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Endogenous Regime-Switching representation of Ambiguity Attitudes in a model of Sudden Stops

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Abstract

This thesis studies the implications of ambiguity attitudes as a result of subjective beliefs formation in a model of sudden stopt. To represent the ambiguity attitudes I introduce an endogenous switching mechanism (based on the scaled Lagrange multiplier on the collateral constraint and the continuation value/consumption ratio) which affects the switching of key ambiguity parameters between the averse and seeking regime, introducing an implied control mechanism for subjects' behaviour.

The model is then solved using global methods (Chebyshev collocation).

As a final result the model should shows how the introduction of such dynamics is able to better characterize some cycle facts, specifically, how pessimism (optimism) can decrease (increase) the value of collateral and thus affect the ability to borrow and leverage (??).

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The importance of financial channels as sources of shocks to the macroeconomy has been established in a wide variety of contextes.

1.1 Related literature

My work is connected with serveral strands of the literature, in particular, models of sudden stops introduce an occasionally binding collateral constraint, which when binding produces sharp reductions in asset prices following Fisher (1933) mechanism as in Enrique G. Mendoza (2010), Benigno et al. (2013); J. Bianchi and Enrique G Mendoza (2010), Jeanne and Korinek (2010) study the effect of the introduction of a macroprudental tax, again J. Bianchi and E. Mendoza (2015) study the role of commitment of the financial regulator on the magnitude of crises while Schmitt-Grohé and Uribe (2020) study the possible existence of multiple equilibria with both overborrowing and underborrowing. On the other hand, the role of beliefs in macroeconomy has been extensively studied since the work of Gilboa and Schmeidler (1989), which first developed the multiple priors framework, carried over by Epstein and Wang (1994), Epstein and Schneider (2003) and in parallel Hansen and Sargent (2001) and Hansen and Sargent (2005), which borrowing from robust control have introduced an ambiguity adjusted expectation operator which accounts for uncertainty about the probability measure which drives the stochastic processes (multiplier preferences), Maccheroni, Marinacci, and Rustichini (2006) link these two frameworks giving a general representation of preferences which account for ambiguity adversion (Knightian Uncertainty)

The intersection between ambiguity aversion and asset pricing has been studied by, among others, Epstein and Schneider (2008), Barillas, Hansen, and Sargent (2009), Ilut and Schneider (2014), F. Bianchi, Ilut, and Schneider (2017), Adam and Woodford (2021), these works were mainly concerned on the monetary policy implications of such departures from the rational expectation paradigm. Few works in this context have introduced also seeking attitudes as in Bassanin, Faia, and Patella (2021), where the endogenization of attitudes is achieved by threshold switching and Bhandari, Borovička, and Ho (2024) who define attitudes both as an exogenous AR(1) process and as function of TFP.

Chapter 2

The model

The economic environment is represented by a small open economy, where individuals face an occasionally binding borrowing constraint. The model deviates from the Sudden Stops framework in two directions. Drawing from Hansen and Sargent (2001), I introduce a martingale that distorts expectations to account for the subjects' ambiguous attitudes toward the probability measure of the shocks driving the economy. Additionally, I develop an endogenous switching mechanism to account for the agents' differing beliefs in the relevant states of the economy.

2.1 Multiplier preferences

A model-consistent representation of agents' deviations from the rational expectations paradigm is provided by the multiplier preferences framework, developed by Hansen and Sargent (2001), hereafter referred to as HS.

The model is driven by a shock to aggregate income, modeled as an AR(1) process, $\{y_t\}_{t\in\mathbb{N}}$, with $y_t \mid y_{t-1} \sim \pi(y_t \mid y_{t-1})$. Following HS, agents consider an alternative probability measure $\tilde{\pi}(y_t \mid y_{t-1})$ (where $\tilde{\pi} \ll \pi$) relative to the approximating one, in order to obtain robust decision rules. This occurs because agents do not fully trust the approximating measure and instead consider distributions within its neighborhood, leading to decision rules that are adjusted for the worst- or best-case scenarios. The deviation from the rational expectations paradigm is introduced as a distortion in expectations, represented by a non-negative, $\mathcal{F}_t$-measurable martingale $\{M_t\}_{t\in\mathbb{N}}$, which expresses the change of measure (Radon-Nikodym derivative) between the two probability distributions. This martingale overweights (underweights) tail events, leading to expectations that (overestimate) underestimate the outcomes of the process.

$$\mathbb{E}[M_{t+1}x_{t+1} \mid \mathcal{F}_t] = \tilde{\mathbb{E}}[x_{t+1} \mid \mathcal{F}_t], \qquad \mathbb{E}[M_{t+1} \mid \mathcal{F}_t] = M_t, \qquad \mathbb{E}[M_t] = 1$$

To construct a recursive representation starting with the following formulation we can decompose[1] M_{t+1} to isolate distortions to the probability measure of events in $t+1$ conditional on date t information, that is:

$$m_{t+1} = \frac{M_{t+1}}{M_t}, \qquad M_{t+1} = \prod_{j=1}^{t+1} M_0 m_j \tag{2.1}$$

The robustness adjustment is introduced into the decision rule by defining an operator which accounts for the discrepancy between the two probability distributions, measured by the entropy.

$$R_t(V_{t+1} \mid \theta) = \min_{m_{t+1}\in\mathcal{M}_{t+1}} \mathbb{E}\left[m_{t+1}V_{t+1} \mid \mathcal{F}_t\right] + \theta\mathbb{E}\left[m_{t+1}\log m_{t+1} \mid \mathcal{F}_t\right] \tag{2.2}$$

The minimazion problem is carried over the probability distortion increment m_{t+1} onto the space $\mathcal{M}_{t+1}$ of non-negative, $\mathcal{F}_{t+1}$-measurable random variables such that $\mathbb{E}[m_{t+1} \mid \mathcal{F}_t] = 1$.

