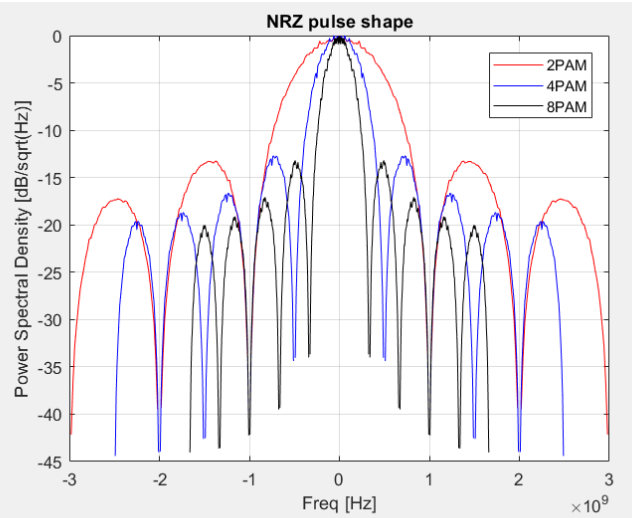
Course: Digital Communication Laboratory

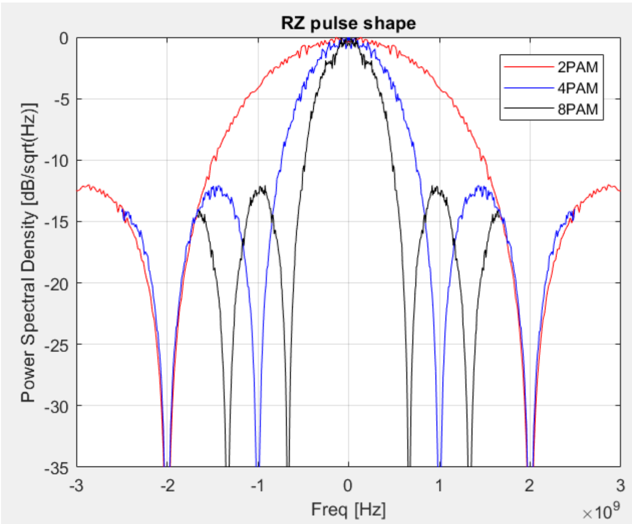
PROJECT #02

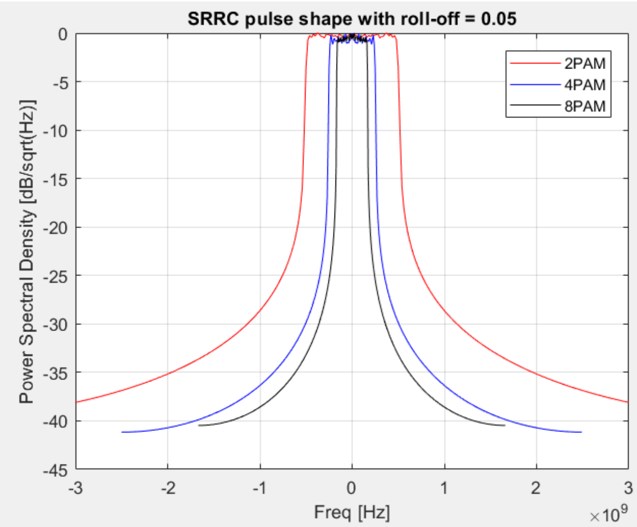
The aim of the project is to build a numerical simulator in order to analyse MPAM systems with antipodal symbols.

