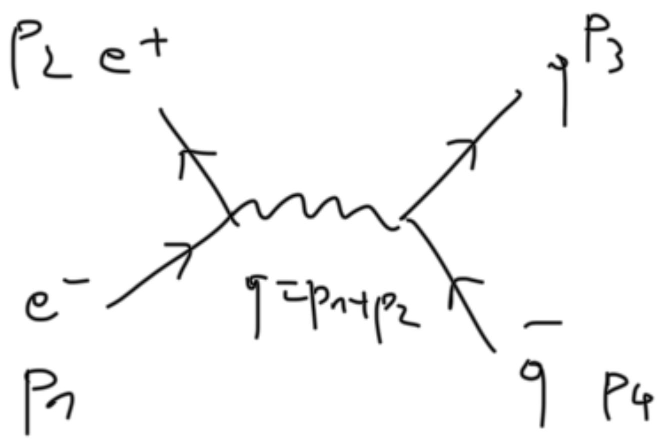
$e^+e^- \longrightarrow \gamma \longrightarrow q\bar{q}$



only Feynman diagram for photon exchange in $e^-e^+ \to q\bar{q}$

Using QED Feynman rules assuming quark with flavor f we get the following matrix element

$$\mathcal{M}_f = \bar{v}(p_2)\, ie\gamma^\mu u(p_1)\, \frac{-i g_{\mu\nu}}{(p_1+p_2)^2}\, \bar{u}(p_3)\, ieQ_f \gamma^\nu v(p_4)$$

$$\overline{|M|^2} = \frac{1}{4}\frac{e^4 Q_f^2}{q^4} g_{\mu\nu} g_{\alpha\beta} T^{\mu\nu\alpha\beta}$$

where $T^{\mu\nu\alpha\beta} = \sum_{\text{all spins}} \bar{v}(p_2)\gamma^\mu \underbrace{u(p_1)\bar{u}(p_1)}_{\not{p}_1 \text{ (massless limit)}} \gamma^\alpha v(p_2)\, \bar{u}(p_3)\gamma^\nu \underbrace{v(p_4)\bar{v}(p_4)}_{\not{p}_4 \text{ (massless limit)}} \gamma^\beta u(p_3)$

$$T^{\mu\nu\alpha\beta} = \sum_{s_2} \bar{v}(p_2)\gamma^\mu \not{p}_1 \gamma^\alpha v(p_2) \sum_{s_3} \bar{u}(p_3)\gamma^\nu \not{p}_4 \gamma^\beta u(p_3)$$

$$= \underbrace{\mathrm{Tr}\left[\gamma^\mu \not{p}_1 \gamma^\alpha \not{p}_2\right]}_{A} \underbrace{\mathrm{Tr}\left[\gamma^\nu \not{p}_4 \gamma^\beta \not{p}_3\right]}_{B}$$

$$A = p_{1\eta}\, p_{2\varepsilon}\, \mathrm{Tr}\left[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^\varepsilon\right] = p_{1\eta}\, p_{2\varepsilon}\, 4\left(g^{\mu\eta} g^{\alpha\varepsilon} - g^{\mu\alpha} g^{\eta\varepsilon} + g^{\mu\varepsilon} g^{\eta\alpha}\right)$$

$$= \left(p_1^\mu p_2^\alpha - g^{\mu\alpha}(p_1 p_2) + p_1^\alpha p_2^\mu\right) 4$$

$$B = p_{4\eta}\, p_{3\varepsilon}\, \mathrm{Tr}\left[\gamma^\nu \gamma^\eta \gamma^\beta \gamma^\varepsilon\right] = [\cdots] = 4\left(p_4^\nu p_3^\beta - g^{\nu\beta}(p_4 p_3) + p_4^\beta p_3^\nu\right)$$

$$g_{\mu\nu} g_{\alpha\beta} T^{\mu\nu\alpha\beta} = 16\, g_{\mu\nu} g_{\alpha\beta} \left(p_1^\mu p_2^\alpha - g^{\mu\alpha}\, p_1 \cdot p_2 + p_1^\alpha p_2^\mu\right)\left(p_4^\nu p_3^\beta - g^{\nu\beta}(p_4 p_3) + p_4^\beta p_3^\nu\right)$$

$$= \left(p_{1\nu} p_{2\beta} - g_{\nu\beta}\, p_1 p_2 + p_{1\beta} p_{2\nu}\right)\left(p_4^\nu p_3^\beta - g^{\nu\beta}(p_4 p_3) + p_4^\beta p_3^\nu\right) 16 =$$

$$-\{(p_1 p_4)(p_2 p_3) - (p_1 p_2)\cancel{(p_3 p_4)} + (p_1 p_3)(p_2 p_4) +$$

$$-(p_1 p_2)\cancel{(p_3 p_4)} + 4(p_1 p_2)\cancel{(p_3 p_4)} - (p_1 p_2)\cancel{(p_3 p_4)}$$

$$+(p_1 p_3)(p_2 p_4) - (p_1 p_2)\cancel{(p_3 p_4)} + (p_1 p_4)(p_3 p_2)\}\,16 =$$

$$= 32\left[(p_1 p_4)(p_2 p_3) + (p_1 p_3)(p_2 p_4)\right] = \circledast$$

Mandelstam variables (massless limit)

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = 2 p_1 p_2 = 2 p_3 p_4$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 = -2 p_1 p_3 = -2 p_2 p_4 = -\frac{s}{2}(1 - \cos\theta)$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 = -2 p_1 p_4 = -2 p_2 p_3 = -\frac{s}{2}(1 + \cos\theta)$$

$$\circledast = 32\left(\frac{u^2}{4} + \frac{t^2}{4}\right)$$

$$\overline{|M|^2} = \frac{e^4 Q_f^2}{\cancel{4}\, s^2} \, \frac{1 \cdots}{\cancel{2}} \, \frac{\cdots}{\cancel{4}} = \frac{\cdots}{s^2} \left(u^2 + t^2\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{c.m.} = \frac{1}{64\pi^2 s} |M|^2 = \frac{e^4}{16\pi^2} \, \frac{Q_f^2}{4 s^2} \left(u^2 + t^2\right)$$

$$= \left(Q_f^2\right) \frac{\alpha^2}{8 s} \left(1 + \cos^2\theta\right)$$

$$\frac{d\sigma}{d\cos\theta} = \left(Q_f^2\right) \frac{\alpha^2 \pi}{4 s} \left(1 + \cos^2\theta\right)$$

Consider other flavour and charges $Q_f^2 \to N_c \sum_f Q_f^2$ where the sum can be performed over quarks with mass m, $2m < s$