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Ramon Llull: from 'Ars electionis' to social choice theory

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Abstract Ramon Llull (Majorca c.1232–1316) is one of the earliest founding fathers of voting theory and social choice theory. The present article places Llull's contributions and discussion in the historical context of elections in the medieval Church and the emergence of majority rule as a new general principle for making enforceable collective decisions in replacement of traditional unanimous requirements. To make the majority principle operational, Llull initially proposed a system of exhaustive binary comparisons that is more efficacious in producing a winner than the Condorcet system, in anticipation to the so called Copeland procedure. In contrast to some previous tentative suggestions, careful reading of Llull's papers demonstrates that he did not propose a rank-order count system, such as those proposed later on by Cusanus and Borda. A new hypothesis is presented to explain Llull's later proposal of an eliminatory system of partial binary comparisons. Some performance of Llull's voting systems is estimated by innovative analysis of results in certain modern sports tournaments.

1 Introduction

Ramon Llull (Majorca c.1232–1316) should be considered the earliest founding father of voting theory and social choice theory. To give ground to this statement, the present article places Llull's contributions and discussion in the historical context of elections in the medieval Christian Church and the emergence of majority rule as a new general principle for making enforceable collective decisions in councils and parliaments, in replacement of traditionally inefficacious unanimous requirements. As will be shown, in order to make the majority principle operational, Llull initially proposed a system of exhaustive binary comparisons that is more efficacious in producing a winner than

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the Condorcet procedure. A new hypothesis is also presented below to explain Llull's later proposal of an eliminatory system of partial binary comparisons, or "successive procedure". Some performance of Llull's voting systems will be innovatively estimated.

In the late 13th century Ramon Llull had the ambition to account for all wisdom in every branch of human knowledge of the time. For that purpose he developed a general method or 'Ars generalis' based on binary combinations of a number of simple basic principles or categories (see Bonner 2007, http://lullianarts.net). In consistency, a voting method or 'Ars electionis' was devised along the same lines, that is on the basis of binary comparisons of candidates. Llull wrote (at least) three seminal pieces on voting and elections, to be cited using the abbreviations given in the parentheses:

- *Artifitium electionis personarum* (c. 1274–83) (The method for the elections of persons) (AEP)
- *En qual manera Natana fo eleta a abadessa* (c. 1283) (In which way Natana was elected abbess) (B24)
- *De arte electionis* (On the method of elections) (1299) (DAE)

In AEP Llull presents his initial voting proposal in scholarly form while introducing a graphic representation of binary comparisons of candidates. B24 is the 24th chapter of *Blanquerna*, originally written in Catalan and considered one of the first novels written in a Romance language, where the election of the abbess of a convent is used as an occasion for divulgation. A variant of the system with significant differences in likely performance is scholarly presented in DAE. (The three texts with translations into English are available entirely in http://uni-augsburg.de/llull, Hägele and Pukelsheim 2001; Drton et al. 2004, and partly in McLean 1990; McLean and Urken 1995; Colomer et al. 2007a).

2 The emergence of majority rule

The immediate context of these Lull writings was a period of frequent and dramatic conflicts in the Christian Church on the occasion of elections of bishops, abbots, abbesses, priors, and other prelates including popes. These elections were traditionally guided by the principle of unanimity, as voting was conceived as a way to reveal God's will and discover the truth. In the 5th century the Church had formally adopted from the Justinian code of Rome, which established that "What concerns similarly all ought to be approved by all", the principle that "He who governs all should be elected by all". But very often a unanimity agreement was not reached by the faithful, the clergy or the friars with voting entitlements, thus provoking recurrent conflicts and schisms. As the arbitral power of the Roman-German emperor and other traditional local rulers began to dwindle, the Church was moved to choose more effective voting rules on its own.

More specifically, a less-than-unanimity rule, namely 2/3, was generally established for Church elections in 1179. Three specific systems were devised in 1215: "acclamation" (by at least 2/3 of the voters participating in any demonstration for a candidate), "compromise" (involving the transfer of the election to a previously elected committee, which should also decide by 2/3), and "scrutiny" (where in the case of no

clear decision, the "sanior [actually senior or sounder] part" of the electorate should prevail). Furthermore, for the election of the pope by 2/3 a conclave of cardinals was designed in 1276. At its establishment, pope Gregory X acknowledged that "Not zeal to zeal, nor merit to merit, but solely numbers to numbers are to be compared" (in Gregory X 1991: 1, VI, 9; more details in Colomer and McLean 1998; also Ruffini 1925, 1927/1976; Moulin 1953; Colomer et al. 2007a,b; Uckelman and Uckelman 2010).

