Bayesian spatio-temporal modelling of economic activity in Italy by means of the information from nighttime lights

UNIVERSITÀ CATTOLICA del Sacro Cuore

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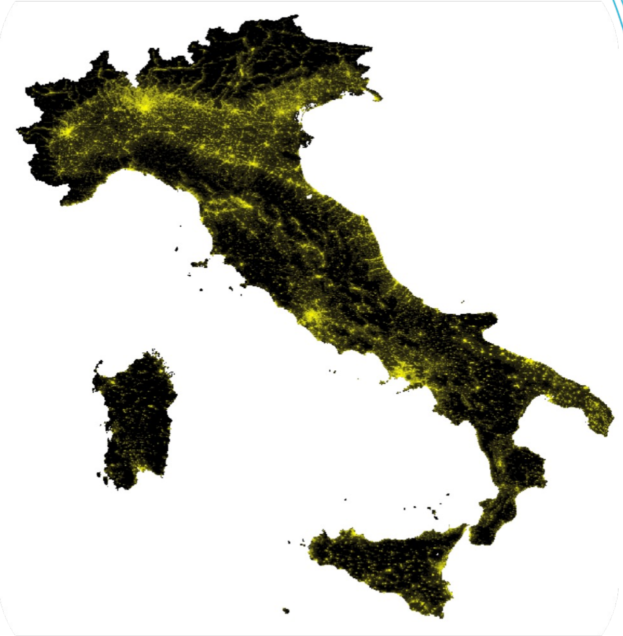
Research Questions:

How can ***Nighttime Lights*** explain and predict sub-national economic activity in Italy?

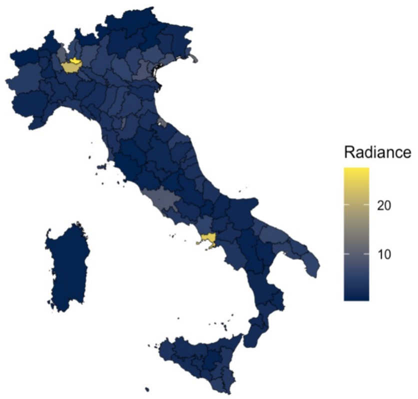
- **Why it matters:** NTL provides an objective, high-resolution alternative to inconsistent, delayed, or biased traditional economic data.

How do ***spatial and temporal dependencies*** shape economic geography?

- **Why it matters:** Economic activity is not independent across nearby areas and ignoring this can cause biased estimates and misleading conclusions.

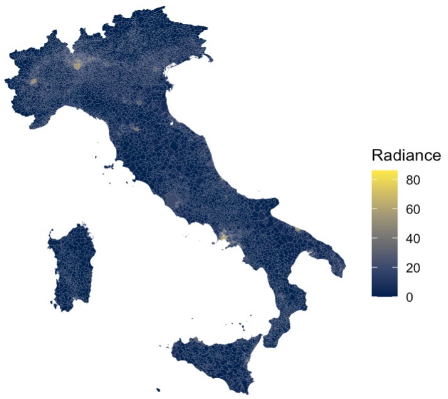


Nighttime lights Data: NASA-Black Marble (VNP46A4)



Provincial-Level NTL

Spatial Resolution: 110 *provinces*
Temporal Scope: 12-year annual (2012 – 2023)



Municipal-Level NTL

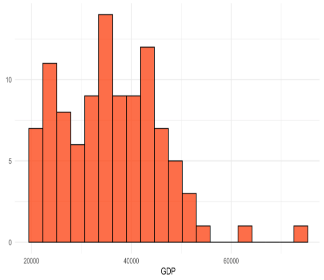
Spatial Resolution: 8100 *comuni*
Temporal Scope: Single year (2023)

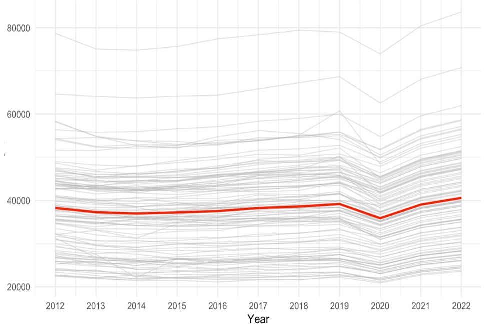
Measuring the Economic Activity

Provincial-Level Analysis:

✓ **Measure:** Harmonized Gross Domestic Product (GDP-PPP) (Kummu et al., 2025) - Downscaled global dataset
✓ **Period:** 1990-2022 (33-year panel)
✓ **Characteristics:**

- Measure of aggregate economic production
- Model-based allocations with statistical limitations
- Suitable for **temporal trend analysis**

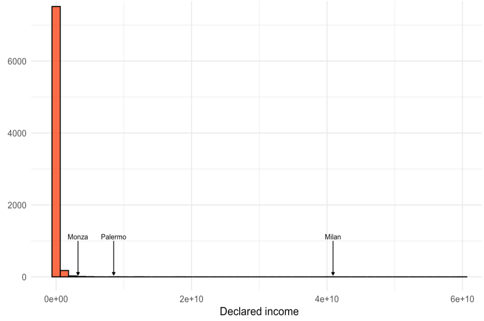


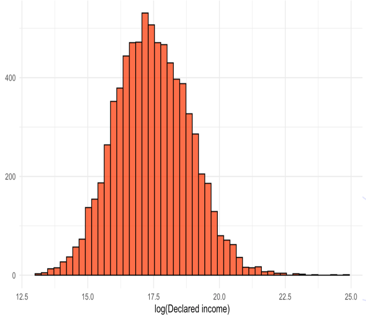


Municipal-Level Analysis:

✓ **Measure:** Total Declared Income (IRPEF) (Ministry of Finance, 2023)
✓ **Period:** 2023
✓ **Characteristics:**

- Direct administrative records
- Captures reported economic capacity of physical persons
- Reveals **fine-grained local disparities**





Spatial Statistics: When Location Matters

«Everything is related to everything else, but near things are more related than far things.»

Waldo Tobler, First Law of Geography

The adjacency matrix $\boldsymbol{W}$ is a binary $n \times n$ matrix defining neighboring spatial units to encode local dependence. From this, we adopt the ***Spatial Markov Property*** which assumes that the value at any location, given the values of its immediate neighbors, is *conditionally independent* of all other locations.

$$W_{ij} = \begin{cases} 1 & if\ units\ \boldsymbol{i}\ and\ \boldsymbol{j}\ are\ neighbors \\ 0 & otherwise \end{cases}$$

Spatial dependences are localized to units immediate neighbors. This critical assumption allows us to treat the spatial problem as a ***Markov Random Field.*** *The latent field captures residual spatial dependence after accounting for NTL.*

Hierarchical Bayesian Modelling approach

Overview

A multi-level statistical model that incorporates data, parameters, and hyperparameters in a structured hierarchy to improve estimation, share strength across groups, and quantify uncertainty.

Core idea

It combines **prior knowledge** (*prior distribution*) with **observed data** (*likelihood*), through a **latent process** (e.g., spatial, temporal, or group-level structure), to form updated probabilistic beliefs (*posterior distribution*) via Bayes' theorem.

$$p(\boldsymbol{\theta}|y) = \frac{p(\boldsymbol{y}|\boldsymbol{\theta}) \times p(\boldsymbol{\theta})}{p(\boldsymbol{y})}$$

Model levels

I. **Data Level** (*Likelihood*):

$$y_i|\theta \sim f(y_i|\theta)$$

The observed data y_i are modeled as random variables drawn from a distribution f, conditional on a set of parameters.

II. **Parameter/Process Level:**

$$\theta|\lambda \sim g(\theta|\lambda)$$

Parameters are assumed to be drawn from a population distribution g, governed by a set of higher-level parameters λ.

III. **Hyperparameter Level:**

$$\lambda \sim h(\lambda)$$

Prior distributions on hyperparameters.

