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Learning to run the number line: the development of attentional shifts during single-digit arithmetic

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Solving single-digit subtraction and addition problems is associated with left and right shifts of attention in adults. Here, we explored the development of these spatial shifts in children from the third to fifth grade. In two experiments, children solved single-digit addition (Experiments 1 and 2), subtraction (Experiment 1), and multiplication (Experiment 2) problems in which operands and the arithmetic sign were shown sequentially. Although the first operand and the arithmetic sign were presented on the center of a screen, the second operand was presented either in the left or the right visual field. In Experiment 1, we found that subtraction problems were increasingly associated with a leftward bias by the fifth grade, such that problem solving was facilitated when the second operand was in the left visual field. In Experiment 2, we found that children can also associate addition problems with the right side of space by the fourth grade. No developmental increase in either leftward or rightward bias was observed for multiplication problems. These attentional shifts might be due to the increasing reliance on calculation procedures that involve mental movements to the left or right of a sequential representation of numbers during subtraction and addition.

Keywords: arithmetic; spatial attention; numerical cognition; development

Introduction

Increasing evidence indicates that our ability to process numbers is grounded in spatial representations.¹ For example, small numbers are associated with the left side of space and large numbers with the right side of space.^{2–4} Numbers also automatically bias spatial attention, such that targets are detected faster in the left visual field (LVF) when they follow small numbers, and faster in the right visual field (RVF) when they follow large numbers.^{5,6} Taken together, this suggests

that numerical magnitudes may be represented on a mental number line (MNL) that is organized horizontally and in ascending order from left to right.^{3,7}

Recent studies suggest that such spatial associations may not be limited to number processing per se, but might also be observed during arithmetic calculation in adults. For example, when adults are asked to estimate the result of a subtraction or an addition problem, they tend to underestimate the result of a subtraction and overestimate the result of an addition. Several explanations for this effect, termed the *operational momentum effect* (OME)) have been proposed. Nevertheless, one major explanation is that participants rely

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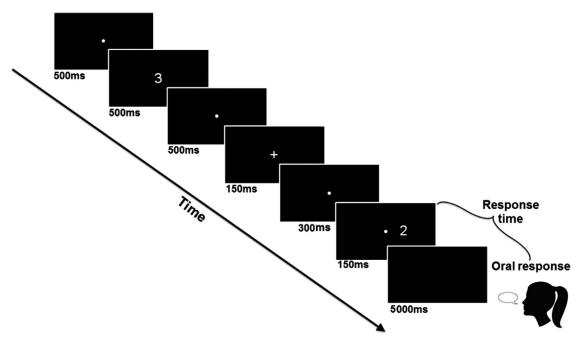


Figure 1. Sequence and timing of a sample trial.

on attentional shifts along the MNL to estimate results of problems. In a recent study, we argued that such attentional shifts occur during exact symbolic arithmetic as well. 13 Specifically, we asked adults to solve single-digit arithmetic problems presented on a computer screen. While the first operand and the arithmetic sign were presented sequentially at the center of the screen, the second operand was presented either in the LVF or the RVF (Fig. 1). Results indicated that participants solved addition problems faster when the second operand was presented in the RVF than the LVF, while they solved subtraction problems faster when the second operand was presented in the LVF than the RVF. Thus, spatial shifts of attention are elicited during exact single-digit arithmetic in adults. Importantly, no spatial bias was observed in multiplication problems (which are explicitly learned by rote in school). It is possible that these horizontal shifts of attention reflect calculation procedures relying on left-right movements along the MNL that have been automatized after years of practice with arithmetic calculation.¹⁴

To date, a few studies have investigated the associations between arithmetic and space in chil-

dren. However, available evidence suggests that these associations may be relatively late developing. For example, studies suggest that the OME does not emerge until the age of 9.15-17 It is possible that associations between space and arithmetic might be the result of years of education and that an "unconscious shift of attention on the MNL becomes evident only with increasing expertise and automatization."16 This proposal is consistent with the idea that automatic leftward and rightward movements along a sequential representation of numbers might stem from the repeated use of deliberate counting during arithmetic learning in elementary school. 14,18-20 Another nonmutually exclusive possibility is that the emergence of associations between arithmetic and space is driven by the maturation of cognitive mechanisms that are necessary to navigate along the MNL. For example, a recent neuroimaging study showed that associations between arithmetic operators and space are supported by grade-related increases of activity in the hippocampus, suggesting that the maturation of hippocampal mechanisms may support the emergence of the link between arithmetic and space. 21 Yet, to our knowledge, there is no evidence

that either development or increase in arithmetic fluency is associated with the emergence of shifts of attention along the MNL during mental calculation in children.

