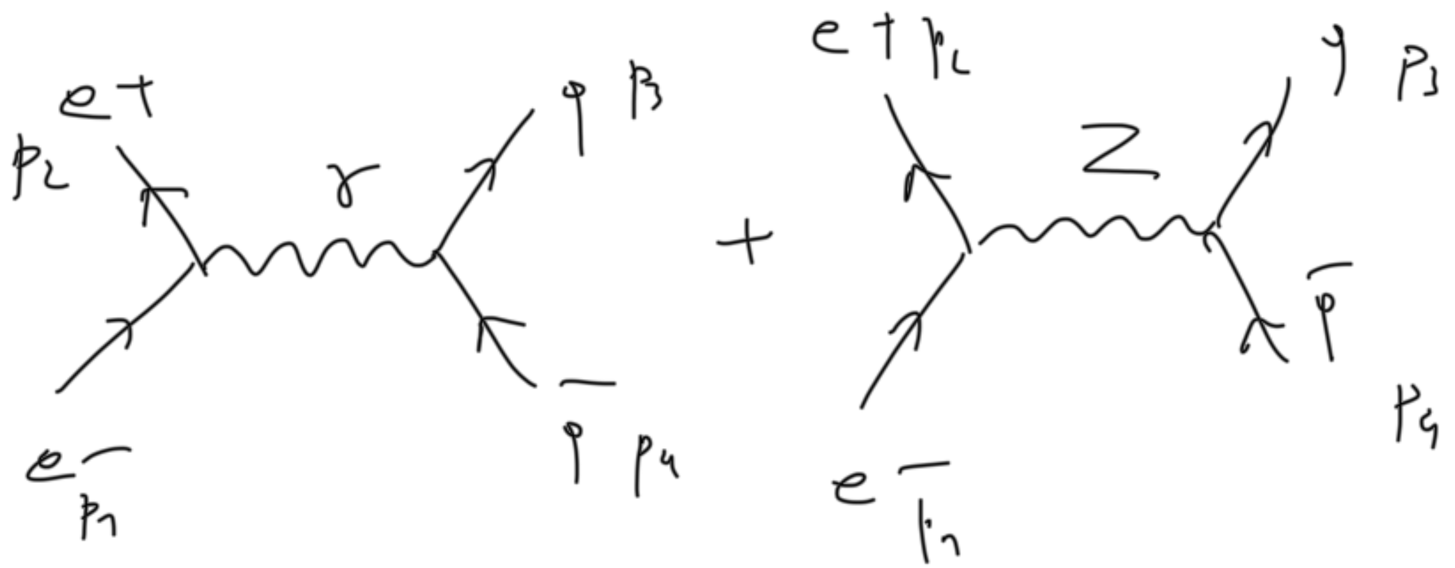
$$e^+e^- \longrightarrow \gamma / Z \longrightarrow f\bar{f}$$



$$\mathcal{M}_\gamma = i\frac{e^2 Q_f}{q^2}\,\bar{v}(p_2)\gamma^\mu u(p_1)\, g_{\mu\nu}\,\bar{u}(p_3)\gamma^\nu v(p_4)$$

$$\mathcal{M}_Z = \bar{v}(p_2)\,\frac{\cancel{-i} g_w}{2\sqrt{2}}\gamma^\mu(V_e - \gamma^5 A_e)u(p_1)\;\frac{\cancel{-i}\, g_{\mu\nu}}{q^2 - M_Z^2}\;\bar{u}(p_3)\,\frac{\cancel{-i}\, g_w}{2\sqrt{2}}\gamma^\nu(V_f - A_f\gamma_5)v(p_4)$$

$$= \frac{i g_w^2}{8(q^2 - M_Z^2)}\quad \bar{v}(p_2)\gamma^\mu(V_e - \gamma^5 A_e)u(p_1)\;\frac{g^{\mu\nu}}{q^2 - M_Z^2}\;\bar{u}(p_3)\gamma^\nu(V_f - \gamma^5 A_f)v(p_4)$$

$$|\mathcal{M}_\gamma + \mathcal{M}_Z|^2 = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + 2\,\mathrm{Re}(\mathcal{M}_\gamma^* \mathcal{M}_Z)$$

