Mass logarithms effects in DIS and their resummation to all orders

Master Thesis in Physics at "La Sapienza" University of Rome

Andrea Barontini
Supervisor: Dr. Marco Bonvini

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Introduction

- Collinear (mass) logarithms:

$$L \equiv \int_{m_q^2}^{Q^2} \frac{dk_t^2}{k_t^2} = ln(\frac{Q^2}{m_q^2})$$

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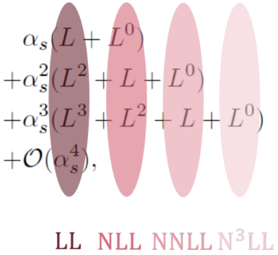
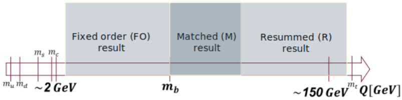


- Single log enhancement form:

$Q^2 \gg m_q^2 \rightarrow \alpha_s L \sim 1 \rightarrow$ **resummation**

- **Goal**: construct predictions that are reliable in a wide kinematic region: $\rightarrow$ **VFNS**



Before starting...

Remark

For clarity sake, all the methods have been applied to DIS (*Deep inelastic scattering*) with heavy-quark production but they can be generalized for a general hadron initiated process.

Choice of massless and massive quarks

The u, d, s and c quarks have been considered massless. The b quark is the one considered massive while the t is completely ignored.

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Fixed order and resummed results

- **Fixed order (4FS)**:

$$F^{FO} = \sum_{i,j=q,\bar{q},g} C_j^{[4]}(\epsilon, m_b)\Gamma_{ji}^{[4]}(\epsilon) f_i^{B[4]} = \sum_{i,j=q,\bar{q},g} C_j^{[4]}(m_b) f_j^{[4]}$$

- **Resummed (5FS)**:

$$F^{R} = \sum_{i,j=q,\bar{q},g,b,\bar{b}} C_j^{[5]}(\epsilon)\Gamma_{ji}^{[5]}(\epsilon) f_i^{B[5]} = \sum_{i,j=q,\bar{q},g,b,\bar{b}} C_j^{[5]} f_j^{[5]}$$

Notation

- From now on the products are in Mellin space.
- The B superscript denotes a *bare* quantity.
- The $[n]$ superscript denotes the number of *active flavors* ([4]=4FS, [5]=5FS), while the (n) superscript will denote the perturbative order of the term.

Matching matrix

The *matching matrix* $K_{ij}^{[5]\leftarrow[4]}$ links the PDFs in the two VFNS

$$f_i^{[5]}(Q, m_b) = \sum_{j=q,\bar{q},g,b,\bar{b}} \sum_{k=q,\bar{q},g} U_{ij}^{[5]}(Q, \mu_b) K_{jk}^{[5]\leftarrow[4]}(m_b, \mu_b) f_k^{[4]}(\mu_b)$$

with $U_{ij}^{[5]}$ solutions of *DGLAP evolution equations*. Therefore

$$C_j^{[4]}(Q, m_b) = \sum_{k=q,\bar{q},g,b,\bar{b}} C_k^{[5]} K_{kj}^{[5]\leftarrow[4]}(m_b, \mu_b) + \mathcal{O}\left(\frac{m_b^2}{Q^2}\right)$$

This allows us to include the mass power-corrections of the 4FS in the 5FS.

Remark

The scale μ_b is called *threshold scale* and must be chosen such that $m_b \sim \mu_b$. In the numerical results the variations of μ_b near m_b have been used to estimate the theoretical uncertainties.