To explore this question, we presented children from the third to fifth grade with the same paradigm as Mathieu and colleagues. ¹³ Specifically, children were asked to solve single-digit addition and subtraction problems that were presented sequentially, with the last operand shown in either the LVF or RVF. Children also completed a timed test of arithmetic fluency, indicating their degree of knowledge and practice with arithmetic. We expected that either increasing grade or arithmetic fluency would be associated with emerging associations between addition and subtraction problems and rightward and leftward shifts of attention, respectively.

Experiment 1

Materials and methods

Participants. Our previous study using the same paradigm indicated that the difference in spatial biases between small addition and subtraction problems was associated with an effect size of d = 0.55. Within one group, a power analysis indicates that a minimum sample size of n = 22 is necessary to detect an effect of that size in the expected direction at P = 0.05 (with a power of 80%). Because we expected to study children in the third to fifth grades, we planned on recruiting a minimum of 22 children in each grade (i.e., 66 children total). This target sample was reached after two waves of data collection, one between December 2014 and March 2015 (data collection was stopped then because of lack of resources) and the other between November and December 2019 (when new resources became available).

Participants in Wave 1 were recruited from three private elementary schools in the area of Lyon in France. They consisted of 63 children aged between 7 and 11 years old. Data from six children were excluded from the analysis for medical (dyscalculia and deafness) and behavioral (lack of answers for one of the two types of operations) reasons. Thus, data from Wave 1 came from the remaining 57 children (32 females, mean age = 9.44 years old, SD = 0.81). Participants in Wave 2 were recruited from one private primary school in the area of Lyon (n = 29) and via a social media website (n = 18). Thus, 47 children aged between 7 and 10 years old

were in Wave 2 (20 females, mean age = 8.75 years old, SD = 0.61). Data from three children were further removed because they were outliers (see below). As such, final data came from 101 children (50 females) with ages between 7 and 11 years old (mean age = 9.10, SD = 0.80). All children were native French speakers. Overall, 47 children were in third grade, 33 children were in fourth grade, and 21 children were in fifth grade. The experiment was performed in accordance with the recommendations of the Ethics Committee of the CNRS, as well as in accordance with the Declaration of Helsinki.

Procedure. All children were tested in a single session that lasted approximately 30 min, either in a quiet area of their school or in the laboratory. Each child was tested individually and gave verbal consent to participant. Parent's written consents were also obtained. The testing session started with the Math Fluency test from the Woodcock–Johnson battery²² to assess children's arithmetic skills (see below), followed by the experimental task.¹³

Measures.

Math fluency. Children's arithmetic fluency was first assessed with the Woodcock–Johnson III Math Fluency subtest. The Math Fluency subtest is a paper-and-pencil test that includes single-digit addition, subtraction, and multiplication problems presented in a mixed manner. The children's performance was calculated based on the number of correctly responded items in less than 3 minutes.²² To facilitate analyses and interpretation of the results, the whole sample was evenly split into three groups as a function of their math fluency scores: 31 children were from the "lower fluency" group, 38 children were from the "intermediate fluency" group, and 32 children were from the "higher fluency" group (Table 1).

Experimental task. The experimental task was computer based and adapted from Mathieu and colleagues. Small arithmetic problems included pairs of nonidentical operands between 1 and 5 ((2, 1); (3, 1); (3, 2); (4, 1); (4, 2); (4, 3); (5, 1); (5, 2); (5, 3); (5, 4)), and large arithmetic problems included pairs of nonidentical operands between 5 and 9 ((6, 5); (7, 5); (7, 6); (8, 5); (8, 6); (8, 7); (9, 5); (9, 6); (9, 7); (9, 8)). Both small and large problems contained the number 5 in order to have the same number of problems in both categories. These pairs of nonidentical operands were used to construct 20 addition