To extend the framework to the case of ambiguity averse and seeking behaviours I follow Bassanin, Faia, and Patella (2021) and assume $\theta(a_t) \in \{\theta^+, \theta^-\}$.

This parameter is central to the model, as it governs the subjects' pessimistic or optimistic attitudes. The original formulation proposed by HS assumes $\theta > 0$, implying agents' pessimistic attitudes toward model ambiguity, represented as $\theta = \theta^-$. To incorporate optimistic beliefs, it is necessary to assume $\theta < 0$, that is $\theta = \theta^+$. It is important to assume $\theta \neq 0$ always, as this is required to prevent the robustness adjustment from collapsing into the standard Rational Expectations framework.

[1] By assumption $M_0 = 1$

The switching between the two parameters is driven by movements in the ambiguity process[2] $\{a_t\}_{t\in\mathbb{N}}$ and $a_t \in \{\text{Pessimistic} = 0, \text{Optimistic} = 1\}$, introducing an indirect control in agents beliefs.

At the optimum the distortion increment is

$$m_{t+1} = \frac{\exp\{-1/\theta(a_t)V_{t+1}\}}{\mathbb{E}\left[\exp\{-1/\theta(a_t)V_{t+1}\}\right]} \tag{2.3}$$

Substituting into the adjusted expectations $R_t(V_{t+1} \mid \theta(a_t))$ we obtain the minimized[3] adjusted continuation value

$$R_t(V_{t+1} \mid \theta(a_t)) = -\theta(a_t)\log\left(\mathbb{E}\left[\exp\left\{-1/\theta(a_t)V_{t+1}\right\} \mid \mathcal{F}_t\right]\right) \tag{2.4}$$

2.2 Optimization problem

Agents optimize their lifetime utiliy:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right] \tag{2.5}$$

The specification is a simple CRRA constant-relative-risk-aversion utility function where c_t represents consumption, γ is the risk-aversion coefficient and β is the discount factor.

Agent's face two different constraints, their budget constraint

$$c_t + q_t x_{t+1} + \frac{b_{t+1}}{R} = (1-\phi)y_t + x_t(q_t + d_t) + b_t \tag{2.6}$$

where q_t denotes the price of assets, x_t and b_t represent holdings of one-period non-state-contingent foreign bonds and domestic capital, $(1-\phi)y_t$ and $d = \phi y_t$ denotes respectively the endowment income and the dividend derived from the the risky

[2]This process is better characterized in SECTION

[3]It's easy to notice that since $\theta(a_t) \in \left\{\theta^+, \theta^-\right\}$ and $\text{sign}(\theta^+) = -\text{sign}(\theta^-)$ the original minimization problem studied by Hansen and Sargent (2001) becomes a maximization, which is coherent with the framework I propose

asset finally R is the exogenous foreign real interest rate[4].

$$-\frac{b_{t+1}}{R} \leq \kappa q_t x_{t+1} \tag{2.7}$$

Eq (2.7) represents the collateral constraint, limiting the fraction of debt issued by the agents at a fraction of the market value of capital.

The only stochastic components in the model are the income shock, defined as a log-normal AR(1) process

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon) \tag{2.8}$$

and the ambiguity process $\{a_t\}_{t\in\mathbb{N}}$ which follows an inhomogeneous Markov Chain (PARAGRAFO).

From these two processes, a joint process $z_t = (a_t, y_t)$ defined on $(Y \times A, \mathcal{F}_t, \pi \times \zeta)$ can be derived over which expectations will be taken.

2.3 Recursive optimization

To obtain a recursive formulation for the optimization problem I make use of the results on the probability distortion increment m_{t+1} obtained in the previous section. Substituting the minimized value of the inner problem the optimization becomes[5]:

$$\begin{aligned} V = \max_{c,x',b'} \Bigg\{ \frac{c^{1-\gamma}}{1-\gamma} - \beta\theta(a_t) \log \left(\mathbb{E}_{z'|z} \left[\exp\left\{ -\frac{1}{\theta(a_t)} V_{t+1} \right\} \right] \right) \\ +\lambda \left[y + q(x + \phi y) + b - qx' - c - \frac{b'}{R} \right] \\ +\mu \left[\kappa q x' + \frac{b'}{R} \right] \Bigg\} \end{aligned} \tag{2.9}$$

The max − min problem agents face is reduced to the maximization of the recursion with respect to consumption at time t, bonds and assets at time $t + 1$.

[4]Assuming the economy small enough to be price taker

[5]I use the convention $x = x_t$ and $x' = x_{t+1}$

The set of optimality conditions for this problem is:

$$c^{-\gamma} = \beta R \mathbb{E}_{z'|z}\left[m' c'^{-\gamma}\right] + \mu \tag{2.10}$$

$$q = \beta \frac{\mathbb{E}_{z'|z}\left[m' c'^{-\gamma}[q' + \alpha y']\right]}{c^{-\gamma} - \kappa\mu} \tag{2.11}$$

Condition (2.10) is the Euler equation for bonds, this equation suggests that when the borrowing constraint binds, the Lagrange multiplier is positive, implying that the marginal benefit of reallocating consumption to the present exceeds the marginal benefit of borrowing by a value equals to μ.

Condition (2.11) represents the Euler equation for assets, equating the marginal gain of an additional unit of capital with its marginal cost.

Both expectational terms are distorted by m', implying differing marginal benefits compared to the Rational expectations benchmark.

