Formularium

Riccati equation:

$$
\begin{aligned}
P &= Q + A^\top PA - A^\top PB(R + B^\top PB)^{-1}B^\top PA \\
K &= -(R + B^\top PB)^{-1}B^\top PA
\end{aligned}
$$

Gradient projection

$$z^{\nu+1} = P_C(z^\nu - \gamma_k \nabla f(z^\nu)),$$

where the "best" γ_k is $\frac{2}{L_f+\mu_f}$. It gives

$$\|z^\nu - z^*\| \leq \left(\frac{\kappa_f - 1}{\kappa_f + 1}\right)^\nu \|z^0 - z^*\|.$$

Convex optimal control problems:

Primal problem

$$
\begin{aligned}
&\underset{z,w}{\textbf{minimize}} \quad f(z) + g(w) \\
&\textbf{subject to} \quad Lz = w.
\end{aligned}
$$

Dual problem

$$
\begin{aligned}
&\underset{y}{\textbf{minimize}} \quad f^*(-L^\top y) + g^*(y), \\
&\text{with } f^*(x) = \sup_z \{x^\top z - f(z)\}.
\end{aligned}
$$

Alternating Minimization Algorithm (AMA):

$$
\begin{aligned}
z^{\nu+1} &= \arg\min_z \mathcal{L}(z, w^\nu, y^\nu) \\
w^{\nu+1} &= \arg\min_w \mathcal{L}(z^{\nu+1}, w, y^\nu) + \tfrac{\gamma}{2}\|Lz^{\nu+1} - w\|^2 \\
y^{\nu+1} &= y^\nu + \gamma(Lz^{\nu+1} - w^{\nu+1}).
\end{aligned}
$$

Alternating Direction Method of Multipliers (ADMM):

$$
\begin{aligned}
z^{\nu+1} &= \arg\min_z \mathcal{L}(z, w^\nu, y^\nu) + \tfrac{\gamma}{2}\|Lz - w^\nu\|^2 \\
w^{\nu+1} &= \arg\min_w \mathcal{L}(z^{\nu+1}, w, y^\nu) + \tfrac{\gamma}{2}\|Lz^{\nu+1} - w\|^2 \\
y^{\nu+1} &= y^\nu + \gamma(Lz^{\nu+1} - w^{\nu+1}).
\end{aligned}
$$