Table 1. Descriptive statistics of Experiment 1

Variable	Mean (SD)	Range (min-max)	Skewness	Kurtosis
Age (years)	9.10 (0.80)	7.5–11	0.52	-0.61
WJ Math Fluency				
Lower fluency	30.87 (3.30)	23-36	0.00	-0.26
Intermediate fluency	41.29 (3.00)	37-46	0.19	-1.17
Higher fluency	58.72 (9.67)	47-82	1	0.50
All children	43.61 (12.69)	23-82	0.94	0.69
dRT small addition problems				
3rd grade	34.49 (257.91)	-564.76 to 639.30	-0.11	0.83
4th grade	-64.85 (254.44)	-651.87 to 641.96	0.23	0.96
5th grade	8.06 (272.17)	-620.12 to 539.06	0.02	0.52
All children	-3.46 (260.92)	-651.87 to 621.14	0.03	0.47
dRT small subtraction problems				
3rd grade	67.20 (239.39)	-423.25 to 621.14	0.44	-0.31
4th grade	33.31 (272.71)	-679.18 to 569.65	-0.25	0.16
5th grade	-80.13 (155.87)	-329.57 to 250.33	0.26	-0.52
All children	25.49 (241.25)	-679.18 to 621.14	0.25	0.09
RT small addition problems	1710.19 (382.87)	929.45 to 2661.13	0.13	-0.40
RT small subtraction problems	1839.16 (460.59)	794.59 to 3195.25	0.51	0.23

problems (10 small and 10 large problems) and 20 subtraction problems (10 small and 10 large problems) with their second operand presented once to the right and once to the left, resulting in a total of 80 problems (i.e., 40 addition and 40 subtraction problems). For both addition and subtraction problems, the larger of the two operands was always presented first. This ensured that results from the subtraction problems were positive and that children could not anticipate the type of problem to come. The presentation of the trials was pseudorandomized, so that no more than three problems of the same type (i.e., same size and spatial location of the second operand) would appear consecutively. Four scenarios were created by generating four random lists of trials. Each scenario was separated into two runs of 40 operations. The experiment systematically started with a practice run of eight trials, including tie problems (e.g., 7-7), problems with 0 (e.g., 3 + 0), and problems with small and large operands (e.g., 7 + 3).

In each trial, operands and the arithmetic sign were displayed in white Times New Roman 36-point font on a black background (Fig. 1). Each trial started with the presentation of a white fixation dot for 500 ms followed by the presentation of the first operand that lasted an additional 500 ms on screen. After a first delay of 500 ms, either a "+" or a "-" sign

appeared on screen for 150 ms at the center of the screen. A second delay of 300 ms separated the disappearance of the arithmetic sign from the second operand. This delay was chosen because it was the delay for which arithmetic shifts were maximal in Mathieu *et al.*¹³ The second operand was then displayed for 150 ms, either 5° to the left or 5° to the right of the center of the screen. Children needed to solve the calculation verbally in less than 5 s, otherwise the software automatically moved on to the next arithmetic problem.

All response times (RTs) were recorded through a headset microphone and corresponded to the period between the presentation of the second operand and the onset of the answer. The experiment was controlled by the DmDX software, 23 and RTs were checked offline and manually adjusted with CheckVocal²⁴ for each participant. Before the experiment started, children were given instructions printed on an A4 white sheet and were able to ask any questions if needed. Children were located 44 cm from the 15-inch computer screen. A chin rest was used to avoid head movements.

Data analysis. For each participant and operation, we subtracted for our main analyses the mean RT of trials in which the second operand appeared on the right from the mean RT of trials in which the second operand appeared on the

left. This difference in RT (dRT) served as the dependent variable. Following the procedure of Ref. 13, we excluded from the analyses data from participants who were outliers because their average dRTs differed by more than 2.5 SDs from the sample mean (this corresponded to three children). dRTs were then analyzed in linear mixed effect analyses. Because our hypotheses were unidirectional (i.e., we only expected dRTs for addition to be positive and larger than dRTs for subtraction, while we only expected dRTs for subtraction to be negative and smaller than dRTs for addition), *P* values for all *t*-tests are one-tailed.

Results

Only correct responses were analyzed. Correct trials constituted 90.33% of trials for small problems, but only 62.78% of trials for large problems. To maximize power and ensure that our results would not be confounded by differences in accuracy between older and younger children, we exclusively focused our analyses on small problems. Across all participants, an intraclass correlation (ICC) analysis indicated that reliability of RTs between runs was moderate to strong (ICC = 0.6; 95% CI: 0.5-0.7, P < 0.001). On average, small addition problems were answered more accurately (91% versus 89%; t(100) = 2.15, P = 0.03, d = 0.21) and faster (1710 versus 1839 ms; t(100) = -3.98, P < 0.001,d = -0.40) than small subtraction problems. There was no difference in RTs between problems with the second operand on the right compared with problems with the second operand on the left, either for addition (1713.72 versus 1805.67 ms; t(100) = 0.76, P = 0.45, d = 0.08) or subtraction (1857.09 versus 2017.28 ms; t(100) = 0.85, P = 0.40,d = 0.08).