Matched result

The **matched result** can be written as

$$F^M = F^R + F^{nons}$$

with F^{nons} containing only the mass power-terms. So

$$F^{nons} = F^{FO} - F^R\big|_{FO} = F^{FO} - F^R(\mu_b = Q) = \sum_{i,j=q,\bar{q},g} \left[C_i^{[4]} - C_j^{[5]} K_{ji}^{[5]\leftarrow[4]}(Q, m_b) - C_b^{[5]} K_{bi}^{[5]\leftarrow[4]}(Q, m_b)\right] f_i^{[4]}(Q)$$

Since F^R is naturally written in terms of the 5FS PDFs, we want to write F^{nons} in terms of $f_i^{[5]}$ as well:

$$F^{nons} = \sum_{j=q,\bar{q},g} \left[C_j^{[4]} - \sum_{k=q,\bar{q},g,b,\bar{b}} C_k^{[5]} K_{kj}^{[5]\leftarrow[4]}\right] (K^{[5]\leftarrow[4]})^{-1}_{ji} f_i^{[5]}(Q)$$

Finally, it is possible to include the

$$F^{nons} = \sum_{i=q,\bar{q},g,b,\bar{b}} \delta C_i^{nons}(Q, m_b) f_i^{[5]}(Q)$$

contribution at coefficients functions level as

$$\tilde{C}_i(Q, m_b) = C_i^{[5]}(Q) + \delta C_i^{nons}(Q, m_b), \quad i = q, \bar{q}, g, b, \bar{b}$$

in such a way the final matched result is written as

$$F^M = \sum_{i=q,\bar{q},g,b,\bar{b}} \tilde{C}_i(Q, m_b) f_i^{[5]}(Q, m_b)$$

Ambiguity of the matching matrix inverse

Since K is a rectangular matrix, its inverse $(K^{[5]\leftarrow[4]})^{-1}$ is ambiguous. One can exploit this ambiguity to fix $\delta C_b^{nons}(Q, m_b) = \delta C_{\bar{b}}^{nons}(Q, m_b) = 0$ in such a way

$$\tilde{C}_b(Q, m_b) = C_b^{[5]}(Q) + \delta C_b^{nons}(Q, m_b) = C_b^{[5]}(Q)$$

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The perturbative nature of the heavy PDF

The heavy-quark PDF is

$$f_b^{[5]}(Q, m_b) = \sum_{k=q,\bar{q},g} \left[U_{bb}^{[5]} K_{bk}^{[5]\leftarrow[4]} + \sum_{i=q,\bar{q},g} U_{bi}^{[5]} K_{ik}^{[5]\leftarrow[4]} \right] f_k^{[4]}(\mu_b)$$

But:

- $K_{jk} = \delta_{jk} + \alpha_s K_{jk}^{(1)} + \mathcal{O}(\alpha_s^2) \longrightarrow K_{bk} \sim \mathcal{O}(\alpha_s)$
- $U_{bi} = U_{bi}^{LL}(\alpha_s L) + \alpha_s U_{bi}^{NLL}(\alpha_s L) + \cdots$
  1. $Q \sim 1\text{TeV}$: $\alpha_s L \sim 1 \rightarrow U_{bi}^{[5]} \sim \mathcal{O}(1)$
  2. $Q \lesssim 100\text{GeV}$: $\alpha_s L \sim \alpha_s \rightarrow U_{bi}^{[5]} \sim \mathcal{O}(\alpha_s)$[1]

Therefore, in the two energy regions we have:

1. **High Energy (HE) counting** $\rightarrow f_b^{[5]} \sim \mathcal{O}(1)$
2. **Intermediate energy (IE) counting** $\rightarrow f_b^{[5]} \sim \mathcal{O}(\alpha_s)$

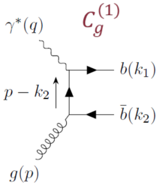
[1] Bonvini, Papanastasiou, Tackmann "Resummation and Matching of b-quark Mass Effects in bb⁻H Production", 2015

Intermediate energy counting

$F_2^{IE} =$

LO

NLO $\quad + \alpha_s^{[5]} \tilde{C}_{g,2}^{(1)} f_g^{[5]} + C_{b,2}^{(0)[5]} f_b^{[5]}$

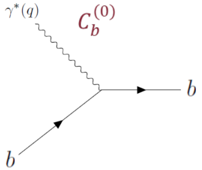


High energy counting

$F_2^{HE} =$

LO $\quad C_{b,2}^{(0)[5]} f_b^{[5]}$

NLO $\quad + \alpha_s^{[5]} C_{g,2}^{(1)} f_g^{[5]}$



Remark

For our notation, leading-order (LO) denotes the $\mathcal{O}(1)$ contribution, next-to-leading order (NLO) denotes the $\mathcal{O}(\alpha_s)$ contribution and so on. Therefore, the FO counting starts at NLO.