Echoing these practices, Llull made explicit references in his writings to the aim of preventing "fraud and simony" (AEP), the advantages of his "new electoral method" in contrast to "the method to which they were accustomed for an election" (B24), and his intention to be "far from the secret scrutiny and the election by compromise wherein more fraud can be committed than in [his] proposed method" (DAE). "Evidently—he stated—a good electoral system in the Holy Church is very necessary for electing office holders, since by them the church is governed and they fight against the church's enemies who commit sins like the infidels and schismatics" (DAE).

More generally, highly innovative elections by majority rule were gradually accepted in late medieval times as an expedient system when unanimous and consensual decisions resulted impossible. Some forms of majority decisions had been used in ancient Roman comitia and councils, but actually they involved mixed procedures of indirect elections, lots and shows of allegiance by acclamation. Traditional unanimous or broadly consensual decisions in old local communities in disparate places had been achieved by such procedures as silent acquiescence, clashing spears against shields, shouts of commendation, or acclamation at harangues, murmurs in favor, or cries against the proposer, rising on one's feet, or other 'viva voce' expressions, rather than formal voting sessions. (Colomer 2004, pp. 13–30).

By the time of Llull's writings, voting and elections were regularly held in Christian convents and councils, as well as in many city councils and parliaments all across Europe. Different voting rules and procedures involving people's acclamation, lots, several stage elections, approval voting, and eliminatory methods were innovatively tried (Lines 1986; Coggins and Perali 1998; Mowbray and Gollmann1 2007; Colomer 2011a). In particular, Llull must have been aware that in contrast to most other institutions of the kind, in the Parliament of Catalonia majority rule prevailed among clergy and cities (while the noble estate stuck to unanimous requirements) (Coroleu and Pella 1876/1993; Lord 1930; Ferro 1987; Colomer 2001, pp. 77–79).

The establishment of the majority principle in late medieval and early modern times, especially in medieval Germanic law and the Church's canon law, turned out to be one of the greatest achievements in human civilization. It implies nothing less than the provision that every member of a community ought to consent to collective decisions that he or she may not share. Of course, majority rule is much more effective in making collective decisions than the traditional unanimity rule by which every individual can veto any decision. But, as is well-known, a winner with an absolute majority support is guaranteed only in elections with only two candidates (and an odd number of voters). In order to enforce the majority principle, Llull's luminous idea was to reduce the number of candidates to 'twos' by generating a series of pairwise comparisons. An absolute majority winner in each comparison or voting round was thus guaranteed. According to Llull's initial proposal, the winner by majority rule ought to be the winner

of a majority of majorities (or, more precisely, an atleast relative majority of absolute majorities) in pairwise comparisons—a rather persuasive 'majority' winner indeed. Llull's proposal was the first serious attempt to devise a system able to implement the majority principle in real elections. This is why his work can be considered of foundational and precursory nature. (Heinberg 1926; Colomer 2004).

Llull's voting proposals were, however, forgotten for several centuries. With minor variants, they were somehow reinvented in the process of establishing innovative democratic elections in late 18th century revolutionary France. While Llull's B24 was available in Catalan and other languages for a long term, DAE was discovered in 1937 (Honecker 1937) and AEP in 1959 (Pérez Martínez 1959), but nobody with interest in voting and elections paid attention to these works for a while. Actually Llull's basic voting procedure was reinvented by mid-20th century. And his works were eventually rediscovered in the context of worldwide diffusion of electoral democracy and the development of social choice theory. (Especially by McLean 1990; McLean and London 1992; McLean and Urken 1995, and Hägele and Pukelsheim 2001. Further references in Felsenthal and Machover 1992; Colomer 2001, 2004, 2010. See, however, how encompassing analyses of some of those procedures refer only to Copeland, but not to Llull, as, for example, in Saari and Merlin 1996, 1997).

3 Exhaustive binary comparisons

Ramon Llull's initial proposal (as presented in AEP and B24) is a voting system based on decisions by majority rule in exhaustive pair-wise or binary comparisons of all candidates. In his writings on voting, Llull, nicknamed Doctor Illuminatus, uses tables with graphic representations of all exhaustive combinations in a similar way to many others of his works in which diagrams, connecting lines and rotating circles abound. For this (and his influence on Gottfried Leibniz) he has also been considered a precursor of computer science. (Gardner 1982; Fidora and Sierra 2011).