Descriptive statistics for age, math fluency scores, dRTs for small addition and subtraction problems, and RTs for small addition and subtraction problems are shown in Table 1. First, we examined the proportion of the children with a rightward bias (i.e., positive dRT) versus a leftward bias (i.e., negative dRT) in addition and subtraction problems as a function of grade. For addition problems, positive dRTs were evenly distributed across grades (55% in third grade, 42% in fourth grade, and 52% in fifth grade). Specifically, subjects with rightward biases were not more frequent in the fifth grade than in the third grade ($\chi^2(1, n = 68) = 0.05, P = 0.82$,

V=0.02) or in the fifth grade than in the fourth grade ($\chi^2(1, n=54)=0.51, P=0.47, V=0.07$). For subtraction problems, however, the proportion of children with a negative dRT increased from the third to fifth grade (40% in third grade, 48% in fourth grade, and 71% in fifth grade). That is, children with leftward biases were more frequent in the fifth grade than in the third grade ($\chi^2(1, n=68)=5.58, P=0.02, V=0.24$) and tended to be more frequent in the fifth grade than in the fourth grade ($\chi^2(1, n=54)=2.76, P=0.09, V=0.17$).

Second, a linear mixed-model analysis with dRT (i.e., difference in RT between problems with the second operand on the left and problems with the second operand on the right) as dependent variable was performed using GAMLj²⁵ in jamovi.²⁶ Fixed factors included the between-subject factors grade (third, fourth, and fifth) and arithmetic fluency (lower, intermediate, and higher fluency), as well as the within-subject factor operation (addition and subtraction). Subject was entered as a random factor. We also included wave, gender, and overall RTs as covariates to control for these potential confounds. There was no main effect of operation (F(1,181) = 0.75, P = 0.75, f < 0.1) or arithmetic fluency (F(2,181) = 0.44, P = 0.64, f < 0.1). Arithmetic fluency also did not interact with operation (F(1,181) = 1.89, P = 0.15, f < 0.1) or with any other factor. However, there was a tendency for a main effect of grade (F(2,181) = 2.91, P = 0.06, f = 0.14), which was qualified by an interaction with operation (F(2,181) = 3.01, P = 0.05, f = 0.15). Analyses of simple effects revealed that whereas dRT was not larger for addition than subtraction in the third grade (t(181) = -0.47, P = 0.68, d = -0.09) and the fourth grade (t(181) = -1.64, P = 0.95, d = -0.24), it was larger for addition than subtraction in the fifth grade (t(181) = 1.84, P = 0.03, d = 0.28). As can be seen in Figure 2, the effect was driven by a decrease in dRT with grade for subtraction problems. Indeed, simple effect analyses also showed that dRTs for subtraction problems were not smaller than 0 in the third (t(92) = -1.89, P = 0.97, d = -0.28) and fourth grades (t(92) = -0.95, P = 0.83, d = -0.12). dRTs for subtraction problems were, however, significantly smaller than 0 in fifth grade (t(92) = 1.94, P = 0.03, d = 0.51). dRTs for addition problems were larger than 0 in none of the grades (all t's < 1.09, all P's > 0.14).

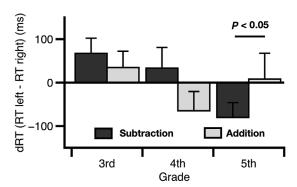


Figure 2. dRT as a function of operation (subtraction and addition) and grade (third, fourth, and fifth) in Experiment 1. Error bars represent standard error of the mean (SEM).