The competitive equilibrium is characterized also by the market clearing conditions for the goods and stock market

$$c + \frac{b'}{R} = y + b \tag{2.12}$$

$$x = 1 \tag{2.13}$$

and the complementary slackness condition

$$\mu\left[\frac{b'}{R} + \kappa q\right] = 0 \tag{2.14}$$

2.4 Endogenous Switching mechanism

The model assumes that agents' behavior is shaped by both ambiguity-averse and ambiguity-seeking tendencies. Endogenizing the process that generates these dynamics is non-trivial. However, by assuming that this process follows an inhomogeneous Markov chain, it establishes a link between agents' concerns—represented by two distinct regimes—and the variables driving those concerns. This framework is closely related to the theory of Markov Chains in Random Environments (MCRE), where the environmental process that governs the evolution of the chain is represented by the state variables (y_t, b_t), i.e. the income shock process and the debt, which indirectly defines the value of the endogenizing variables. A detailed discussion of this framework is provided in Appendix A.1.

To the ambiguity process $\{a_t\}_{t\in\mathbb{N}}$ is linked the following kernel[6]:

$$\mathbb{K}_{a'|a}\left(\mu, \frac{V}{c}\right) = \begin{pmatrix} 1 - \Lambda_{\theta_{pos}}(\mu) & \Lambda_{\theta_{pos}}(\mu) \\ \\ \Lambda_{\theta_{neg}}\left(\frac{V}{c}\right) & 1 - \Lambda_{\theta_{neg}}\left(\frac{V}{c}\right) \end{pmatrix} \tag{2.15}$$

To study the switching mechanism we can look at the transition kernel from two different point of view, the transition probabilities as functions of the endogenizing variables V/c, $\mu/c^{-\gamma}$ and as functions of the state variables y,b.

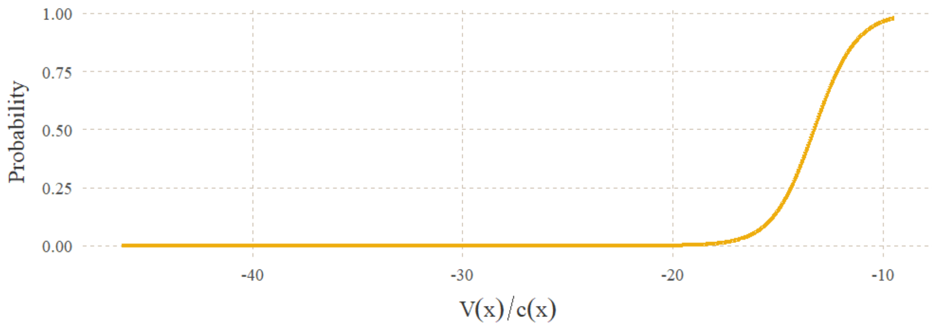
The dependence on the scaled Lagrange multiplier $\frac{\mu}{c^{-\gamma}}$ and on the continuation value/consumption ratio $\frac{V}{c}$ is made clear by the following reasoning: the switch to the optimistic regime is triggered by an increase in the ratio between the lifetime utility and the current value of consumption, The higher this ratio, the more agents perceive themselves as wealthy. This is because a larger value function, relative to current consumption, suggests that agents expect a greater lifetime utility or future wealth, anticipating the ability to sustain or improve their consumption levels over time, reinforcing their perception of being better off financially.

On the contrary, the switching to the pessimistic regime is triggered by an increase in the value of the scaled Lagrange multiplier, as representative of the tightening in the possibilities for consumption and debt as the constraint becomes binding. The functional forms of the transition probabilities can be observed in Figure 2.1.

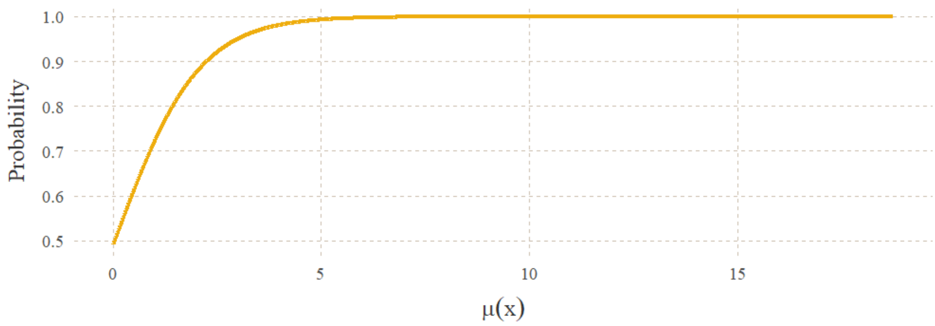
As one can notice the probability of switching to the pessimistic regime does not span the entire $[0, 1]$ range, but instead starts from a fixed value of 0.5. The reasoning behind this is based on the assumption that, even when the constraint is non-binding, agents can still experience pessimistic behavior. However, this does not hold for the transition probability to the optimistic regime, where for lower

[6] The function $\Lambda_\alpha(x) = \dfrac{1}{1 + e^{-(x-\mu)/s}}$ is the CDF of a logistic distribution.

values of the value function/consumption ratio, the possibility of switching to the optimistic regime is ruled out. Another interesting property is that both states are allowed to be absorbing[7]. This assumption is particularly important for explaining crisis events. During the build-up of a crisis, agents reach higher values of the value function/consumption ratio, and as soon as they enter in a deflationary dynamic, the probability of switching to the pessimistic regime becomes one, making this state absorbing, until the deflation subsides.



(a)



(b)

Figure 2.1. Transition probabilities for the two regimes, (a) probability of switching to the optimist regime, b) probability of switching to the pessimist regime.

Another interesting perspective emerges when inspecting the transition probabilities as functions of the state variables y and b. Figure 2.2 clearly shows how the switching rule is unevenly affected by the behavior of the two state variables. Naturally, an increase in income reduces the influence of the Lagrange multiplier, as higher income is sufficient to offset the weight of an of increased debt on agents' concerns,

[7]The theoretical background is given in Appendix A.1

(I grafici sono sotto sopra... cambiare) However, the probability of switching to the optimistic regime increases more rapidly, while the increase in the probability to switch to the pessimistic regime occurs at a much slower rate. This suggests that agents are more sensitive to the level of debt when considering pessimistic outcomes than they are when transitioning to optimistic ones.

