Hoja de trabajo #3

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Ejercicio #1

Utilizando la definicion de suma ($\oplus$) para los numeros naturales unarios, llevar a cabo la suma entre tres [$s(s(s(0)))$] y cuatro [$s(s(s(s(0))))$].

$$s(s(s(0))) \oplus s(s(s(s(0))))$$

a=s(x)
x=s(s(0))
b=s(s(s(s(0))))

$$s[s(s(0)) \oplus b]$$

a=s(x)
x=s(0)
b=s(s(s(s(0))))

$$s[s(s(0) \oplus b)]$$

a=s(x)
x=0
b=s(s(s(s(0))))

$$s[s(s(0 \oplus b))]$$

0+b=b

$$s[(s(s(b))]$$

$$[s(s(s(s(s(s(s(0)))))))]$$

$$= 7$$

Ejercicio #2

Definir inductivamente una función para multiplicar ($\otimes$) numeros naturales unarios.

$$a \otimes b := \begin{cases} 0 & \text{si } a = o \\ 0 & \text{si } b = o \\ a & \text{si } b = 1 \\ b & \text{si } a = 1 \\ b \oplus (x \otimes b) & \text{si } a = \sigma(x) \end{cases}$$

Ejercicio #3

Verifique que su definición de multiplicación es correcta multiplicando los siguientes valores:

- $\sigma(\sigma(\sigma(0))) \otimes 0$

$$\sigma(\sigma(\sigma(0))) \otimes 0$$

a=$\sigma(\sigma(\sigma(0)))$

$$a \otimes 0 = 0$$

- $\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$

$$\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$$

a=$\sigma(x)$
x=$\sigma(\sigma(0))$
b=$\sigma(0)$

$$b \oplus (x \otimes b)$$
$$= b \oplus (\sigma(\sigma(0)) \otimes b)$$

a=$\sigma(x)$
x=$\sigma(0)$

$$= b \oplus b \oplus (\sigma(0) \otimes b)$$

a=$\sigma(x)$
x=0

$$= b \oplus b \oplus b \oplus (0 \otimes b)$$

0 $\otimes$ b=0

$$= b \oplus b \oplus b \oplus (0)$$
$$= b \oplus b \oplus b$$
$$= \sigma(0) \oplus \sigma(0) \oplus \sigma(0)$$
$$= \sigma(\sigma(\sigma(0)))$$
$$= 3$$

- $\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$

$$\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$$

a=$\sigma(x)$
x=$\sigma(\sigma(0))$
b=$\sigma\sigma((0))$

$$b \oplus (x \otimes b)$$
$$= b \oplus (\sigma(\sigma(0)) \otimes b)$$

a=$\sigma(x)$
x=$\sigma(0)$

$$= b \oplus b \oplus (\sigma(0) \otimes b)$$

a=$\sigma(x)$
x=0

$$= b \oplus b \oplus b \oplus (0 \otimes b)$$

0 ⊗ b=0

$$= b \oplus b \oplus b \oplus (0)$$

$$= b \oplus b \oplus b$$

$$= \sigma(\sigma(0)) \oplus \sigma(\sigma(0)) \oplus \sigma(\sigma(0))$$

$$= \sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0))))))$$

$$= 6$$

Ejercicio #4

Demostrar utilizando inducción:

- $a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$
  Caso Base: a=0

$$0 + \sigma(\sigma(0)) = \sigma(\sigma(0))$$

$$\sigma(\sigma(0)) = \sigma(\sigma(0))$$

  Caso Inductivo: a=$\sigma(x)$
  Hipótesis Inductiva: x+$\sigma(\sigma(0)) = \sigma(\sigma(x))$

$$\sigma(x) \oplus \sigma(\sigma(0)) = \sigma(\sigma(\sigma(x)))$$

$$\sigma(x \oplus \sigma(\sigma(0))) = \sigma(\sigma(\sigma(x)))$$

$$\sigma(\sigma(\sigma(0)) \oplus x) = \sigma(\sigma(\sigma(x)))$$

$$\sigma(\sigma(\sigma(0) \oplus x)) = \sigma(\sigma(\sigma(x)))$$

$$\sigma(\sigma(\sigma(0 \oplus x))) = \sigma(\sigma(\sigma(x)))$$

$$\sigma(\sigma(\sigma(x))) = \sigma(\sigma(\sigma(x)))$$

- $a \otimes b = b \otimes a$
  Caso Base: a=0

$$0 \otimes b = b \otimes 0$$

$$0 = 0$$

  Caso Inductivo: a=$\sigma(x)$
  Hipótesis Inductiva: $x \otimes b = b \otimes x$

$$\sigma(x) \otimes b = b \otimes \sigma(x)$$

  $b = \sigma(i)$

$$b \oplus (x \otimes b) = b \otimes \sigma(x)$$

$$\sigma(i) \oplus (x \otimes \sigma(i)) = \sigma(i) \otimes \sigma(x)$$

$$\sigma(i) \oplus (x \otimes \sigma(i)) = \sigma(x) \oplus (i \otimes \sigma(x))$$

$$\sigma(i \oplus (x \otimes \sigma(i))) = \sigma(x \oplus (i \otimes \sigma(x)))$$

$$\sigma(i \oplus (x \otimes b)) = \sigma(x \oplus (i \otimes a))$$

Por Hipótesis inductiva:
$x \otimes b = b \otimes x$
$i \otimes a = a \otimes i$

$$\sigma(i \oplus (b \otimes x)) = \sigma(x \oplus (a \otimes i))$$

$$\sigma(i \oplus (\sigma(i) \otimes x)) = \sigma(x \oplus (\sigma(x) \otimes i))$$

$$\sigma(i \oplus x \oplus (i \otimes x)) = \sigma(x \oplus i \oplus (x \otimes i))$$