Discussion

Experiment 1 reveals two main findings. First, we found evidence for an increase in leftward bias (i.e., a decrease in dRT) when children solve subtraction problems over the course of elementary school, such that children in fifth grade exhibit shorter RTs when the second operand is on the LVF (as compared with the RVF). Critically, because dRT for subtraction varied as a function of grade and not arithmetic fluency, our findings suggest that associations between subtraction problems and the left side of space are more likely to emerge as a result of development rather than increase in arithmetic fluency per se. Second, contrary to our hypothesis, we did not find any evidence for an increase in rightward bias for addition problems in this sample of children. One possibility is that these children may have relied on a strategy involving direct retrieval of the answer from long-term memory when solving small addition problems. For instance, the French math curriculum emphasizes that small addition problems may be memorized in tables.²⁷ This would arguably (1) prevent children from using strategies relying on movements along the MNL and (2) make addition problems more similar to multiplication problems (which are largely learned by rote in school) than to subtraction problems (which are never learned by rote in the French curriculum²⁷). Therefore, in Experiment 2, we directly compared the spatial biases (or lack thereof) associated with single-digit addition and multiplication problems in another group of children. This design was also adapted from Ref. 13.

Experiment 2

Materials and methods

Participants. As with Experiment 1, data collection was conducted in two different time waves to reach the target sample size. Participants in Wave 1 were recruited from one private primary school in the area of Lyon, France. They consisted of 38 children aged between 8 and 11 years old (21 females, mean age = 10 years old, SD = 0.65). Participants in Wave 2 were recruited from another private elementary school in the same area (n = 22) and via a social media website (n = 9). Thus, 31 children aged between 8 and 10 years were in Wave 2 (20 females, mean age = 9.42 years old, SD = 0.48). Data from one child were removed because they were outliers (see below). As such, the overall sample came from 68 children (41 females) with ages between 8 and 11 years old (mean age = 9.74 years old, SD = 0.65). All children were native French speakers. Overall, 42 children were in the fourth grade and 26 children were in the fifth grade. The experiment was performed in accordance with the recommendations of the Ethics Committee of the CNRS, as well as in accordance with the Declaration of Helsinki.

Procedure. Experiment 2 followed the same procedure as Experiment 1, with all children being tested in a single session. However, the testing session lasted between 30 and 40 min because the experimental task included two more runs (see below). The testing session started with the Math Fluency test from the Woodcock–Johnson battery, ²² followed by the experimental task.

Measures. The Woodcock–Johnson Math Fluency subtest used in Experiment 2 was the same as in Experiment 1. To facilitate analyses and interpretation of the results, the whole sample was evenly split into three groups as a function of math fluency score: 24 children were from the lower fluency group, 21 children were from the intermediate fluency group, and 22 children were from the higher fluency group (Table 2). Regarding the experimental task, the pairs of nonidentical operands, the classification of problems (i.e., small and large), the apparatus, and the stimulus timing were the same as in Experiment 1. Hence, only full details for the stimuli and the experimental procedure of Experiment 2 are provided next.

Table 2. Descriptive statistics of Experiment 2

Variable	Mean (SD)	Range min-max	Skewness	Kurtosis
Age (years)	9.74 (0.65)	8.33-11	0.10	-0.97
WJ Math Fluency ($n = 67$)				
Lower fluency	39.58 (4.57)	27-45	-0.95	0.99
Intermediate fluency	50 (2.65)	46-53	-0.32	-1.40
Higher fluency	67.36 (9.20)	54-86	0.57	-0.79
All children	51.97 (13.13)	27-86	0.72	0.06
dRT small addition problems				
4th grade	84.81 (185.85)	-316.51 to 455.38	0.21	-0.38
5th grade	102.23 (155.70)	-170.47 to 452.71	0.62	-0.31
All children	91.47 (173.94)	-316.51 to 455.38	0.29	-0.33
dRT small multiplication problems				
4th grade	-27.63 (212.80)	-496.96 to 460.62	0.13	-0.04
5th grade	6.44 (207.49)	-500.64 to 452.27	-0.16	0.71
All children	-14.61 (209.89)	-500.64 to 460.62	0.02	0.06
RT small addition problems	1629.33 (388.27)	813.90 to 2600.42	0.47	0.10
RT small multiplication problems	1727.22 (355.62)	900.99 to 2771.22	0.30	0.56

For Experiment 2, the pairs of nonidentical operands were used to construct 20 addition problems (10 small and 10 large problems) and 20 multiplication problems (10 small and 10 large problems), with their second operand presented once to the right and once to the left, resulting in 40 addition and 40 multiplication problems. The larger operand could be presented as either the first or the second operand, such that each problem was presented once for each order of presentation of operand (first versus second position). Therefore, there were a total of 160 problems, which were separated into 4 runs of 40 operations. The trials were pseudorandomized and four different scenarios based on four different lists were created (with four runs in each scenario).