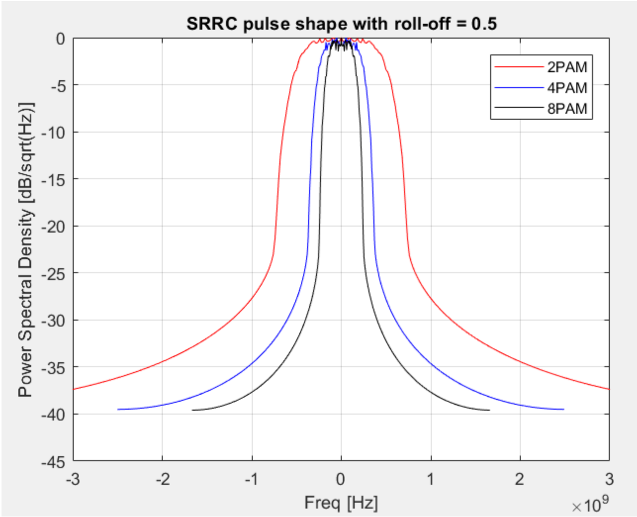
1. Transmitted spectra

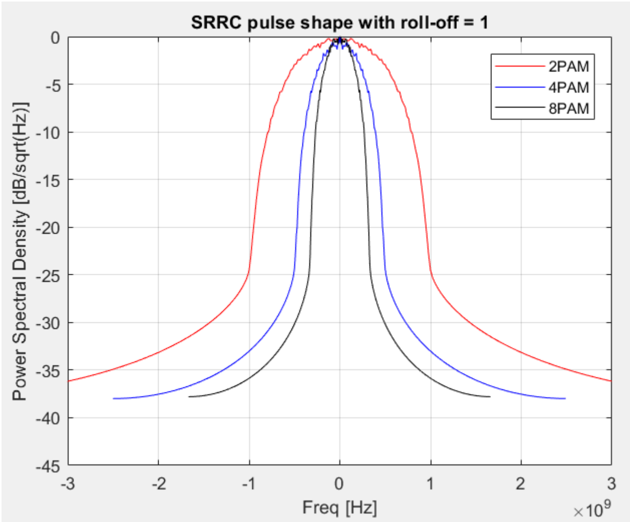
Three possible modulation formats are analysed (2PAM, 4PAM and 8PAM) and three different pulse shapes are applied at the transmitter (NRZ, RZ and Square Root Raised Cosine with different roll-off). The following pictures show the transmitted spectra for the different cases. The spectra are made smoother by means of the Bartlett periodogram. All simulations are run at the same bit rate.











2. Transmission in AWGN channel with matched filter

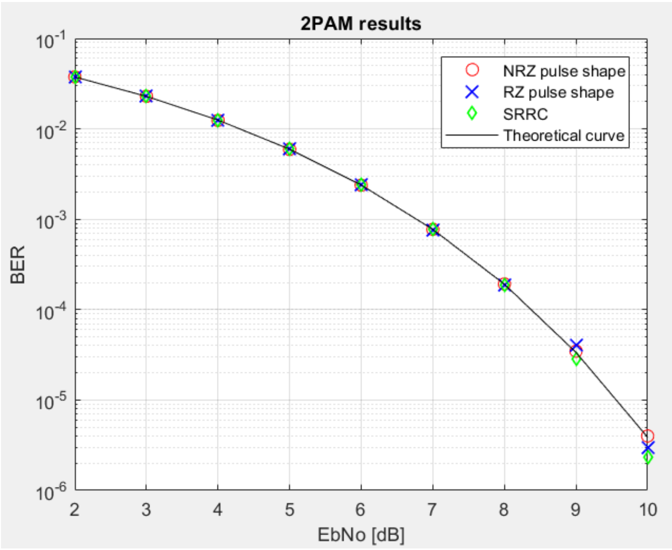
The second part of the project consists in simulating an AWGN channel using a matched filter at the receiver. To evaluate the performance of the different modulation formats (2PAM, 4PAM and 8PAM) and the different pulse shapes, BER (Bit Error Rate) and SER (Symbol Error Rate) are evaluated for different cases. The experimental results (dots) are evaluated through error counting, whereas the theoretical curves are evaluated using the following formulas.

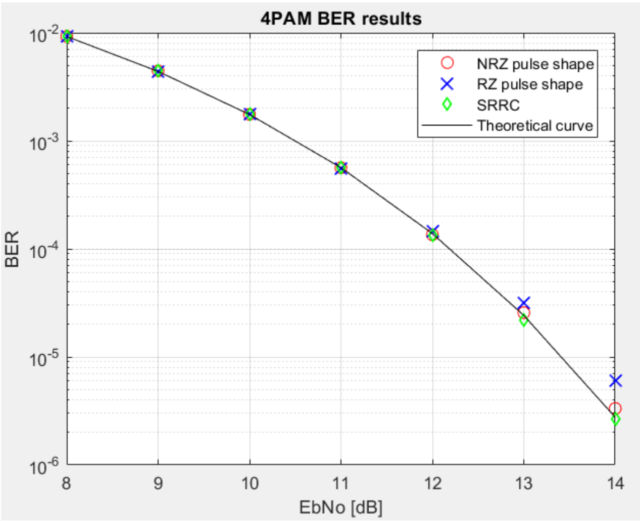
$$SER = \frac{M-1}{M} erfc\left(\sqrt{\frac{3\log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right)$$

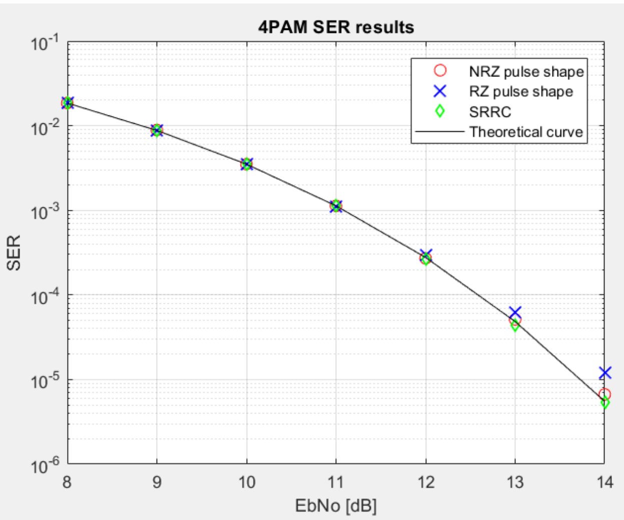
$$BER = \frac{1}{mM}\sum_{j=0}^{M-2} d_H(j+1,j)\, erfc\left(\sqrt{\gamma\frac{E_b}{N_0}}\right) + \frac{1}{mM}\sum_{j=0}^{M-3}\sum_{k=j+1}^{M-2}\left(d_H(k+1,j)-d_H(k,j)\right) erfc\left(\sqrt{(2k-2j+1)^2\gamma\frac{E_b}{N_0}}\right)$$

