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Optimization of Mechatronic Systems
Semiactive Quarter Car

Report

Group 7
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Contents

1.0 Introduction

This project focuses on modeling, simulating, and optimizing the suspension system of a quarter car vehicle. The quarter car model, shown in Figure 1.1, consists of three main components: the car body, the tire, and the suspension system connecting the two. The objective is to analyze the vehicle's response to road disturbances, $p(t)$, and optimize a suspension system that minimizes the deviation of the car body, $x_1(t)$, from its steady-state position. Two types of suspension are considered: a passive suspension with a fixed damping coefficient, and a semiactive suspension with a skyhook control policy that adjusts the damping in real-time. The study includes deriving equations of motion, simulating the system under various conditions, and solving optimization problems to design the optimal suspension for different road disturbances.

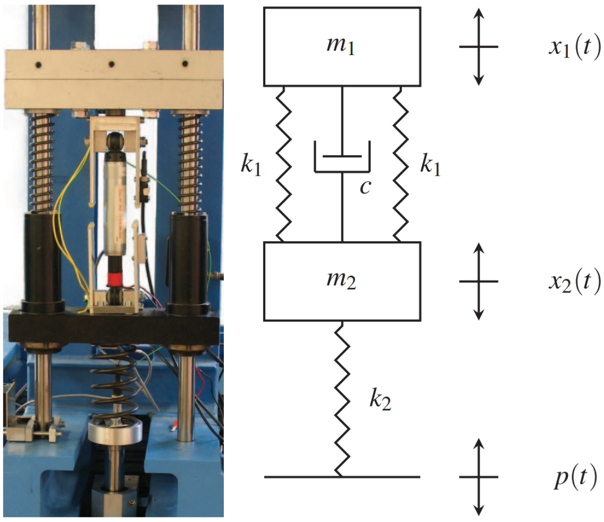

Figure 1.1: Semiactive quarter car system

Parameter	Value
m_1	40 kg
m_2	5 kg
k_1	4500 N/m
k_2	15000 N/m
c_{min}	0 Ns/m
c_{max}	2000 Ns/m

1.1 Question 1 - Equations of Motion with Road Disturbance

The equations of motion of the quarter car, considering the road displacement $p(t)$ as a disturbance, are given by:

$$\begin{cases} m_1\ddot{x}_1(t) = -2k_1(x_1(t) - x_2(t)) - c(\dot{x}_1(t) - \dot{x}_2(t)) \\ m_2\ddot{x}_2(t) = -k_2(x_2(t) - p(t)) + 2k_1(x_1(t) - x_2(t)) + c(\dot{x}_1(t) - \dot{x}_2(t)) \end{cases}$$

where:

- $x_1(t)$ is the car body displacement,
- $x_2(t)$ is the tire displacement.

1.2 Question 2 - Simulation with Fixed Damping

The behavior of the quarter car, for a damping $c = 200$ Ns/m, starting from rest, subjected to a step disturbance of 5 cm, was simulated from $t = 0s$ to $t = 5s$ by solving the ODEs describing the system dynamics. The resulting behavior is shown in Figure 1.2. The system in these conditions turns out to be underdamped, resulting in a stabilization time greater than the simulation time.

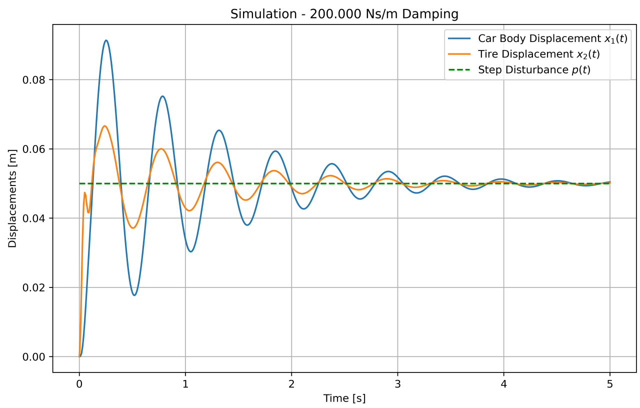

Figure 1.2: Simulation of the system dynamics

1.3 Question 3 - Skyhook Control Policy Implementation

Considering that the quarter car is equipped with a semiactive suspension, the damping coefficient c is no more a fixed value but it varies over time, $c(t)$. To control this variable, the skyhook policy has been implemented as follows:

$$c(t) = \begin{cases} \text{clip}\left(K\frac{\dot{x}_1}{\dot{x}_1 - \dot{x}_2}, c_{\text{min}}, c_{\text{max}}\right) & \text{if } \dot{x}_1(\dot{x}_1 - \dot{x}_2) > 0 \\ c_{\text{min}} & \text{otherwise} \end{cases}$$