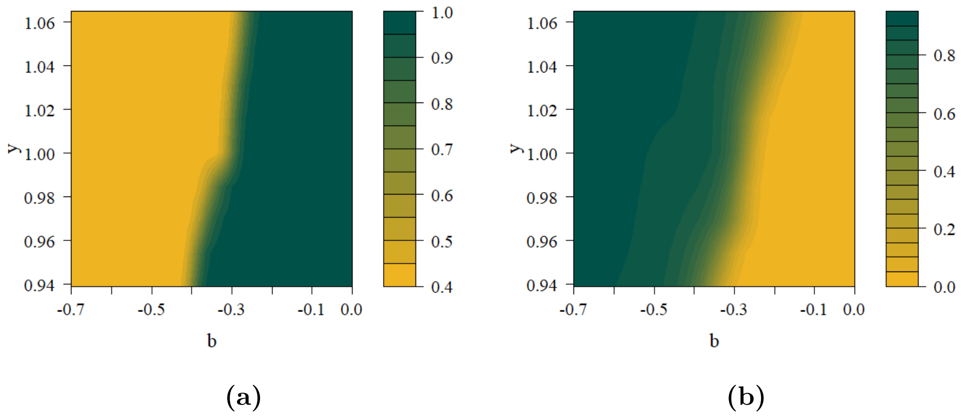


Figure 2.2. Transition probabilities for the two regimes, (a) probability of switching to the optimist regime, b) probability of switching to the pessimist regime.

Chapter 3

Quantitative analysis

In this chapter, I show the results obtained from the model solution. The model is calibrated at an annual frequency and solved with global methods (APPENDIX). I first explore the policy functions for the different regimes, then simulate and show how the model behaves during crisis events, lastly, I characterize the unconditional probabilities for the ambiguity process, which implicitly defines a control for agents' choices.

3.1 Calibration

The model parameters are calibrated partially following Bassanin, Faia, and Patella (2021), partially matching relevant cycle moments and steady state values.

Parameter	Meaning	Strategy	Value
β	Discount factor	Standard DSGE value	0.94
R	Risk-free rate	3 month T-bill rate	1.025
ϕ	Share of dividend	Bassanin, Faia, and Patella 2021	0.133
θ^{-}	Pessimism	Bassanin, Faia, and Patella 2021	2.496
θ^{+}	Optimism	Bassanin, Faia, and Patella 2021	-1.892
$\overline{x}-$	Logistic location Pessimism	Implied by steady state	a
$\overline{x}^{+}$	Logistic location Optimism	Implied by steady state	-12.5
s^{-}	Logistic scale Pessimism	Implied by steady state	0.1
s^{+}	Logistic scale Optimism	Implied by steady state	1
ρ_y	Income Persistence	Matching Moments	0.3
σ_y	Income Volatility	Matching Moments	0.02
κ	Collateral coefficient	Matching Moments	0.15
γ	Risk aversion	Matching Moments	2.5

Table 3.1

The parameters for the two logistic distributions are calibrated such that when the variables are in steady state the transition kernel matches its steady state counterpart[1]. The optimal parameters are obtained by solving for $\overline{x}$

$$\begin{aligned} p_{ss} = \frac{1}{1+\exp\{-(x_{ss}-\overline{x})/s\}} \rightarrow 1/p_{ss} - 1 &= \exp\{-(x_{ss}-\overline{x})/s\} \\ \overline{x} &= \log(1/p_{ss}-1) + x_{ss} \end{aligned} \tag{3.1}$$

To ensure that the distribution for the transition between the optimistic and pessimistic state spans the whole range $[p_{ss}, 1]$ the scale parameter is set to $s^{-} = 0.1$. This allows the conditional distribution to become degenerate, allowing for temporary absorbing states in the relevant regimes.

The parameters concerning the ambiguity attitudes and the share of dividends are calibrated following Bassanin, Faia, and Patella (2021). The authors estimated the first set of parameters by GMM matching the pricing kernel of a model whose

[1] The estimation process for the steady state kernel it's explained in APPENDIX

dynamics closely follow mine, while the share of dividends is estimated following Lettau and Ludvigson (2001).

The other parameters in the model are calibrated using a moment matching technique, in particular moments related to the financial cycle and its connections with the real cycle are used to define the combination of optimal parameters such that the moments generated by the model minimize the L^2 norm of the empirical moments.

The parameter space I explored it's tight, assuming parameters not to deviate much from standard values used elsewhere in the literature $(\rho_y, \sigma_y, \kappa, \gamma) \in \Theta = [0.2, 0.7] \times [0.015, 0.03] \times [0.1, 0.25] \times [1.5, 2.5]$, the moment matched in this step are

Parameter	Meaning	Strategy	Value
β	Discount factor	Standard DSGE value	0.94
R	Risk-free rate	3 month T-bill rate****	1.025
ϕ	Share of dividend	Fraction of financial wealth****	0.133
θ^+	Pessimism	Bassanin, Faia, and Patella 2021	2.496
θ^-	Optimism	Bassanin, Faia, and Patella 2021	-1.892
α^+	Log-logistic scale Pessimism	Matching Moments	a

3.2 Policy functions

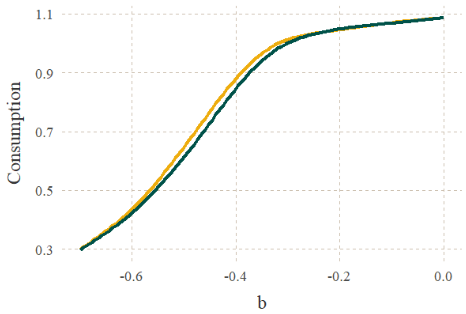
To begin the analysis of the switching mechanism I analyze the policy functions generated by the solution method. Those are defined as as the combination of solutions[2] in the two regimes:

$$d(b, a \mid y) = \mathbb{1}_{\{a_t=1\}} \boldsymbol{\psi}(b) \boldsymbol{\rho}_{\text{Opt}} + (1 - \mathbb{1}_{\{a_t=1\}}) \boldsymbol{\psi}(b) \boldsymbol{\rho}_{\text{Pes}}$$