According to Llull's voting system, an election is to be made by holding multiple rounds of voting by majority rule between all possible pairs of candidates. For n candidates, this requires $n(n-1)/2$ comparisons. The winner is the candidate winning by majority in the greatest number of binary comparisons. In Llull's words:

> In each voting round or comparison between two candidates, "a point is given… to the person who has the greatest number of votes"; after all rounds, "the person with a greater number of points than any other is elected" (AEP); "let be elected the one who will have the greatest number of votes in the greatest number of chambers [cells or comparisons]" (B24).

These sentences should help clarify some previous confusion with other procedures such as rank-order count. In the present author's translations from Latin and Catalan, respectively, "plura puncta" in the first text and "més cambres" in the second are not translated as "the most" points or chambers but as "a greater number of points than any other" and "the greatest number of chambers". Llull proposes to make a winner the candidate winning a greater number of binary comparisons (points or chambers)

than any other candidate, not the one winning the most votes or the greatest absolute number of votes in all comparisons added together.

This Llull's basic system was reinvented by American mathematician Arthur H. Copeland by the mid-20th century (Copeland 1951).[1] In contrast, the rank-order count system requires the voters to rank all the candidates and award them 0, 1, 2, etc. points or votes from the least to the most preferred; the winner is the candidate with the highest number of points or votes altogether. This system was not proposed by Llull but firstly devised by Nicholas of Cusa (1401–1464), apparently after reading Llull's *De arte electionis*. The rank-order count system was also reinvented a few centuries later in France by Jean-Charles de Borda (1733–1799).

A very similar system to the original Llull's system is the better-known Condorcet system, as proposed by Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet (1743–1794), who was chairman of the revolutionary French National Assembly and author of the Girondin party's constitutional project in the Convention (Condorcet 1785). As is well known, by the Condorcet system a candidate is required to win by majority all binary or pair-wise comparisons, that is, the Condorcet-winner is the candidate able to win by majority against each and every other candidate (a kind of unanimity of majorities, in contrast to the previous Llull's proposal).

Obviously, the Llull system produces the same winner as the Condorcet system when the latter exists. It has been noted that when the voters vote on the basis of their relative "distances" to the candidates and the latter are perceived as ordered along a single linear dimension (such as the left-right axis that emerged as a representational device in the French National Assembly), the Llull- and Condorcet-winner is always the candidate preferred by the median voter. As the median voter's preference minimizes the sum of distances from all voters' preferences, the Llull- and Condorcet-winner can be considered highly satisfactory for the electorate and is frequently used in social choice theory and voting analyses as a positive reference for comparison with the winner by other voting systems. (See extensive discussion in Colomer 2001, 2011c).

However, in multidimensional issue spaces a candidate capable of winning against every other candidate, as required by the Condorcet system, may not exist. A Condorcet "cycle" can occur by which every candidate can be defeated by some other, such as in $X > Y > Z > X \ldots$ (where each letter represents a candidate and ">" means "defeats"). Thus, the Condorcet system can be inefficacious in producing a single winner. In general, the Llull system should be more efficacious in producing a winner than the Condorcet system, as it does not require winning all but only a higher number of binary comparisons than the other candidates.

To facilitate discussion, let us see a simple example of how these systems work in Table 1. There are five candidates, V, W, X, Y, Z, and five voters, A, B, C, D, E, with complete orders of preference over the candidates. By casting all $5(5-1)/2 = 10$

[1] A minor difference exists. In every binary comparison, Llull proposed giving one point to the winner by majority, as well as one point to each of the two candidates if a tie of votes arises, and add up the points for each candidate. Copeland proposed giving one point to the winner and half a point to each of the two candidates in a tie. Llull, nevertheless, advises gathering an odd number of voters to prevent ties in any pairwise comparison.