Summary of FONLL results

- **LO**:
  - FONLL-00
  - FONLL-0 (**impr.**)
- **NNLO**:
  - FONLL-C
  - FONLL-D (**impr.**)
- **NLO**:
  - FONLL-A
  - FONLL-B (**impr.**)
- **N^{3}LO**:
  - FONLL-E
  - FONLL-F (**impr.**)

Notation

- The red results are the results proposed in the original paper [2], while the others can be obtained generalizing their method of construction.
- **impr.** denotes that the result is an improved version for its perturbative order.

[2] Forte, Laenen, Nason, Rojo, "Heavy quarks in deep-inelastic scattering", 2010

Equivalence to FONLL results

It can be shown that:

- IE counting $\equiv$ **impr.** FONLL countings
- HE countings $\equiv$ standard FONLL countings

However, our IE results are used with higher order PDFs. So we argue that:

- **FONLL-0 $\equiv$ natural IE NLO result**
- **FONLL-B $\equiv$ natural IE NNLO result**

Therefore, our proposal for the final combined result is:

	FONLL	Impr. FONLL	Our proposal
NLO	FONLL-A	FONLL-B	FONLL-0 $+\chi(Q)$ [FONLL-A − FONLL-0]
NNLO	FONLL-C	FONLL-D	FONLL-B $+\chi(Q)$ [FONLL-C − FONLL-B]
N^3LO	FONLL-E	FONLL-F	FONLL-D $+\chi(Q)$ [FONLL-E − FONLL-D]

with $\chi(Q)$ a certain *damping* function.

Another consequence: alternative PDF evolution

Considering $U_{bi}^{[5]} \sim U_{ib}^{[5]} \sim \mathcal{O}(\alpha_s)$ also have consequences on the PDF evolution:

- Standard PDFs:
  - $f_b^{(0)} = U_{bg}^{[5]} f_g^{[5]} + U_{bq}^{[5]} f_q^{[5]}$
  - $f_b^{(1)} = \alpha_s(\mu_b) U_{bb}^{[5]} K_{bg}^{(1)} f_g^{[4]}$
- Alternative PDFs:
  - $\tilde{f}_b^{(0)} = 0$
  - $\tilde{f}_b^{(1)} = U_{bg}^{[5]} f_g^{[5]} + U_{bq}^{[5]} f_q^{[5]} + \alpha_s(\mu_b) U_{bb}^{[5]} K_{bg}^{(1)} f_g^{[4]}$

Remark

In the numerical results we have used both the standard and the alternative PDFs. Both of them have been obtained evolving with APFEL++[3] the starting *PDF4LHC15* PDFs at 2GeV.

[3] Bertone,Carrazza,Rojo "APFEL: A PDF Evolution Library with QED corrections", 2014

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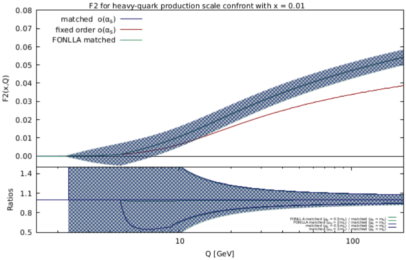
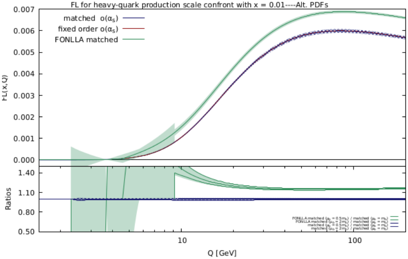
Comparison between FONLL-A and NLO IE result