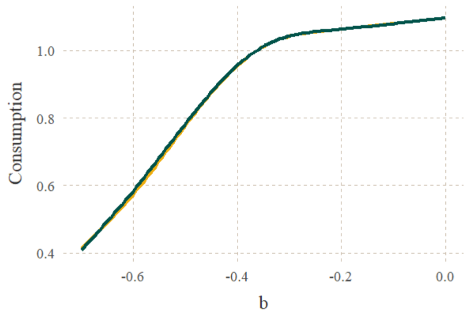
This allows me to look at the different shapes conditioning both on the realization of the income and ambiguity processes.

Figure (NUM) shows the policy functions for consumption, debt and asset prices for low (left) and high (right) income states both in the optimistic (yellow) and pessimistic (green) regimes.

[2]The solution mechanism is discussed in APPENDIX.



(a)



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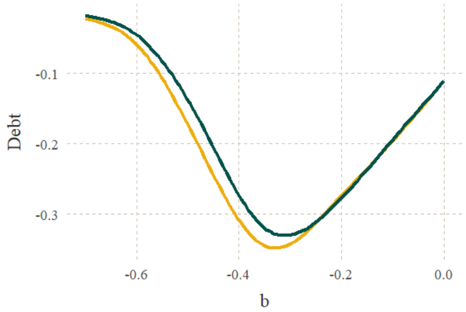
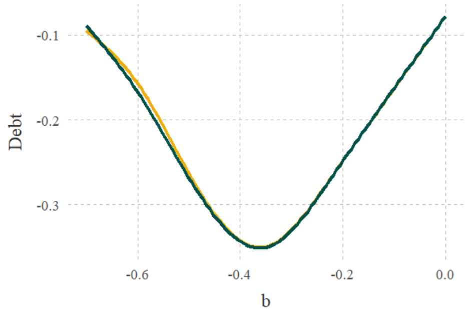


(c)



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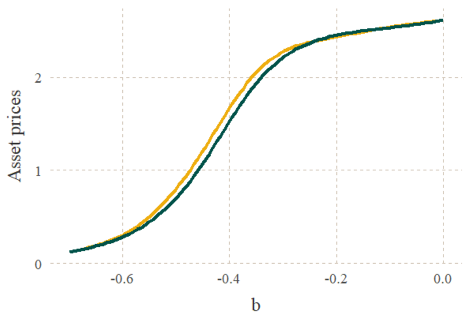

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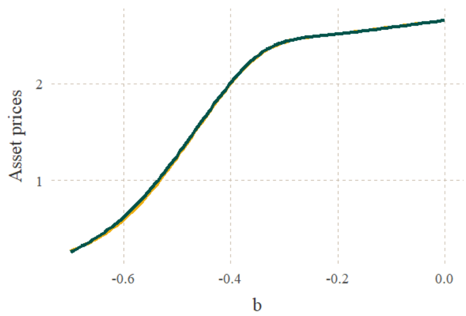


(f)

Figure 3.1. Policy func

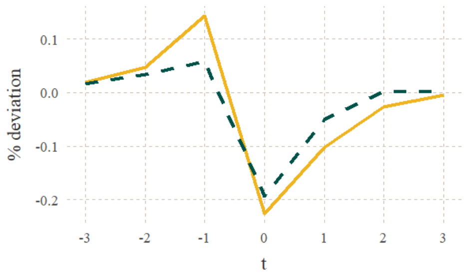
Two properties are especially noteworthy. First, as expected, the policy functions in the pessimistic regime reflect more cautious behavior of agents for low-income states, while they overlap for higher-income, where there is no distinction between pessimistic and optimistic behavior. Additionally, it is interesting to observe that, to the right of the kink (in the region where the constraint is marginally binding), the policy functions for the high-income state exhibit a flatter shape. This implies a faster and higher increase in both consumption and asset prices, as well

as a slower decrease in the debt level.
The second notable property is in the region where there is a meaningful distinction between the policy functions in the optimistic and pessimistic regimes. These functions diverge only in the region where the constraint is binding and differ in the starting point of the binding region. However, they converge again at the lower boundary of the debt domain, where the debt is sufficiently low that optimistic and pessimistic behaviors converge once again.

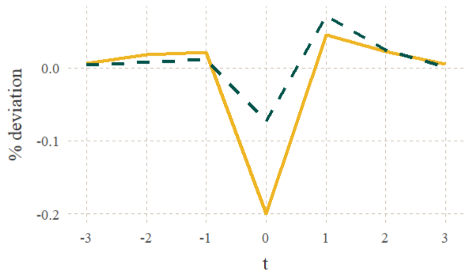
3.3 Crisis Event

RESULTS. The dynamics for the crisis event are obtained by simulating[3] series conditional on each regime and retaining the simulated variable by sampling the current state from the transition probability given the previous realization. The dynamics for the simulated data (yellow) showed in FIGURE are compared to the results obtained from the baseline Rational expectations model (green).
The model with ambiguity attitudes shows how the build up of debt (a) is enhanced by the increasingly optimistic behaviour of agents before the crisis episode producing a sharp increase in the deviation of leverage from its unconditional mean of about 250%. The effect of the deleverage are also affected positively by the pessimistic behaviour during the crisis episode where the reduction in debt consumption and asset prices twice the reduction in the baseline model.