Table 1 Condorcet and Llull Systems

Voters	A	B	C	D	E
First preference	V	Z	Z	X	V
	W	V	V	W	X
	Y	W	W	Y	Y
	X	X	X	Z	Z
Least preference	Z	Y	Y	V	W

Binary comparisons	Number of comparisons won
V(4) > W(1); V(4) > X(1); V(4) > Y(1); V(2) < Z(3)	V : 3
W(3) > X(2); W(4) > Y(1); W(2) < Z(3)	W : 2
X(4) > Y(1); X(3) > Z(2)	X : 2
Y(3) > Z(2)	Y : 1
	Z : 2

Total: 10

The numbers in parenthesis indicate votes, according to the voters' preferences; ">" means "defeats"

Results:

No Condorcet winner, but Condorcet cycle: V > W > X > Y > Z > V . . .

Llull-Copeland winner: V

binary comparisons of all candidates and assuming that the voters vote for their most preferred candidate within each pair, candidate V wins in 3 comparisons (against W, X, Y), W wins in 2 (against X, Y), X wins in 2 (against Y, Z), Y wins in 1 (against Z), and Z wins in 2 comparisons (against V, W). So, no candidate wins against every other candidate, thus there is no Condorcet-winner, but a cycle by which V > W > X > Y > Z > V . . . The Condorcet system does not produce a winner. The Llull system, in contrast, does produce a winner, V, who wins in 3 comparisons, with no ties.

However, the Llull system is more effective in producing a winner than the Condorcet system only in elections with five or more candidates, such as in the example just presented. Let us assume, in contrast, that the election is contested by $n = 4$ candidates: W, X, Y, Z. There are, thus, $4(4-1)/2 = 6$ pairs of comparisons: W–X, W–Y, W–Z, X–Y, X–Z, Y–Z. In order to win by the Condorcet system, a candidate should win in 3 pairs (that is, against each and every one of the other 3 candidates). If there is no Condorcet-winner, it must be because no candidate wins in more than 2 pairs. But then, with 4 candidates and 6 comparisons, at least two candidates must tie by winning in 2 pairs each (for example, W may win in 2 pairs, X in 2 pairs, Y in 1 pair, and Z in 1 pair, or three candidates may win in 2 pairs each).

Then, for 4 or fewer candidates, if no Condorcet-winner exists, the Llull system does not produce a winner either, but a tie in the number of comparisons won by more than one candidate, thus not solving the inefficacy of the Condorcet system. Actually, with five or more candidates, the Llull system, as it does not require winning all comparisons, can also produce a tie in the number of comparisons won by more than one candidate. The Llull system can, therefore, produce relatively frequent ties in relatively complex electorates. "Complexity" may imply either a multidimensional issue space

prone to producing a Condorcet cycle (even with a low number of candidates) or a high number of candidates. Otherwise, with moderate numbers of issue dimensions and candidates the Llull system tends to match the Condorcet results.

In his writings Llull presented efficacious examples of elections with more than four candidates (16, 7 [as previously selected by a broader electorate] and 9 candidates, respectively in AEP, B24, and DAE). In AEP, nevertheless, Llull provided a subsidiary system to break ties: "lots are drawn… and the one [candidate] whose lot wins is elected".

The election of officers by lots had long tradition in classical Athens and a number of medieval cities, as well as in the early days of the Church. The first apostles of Jesus drew lots to select the replacement for the traitor Judas (Acts of Apostles I: 23–26). They were imitated by early non-orthodox Gnostic Christians, who drew lots at each of their meetings to elect priests, bishops, and other officers. But the Christian Church condemned such a practice as blasphemy and solemnly forbade the choice of prelates by lots. In the 13th century, just a few years before Llull's writings on voting, lots were formally prohibited in a new canon law collection formed by Raymond of Penyafort which was given force by pope Gregory IX's bull Rex Pacificus (1234): "Not only should election by lots not be done, but the delegates in elections by compromise should not be selected by lots" (in Gregory IX 1234/1959: 5, XXI, III, p. 823; author's translation; related discussion in Colomer 2004, pp. 22–25 and Colomer 2011b).

4 Non-exhaustive comparisons

In the days before printing and other modern communication techniques were available Ramon Llull may have been inadvertent for a little while of the formal papal prohibition of lots for elections of prelates in the Church. Actually he contemplated lots only to break ties between candidates, not to elect them in first instance. Nonetheless, in further works he withdrew this proposal. In B24, Llull resolved that in the event of a tie "one decides solely by means of the system", that is, without making recourse to another system, such as lots, and ought to choose the candidate with best love of God, best virtues, least vices, and being most suitable person.