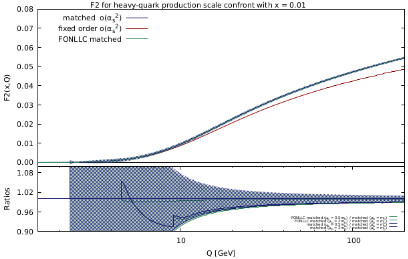
- F_2 and F_L results in FONLL-A scheme and with the IE counting at NLO (FONLL-0) are presented for $x = 0.01$.
- The uncertainty region is obtained symmetrizing the larger variation of the $\mu_b = 0.5 m_b$ and $\mu_b = 2 m_b$ results with respect to the standard $\mu_b = m_b$ version.

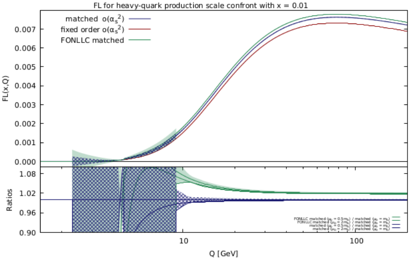




Comparison between FONLL-C and NNLO IE result

- F_2 and F_L results in FONLL-C scheme and with the IE counting at NNLO (FONLL-B) are presented for $x = 0.01$.





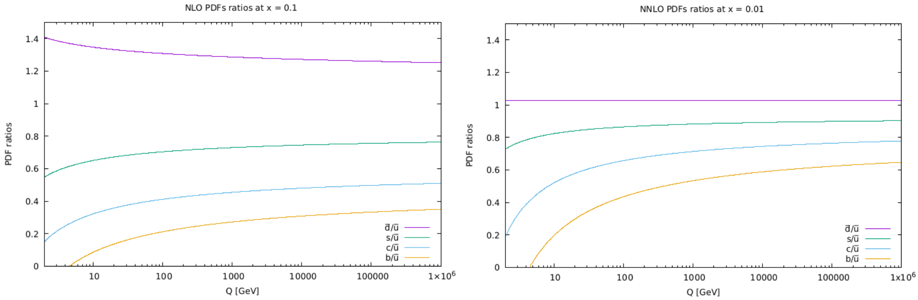
- The two results have a very similar quality from both an uncertainty and a slope discontinuity point of views

Final combined result at NLO and NNLO

The **NLO** and **NNLO** combined result are

- $F^C_{NLO} = \text{FONLL-0} + \chi(Q)\left[\text{FONLL-A} - \text{FONLL-0}\right]$
- $F^C_{NNLO} = \text{FONLL-B} + \chi(Q)\left[\text{FONLL-C} - \text{FONLL-B}\right]$

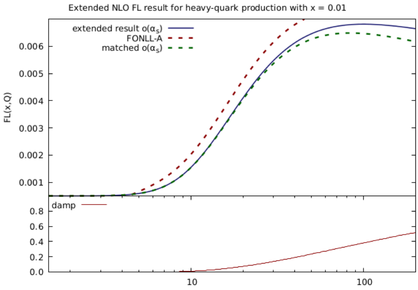
To choose the *damping* functional form, we look to the ratio on the $\bar{u}$ PDF of the heavy PDF.

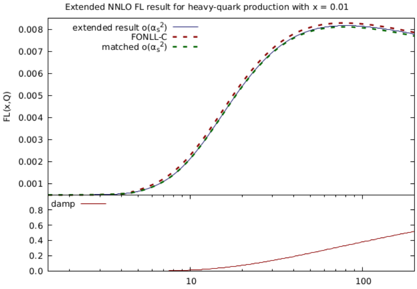


Therefore, the chosen functional form has been

$$\chi(Q) = (1 - \sqrt{m_b/Q})^4$$

which follows the behaviour of the b PDF.
With this choice, the NLO and NNLO combined result for F_L are