Data analysis. As in Experiment 1, dRT was calculated for each participant and operation by subtracting the mean RT of trials in which the second operand appeared on the right from the mean RT of trials in which the second operand appeared on the left. Outlier participants (i.e., participants with an average dRT that was smaller or larger than 2.5 SDs from the sample mean in either small addition or multiplication problems) were removed from the analyses (this corresponded to one child). dRTs were then analyzed in linear mixed effect analyses. *P* values for all *t*-tests are one-tailed, as we only expected either null or positive dRTs (as well as

either similar or larger dRTs for addition than multiplication).

Results

As in Experiment 1, only correct responses in small arithmetic problems were analyzed. Correct trials constituted 89.4% of trials for small problems, but only 56.89% of trials for large problems. There was no accuracy difference between small addition problems (89.75%) and small multiplication problems (89.05%) (t(67) = 0.78, P = 0.50, d = 0.09). Additionally, data from one child on the WJ Math Fluency task was considered as missing in further statistical analyses due to failure to understand task instructions. Therefore, this participant was dropped from all analyses that involved the math fluency scores.

Descriptive statistics for age, math fluency scores, dRTs for small addition and multiplication problems, and overall RTs for all small addition and multiplication problems are shown in Table 2. First, we examined the proportion of the children with a rightward bias (i.e., positive dRT) versus a leftward bias (i.e., negative dRT) in addition and multiplication problems as a function of grade (Fig. 3). For both addition and multiplication problems, positive dRTs were evenly distributed across grades (addition: 59.52% in fourth grade, and 40.48% in fifth grade; multiplication: 65.38% in fourth grade, and 57.69% in fifth grade). Specifically, subjects

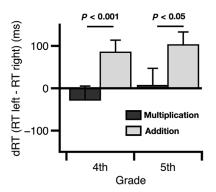


Figure 3. dRT as a function of operation (multiplication and addition) and grade (fourth and fifth) in Experiment 2. Error bars represent standard error of the mean (SEM).

with rightward biases in addition and multiplication problems were not more frequent in the fifth grade than in the fourth grade ($\chi^2(1, n = 68) = 0.23$, P = 0.41, V = 0.06, for addition problems; and $\chi^2(1, n = 68) = 1.91$, P = 0.13, V = 0.17 for multiplication problems).

Second, dRTs were analyzed in a linear mixedmodel with grade (fourth and fifth), arithmetic fluency (lower fluency, intermediate fluency, and higher fluency), and operation (addition and multiplication) entered as fixed factors, and subject entered as random factor. We also included wave, gender, and overall RTs as covariates. Results showed no significant main effect of grade (F(1,119) = 0.54, P = 0.46, f < 0.1) or arithmetic fluency (F(2,119) = 0.87, P = 0.42, f < 0.1). Neither grade nor arithmetic fluency also interacted with operation. There was, however, a significant main effect of operation (F(1,119) = 10.52, P = 0.002,f = 0.28), indicating that dRTs were on average larger for addition than multiplication problems. Simple effects revealed that this was the case both in the fourth grade (t(119) = 2.94, P < 0.001, d = 0.39) and the fifth grade (t(119) = 1.81, P = 0.04,d = 0.40). Simple effect analyses also showed that dRTs for addition problems were larger than 0 in both the fourth grade (t(61) = 3.41, P = 0.001, d = 0.53) and the fifth grade (t(61) = 3.10, P = 0.003, d = 0.66), whereas dRTs for multiplication problems were larger than 0 in none of the grades (all t's < 0.21, all P's > 0.37).

Discussion

Overall, two main findings can be highlighted from Experiment 2. First, we did not find any evidence for either a leftward or rightward bias during multiplication problem solving in elementary school children. This is consistent with the fact that answers of multiplication problems are mainly learned by rote in school and, therefore, never really calculated by children. Thus, multiplication problems may never be clearly associated with the MNL and never show spatial associations. Second, Experiment 2 shows that an association between addition problems and the right side of space can be present in children as early as in fourth grade. Critically, the lack of spatial bias with multiplication problems also makes it unlikely that this rightward bias observed with addition problems in Experiment 2 is due to the size of the answer. Indeed, addition outcomes are always larger than both of the operands involved. Thus, it could be argued that the rightward bias observed may simply result from an association between relatively large numbers and the right space of the MNL (e.g., Refs. 2-4 and 28), rather than a calculation procedure per se. However, outcomes of multiplication problems in Experiment 2 were larger, or as large as, the operands involved (because there were no multiplications involving 0). Thus, the rightward bias observed with addition problems is more likely due to specific aspects of addition problems (including the fact that these problems may rely on movements to the right of the MNL) than to the size of the results involved. Nonetheless, results from Experiment 2 are partly inconsistent with our own findings in Experiment 1, since neither a rightward bias nor a developmental increase in rightward bias was found in addition problems in Experiment 1. Potential reasons for the inconsistencies between experiments are discussed in the General discussion.