Demostrar que $i \oplus x = x \oplus i$
para $i = \sigma(j)$
Hipótesis inductiva: $j \oplus x = x \oplus j$
$\sigma(j) \oplus x = x \oplus \sigma(j)$
$\sigma(j \oplus x) = x \oplus \sigma(j)$
$\sigma(x \oplus j) = x \oplus \sigma(j)$
$\sigma(x) \oplus j = x \oplus \sigma(j)$
Demostrar para el sucesor:
Hipótesis inductiva: $\sigma(x) \oplus j = x \oplus \sigma(j)$
$\sigma(\sigma(x)) \oplus j = \sigma(x) \oplus \sigma(j)$
$\sigma(\sigma(x) \oplus j) = \sigma(x \oplus \sigma(j))$
Por Hipótesis inductiva:
$\sigma(\sigma(x) \oplus j) = \sigma(\sigma(x) \oplus j)$
Por lo tanto $i \oplus x = x \oplus i$

$$\sigma(x \oplus i \oplus (i \otimes x)) = \sigma(x \oplus i \oplus (x \otimes i))$$

Por Hipótesis inductiva:
$x \otimes i = i \otimes x$

$$\sigma(x \oplus i \oplus (x \otimes i)) = \sigma(x \oplus i \oplus (x \otimes i))$$

k=$x \oplus i \oplus (x \otimes i)$

$$\sigma(k) = \sigma(k)$$

- $a \otimes (b \otimes c) = (a \otimes b) \otimes c$
  Caso Base: a=0

$$0 \otimes (b \otimes c) = (0 \otimes b) \otimes c$$

$$0 \otimes (b \otimes c) = 0 \otimes c$$

$$0 = 0$$

Caso Inductivo: a=$\sigma(x)$
Hipótesis Inductiva: $x \otimes (b \otimes c) = (x \otimes b) \otimes c$

$$\sigma(x) \otimes (b \otimes c) = (\sigma(x) \otimes b) \otimes c$$

$$(b \otimes c) \oplus (x \otimes (b \otimes c)) = (\sigma(x) \otimes b) \otimes c$$

$$(b \otimes c) \oplus (x \otimes (b \otimes c)) = (b \oplus (x \otimes b)) \otimes c$$

Distribuir la multiplicación $b \oplus (x \otimes b)) \otimes c$ (ver siguiente demostración $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$):

$$(b \otimes c) \oplus (x \otimes (b \otimes c)) = (b \otimes c) \oplus ((x \otimes b) \otimes c)$$

Aplicar propedad conmutativa de la suma, explicada en la demostración anterior

$$(b \otimes c) \oplus [(x \otimes (b \otimes c)) = ((x \otimes b) \otimes c)] \oplus (b \otimes c)$$

Asumiendo que $[(x \otimes (b \otimes c)) = ((x \otimes b) \otimes c)]$ es verdadero (Hipótesis Inductiva):

$$(b \otimes c) = (b \otimes c)$$

- $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$
  Caso Base: a=0

$$(0 \oplus b) \otimes c = (0 \otimes c) \oplus (b \otimes c)$$

$$(b) \otimes c = (0) \oplus (b \otimes c)$$

$$b \otimes c = b \otimes c$$

Caso Inductivo: a=$\sigma(x)$
Hipótesis Inductiva: $(x \oplus b) \otimes c = (x \otimes c) \oplus (b \otimes c)$

$$(\sigma(x) \oplus b) \otimes c = (\sigma(x) \otimes c) \oplus (b \otimes c)$$

$$\sigma(x \oplus b) \otimes c = (\sigma(x) \otimes c) \oplus (b \otimes c)$$

$$c \oplus ((x \oplus b) \otimes c) = (\sigma(x) \otimes c) \oplus (b \otimes c)$$

$$c \oplus ((x \oplus b) \otimes c) = (c \oplus (x \otimes c)) \oplus (b \otimes c)$$

Aplicar propedad conmutativa de la suma, explicada dos demostraciones atrás

$$c \oplus [(x \oplus b) \otimes c) = (x \otimes c)) \oplus (b \otimes c)] \oplus c$$

Asumiendo que $[(x \oplus b) \otimes c) = (x \otimes c)) \oplus (b \otimes c)]$ es verdadero (Hipótesis Inductiva):

$$c = c$$