Conclusions

- In the intermediate energy region FONLL-B should be used as the natural NNLO result (and FONLL-0 for NLO).
- In the high energy region FONLL-C should be used as the natural NNLO result (and FONLL-A for NLO).
- The N^3LO prediction in the intermediate energy region is more easily accessible using the proposed prescription $\rightarrow$ only some ingredients, such massive $C_g^{(3)}$ and $C_q^{(3)}$, are still in progress but there exist reliable approximations[4].

$$\boldsymbol{F^{C}_{N^3LO} = \textbf{FONLL-D} + \chi(Q)\left[\textbf{FONLL-E} - \textbf{FONLL-D}\right]}$$

[4] Kawamura, Lo Presti, Moch, Vogt "On the next-to-next-to-leading order QCD corrections to heavy-quark production in deep-inelastic scattering", 2012

Thanks for your attention!

Equivalence to FONLL-0

The F_2 prediction in FONLL-0 scheme is

$$F_2^{(0)} = C_{b,2}^{(0)} f_b + \alpha_s [B_{g,2}^{(1)} - \overline{B}_{g,2}^{[0](1)} \left(\frac{m_b}{Q} \right)] f_g$$

where the massive-zero coefficients function is redefined as

$$\overline{B}_{g,2}^{[0](1)} \left(\frac{m_b}{Q} \right) = B_{g,2}^{[0](1)} (\frac{m_b}{Q}) - B_{g,2}^{[0](1)} (1)$$

But

$$B_{g,2}^{[0](1)} (\frac{m_b}{Q}) = C_{g,2}^{(1)} + 2C_{b,2}^{(0)} K_{bg}^{(1)} \left(\frac{m_b}{Q} \right)$$

So one gets

$$F_2^{(0)} = C_{b,2}^{(0)} f_b + \alpha_s [B_{g,2}^{(1)} - 2C_{b,2}^{(0)} K_{bg}^{(1)} \left(\frac{m_b}{Q} \right)] f_g = C_{b,2}^{(0)} f_b + \alpha_s \tilde{C}_{g,2}^{(1)} f_g$$

Running coupling in different VFNS

- Fixed order expressions are naturally written in terms of $\alpha_s^{[4]}$
- Resummed expressions are naturally written in terms of $\alpha_s^{[5]}$

So we need a way to express everything in terms of the same coupling.

$$\frac{\alpha_s^{[n_f+1]}(\mu^2)}{\alpha_s^{[n_f]}(\mu^2)} = 1 + \frac{\alpha_s^{[n_f]}}{6\pi} \log \frac{\mu^2}{m_{n_f+1}^2} + \mathcal{O}(\alpha_s^2)$$

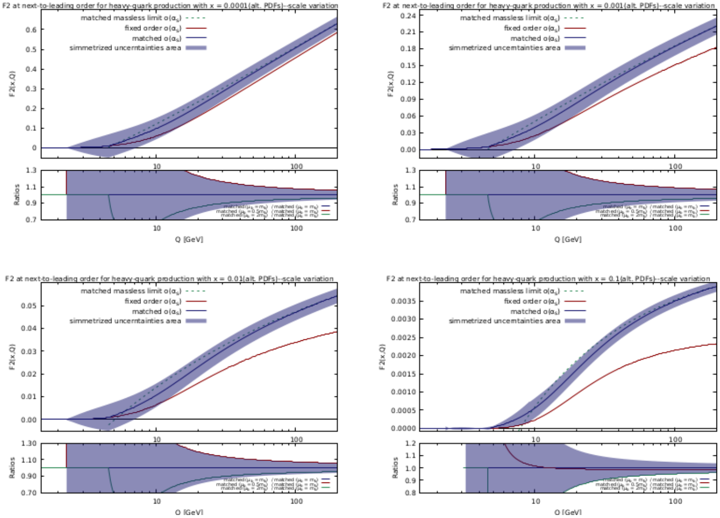
Complete definition of alternative PDFs

$$\tilde{f}_g^{(0)}(m_b, Q) = U_{gg}^{[5]}(Q, \mu_b) f_g^{[4]}(\mu_b) + U_{gq}^{[5]}(Q, \mu_b) f_q^{[4]}(\mu_b)$$
$$\tilde{f}_g^{(1)}(m_b, Q) = \alpha_s(\mu_b) U_{gg}^{[5]}(Q, \mu_b) K_{gg}^{(1)}(m_b, \mu_b) f_g^{[4]}(\mu_b)$$