Figure 1.3, 1.4, and 1.5 illustrate the system behavior for different values of $K = [500, 1000, 2000]$. It can be observed that as the value of K increases, the damping increases and thus the overshoot decreases, and the transient duration becomes shorter.

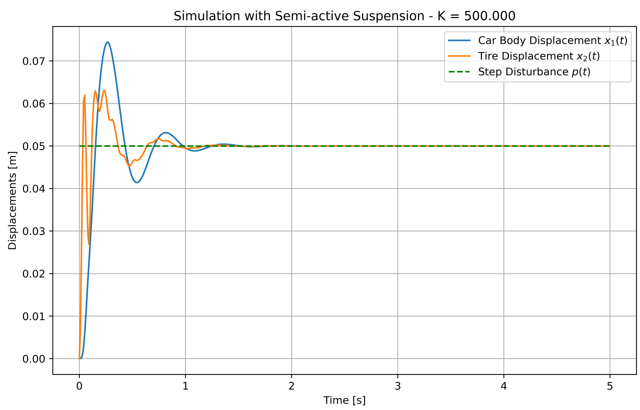

Figure 1.3: System simulation - K = 500 $\frac{Ns}{m}$

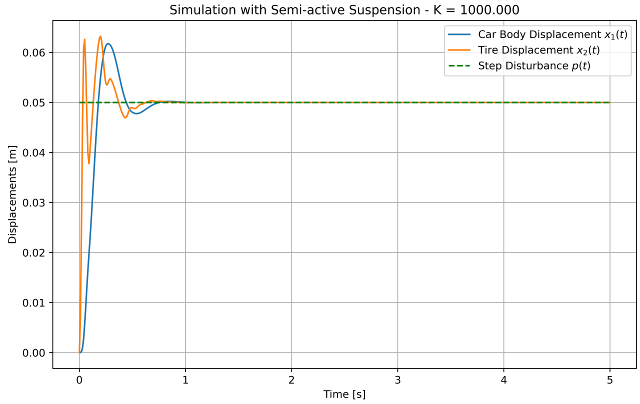

Figure 1.4: System simulation - K = 1000 $\frac{Ns}{m}$

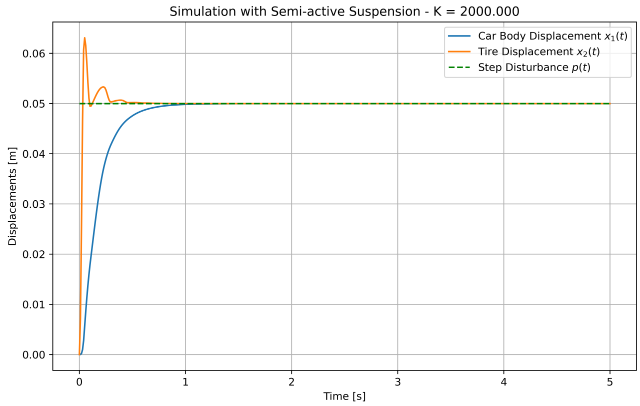

Figure 1.5: System simulation - K = 2000 $\frac{Ns}{m}$

1.4 Question 4 - Formulating Optimization Problems

Two optimization problems were formulated to optimize the suspension performance for different road displacements. The first problem aimed to determine the optimal passive suspension damping coefficient, c, while the second focused on finding the optimal gain, K, for the skyhook control policy. Since the system represents a quarter of a car's suspension, optimality is defined as minimizing the deviation of $x_1(t)$, the car body position, from its steady-state value, $x_{1,\text{ss}}$.

