THE UNIVERSITY OF MELBOURNE

SEMESTER 1, 2014

DEPARTMENT OF MATHEMATICS AND STATISTICS

MAST90053 EXPERIMENTAL MATHEMATICS

EXAM

Exam due date: ***6pm****, Tuesday 17 June 2014*

This paper consists of 2 pages.

Advice and instructions

- Answers can be given in `Mathematica` notebooks, PDFs or scanned handwriting.
- All answers should be submitted electronically on LMS before the exam deadline.
- Explanations and annotations are more important than final answers.
- You are required to use your own, self-written computer programs.

1. Write a program in Mathematica that, given a hypergeometric term t_k, returns a hypergeometric term s_k such that

$$s_{k+1} - s_k = t_k$$

if it exists or otherwise returns the message "not summable".

Clearly annotate your program, and explain the various steps needed to produce the desired output.

Your program **must not** make use of Mathematica's built-in functions for equation solving (such as `Solve`, `RSolve`, `NSolve`, etc) with the exception of `LinearSolve`. In other words, you can only ask Mathematica to solve linear systems. Any other kind of algebraic manipulation is allowed.

Your program should at least be able to give the correct answer to all sums in Exercise 5.7.1, and 5.7.2 of the book $A{=}B$. Your annotations may be included in your `Mathematica` notebook or submitted in a separate PDF file. **50 marks**

2. Prove the following statement. Let m be a positive integer, $p(k)$ a non-zero polynomial in k of degree $m-1$, and a a constant; then

$$\sum_k \frac{p(k)}{\prod_{j=0}^{m-1}(k+a+j)}$$

is not Gosper-summable. Your proof may be computer assisted, but has to constitute a full, rigorous proof. **25 marks**

3. Consider the following term

$$t_k = \frac{1}{(k+1)(k+2)(k+y)}.$$

Determine all values of y for which t_k is Gosper summable. Prove your answer. Your proof may be computer assisted, but has to constitute a full, rigorous proof.

25 marks