$$\tilde{f}_q^{(0)}(m_b, Q) = U_{qq}^{[5]}(Q, \mu_b) f_q^{[4]}(\mu_b) + U_{qg}^{[5]}(Q, \mu_b) f_g^{[4]}(\mu_b)$$
$$\tilde{f}_q^{(1)}(m_b, Q) = \alpha_s(\mu_b) U_{qg}^{[5]}(Q, \mu_b) K_{gg}^{(1)}(m_b, \mu_b) f_g^{[4]}(\mu_b)$$

$$\begin{aligned}
\tilde{f}_b^{(1)}(m_b, Q) &= \left[U_{bg}^{[5]}(Q, \mu_b) + \alpha_s(\mu_b) U_{bb}^{[5]}(Q, \mu_b) K_{bg}^{(1)}(m_b, \mu_b)\right] f_g^{[4]}(\mu_b) + U_{bq}^{[5]}(Q, \mu_b) f_q^{[4]}(\mu_b) \\
\tilde{f}_b^{(2)}(m_b, Q) &= \alpha_s(\mu_b)\left[U_{bg}^{[5]}(Q, \mu_b) K_{gg}^{(1)}(m_b, \mu_b) + \alpha_s(\mu_b) U_{bb}^{[5]}(Q, \mu_b) K_{bg}^{(2)}(m_b, \mu_b)\right] f_g^{[4]}(\mu_b) \\
&\quad + \alpha_s^2(\mu_b) U_{bb}^{[5]}(Q, \mu_b) K_{bq}^{(2)}(m_b, \mu_b) f_q^{[4]}(\mu_b),
\end{aligned}$$

F2 at NLO


F2 at next-to-leading order for heavy-quark production with x = 0.0001(alt. PDFs)--scale variation
F2 at next-to-leading order for heavy-quark production with x = 0.001(alt. PDFs)--scale variation
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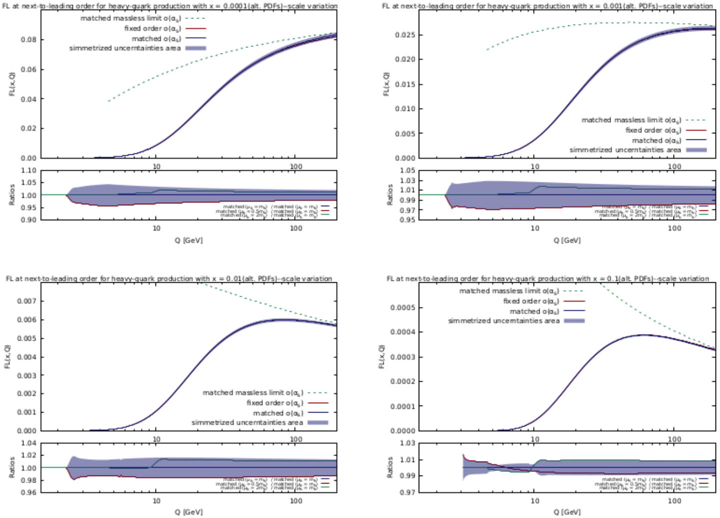
F2 at next-to-leading order for heavy-quark production with x = 0.1(alt. PDFs)--scale variation
matched massless limit o(αs)
fixed order o(αs)
matched o(αs)
simmetrized uncertainties area
F2(x,Q)
Ratios
Q [GeV]