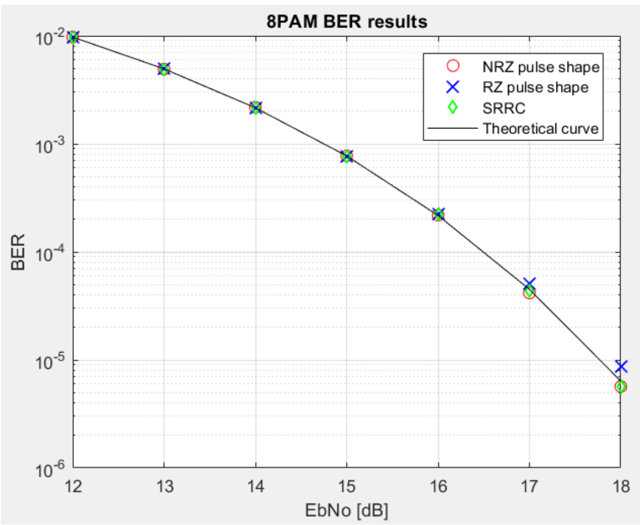
where d_H is the Hamming distance between different codes. Even though this formula is valid for all possible codings, Gray-coding is used for 4PAM and 8PAM. The penalty introduced by non-Gray coding will be discussed later.

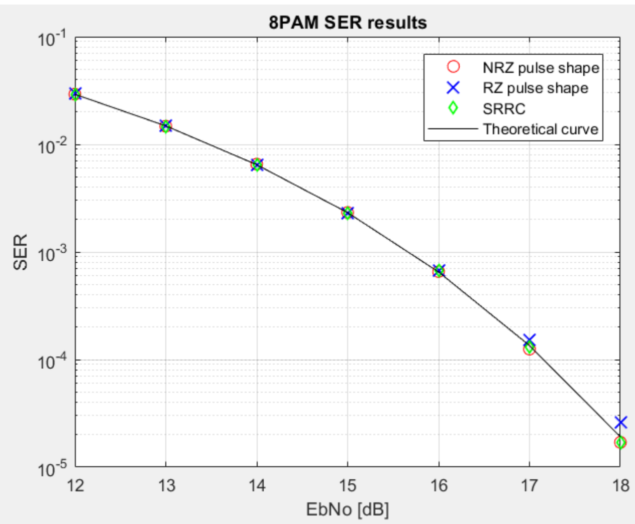
The pictures below show BER and SER results for different pulse shapes (the SRRC used has 0.5 roll-off) on the same modulation format. Dots are simulation results, continuous lines are theoretical results. It is clear from the plots that the results are the same for every pulse shape, which leads to conclude that BER and SER are independent on the pulse shape.



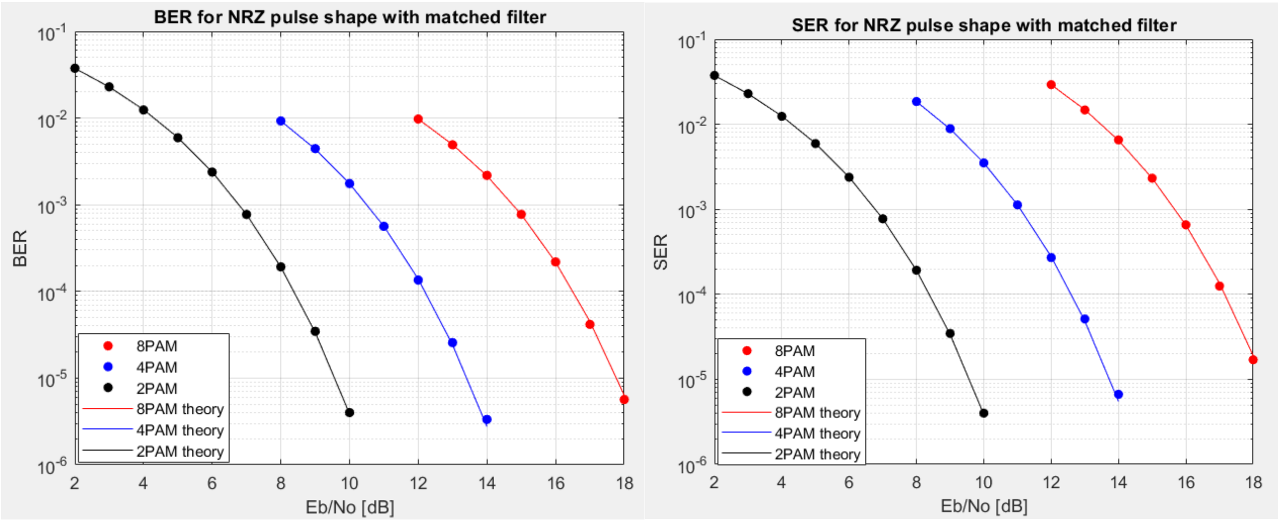








The following plots show a comparison between different modulation formats using the same pulse shape (NRZ in this case). It can be seen how changing the modulation format (which means increasing the number of bits per symbol) affects BER and SER by shifting the curves to the right.