General discussion

Several studies suggest that arithmetic calculation is associated with attentional shifts in adults. ^{13,29-34} To our knowledge, however, no study has investigated how and when these shifts emerge in children, as well as whether they result from development or increase with arithmetic fluency. The aim of the two experiments presented in this paper was to investigate the emergence of spatial biases during mental calculation in children from the third to fifth grades. Children were asked to verbally solve

single-digit addition, subtraction, and multiplication problems. Operands and arithmetic signs were presented sequentially. Although the first operand and the arithmetic sign were presented in the center of the screen, the presentation of the second operand was displayed either in the left or the right side of the fixation.

Why may children develop associations between arithmetic problems and space?

In Experiment 1, we found that the difference in RT between problems with a second operand on the left versus the right side of space decreased from third to fifth grade, such that only children at the end of elementary school (i.e., grade 5) showed an association between subtraction problems and the left side of space. We also found in Experiment 2 that at least some children may show an association between small addition problems and the right side of space as early as grade 4. We can see at least three potential explanations for the emergence of these associations.

First, it is possible that the arithmetic signs themselves may bias spatial attention. In other words, while subtraction signs may trigger leftward shifts of attention, addition signs may trigger rightward shifts of attention. For example, it has been proposed that arithmetic operators might be associated with simple heuristics, such as "the result of a subtraction should always be smaller than the first operand" and "the result of an addition should always be larger than the first operand."11,28,35 Although this would be broadly consistent with neuroimaging evidence that arithmetic operators are associated with activity in brain region underlying spatial attention, 21,36 we believe that this is explanation is unlikely for two reasons. The first one is that previous behavioral studies have failed to find that arithmetic operators (devoid of a problemsolving context) can bias spatial attention by themselves. For example, both Pinhas et al.37 and Liu et al.29 presented participants with addition and subtraction signs that were followed by targets that participants had to detect in either the LVF or RVF. None of these studies showed that arithmetic operators differentially affected target detection. The second reason is that we did not find any rightward bias associated with multiplication problems (neither in the present study nor in our previous study with adult participants, see Ref. 13). Yet, if arithmetic

signs were associated with spatial biases because they are associated with heuristics (see above), one could also expect an association between the multiplication sign and the right side of space, given that results of multiplication problems are systematically larger than the first operand (at least when it is greater than 1).

Second, because operands were kept constant across operations, results were overall smaller for subtraction than for addition problems. It could then be argued that number–space associations might have contributed to the difference in spatial bias between subtraction and addition. However, this is unlikely because number–space associations have been demonstrated before the third grade in previous studies, ^{38,39} and the association between subtraction and the left side of space was only observed in the fifth grade in Experiment 1. Furthermore, no spatial association was observed for multiplication problems in Experiment 2, despite the fact that multiplication problems lead to results that are even greater than addition problems.

Third, a number of recent studies have suggested that the repeated use of counting when young children solve basic addition and subtraction problems might lead to an automatization of these counting procedures, rather than to the construction of a network of arithmetic facts in memory as posited by prior literature. 14,18-20 For example, studies in adults^{18,20} and 10-year-old children¹⁹ show that the time participants take to solve very small addition problems is not constant but increases linearly as a function of the distance between the original operand and the sum. This suggests that, even if they might not be aware of it, adults and skilled children might solve these basic problems by rapidly "moving" from a source to a target number along the MNL. Given the left-to-right orientation of that MNL in children and adults, 1,38,40 these forward and backward movements are likely to resemble rightward and leftward shifts of attention.