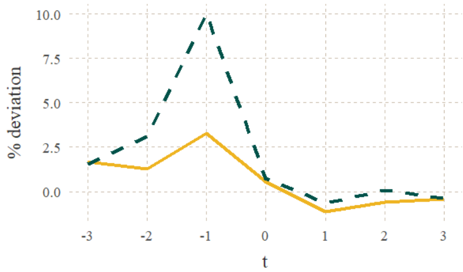
[3]The simulated path have length of 100000 periods. The crisis episode is defined as in Enrique G. Mendoza (2010) as the situation in which the current account is two standard deviations greater than its ergodic mean. The extracted path are then averaged across all crisis events.



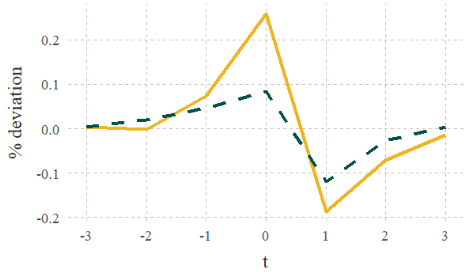
(a) Debt



(b) Consumption



(c) Leverage



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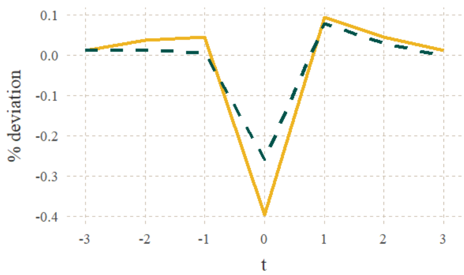
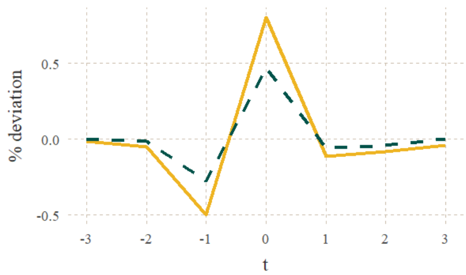


(e) Asset price



(f) Returns

Figure 3.2. Deviation from unconditional means during crisis episode

3.4 Probability

One way to study the effects of endogenizing the ambiguity process on subjects' attitudes towards optimism and pessimism is by examining their unconditional probabilities. In the setup I propose, the variables can influence the transition probabilities, making the unconditional distribution of the process more likely to favor certain states if the control variables move in a specific direction.

To analyze their behavior, we can distinguish between one-step probabilities and limit probabilities.

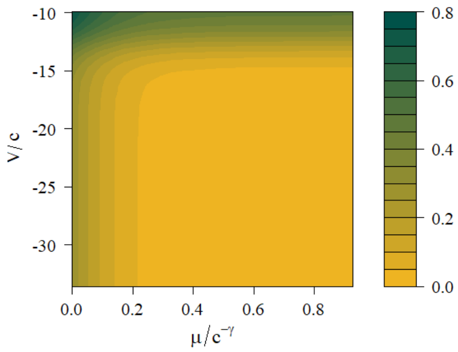
To simplify this task, I will examine the probability of being in the optimistic state a^+ and observe how it is influenced by the control variables V/c and $\mu/c^{-\gamma}$.

Assume the vector of inital conditions $\zeta_0 = \left(\zeta_0^+, \zeta_0^-\right)$ be defined by $\zeta_0^+ = \zeta_0^- = 1/2$. The one step probability is defined as usual by $\zeta_1 = \zeta_0 \mathbb{K}_{a_1|a_0}$; however, since the ambiguity process is inhomogeneous its k-th step transition kernel is given by the product of kernels given the values of the endogenous variables $\mathbb{K}_{a_n|a} = \prod_{i=1}^{n} \mathbb{K}_{a_i|a}$. Because this kernel is entirely described by the two state variables, we can derive it's shape at the limit, and thus, the vector of unconditional probabilities given all the possible outcomes of the two control variables.

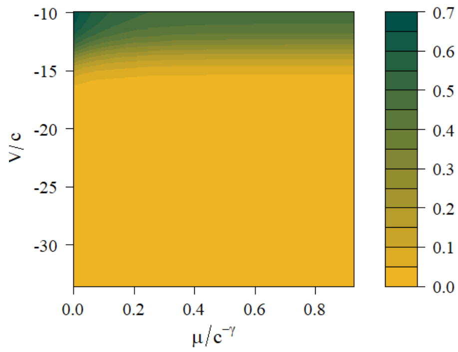
$$\zeta_\infty = \lim_{n\to\infty} \zeta_0 \prod_{i=1}^{n} \mathbb{K}_{a_i|a} \tag{3.2}$$

These results are summarized in the two contour plots below. It is interesting to note that, in the limit, the role of the Lagrange multiplier becomes less significant in influencing subjects' expectations compared to the 1-step unconditional probability, where it shows strong effects even when the continuation value is very low.

This aligns with the structure of the model and the underlying idea behind the definition of the transition kernel. In the short term, subjects are influenced by both their continuation value and the tightening of the collateral constraint. This results in a faster increase in the unconditional probability in states with a low continuation value when the constraint is non-binding. However, over the long term, the effect of the collateral constraint becomes less significant, as agents focus more on their lifetime utility. As a result, the impact diminishes in regions where the multiplier



(a)



(b)

Figure 3.3. One step (a) and limit probability (b) distribution for optimist state.

does not bind.

This outcome suggests that outside crisis events, the probabilities of adopting optimistic or pessimistic behavior are balanced when the continuation value is low. However, in the long term, pessimistic behavior tends to prevail. For higher values of the value function/consumption ratio, the probability of being optimistic remains steady, as this variable dominates, indicating that agents are less concerned about the constraint binding when their wealth is sufficient to offset its effects.