FL at NLO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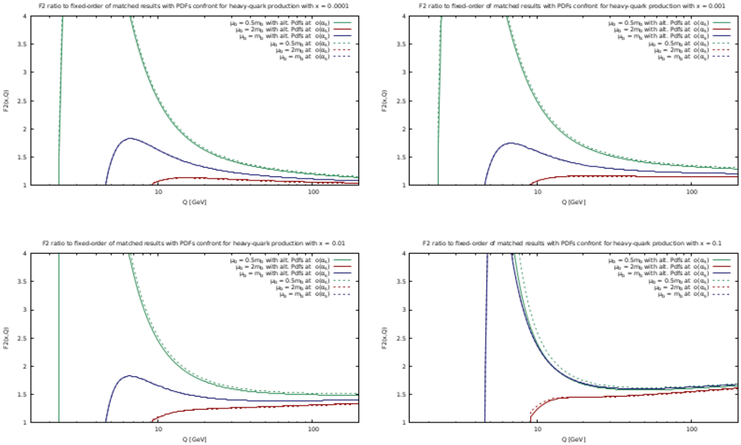
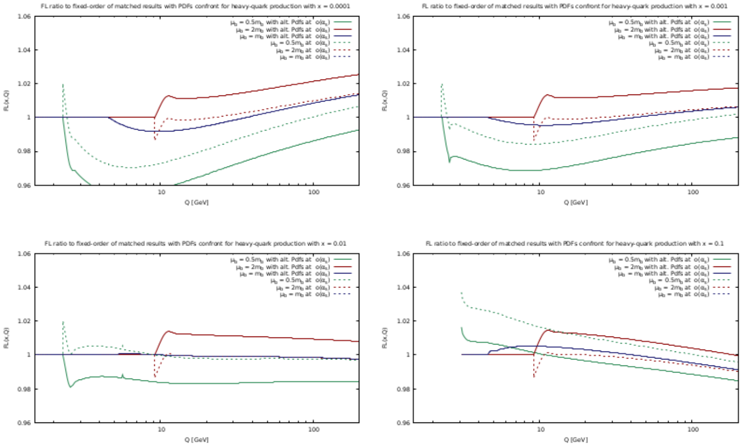
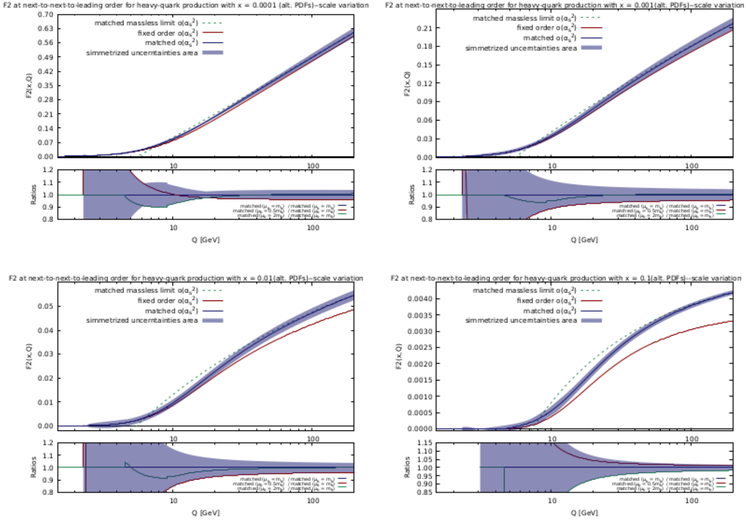
F2 PDFs comparison NLO