1.4.1 Optimal passive suspension

The optimization problem for the passive suspension case was defined as:

$$\begin{aligned} \text{minimize} \quad & J = \int_0^T [x_1(t) - x_{1,\text{ss}}]^2 \, dt \\ \text{subject to} \quad & c \geq c_{\text{min}} = 0 \\ & c \leq c_{\text{max}} \end{aligned}$$

where:

- J is the cost function to be minimized,
- T is the total time of simulation,
- c_{max} is the maximum damping coefficient allowed.

1.4.2 Optimal skyhook control policy

The semiactive suspension problem was formulated as:

$$\text{minimize} \quad J = \int_0^T [x_1(t) - x_{1,\text{ss}}]^2 \, dt$$
$$\text{subject to} \quad K \geq 0$$

where:

- J is the cost function to be minimized.

1.5 Question 5 - Optimization Problem Solutions

1.5.1 Optimal passive suspension

The optimization problem was solved for three different step disturbance values: 1 cm, 5 cm, and 10 cm. The results indicated that all three disturbances yielded the same optimal damping coefficient of 1165 Ns/m, as illustrated in Figure 1.6, 1.7, and Figure 1.8.

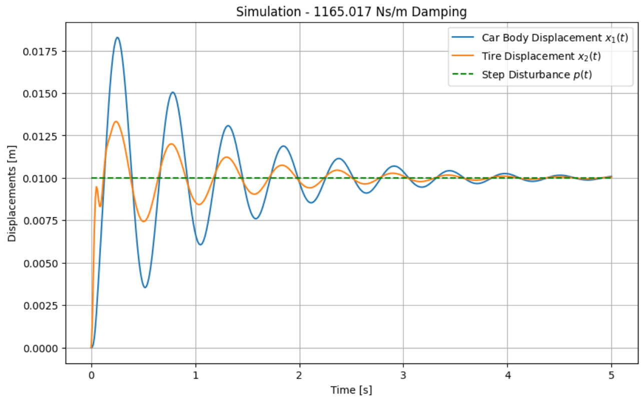

Figure 1.6: System simulation with passive suspension - step-size = 1 cm

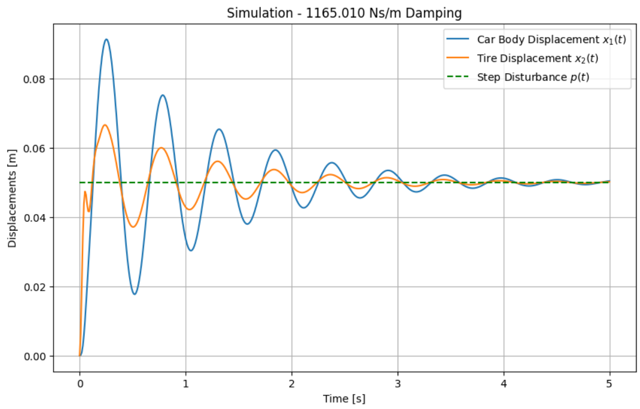

Figure 1.7: System simulation with passive suspension - step-size = 5 cm

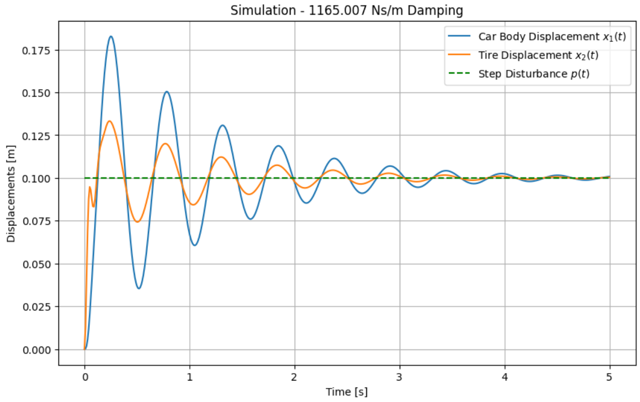

Figure 1.8: System simulation with passive suspension - step-size = 10 cm

1.5.2 Optimal skyhook control policy

Similarly, the second optimization problem was also solved to determine the optimal value of K for the same three step disturbances (1 cm, 5 cm, and 10 cm). The results consistently yielded the optimal value of K to be 1180 Ns/m, as depicted in Figure 1.9, 1.10, and 1.11. Comparing the results with the passive one, skyhook control has better performance as it converges much faster.