Chapter 4

Conclusions

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Appendix A

Appendix

A.1 Probabilistic setup

From a probabilistic point of view, the setup I propose for the ambiguity process $\{a_t\}_{t\in\mathbb{N}}$ belongs to the family of Markov Chains in random environment (MCRE). A formal definition borrowing from Cogburn (1980) for this process is given:
Consider a family of transition kernels $\{K(\varphi) : \varphi \in \Phi\}$ on the space $(\Omega, \mathcal{F})$ and a process $\{X_t\}_{t\in\mathbb{N}}$ taking values in Ω satisfying

$$\mathbb{P}\left(X_{t+1} \in A \mid X_t, \ldots, X_0\right) = \mathbb{P}\left(\varphi_t : X_t, A\right) \qquad a.s. \tag{A.1}$$

If we assume φ to be a random variable in the space $(\Phi, \mathcal{A})$, conditioning on $\varphi_0, \ldots, \varphi_t$

$$\mathbb{P}\left(X_{t+1} \in A \mid \varphi_t, \ldots, \varphi_0, X_t, \ldots, X_0\right) = \mathbb{P}\left(\varphi_t : X_t, A\right) \qquad a.s. \tag{A.2}$$

Then $\forall t \in \mathbb{N}$ and $\forall A \in \mathcal{F} \implies \{X_t\}_{t\in\mathbb{N}}$ is a Markov Chain in random environment (MCRE), the process $\{\varphi_t\}_{t\in\mathbb{N}}$ is called the environmental process.
In what follows $\{\varphi_t\}_{t\in\mathbb{N}}$ is assumed to be time homogeneous and Φ countable.
Condition (A.2) completely defines the model, stating that given the history of the environmental process the MCRE evolves as a time non-homogeneous Markov Chain. It's easy to notice how this setup simply transalte to the framework I propose where the enivornmental process is represented by the income/debt process

$\{\varphi_t\}_{t\in\mathbb{N}} \longrightarrow \{(z_t, b_t)\}_{t\in\mathbb{N}}$ and the MCRE $\{X_t\}_{t\in\mathbb{N}} \longrightarrow \{a_t\}_{t\in\mathbb{N}}$ is represented by the ambiguity process.

To obtain the complete kernel for the evolution of the joint process we need to define the bichain $\{\varphi_t, X_t\}_{t\in\mathbb{N}}$ on the product space $(\Phi \times \Omega, \mathcal{A} \times \mathcal{F})$, its one step transition kernel is then given by

$$\mathbb{P}(\varphi, x; B \times \{y\}) = \Gamma(\varphi, B)K(\varphi; x, y) \tag{A.3}$$

An intresting property can be derived by looking at the behaviour of the environmental process, when $\{\varphi_t\}_{t\in\mathbb{N}}$ is an irreducible, recurrent Markov chain on a countable state space, every $\varphi \in \Phi$ is visited infinitely often with probability one. If we define the set of stopping times $\{\tau_0, \ldots\}$ for the n-th return of the environmental process at φ, the subsequence $\{X_t\}_{t\in\{\tau_0,\ldots\}}$ is a time-homogeneous Markov chain. This property becomes useful when studying the behaviour of the ambiguity process as in Section 3.3, as it allows to look at the realizations of time-homogeneous sub-chain during crisis events for fixed values of the environmental process. To this sub-chain is associated a kernel $R_\varphi(x, y)$ for $\{\varphi\} \times \Omega$.

This property can be used also to study other types of behaviours of the sub-chain. Let C be a non-empty, closed set and define the probability of absorption

$$\rho_C(\varphi, x) = \mathbb{P}_{\varphi,x}\left(\bigcup_{t=1}^{\infty} X_t \in C\right) \tag{A.4}$$

Using the previous result on the homogeneity of the sub-chain, Bourgin and Cogburn (1981) shows how $X_t \in C$ if, and only if some of the states of the sub-chain $Y_t = X_{\tau_n} \in C$. Thus $\rho_C(\varphi, x)$ will be the same as the probability that the chain $\{X_t\}_{t\in\{\tau_0,\ldots\}}$ is absorbed in C. This property becomes useful when one enables the chain to have closed sets as it allows to study it's behaviour during particular events, for example in my work both states are allowed to become absorbing for certain combinations of the state variables.

One last major difference that arises in this setup relates to the classification of states. In general the classes of states for MCRE mimics the classification for standard Markov chains. Specifically, in the MCRE framework, (1) transient states are referred to as inessential states, while (2) recurrent states are called essential. A state is considered inessential if, for every X, $\mathbb{P}(X_t \in B \text{ i.o.}) = 0$ (where i.o. stands for infinitely often), otherwise this state is called essential. Essential states can be carefully decomposed in another sub-class of states, that is, if B is essential, and contained in a countable union of inessential states, then is called improperly essential, otherwise is properly essential.

This framework becomes particularly valuable when examining long-term behavior, crisis scenarios, or conditions where certain states may become absorbing. By enabling deeper exploration of state transitions and their probabilistic properties, this classification helps capture the key aspects of decision-making under ambiguity in the presence of environmental shocks.

A.2 Numerical Method

The model is solved by Cubic Splines collocation method.

Define the aggregate state variable $x = (b, y, a) \in X = B \times Y \times A$, the solution to the optimization problem is given by the set of r decision rules $\{c(x), m(x), \mu(x), q(x), b'(x), V(x)\}_{x \in X}$ which solve the equilibrium conditions:

$$c(x)^{-\gamma} = \beta R \mathbb{E}_{z'|z}\left[m(x')c(x')^{-\gamma}\right] + \mu(x)$$

$$q(x) = \beta \frac{\mathbb{E}_{z'|z}\left[m(x')c(x')^{-\gamma}[q(x') + \alpha y']\right]}{c(x)^{-\gamma} - \kappa\mu(x)}$$

$$\mu(x)\left[\frac{b'(x)}{R} + \kappa q(x)\right] = 0$$

$$c(x) + \frac{b'(x)}{R} = y + b$$

$$V(x) = \frac{c(x)^{1-\gamma} - 1}{1-\gamma} - \beta\theta(a)\ln\left(\mathbb{E}_{z'|z}\left[\exp\left\{-V(x')/\theta(a)\right\}\right]\right)$$

$$m(x') = \frac{\exp\{-V(x')/\theta(a)\}}{\mathbb{E}_{z'|z}\left[\exp\{-V(x')/\theta(a)\}\right]}$$

For each of our state variable I define a grid for the income process $\mathbf{Z} = \{\underline{z}, \dots, \overline{z}\}$ using Tauchen (1986) method, and fix the grid for the current debt level $\mathbf{B} = \{\underline{b}, \dots, \overline{b}\}$, and the set for the ambiguity process $\mathbf{A} = \{0, 1\}$.