The emergence of associations between arithmetic and space in elementary school is broadly consistent with this idea that children activate the MNL when solving simple arithmetic problems. Furthermore, we found that these associations increase with grade rather than arithmetic fluency. On the one hand, this suggests that increasing involvement of the MNL for solving arithmetic problems relies on the maturation of cognitive

systems more than it relies on an increase in fluency per se. For example, neuroimaging studies have found that arithmetic problem solving relies on a network of brain regions supporting spatial attention. 41,36 However, these mechanisms are slow developing and additional structures might be required in children to learn how to navigate along the MNL.21 On the other hand, the lack of relation between arithmetic fluency and associations between problems and space needs to be interpreted with caution. Indeed, our measure of arithmetic fluency was relatively broad and, because problems were presented in a mixed manner, made it impossible to assess fluency in each operation independently (addition, subtraction, and multiplication). Therefore, future studies should investigate how associations between arithmetic and space relate to fluency in different operations.

Addition problems are inconsistently related to rightward biases

It is important to note that our results regarding addition are nonetheless inconsistent. That is, we observed a rightward bias in Experiment 2 but not in Experiment 1. There are a number of possible explanations for this. However, at least two speculations may come to mind (note that these explanations are not mutually exclusive). First, it is possible that children may have used different strategies to solve addition problems in Experiments 1 and 2. For instance, whereas children in Experiment 2 may have relied on rapid calculation supported by movements along the MNL, children in Experiment 1 may have relied on direct retrieval strategies. To some extent, this idea is supported by the fact that the methods used to teach single-digit addition in French classroom are more varied than those used to teach single-digit subtraction. That is, whereas single-digit subtraction is almost never learned by rote (but rather by calculation), single-digit addition problems can be memorized in tables in some classrooms.²⁷ Although rote learning of small addition problem would often be used alongside practice with calculation, this would undoubtedly slow down any automatization of procedures relying on movements along the MNL in children (which would, therefore, fail to exhibit a spatial bias when solving addition problems). Therefore, the use of retrieval strategies for addition problems (or a mixture of retrieval and

calculation strategies) in children in Experiment 1 might explain the lack of association between addition problems and the right side of space. Unfortunately, we did not collect information on the way children were taught addition problems in our study. Thus, this explanation remains speculative and would require further testing.

A second possibility is that automatizing calculation procedures relying on movements along the MNL is likely to require a great deal of practice with counting and arithmetic calculation. Because this may also critically differ between children from different schools and classrooms, it is possible that children from Experiment 1 may have been less exposed to arithmetic problem solving than children from Experiment 2. An examination of arithmetic performance between children from Experiments 1 and 2 might support this idea. For example, the fourth and fifth graders in Experiment 2 had a significantly higher score on the math fluency test than the fourth and fifth graders in Experiment 1 (t(119) = 2.16, P = 0.017, one-tailed). This raises the possibility that we might have failed to observe a rightward bias in children from Experiment 1 because these children were not as fluent with addition problem solving as children from Experiment 2. Note, however, that this possibility is mitigated by the following two reasons. First, math fluency scores were not related to dRTs for small addition problems in either Experiment 1 or 2. Second, we could observe a leftward bias for subtraction problems in Experiment 1, even when performance in these problems was worse than for addition problems. Of course, it is also possible that children from Experiments 1 and 2 might differ with respect to cognitive skills that have been shown to affect the automaticity of counting procedures, such as working memory and processing speed. 14,18-20 Because these skills were not measured here, future studies are needed to better understand individual variability in the development of attentional shifts during addition problem solving.

Conclusion

In sum, the present study reveals that at least some children already associate addition problems with the right side of space by the fourth grade, whereas subtraction problems are associated with the left side of space by the fifth grade. By contrast, no spatial bias was observed for single-digit multiplication problems. To our knowledge, our findings provide the first evidence for the emergence of associations between simple arithmetic problems and space in elementary school children. Although we did not find that these associations relate to a broad measure of arithmetic fluency, future studies are needed to investigate how these associations precisely relate to different aspects of arithmetic skills. Our study also calls for studies investigating how the way arithmetic is taught in school influences spatial biases, given that these were found to be inconsistent between different groups of children in the present study.

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Author contributions

The concept and design of the present research project was developed by C.T. and J.P. Data collection for Wave 1 was conducted by A.C. Data collection for Wave 2 was conducted by A.D.-B.Y., L.L., H.C., and E.L. Interpretation of the data, writing, and editing of the manuscript were conducted by A.D.-B.Y., C.T., and J.P. Statistical analyses were conducted by A.D.-B.Y., A.M., and J.P. A.D.-B.Y. takes responsibility for the integrity of the data analyzed.

Competing interests

The authors declare no competing interests.

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