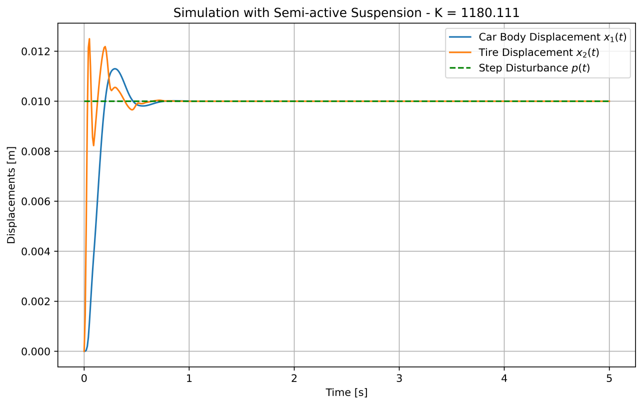

Figure 1.9: System simulation with skyhook control - step-size = 1 cm

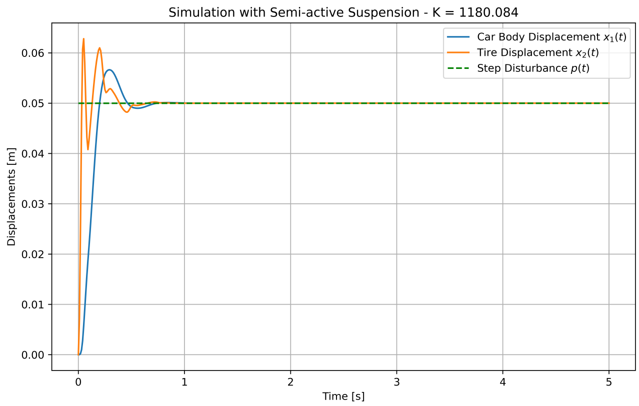

Figure 1.10: System simulation with skyhook control - step-size = 5 cm

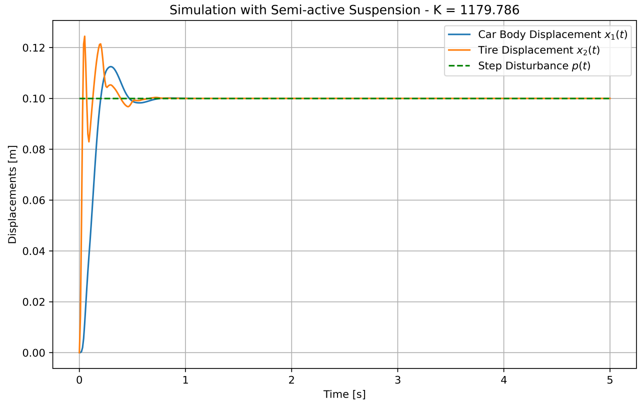

Figure 1.11: System simulation with skyhook control - step-size = 10 cm

1.6 Question 6 - Convexity Analyses of the Optimization Problems

There are several methods to check whether the problem is convex. We could count the number of minima of the objective function within the range of the decision variable, if it is greater than 1, the problem is certainly non-convex, but if it is 1, convexity is not implied. Another method is to approximate the Hessian of the objective function using numerical methods such as finite differences, which is more computationally expensive.

1.6.1 Optimal passive suspension

To assess whether the passive suspension problem is convex or not, the objective function was plotted, Figure 1.12. It is now clear that it is a convex optimization problem characterized by a global minimum at $c = 1166.333$ Ns/m. To further prove this conclusion, we performed a check and verified that the Hessian is positive semi-definite.