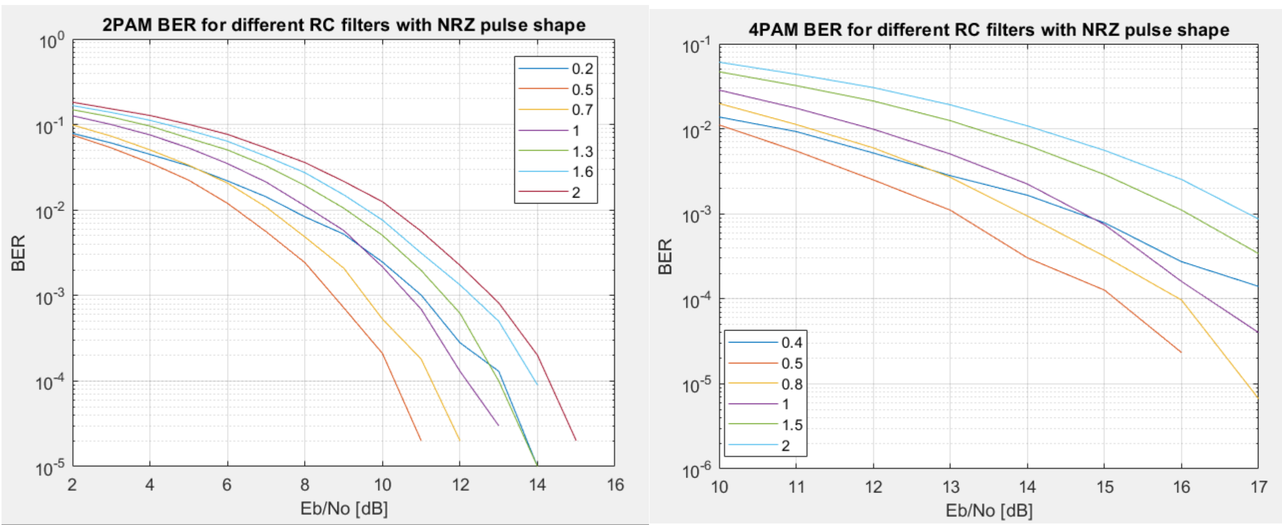
3. Transmission in AWGN channel with RC filter

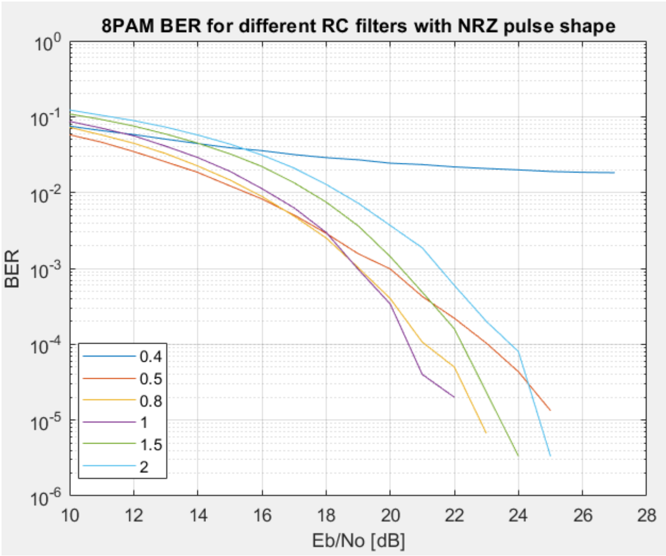
This part of the project aims to evaluate the effects of an RC filter at the receiver and the results related to different bandwidths. This analysis is important for realistic applications because the matched filter used before is an ideal case, whereas the RC filter can be used in real situations.

First of all, NRZ pulse shape is used. The following pictures show different results for BER corresponding to different bandwidths for the RC filter and different modulation formats. The bandwidth is calculated as follows:

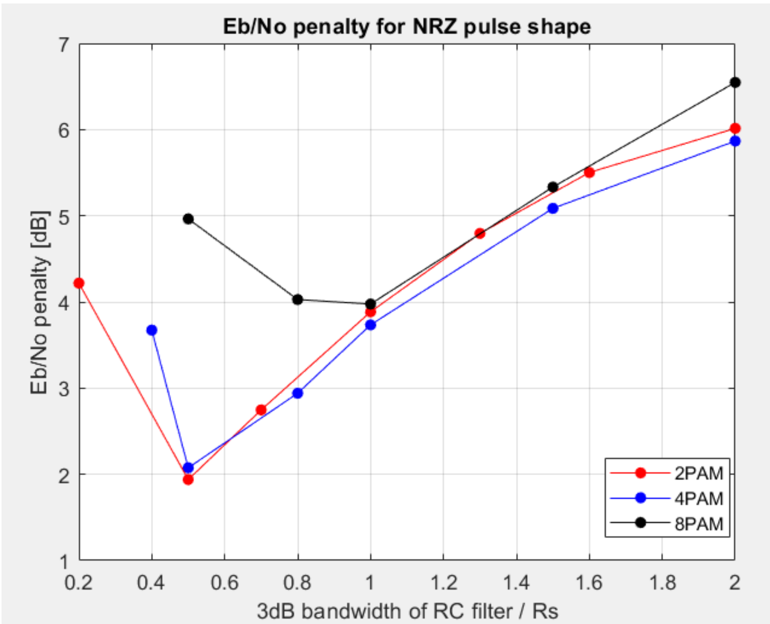
$$f_{3dB} = c * R_S,$$

Where R_S is the symbol rate and the coefficient c can be seen in the legends of the plots.