$$|\mathcal{M}_Z|^2 \quad 1 \quad g_w^4 \quad g_{\mu\nu}\, g_{\alpha\beta}\; T^{\mu\nu\alpha\beta}$$

$$|M_Z| = \frac{1}{4} \cdot \frac{\ldots}{64\,(p^2 - M_Z^2)^2}\, g^{\mu\nu} g^{\alpha\beta} \ldots$$

$$T^{\mu\nu\alpha\beta} = \sum_{\text{spins}} \bar{v}_{p_2} \gamma^\mu (V_e - \gamma^5 A_e)\, u(p_1)\, \bar{u}(p_1)\, \gamma^\alpha (V_f - \gamma^5 A_f)\, v_{p_2}$$

$$\times\, \bar{u}_{p_3} \gamma^\nu (V_f - \gamma^5 A_f)\, v_{p_4}\, \bar{v}_{p_4} \gamma^\beta (V_f - \gamma^5 A_f)\, u_{p_3} =$$

$$= \mathrm{Tr}\left[\gamma^\mu (V_e - \gamma^5 A_e)\, \not{p}_1 \gamma^\alpha (V_f - \gamma^5 A_f)\, \not{p}_2\right] = (A)$$

$$\times\, \mathrm{Tr}\left[\gamma^\nu (V_f - \gamma^5 A_f)\, \not{p}_4 \gamma^\beta (V_f - \gamma^5 A_f)\, \not{p}_3\right] = (B)$$

$$(A) = p_{1\eta}\, p_{2\varepsilon} \Big\{ V_e^2\, \mathrm{Tr}[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^\varepsilon] + A_e^2\, \mathrm{Tr}[\gamma^\mu \gamma^5 \gamma^\eta \gamma^\alpha \gamma^5 \gamma^\varepsilon]$$

$$- V_e A_e \Big[\mathrm{Tr}[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^5 \gamma^\varepsilon] + \mathrm{Tr}[\gamma^\mu \gamma^5 \gamma^\eta \gamma^\alpha \gamma^\varepsilon]\Big]\Big\}$$

$$= p_{1\eta}\, p_{2\varepsilon} \Big\{ (V_e^2 + A_e^2)\, \mathrm{Tr}[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^\varepsilon] + 2 V_e A_e\, \mathrm{Tr}[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^\varepsilon \gamma^5] \Big\} =$$

$$= p_{1\eta}\, p_{2\varepsilon} \left\{ (V_e^2 + A_e^2)\left(g^{\mu\eta} g^{\alpha\varepsilon} - g^{\mu\alpha} g^{\eta\varepsilon} + g^{\mu\varepsilon} g^{\alpha\eta} \right) 4 \right.$$

$$\left. + 2 V_e A_e (-4i)\, \epsilon^{\mu\eta\alpha\varepsilon} \right\}$$

$$B = A\left(\nu \leftrightarrow \mu,\ p_1 \leftrightarrow p_4,\ \alpha \leftrightarrow \beta,\ p_2 \leftrightarrow p_3,\ A_e \leftrightarrow A_f,\ V_e \leftrightarrow V_f \right)$$

$$= p_{4\eta}\, p_{3\varepsilon} \left\{ (V_f^2 + A_f^2)\left(g^{\nu\eta} g^{\beta\varepsilon} - g^{\nu\beta} g^{\eta\varepsilon} + g^{\nu\varepsilon} g^{\beta\eta} \right) 4 \right.$$

$$\left. - 8i\, V_f A_f\, \epsilon^{\nu\eta\beta\varepsilon} \right\}$$

$$g_{\mu\nu} g_{\alpha\beta} T^{\mu\nu\alpha\beta} = (V_e^2 + A_e^2)(V_f + A_f)^2 \text{ (contribution similar to } \gamma)$$

$$+ A_f V_f V_e V_f \times C$$

$$+ D \longrightarrow 0 \text{ for tensor symmetry}$$

$$C \propto p_{1\eta}\, p_{2\varepsilon}\, \epsilon^{\mu\eta\alpha\varepsilon}\, p_{4\rho}\, p_{3\sigma}\, \epsilon^{\nu\rho\beta\sigma}\, g_{\mu\nu}\, g_{\alpha\beta}$$

$$= p_{1\eta}\, p_{2\epsilon}\, p_{4\rho}\, p_{3\sigma}\, \underbrace{\epsilon^{\mu\eta\lambda\epsilon}\,\epsilon^{\mu\rho\lambda\sigma}}_{\delta_{\eta\rho}\delta_{\epsilon\sigma} - \delta_{\eta\sigma}\delta_{\epsilon\rho}} =$$

$$= (p_1 \cdot p_4)(p_2\, p_3) - (p_1\, p_3)(p_2\, p_4)$$

$$= \frac{s^2}{16}\left(1 + \cos\theta - (1 - \cos\theta)\right) = \frac{s^2}{8}\cos\theta$$