The Likelihood: Linking Data to the Model

The likelihood defines the probability distribution of the observed data ($\boldsymbol{Y}$) given the expected structure defined by the linear predictor ($\boldsymbol{\eta}$). For both models, the economic activity variable $\boldsymbol{Y}$ (*positive, continuous and right-skewed*) is assumed to follow a ***Gamma distribution***:

$$\boldsymbol{Y}_{it} \mid \mu_{it}, \nu \sim Gamma(\mu_{it}, \nu)$$

where:

- μ_{it} is the mean parameter.
- ν is the precision parameter controlling dispersion.

Link Function: Connects the expected value (μ_{it}) to the linear predictor (η_{it}) which crucially defines the latent process. By performing this transformation, it ensures the expected value remains positive.

$$\mathrm{g}(E[\boldsymbol{Y}]) = log(\mu_{it}) = \eta_{it}$$

Why Two Models?

- **Municipal data** has high spatial resolution but no temporal dimension.
- **Provincial data** has both spatial and temporal dimensions.

Model 1: Spatial ICAR for **Municipal** Declared Income

Data: Municipal taxable income (IRPEF) – 2023 ($i = 1, \dots, 7{,}742$ municipalities)
Latent Process:

$$\eta_i = \log(\mu_i) = \alpha + \beta \cdot NTL_i + \phi_i^{(m)} + \phi_i^{(r)}$$

- **Municipal spatial random effect** $\phi_i^{(m)}$: Fine-scale spatial smoothing among neighboring municipalities
- **Regional spatial random effect** $\phi_i^{(r)}$: Broad-scale spatial patterns across Italy's regions

We specify the Intrinsic Conditional Autoregressive model **ICAR** for the spatial random effect (Besage, 1974) as:

$$\phi_i \mid \phi_{-i}, \tau_\phi \sim N\left(\frac{1}{n_i} \sum_{i \sim j} \phi_j \,, \frac{1}{n_i \tau_\phi}\right)$$

where $i \sim j$ denotes neighbors of municipality i, and n_i is the number of neighbors. With joint distribution: $\boldsymbol{\phi} \sim N\left(0, \tau_\phi^{-1}(D - W)\right)$
where W is the adjacency matrix and $D = diag(n_i)$.

Model 2: Spatio-Temporal for **Provincial** GDP

Data: Provincial GDP (PPP-adjusted) – data from 2012–2023 ($i = 1, \dots, 107$ provinces and $t = 1, \dots, 12$)
Latent Process:

$$\eta_{it} = \alpha + \psi_r + \beta \cdot NTL_{it} + \phi_i^{(p)} + \delta_t$$

- **Provincial spatial random effect** $\phi_i^{(p)}$: Time-invariant spatial structure
- **Temporal random effect** δ_t: RW(1) process capturing common national trend
- **Regional random intercept** ψ_r: Unstructured heterogeneity across regions

We specify the temporal random effect as a **Random Walk** of order 1 **(RW1):**

$$\delta_t \mid \delta_{t-1}, \tau_\delta \sim N\left(\delta_{t-1}, \frac{1}{\tau_\delta}\right)$$

with initial condition $\delta_1 \sim N\left(0, \frac{1}{\kappa}\right)$. This component models smooth national economic trends and common shocks affecting all provinces simultaneously (e.g., national policy changes, macroeconomic cycles, pandemic effects), in addition to the previously defined ICAR model for the structured spatial random effect.

Bayesian Inference Procedure

Prior Specification: All unknown parameters in our Bayesian models require prior distributions.

- Priors are assumed minimally informative, allowing the data to dominate parameters inference while ensuring computational stability.

Computational Method: The ***Integrated Nested Laplace Approximation*** (**INLA**) offers a computationally efficient alternative to MCMC. This method is specifically designed to handle high-dimensional spatial and temporal fields using sparse matrix algebra for fast, accurate inference.

Model Comparison: We used different measures to asses the performance of models:

- *DIC* (Deviance Information Criterion): A measure of model fit penalized by complexity (effective number of parameters). It balances accuracy with parsimony.
- *WAIC* (Watanabe-Akaike): A fully Bayesian approach to estimating out-of-sample predictive accuracy. It offer a roboust measure by using the entire posterior distribution.