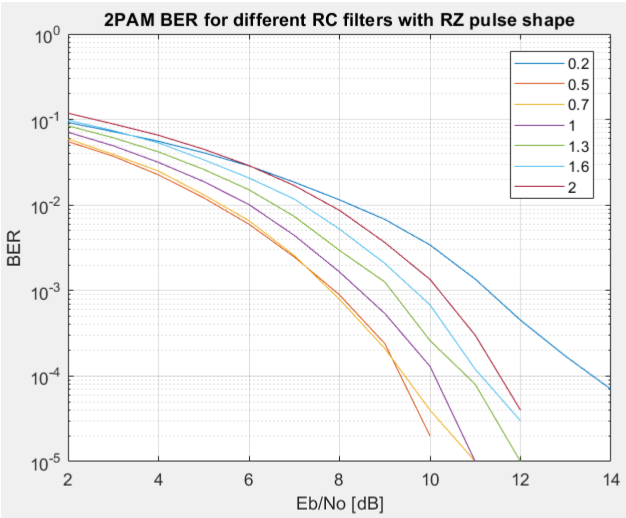


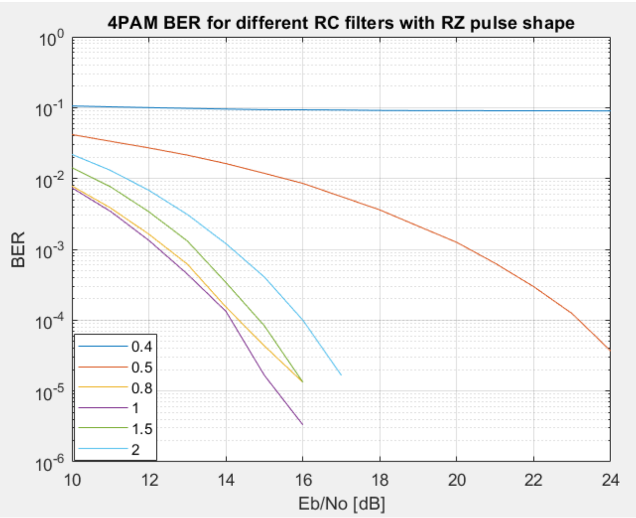


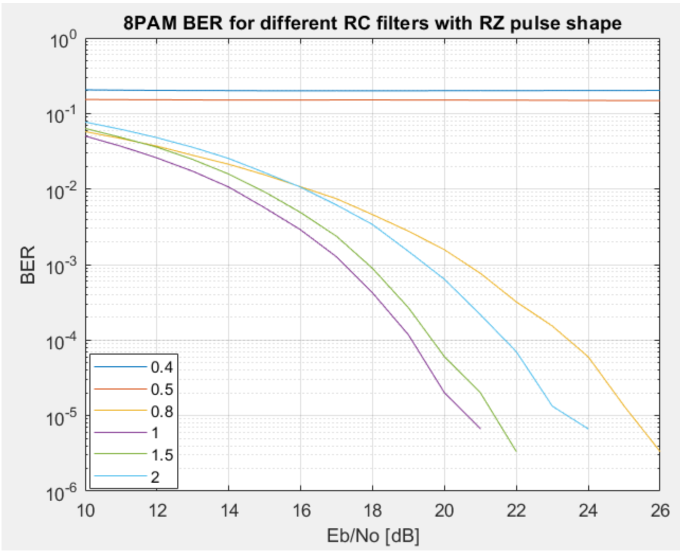
It can be observed that the performances are always worse than the case of a matched filter, but there is an optimum bandwidth which can be found experimentally. The following plot shows the penalty at a target BER of 10^{-3} in terms of Eb/N0 due to the RC filter with respect to the matched filter for different bandwidths. By looking at the graphs, it is possible to notice that there is an optimum bandwidth, which changes depending on the modulation format. For example, when using 2PAM or 4PAM modulation, the optimum bandwidth according to the plot is $f_{3dB} = 0.5 * R_S$, whereas for 8PAM modulation the optimum bandwidth is $f_{3dB} = 0.8 * R_S$.