More innovatively, in DAE Llull presented a new voting system that is able to prevent ties (and thus avoid the need to resort to lots or other problematic formulas). This is a simplified, non-exhaustive binary comparisons system by which after every round of voting the loser candidate is eliminated, the winner by majority is then compared with another candidate, and so on, and the elected is the candidate winning the last comparison. This system needs only $n-1$ binary comparisons. For instance, in the above illustration with four candidates, there would be only 3 comparisons instead of 6, say, for example, W–X, X–Y, Y–Z (in the assumption that $W < X, X < Y$, and then the elected will be the winner in the comparison Y−Z).

Llull apparently expected that this system would produce the same winner as the previously presented exhaustive series of comparisons and, in addition, would prevent ties and be more efficacious. Obviously, if a Condorcet-winner exists, both the initial Llull system and the simplified-eliminatory Llull system produce the same winner. If the Condorcet-winner does not exist, for four or fewer candidates the

simplified-eliminatory Llull system can indeed prevent ties and be more efficacious than the initial Llull system. However, then for any number of candidates the simplified eliminatory method can produce different winners depending on the order in which the candidates are compared.

The previous example in Table 1 with five candidates in which there is no Condorcet-winner can illustrate this point. While the initial Llull system makes V the winner, the simplified-eliminatory Llull system can produce either the same or a different winner. Specifically, if V is paired with Z, the latter will win and V will be eliminated. The final winner can be any of the five candidates depending on the order in which the candidates are compared (since every one of them is capable of winning at least against one other candidate).

The eliminatory voting system, also called "successive procedure", is actually used nowadays in most state parliaments in Europe for regular voting on motions, bills and amendments. By this procedure, each of the amendments is voted 'yes' or 'no' and consequently adopted and added to the bill or eliminated. This system has been critically analyzed by social choice theorists at least since the mid-20th century. It has been established that the order in which the alternatives are presented can change the winner as long as there are fewer voting rounds than alternatives. This is certainly the case of the simplified-eliminatory Llull system, which envisages $n-1$ rounds of voting for n candidates. It is also the case of the parliamentary successive procedure just mentioned which involves n alternatives, counting the initial bill and the bill with each of the proposed amendments, and only $n-1$ voting rounds for accepting or rejecting the amendments and approve the bill, whether amended or not, by default (Rasch 1995). Regarding the potential winner, "the later any motion enters the voting order, the greater its chances of adoption" (Black 1958, p. 40). The organizers of the voting session can, thus, manipulate the system to achieve some preferred outcome.

5 Secret or open ballot

Ramon Llull also discussed the interesting question of whether voters should vote in secrecy or openly to favor his aims of achieving the best electoral result. According to conventional uses in the Church of his time, he admonished that "each person having a vote in the chapter should take an oath by the holy gospels of God" to consider the moral, intellectual, and personal characteristics of the candidates "to always elect the person in whom they are best [embodied]" (AEP). In B24, Llull specified that "an oath be taken by all the [voting] sisters to tell the truth", and then they would vote "in secrecy" for the best candidate. This provision implied some fear of coercion by more powerful voters, whether in hierarchy or perhaps as members of the 'sanior' part of voters, capable of distorting the voters' sincere revelation of preferences. With these concerns, Ramon Llull anticipated arguments in favor of the secret vote by about six centuries.

However, worried about fraud in traditional secret elections in the Church, he eventually leaned towards the side of the open vote. He argued that while "those who elect publicly face great disgrace by their colleagues if they elect badly; those who

elect secretly do not" (DAE). With this twist, Llull also predated further relevant discussions, particularly on the role of the "tribunal of the public opinion" that could be capable of overcoming coercive pressures from the powerful few (as was developed, among others, by Jeremy Bentham and John Stuart Mill in 19th century England; see Lively and Rees 1978).

6 Some sports practices

The performance of different voting systems has been estimated by different methods. For some time, social choice theorists used mathematical calculations and computer simulations under the assumption that all possible voters' preferences are equally probable (an assumption usually called "random society" or "impartial culture"). This assumption has been criticized because it maximizes the probability of majority cycles and paradoxes, as for each preference order it presumes a symmetric or exactly opposed preference order able to prevent a stable majority. A better performance in which cycles tend to disappear has been stipulated on the basis of other assumptions regarding voters' preferences, such as, for example, the one assuming that there is a small group in the electorate with homogeneous preferences. (Regenwetter et al. 2006).