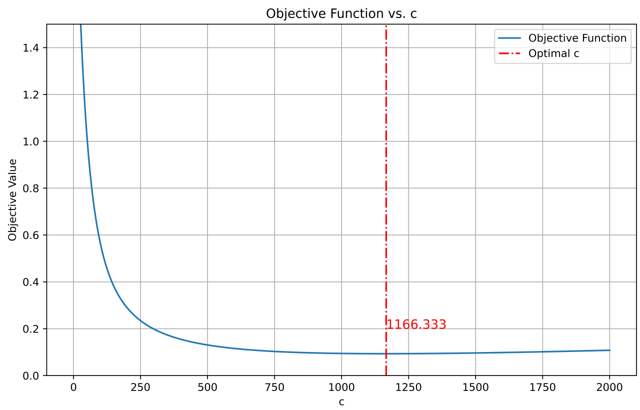

Figure 1.12: Passive Suspension - Objective function for step-size = 10cm

1.6.2 Optimal skyhook control policy

The same analysis method used for the passive suspension, is applied here. Figure 1.13 shows the relation between the objective function and K. It is now evident that the problem is not convex, but it is characterized by only one global minimum therefore the solver is still able to find the optimal solution, $K = 1181.181$ Ns/m. To further prove this result, a check on the semi-positive definiteness of the Hessian has been performed and, as expected, it fails.

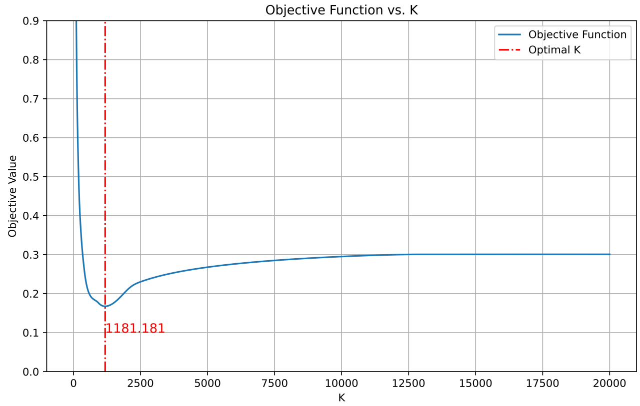

Figure 1.13: Semi-active Suspension - Objective function for step-size = 10cm

1.7 Question 7 - Computational Expense and Improvements

The most expensive parts of both optimization problems are the computation of the Hessian of the Lagrangian and the computation for the gradient of objective function as shown by the logs below.

```
Number of Iterations....: 23
EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |   3.04 s ( 25.54ms)   3.03 s ( 25.49ms)       119
  nlp_grad_f  |   2.23 s ( 89.28ms)   2.25 s ( 89.95ms)        25
  nlp_hess_l  |   9.46 s (411.30ms)   9.46 s (411.09ms)        23
       total  |  14.77 s ( 14.77 s)  14.77 s ( 14.77 s)         1
Optimal gain factor: 1179.7856833503456
```

The computation of the gradient of the objective function is expensive because it involves evaluating how small changes in the control inputs affect the objective function. Due to the complexity of the dynamics which are nonlinear, this can become very time-consuming as the gradient needs to be computed at each iteration for the optimization to proceed.
The Hessian represents the second-order derivatives, which are essential for Newton-like methods used in optimizations like Ipopt. Computing the Hessian involves a more complex operation since it requires calculating the curvature of the function being optimized.
To improve efficiency for both problems, we pass options to Ipopt to indicate that the Jacobian of the constraints are constant and to set the Hessian approximation to limited memory. The latter reduces significantly the computation time because it uses L-BFGS method which instead of storing and updating the full Hessian matrix, maintains a limited number of vectors to represent the curvature information. By doing so, L-BFGS reduces the computational complexity of each iteration. Moreover we simplified the objective function precomputing the square and omitting the constant term. These options and preprocessing lead to faster convergence and shorter computation times, as can be seen from the logs below.

```
Number of Iterations....: 6
EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  | 171.00ms ( 24.43ms) 154.78ms ( 22.11ms)         7
  nlp_grad_f  | 687.00ms ( 85.87ms) 701.03ms ( 87.63ms)         8
       total  | 862.00ms (862.00ms) 863.40ms (863.40ms)         1
Optimal gain factor: 1179.7791383417937
```

Both presented logs were obtained by running the optimal semi-active suspension problem. Similar improvements were also achieved for the optimal passive suspension problem but were omitted for brevity.