The total contribution for this term is

$$|M_Z|^2 = \frac{1}{4}\,\frac{g_W^2}{64(q^2 - m_Z^2)}\Big\{(V_e^2 + A_e^2)(V_f^2 + A_f^2)\, 4 s^2 (1 + \cos^2\theta)$$

$$- 64\,\frac{s^2}{8}\, V_e A_e V_f A_e \cos\theta\Big\}$$

Interference term:

$$\mathcal{M}_\gamma^* \mathcal{M}_Z = \cancel{-i} \frac{e^2 Q_f}{q^2} \frac{\cancel{i} g^2_W}{8(q^2 - m_Z^2)} \left(\bar{v}_{p_4} \gamma^\nu u_{p_3} \, g_{\mu\nu} \, \bar{u}_{p_1} \gamma^\mu v_{p_2} \right)$$

$$\times \; \bar{v}(p_2) \gamma^\alpha (V_e - \gamma^5 A_e) u(p_1) \frac{g_{\alpha\beta}}{q^2 - m_Z^2} \bar{u}(p_3) \gamma^\beta (V_f - \gamma^5 A_f) v(p_4)$$

$$\sum \mathcal{M}_\gamma^* \mathcal{M}_Z = \frac{e^2 Q_f \, g^2_W}{8 q^2 (q^2 - m_Z)^2} g_{\mu\nu} g_{\alpha\beta} \quad T^{\mu\nu\alpha\beta}$$

$$T^{\mu\nu\alpha\beta} = \mathrm{Tr}\left[\gamma^\nu \not{p}_3 \gamma^\beta (V_f - \gamma^5 A_f) \not{p}_4 \right] \times$$

$$\mathrm{Tr}\left[\gamma^\mu \not{p}_2 \gamma^\alpha (V_e - \gamma^5 A_e) \not{p}_1 \right]$$

$$\mathrm{Tr}\left[\gamma^\nu \not{p}_3 \gamma^\beta (V_f - \gamma^5 A_f) \not{p}_4 \right] =$$

$$= V_f \, \mathrm{Tr}\left[\gamma^\nu \not{p}_3 \gamma^\beta \not{p}_4\right] - A_f \, \mathrm{Tr}\left[\gamma^\nu \not{p}_3 \gamma^\beta \gamma^5 \not{p}_4\right]$$

$$g_{\mu\nu} g_{\alpha\beta} T^{\mu\nu\alpha\beta} = g_{\mu\nu} g_{\alpha\beta} \left(V_f \, \mathrm{Tr}\left[\gamma^\nu \not{p}_3 \gamma^\beta \not{p}_4\right] - A_f \, \mathrm{Tr}\left[\gamma^\nu \not{p}_3 \gamma^\beta \gamma^5 \not{p}_4\right]\right)$$

$$\times \left(V_e \, \mathrm{Tr}\left[\gamma^\mu \not{p}_2 \gamma^\alpha \not{p}_1\right] - A_e \, \mathrm{Tr}\left[\gamma^\mu \not{p}_2 \gamma^\alpha \gamma^5 \not{p}_1\right]\right) =$$

$$\propto V_f V_e \underbrace{(\text{photon-like term})}_{\cos^2\theta} + \cancel{(V_f A_e + A_e V_f)(\ldots)} \to 0 \text{ symmetry}$$

$$+ A_e A_f \cos\theta$$

Therefore the cross section will have two different behavior

See ESW ?

$$\frac{d\sigma}{d\cos\theta} \propto A\cos^2\theta + B\cos\theta$$

Compare to the γ-exchange cross section there is an additional contribution $\propto \cos\theta$ which contains the term proportional to the axial current (containing γ_5)