C20220110Mixed

1. **MM-value=13/time=25**

   Due to the Covid pandemic, the number of drivers available for the distribution of goods, drastically decreased. In order to transport primary goods to the population of remote regions, the department of Civil Defense decided to use flying drones and mobile depots. There are n groups of people and each group $i = 1, \ldots, n$ needs to receive W_i kilograms of goods. Temporary depots are installed in h among k possible sites ($h < k$). There are D_s drones available at each location $s = 1, \ldots, k$. Let d_{is} denote the flying distance between group i and site s (the distance is set to $+\infty$ if site s cannot serve group i). Each drone serves a group at a time. The total weight carried by a drone is at most w kilograms, therefore it may happen that more drones are used to serve the same group. The same drone can perform at most one fly in the time horizon of this plan.
   Write a linear program to help the Civil Defense department to locate the temporary depots and define the drone flies with the objective of minimizing the total distance traveled by the drones.

   Notes: (not included in XML)

   - Solution:
     - $x_{isl} = 1$ if the l-th drone by site s serves group i, 0 otherwise.
     - $y_s = 1$ if a temporary depot is installed in site s, 0 otherwise.

$$\min z = \sum_{i=1}^{n}\sum_{s=1}^{k}\sum_{l=1}^{D_s} 2d_{is}x_{isl}$$

$$\sum_{i=1}^{n} x_{isl} \leq 1 \qquad s = 1,\ldots,k, l = 1,\ldots,D_s$$

$$w\sum_{s=1}^{k}\sum_{l=1}^{D_s} x_{isl} \geq W_i \qquad i = 1,\ldots,n$$

$$\sum_{i=1}^{n}\sum_{l=1}^{D_s} x_{isl} \leq D_s y_s \qquad s = 1,\ldots,k$$

$$\sum_{s=1}^{k} y_s = h$$

$$x_{isl} \in \{0,1\} \qquad i = 1,\ldots,n, s = 1,\ldots,k, l = 1,\ldots,D_s$$

$$y_s \in \{0,1\} \qquad s = 1,\ldots,k$$

2. **PLI-value=8/time=15**

Given the following ILP problem:

$$\begin{aligned} \min z = \quad & -2x_1 + 2x_2 \\ & x_1 - 1/2x_2 \leq 3/2 \\ & 2x_1 + 2x_2 \leq 5 \\ & x_1, x_2 \geq 0 \text{ integer.} \end{aligned}$$

Solve the linear relaxation, apply one Gomory's cut, show one further iteration of a simplex method, and stop. Show all the tableaus. Is the solution obtained optimal? Motivate your answer.

Notes: (not included in XML)

- Solution:

	x_1	x_2	y_1	y_2	$-z$
	-2	2	0	0	0
y_1	$\mathbf{1}$	$-1/2$	1	0	$3/2$
y_2	2	2	0	1	5

	x_1	x_2	y_1	y_2	$-z$
	0	1	2	0	3
x_1	1	$-1/2$	1	0	$3/2$
y_2	0	3	-2	1	2

Gomory cut: $1/2x_2 + 0y_1 \geq 1/2 \;\rightarrow\; -1/2x2 + y_3 = -1/2$

	x_1	x_2	y_1	y_2	y_3	$-z$
	0	1	2	0	0	3
x_1	1	$-1/2$	1	0	0	$3/2$
y_2	0	3	-2	1	0	2
y_3	0	**-1/2**	0	0	1	$-1/2$

	x_1	x_2	y_1	y_2	y_3	$-z$
	0	0	2	0	2	2
x_1	1	0	1	0	-1	2
y_2	0	0	-2	1	6	-1
x_2	0	1	0	0	-2	1

The solution is not optimal since there are still variables with negative value. Further iterations would be required.

3. **SP-value=8/time=13**

Consider a directed graph with arc weights given by the following matrix and compute the shortest path from vertex 1 to vertex 5, using the Dijkstra algorithm. Report all the iterations in a table (or two separate tables) showing, in each line, the set of nodes with permanent labels, labels, predecessors. Write the shortest path and its cost.

	1	**2**	**3**	**4**	**5**
1	–	–	–	4	12
2	–	–	1	–	–
3	–	–	–	1	2
4	–	2	–	–	7
5	–	–	–	–	–

Notes: (not included in XML)

- Solution:

S	L_j					P_j				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
$\{1\}$	0	–	–	4	12	–	–	–	1	1
$\{1,4\}$	0	6	–	4	11	–	4	–	1	4
$\{1,2,4\}$	0	6	7	4	11	–	4	2	1	4
$\{1,2,3,4\}$	0	6	7	4	9	–	4	2	1	3

The shortest path is (1, 4, 2, 3, 5) with cost 9.