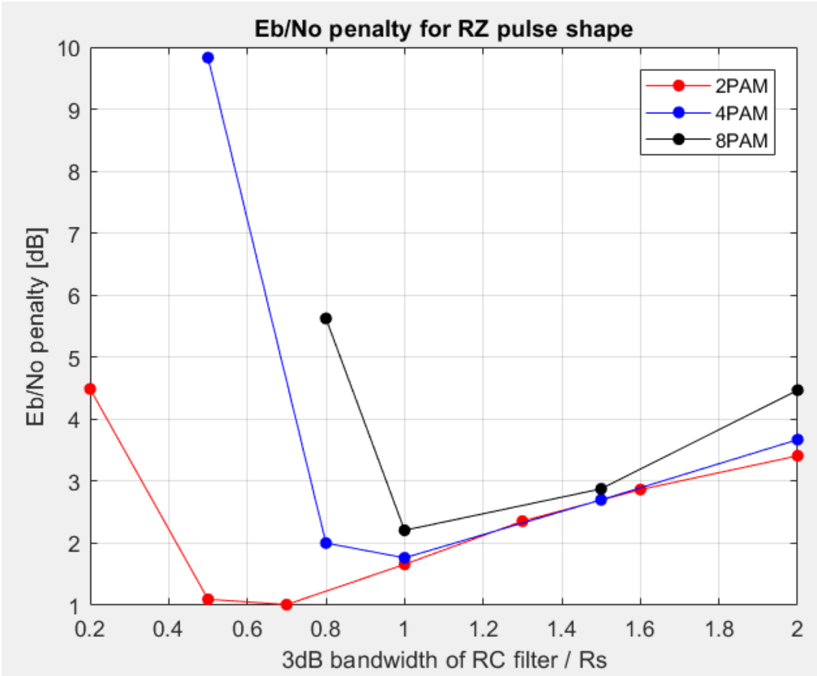
Running another simulation using RZ pulse shape, similar results are found with respect to the case of NRZ pulse shape. The following pictures show BER results for different bandwidths (the legends show the coefficient of Rs in the 3 dB bandwidth of the RC filter).



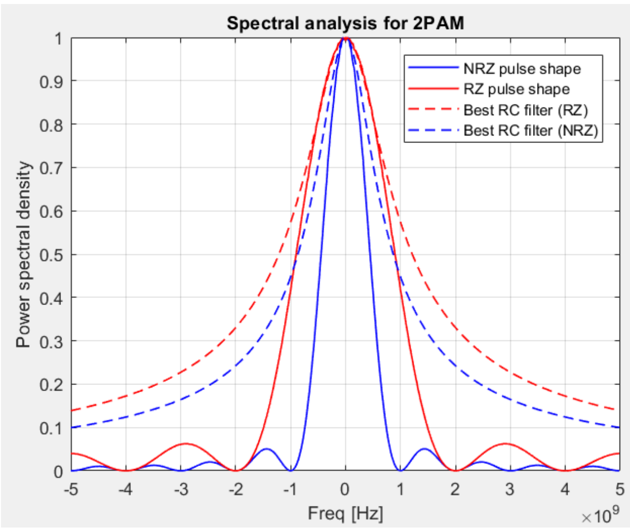


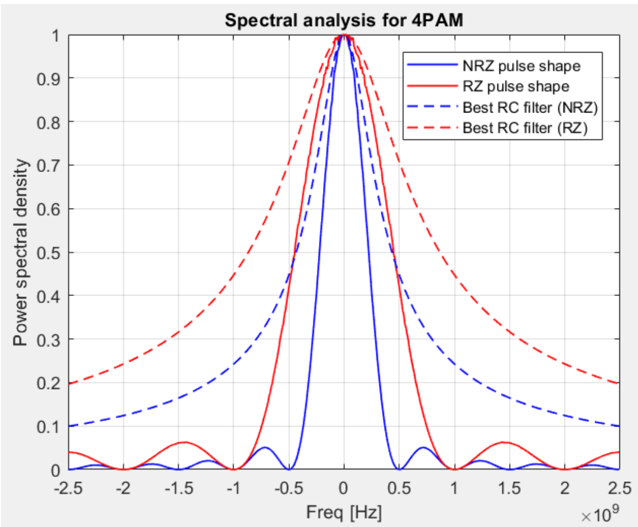


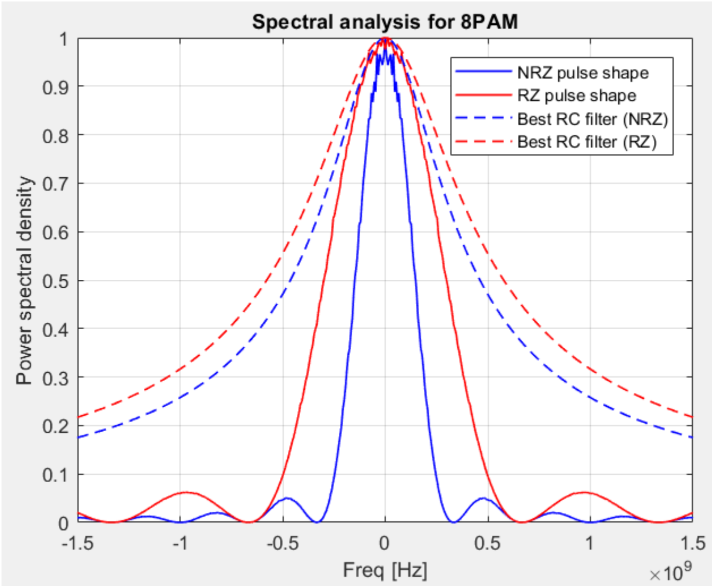
The picture below shows the Eb/No penalty at a target BER of 10^{-3} due to RC filter with respect to matched filter for different bandwidths. Similarly to the NRZ pulse shape case, it is possible to identify an optimum bandwidth for each modulation format, in order to reduce the penalty as much as possible. The optimum bandwidth for 2PAM modulation is now $f_{3dB} = 0.7 * R_S$, whereas the optimum bandwidths for 4PAM and 8PAM are bigger than in the NRZ case ($f_{3dB} = 1 * R_S$ in both cases).