Municipal-level Analysis

Parameter	Mean	SD	2.5% Quantile	97.5% Quantile
Fixed effects:				
Intercept (α)	**17.272**	0.049	17.181	17.383
NTL (β_1)	**0.110**	0.003	0.104	0.116
Precision hyperparameters:				
Municipality effect ($\tau_{\phi}^{(m)}$)	**0.47**	0.019	0.438	0.514
Region effect ($\tau_{\phi}^{(\mathrm{r})}$)	**6.05**	2.832	2.286	13.166
Observation precision (v)	**2.43**	0.087	2.270	2.614

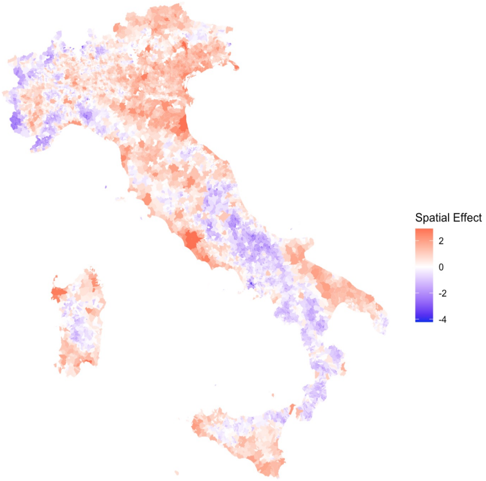
- $\exp(\beta_1) = \exp(0.110) \approx 1.116$
- **Interpretation**: A one-unit increase in NTL is associated with an approximate $11{,}6\%$ increase in mena of the expected municipal declared income.

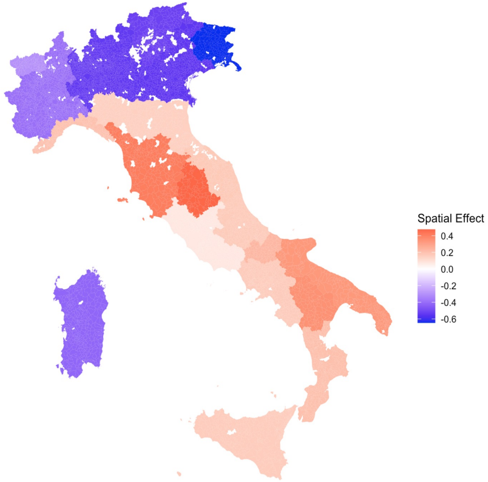
Provincial-level Analysis

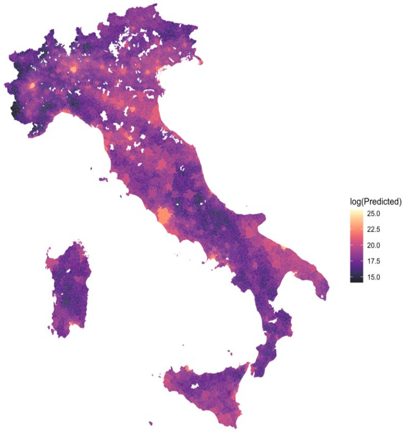
Parameter	Mean	SD	2.5% Quantile	97.5% Quantile
Fixed effects:				
Intercept (α)	**10.496**	0.037	10.422	10.571
NTL (β_1)	**0.015**	0.002	0.010	0.017
Precision hyperparameters:				
Random intercept (τ_ψ)	**12.32**	1.89	8.99	16.39
Province effect ($\tau_\phi^{(\mathrm{r})}$)	**50.24**	20.35	20.93	99.43
Temporal effect (τ_δ)	**718.52**	298.16	282.13	1436.83
Observation precision (υ)	**1906.89**	84.48	1744.64	2077.21

- $\exp(\beta_1) = \exp(0.015) \approx 1.015$
- **Interpretation**: A one-unit increase in NTL is associated with an approximate 1,5% increase in mean of the expected provincial GDP.