However, to estimate actual performances of certain voting procedures, such as those based on exhaustive binary comparisons of candidates as proposed by Llull, Condorcet and Copeland along the centuries, is difficult because some of them have been little used in real elections. Here I propose another approach which involves looking at the use of very similar rules in a number of modern sports tournaments. Just to take a major example, most state-level football championships in European countries are regulated by this kind of rules: exhaustive binary "comparisons" or matches between all competing teams are cast, different numbers of points are given for wins, draws, and losses, and the winner is the candidate winning the greatest number of points.

In order to see whether these uses can enlighten us as to real performances of the above-mentioned procedures, I looked at rules and results in two outstanding football championships, the English premier league and the Spanish league of 1st division, from 1950 to 2010 (which therefore involves 120 tournaments with a total of 8,464 binary "comparisons" or matches). No Condorcet-winner capable of winning all matches has ever existed in these (and presumably other) competitions. In fact, thus, it is the Llull–Copeland system that is basically applied (variants include two or three points for a win, one point for a draw and no points for a loss).

As can be seen in Table 2, the number of draws in points is relatively high, 7% in England and 10% in Spain, so making the above-discussed problem of breaking ties a relevant issue. In these and similar competitions, draws in points based on matches are broken by counting goals (specifically by "goal difference" in favor and against, or by "goal average" in some previous periods). This additional procedure may evoke the proposal of counting votes when binary comparisons are inefficacious in producing a single winner, or the proposal of adopting the Condorcet procedure and the Cusanus–Borda procedure as a supplement when the former does not produce a winner, as suggested by Black (1958).

Table 2 Binary and goal-count procedures in football tournaments, 1950–2010

	Enforced rules (Llull–Copeland) Ties (%)	Potential alternative winners (Cusanus–Borda) Goals-difference (%)
England: Premier league	7	18
Spain: La Liga	10	23
Total	8	21

Source Author's calculations with data for 60 annual tournaments in England and 60 annual tournaments in Spain for the 1950–2010 periods, involving 8,464 binary matches, from http://www.footballstatisticsresults.co.uk and http://www.futbolinSpain.com

The final column in Table 2 is based only on goal differences, which may produce comparable results to those that could be obtained with rank-count procedures such as the one proposed by Cusanus and Borda discussed above. Data for the two football tournaments show that if this procedure were not a supplementary one for the case of ties but the basic rule, in a very significant proportion of cases, around 21% in total, it would have produced a different winner than the winner with the procedures based on binary comparisons that are actually enforced. In other words, more than one fifth of the winners by rules based on binary comparisons would not have been winners (ceteris paribus) by rules based only on numbers of goals.

These data, although not focused on real voting and elections, may support the relevance of two points made in this article. First, the problem of ties in Ramon Llull's and similar systems based on binary comparisons of alternatives is relevant. This problem has grounded the above-presented hypothesis that Llull's innovative proposal of an eliminatory procedure in his third piece, DAE, could be motivated by troubles in proposing acceptable rules for breaking ties. Second, the differences between results by binary comparison methods and by rank-order counts or order procedures taking into account the absolute numbers of votes can be significant.

7 Conclusion

In this article, several voting systems and procedures proposed by Ramon Llull in the 13th century have been examined in their historical context and in the light of modern social choice theory. Llull's basic proposal is a system based on binary comparisons of candidates by which the winner is the candidate winning by majority in the greatest number of comparisons. This system turns out to be more efficacious in preventing cycles and producing a winner than the celebrated Condorcet system requiring the winner to win all binary comparisons, although only for elections with five or more candidates.

The main points of this article regard, first, the importance of Llull's contributions and discussion in the historical context of elections in the Christian Church and the emergence of majority rule as a new general principle for making enforceable collective decisions in replacement of traditional unanimity rule requirements. Second, in contrast to some previous tentative suggestions, careful reading of Llull's papers

in their Catalan and Latin versions demonstrates that he did not propose a rank-order count system, such as those proposed later on by Cusanus and Borda. Third, a new hypothesis has been presented to explain Llull's later proposal of an eliminatory system of partial binary comparisons, or "successive procedure", which is based on the relatively high frequency of ties produced with exhaustive binary comparisons and the condemnation of lots by the Church in Llull's times. Other sights in this article have been cast over Llull's discussion of secret and open voting and on some applications of binary comparison methods in modern sports tournaments. These analyses may give support to the statement that Ramon Llull should be considered one of the earliest founding fathers of voting theory and social choice theory.

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