The pictures below show, for all possible modulation formats, the frequency spectra of the transmitted signal with NRZ and RZ pulse shape and the spectra of the two best RC filters for NRZ and RZ pulse shape. The scale is not logarithmic. It is possible to notice that the spectra of the optimum filters are quite similar to the spectra of the transmitted signals. In addition, it is possible to explain the fact that RZ pulse shape has an optimum bandwidth bigger than NRZ pulse shape, and the reason is that the spectrum of the transmitted signal shaped with NRZ is thinner than the spectrum of the transmitted signal shaped with RZ, and the spectrum of RC filters gets wider as its 3 dB bandwidth increases.

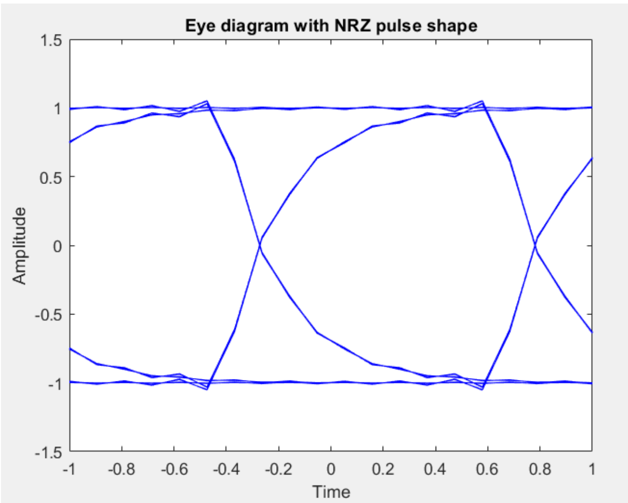


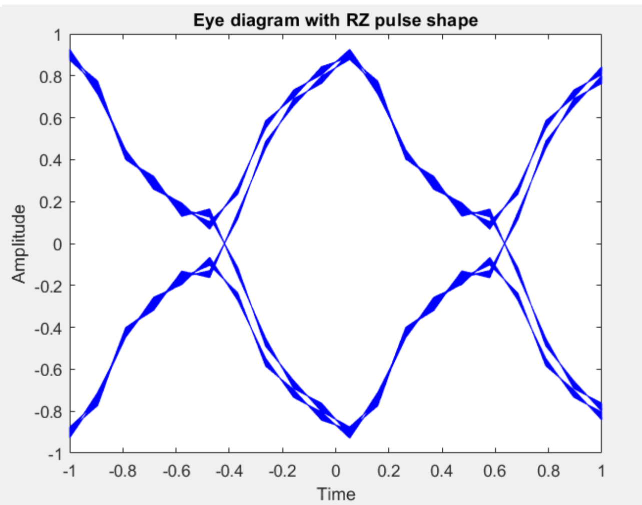




4. BER with ISI by means of the eye diagram

In the previous simulations, the effect of inter-symbol interference was neglected. The goal of this part of the project is to analyse this parameter using eye diagrams. In particular, two different simulations are run for NRZ and RZ pulse shapes with 2PAM modulation, using an RC filter at the receiver with a bandwidth $f_{3dB} = 0.75 * R_S$. The eye diagrams for both cases are shown below.





Then, two different formulas are used to evaluate the BER from the eye diagram, and the theoretical results obtained with these formulas are compared to the experimental results. The first formula is the one reported below:

$$BER = \frac{1}{2} erfc\left(\sqrt{k\left(\frac{E_b}{N_0}\right)_{R_b}}\right)$$

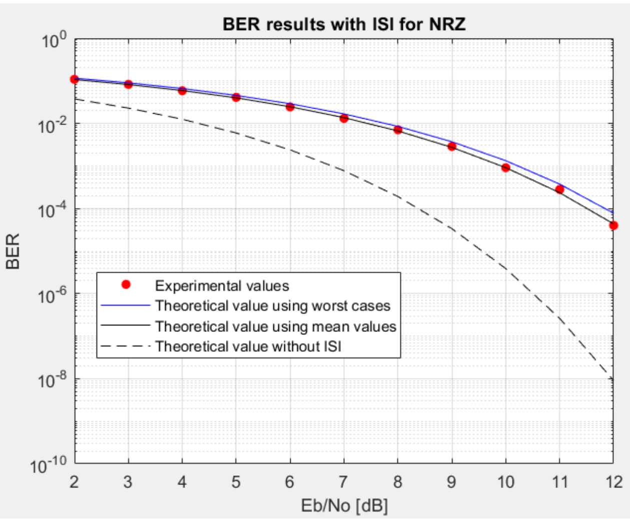
where k is calculated as follows:

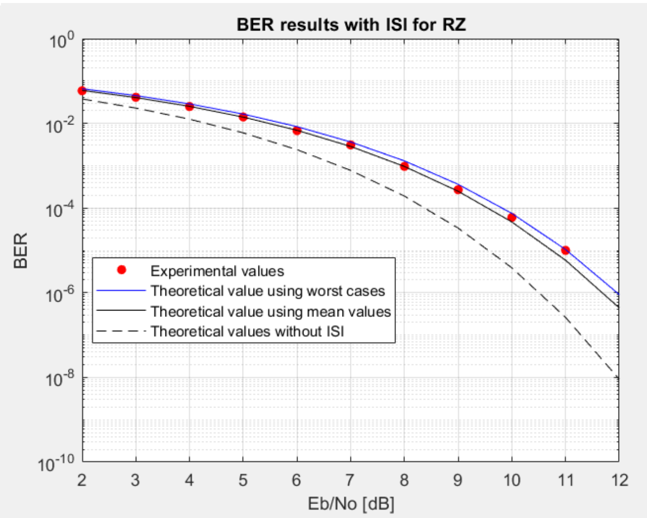
$$k = \frac{(L_1 - H_0)^2}{8} \frac{R_b}{B_{eq,RX}} \frac{1}{A^2 |H_{RX}(0)|^2}$$

where L1 and H0 are, respectively, the minimum of the positive part of the eye diagram at the optimum sampling instant and the maximum of the negative part of the eye diagram at the optimum sampling instant (hence, this method considers the worst cases). In the second method, the coefficient k is evaluated as follows:

$$k = \frac{(\mu_1 - \mu_0)^2}{8} \frac{R_b}{B_{eq,RX}} \frac{1}{A^2 |H_{RX}(0)|^2}$$

where μ_1 and μ_0 are, respectively, the average of the positive part of the eye diagram at the optimum sampling instant and the average of the negative part of the eye diagram at the optimum sampling instant. So, while the previous method considers the worst case, this method takes into consideration the average case. In the following pictures, experimental results are compared with theoretical results obtained by these two formulas. The theoretical value without considering ISI is also reported.

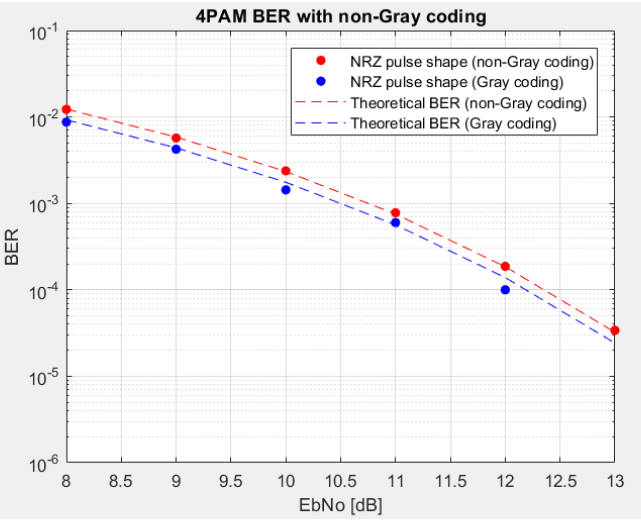


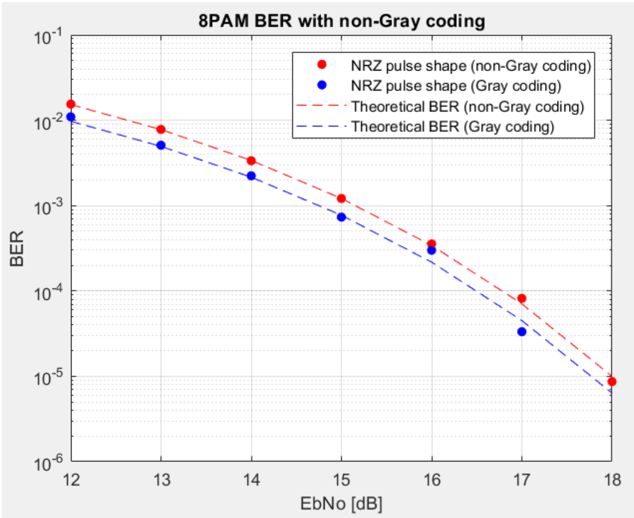


Looking at the plots, it can be observed that the formula considering the average case (black line) provides a better approximation compared to the other one considering the worst case (blue line). It can also be noted that ISI brings a significant difference in terms of BER with respect to the ideal case, especially for NRZ pulse shape. This is due to the fact that RZ pulse shape is less affected by ISI because only the first half of the pulse can change, while the second half is the same for every pulse (for a 50% duty cycle). On the other side, NRZ pulse shape is more prone to ISI.

5. Non-Gray coding

The last part of the projects consists in evaluating the penalty in terms of BER due to non-Gray coding. The simulation uses NRZ pulse shape and a matched filter at the receiver. A binary ordered labelling is used, and it can be observed in the plots below how this coding provides worse results with respect to Gray coding (red dots vs blue dots). The dashed lines are evaluated using formulas while the dots are experimental results. The formula used for BER evaluation is the same used in point 2, since it is a general formula valid also for non-Gray coding.





It is possible to notice that the penalty introduced is bigger for 8PAM than for 4PAM. In general, the more the symbols in the modulation format, the bigger the penalty is, because more symbols mean bigger Hamming distances, which are considered in the formula to evaluate the BER.