FL PDFs comparison NLO



F2 at NNLO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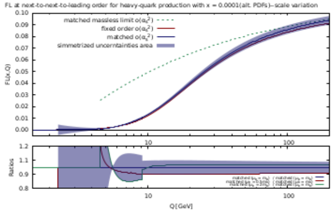
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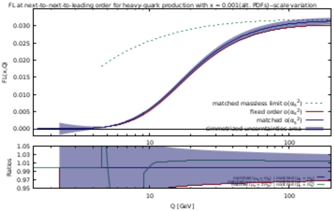
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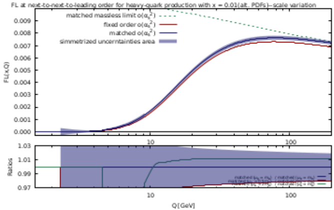
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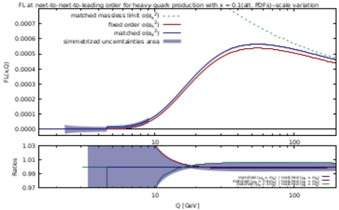
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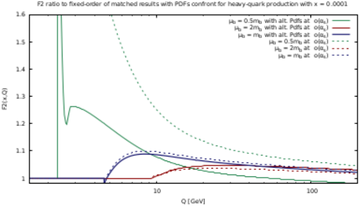
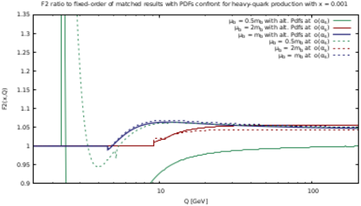
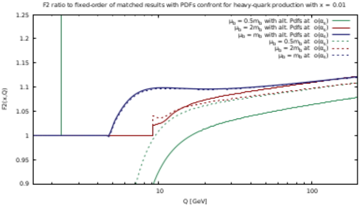
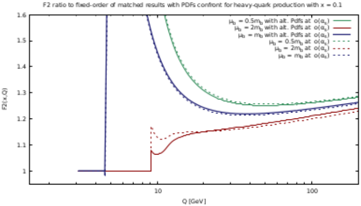
F2 PDFs comparison NNLO

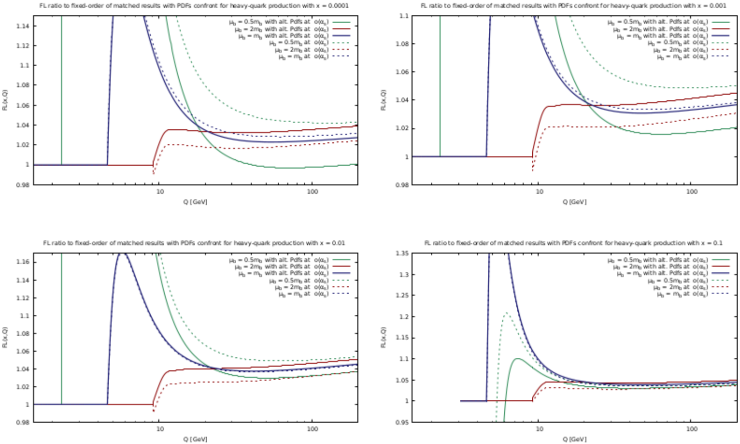




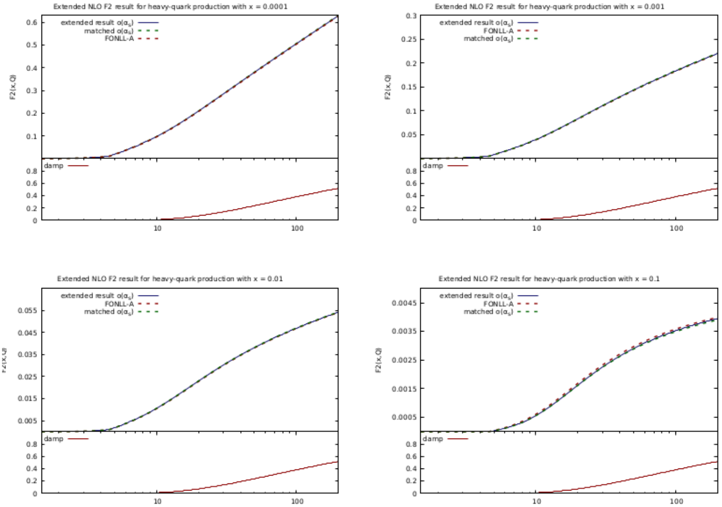




FL PDFs comparison NNLO



F2 extended result NLO



F2 extended result NNLO