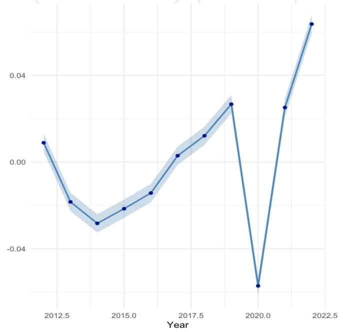
$$\boldsymbol{Model\ 1}: \eta_i = \alpha + \beta \cdot NTL_i + \phi_i^{(m)} + \phi_i^{(r)}$$

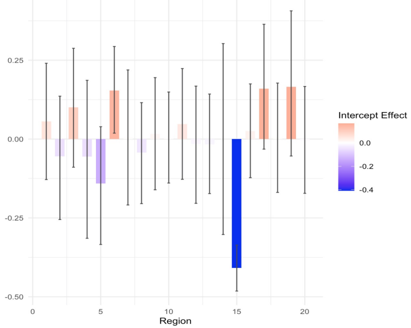


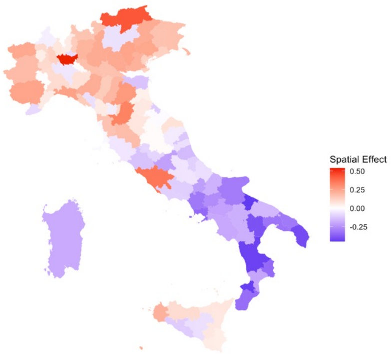


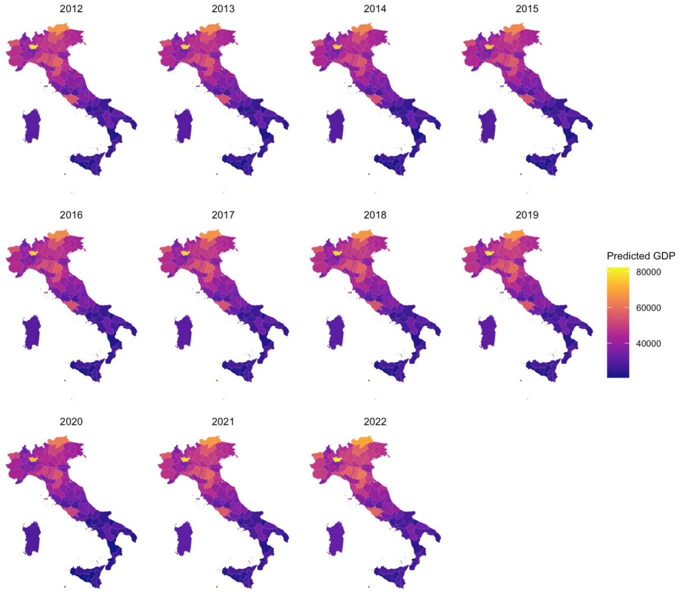


Model 2: $\eta_{it} = \alpha + \psi_r + \beta \cdot NTL_{it} + \phi_i^{(p)} + \delta_t$

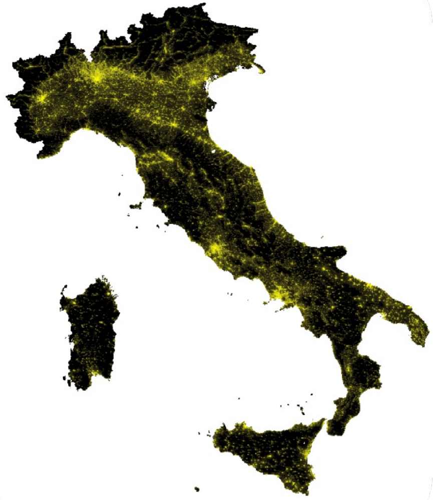








Concluding Remarks:



Valid Proxy for Economic Activity: Our analysis confirms that Nighttime Lights (NASA Black Marble) serve as a robust, high-resolution alternative for estimating sub-national GDP and Income in Italy.

Methodological Efficiency: The Hierarchical Bayesian framework, powered by INLA, proved essential for handling high-dimensional municipal data (~ 8000 units).

Addressing Spatial and Temporal Challenges:

- ***MAUP*** (Modifiable Areal Unit Problem): Mitigated by performing analyses at two distinct scale, municipal and provincial, ensuring results are robust to aggregation levels.
- ***Ecological Fallacy***: We explicitly caution that while aggregate NTL correlates with local wealth, these broad patterns cannot be directly inferred to individuals.

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