The solution is defined by decision rules with the following structure:

$$d(b, s_j, a_k \mid \rho) = \sum_{i}^{n_b} \rho^d_{i,j,k}\psi_i(b) \tag{A.5}$$

the total number of uknowns, defined by the total number of coefficients $\rho^d_{i,j,k}$ is $n_\rho = r \times n_b \times n_y \times n_a$, to speed up the solution method I first fix $n_b = 4$ such that $n_\rho = 4 \times 4 \times 9 \times 2 = 360$ and then increase iteratively $n'_b = n_b + k$ and use the solution previously obtained as initial guess for the next step until $n_b = 10$.

Since the model features an occasionally binding constraint I use Garcia, W. I Zangwill, and Willard I Zangwill (1981) method to transform the inequality constraint into an equality. This method allows me to obtain the residual equation for the

Lagrange multiplier $\mu(x)$.

Define two additional terms $\mu^+ = \max\{0, \tilde{\mu}\}^2$ and $\mu^- = \max\{0, -\tilde{\mu}\}^2$, given this change of variables, the complementary slackness condition (NUM) is subsistuted by

$$\frac{b'(x)}{R} + \kappa q(x) = \mu^- \tag{A.6}$$

we can observe how the Kuhn-Tucker conditions are satisfied by having $\mu^+ \geq 0$, $\mu^- \geq 0$ and $\mu^+\mu^- = 0$. The policy function for the Lagrange multiplier μ is then obtained by μ^+.

The algorithm proceeds as follows:

1) For each $x \in B \times Y \times A$ I find $\{c(x), q(x), \mu(x), V(x)\}$ using their definition as in eq(NUM).

2) Plug $c(x)$ into the resource constraint to find $b'(x)$

$$b'(x) = \begin{cases} R(y + b - c(x)) & \text{if } b'(x) \geq -R\kappa q(x) \\ -R\kappa q(x) & \text{otherwise} \end{cases}$$

3) Given b', the policy functions for the next period $\{c'(x'), q'(x'), V'(x')\}$ are found.

4) Calculate the value of the kernel $\mathbb{K}_{a'|a}\left(\mu(x), \dfrac{V(x)}{c(x)}\right)$ at the nodes.

5) The generic expectation in the discretized space is calculated as $\mathbb{E}[Z' \mid a_i, y_j] = Z' \times \mathbb{K}_{a'|a_i} \otimes \pi(y' \mid y_j)$

5) Define the residuals from our eq. conditions as and solve for them

$$R(x) = \begin{pmatrix} b'(x)/R - \kappa q(x) - \mu^-(x) \\ \left[\beta R \mathbb{E}_{z'|z}\left[m(x')c(x')^{-\gamma}\right] + \mu^+(x)\right]^{\frac{1}{\gamma}} - c(x) \\ \beta \dfrac{\mathbb{E}_{z'|z}\left[m(x')c(x')^{-\gamma}[q(x') + \alpha y']\right]}{c(x)^{-\gamma} - \phi\mu^+(x)} - q(x) \\ \dfrac{c(x)^{1-\gamma} - 1}{1-\gamma} - \beta\theta(a)\ln\left(\mathbb{E}_{z'|z}\left[\exp\left\{-V(b's')/\theta(a)\right\}\right]\right) - V(x) \end{pmatrix} = 0 \tag{A.7}$$

I use a non-linear equation solver routine to minimize the error for each of our n_ρ equations.

A.3 Rational Expectation benchmark model

The recursive optimization problem faced by the agents in the baseline model is similar to the one proposed in this thesis. Agents here do not control for the probability distortion increment, so their continuation value is defined by the usual recursion

$$V = \max\left\{u(c) + \beta\mathbb{E}_{y'|y}\left[V'\right]\right\} \tag{A.8}$$

while the stochastic process and constraints faced in this optimization problem are the same the agents face in the Ambiguity-adjusted case[1].

$$\begin{aligned} c_t + q_t x_{t+1} + \frac{b_{t+1}}{R} &= (1-\phi)y_t + x_t(q_t + d_t) + b_t \\ -\frac{b_{t+1}}{R} &\leq \kappa q_t x_{t+1} \\ \log(y_t) = \rho\log(y_{t-1}) + \varepsilon_t, \qquad & \varepsilon_t \sim \mathcal{N}(0,\sigma) \end{aligned} \tag{A.9}$$

The FOCs for the baseline model are defined by

$$c^{-\gamma} = \beta R\mathbb{E}_{z'|z}\left[c'^{-\gamma}\right] + \mu \tag{A.10}$$

$$q = \beta\frac{\mathbb{E}_{z'|z}\left[c'^{-\gamma}[q' + \alpha y']\right]}{c^{-\gamma} - \kappa\mu} \tag{A.11}$$

together with the market clearing condition

$$\frac{b'}{R} + c = y + b \tag{A.12}$$

and the complementary slackness condition

$$\mu\left[\frac{b'}{R} + \kappa q\right] \tag{A.13}$$

This model is calibrated following Bassanin, Faia, and Patella (2021) (PARAMETRI) and solved again by Cubic Splies collocation.

[1]If the reader wants an explaination of the constraints can read SECTION