AN INTRODUCTION TO THE BISPECTRUM FOR EEG ANALYSIS

D. Wong, D.A. Clifton, L. Tarassenko

Biomedical Signal Processing Group, Institute of Biomedical Engineering, University of Oxford, Oxford, UK

wong@robots.ox.ac.uk

Abstract - This paper provides a tutorial for bispectral analysis, a signal processing technique commonly used for the analysis of the Electroencephalogram (EEG). The use of this technique has been hindered by popular misconceptions deriving from existing tutorial papers.

INTRODUCTION

Analysis of EEG is typically performed using Fourier analysis, which is useful for detecting frequency components that correspond to the mental state of a patient. Typically, such information is obtained from the magnitude spectrum, as the information content of the phase spectrum can be harder to interpret.

Although use of the magnitude spectrum is sometimes sufficient, in many instances, the phase information is critically important. Bispectral analysis offers a way of gaining further phase information by detecting phase relationships between frequency components. Such relationships have been shown to exist in a medical context during periods where a patient has an impaired mental state (for example see [1]).

The rest of this paper outlines the mathematical principles behind the bispectrum, and provides simple examples that demonstrate its use. Using the knowledge gained from the tutorial, we will dispel a commonly-held misconception.

II. BISPECTRUM TUTORIAL

Bispectral analysis makes use of phase information by detecting whether the phase of signal components at frequencies f_1, f_2, and f_3 are interdependent. Furthermore, the degree of dependence is quantified so that a high bispectrum correlates to a signal with highly interdependent frequency components.

Knowing this, it makes intuitive sense that the bispectrum can be calculated from using the Fourier transform of a signal evaluated at f_1, f_2, and f_3. The bispectrum of x can be expressed mathematically as

$$B(f_1, f_2) = \frac{1}{k}\sum_{i=1}^{k} X_i(f_1)X_i(f_2)X_i^*(f_1 + f_2) \quad (1)$$

where X denotes the Fourier transform of x. In this case, the signal has been split into k time segments that we call epochs, and

$$X_i(f_1)X_i(f_2)X_i^*(f_1 + f_2) \quad (2)$$

is the triple product. Using the convolution theorem for Fourier transforms, the bispectrum of a process, x, is formally defined as:

$$B_x(f_1, f_2) = \sum\sum c_3(\tau_1, \tau_2)e^{-j2\pi(f_1\tau_1 + f_2\tau_2)} \quad (3)$$

where $c_3(\tau_1,\tau_2)=E\{x_t,x_{t+\tau 1},x_{t+\tau 2}\}$, and $E\{.\}$ is the expectation operator.

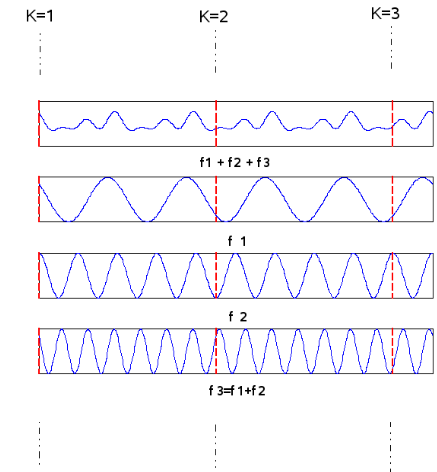
Although other methods of calculating the bispectrum are also used in practice, the examples below use equation 1 for the sake of clarity.

A. Quadratically Phase Coupled Components

Three sinusoid components, x_1,x_2 and x_3, of a signal are quadratically phase coupled (QPC) if the frequency and phase of the x_3 component is equal to the sum of the frequencies and phases of the x_1 and x_2 components. For example, for

$x_1 = \cos(f_1 + \phi_1)$, $x_2 = \cos(f_2 + \phi_2)$,

$x_3 = \cos(f_3 + \phi_3)$ the components are QPC only if $f_1+f_2=f_3$ and $\phi_1 + \phi_2 = \phi_3$



	Phase	Amp.	Phase	Amp.	Phase	Amp.
F1	π/4	π	3π/4	π	5π/4	π
F2	0	π	π	π	0	π
F3	π/4	π	7π/4	π	5π/4	π

Figure 1: A graphical interpretation of the bispectrum calculation for a synthetic QPC signal. The signal is shown above its three component sinusoids.

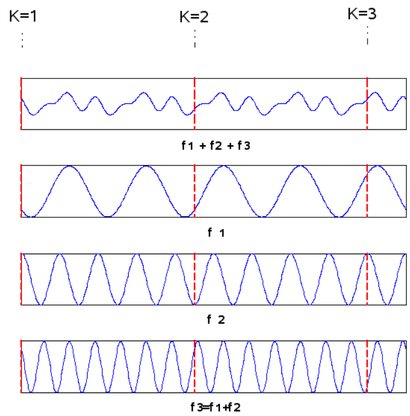
A simple QPC signal, along with its component sinusoids, is shown in Fig. 1. The signal is split into k=3 epochs (where the third epoch is truncated), and the component frequencies and phases are calculated at the start of each epoch. The triple products can then be calculated using eq. 2:

$$K = 1 : \pi e^{\frac{j\pi}{4}} \times \pi e^{j(0)} \times \pi\, e^{-\frac{j\pi}{4}} = \pi^3$$
$$K = 1 : \pi e^{\frac{j3\pi}{4}} \times \pi e^{j\pi} \times \pi\, e^{-\frac{j7\pi}{4}} = \pi^3$$
$$K = 1 : \pi e^{\frac{j5\pi}{4}} \times \pi e^{j(0)} \times \pi\, e^{-\frac{j5\pi}{4}} = \pi^3$$

The mean of the triple products gives the estimate of the bispectrum. In this case, $B(f_1,f_2)=\pi^3$

B. Non-Quadratic Coupling

Figure 2 shows a similar system, but where the components are no longer QPC, as $\phi_1 + \phi_2 \neq \phi_3$. However, the phases are *coherent*, meaning that the relative phase between any of the components does not change over time. Triple products are calculated as before, resulting in $|B(f_1,f_2)|=\pi^3$.



	Phase	Amp.	Phase	Amp.	Phase	Amp.
F1	3π/4	π	5π/4	π	7π/4	π
F2	0	π	π	π	0	π
F3	π/4	π	7π/4	π	5π/4	π

Figure 2: A graphical interpretation of the bispectrum calculation for a non-QPC, coherent, synthetic waveform.

C. Incoherent components

The third example, which is not pictured here for brevity, assumes a non-QPC and non-coherent set of component sinusoids. By non-coherent, it is meant that the relative phases between the sinusoids change over time, so ϕ_1, ϕ_2, ϕ_3 are time-dependent variables.

In this case, the estimate of the bispectrum is once again calculated from equation 2 to give $\frac{\pi^3}{2.08} e^{j(0.6\pi)}$. The magnitude of the bispectrum is much less than in the other examples. In fact, for incoherent signals, the bispectrum tends toward zero when a larger number of epochs are used.

III. DISCUSSION

This tutorial has demonstrated that the bispectrum can be used to identify phase coherence between any three sinusoids that are components of a signal. The coherence is detected for QPC components (example A), for coherent, but non-QPC components (example B), but not for incoherent signals where the relative phase between two fundamentals changes over time (example C).

Previous tutorial papers on the bispectrum, including those by Sigl and Chamoun [2], Nikias and Raghuveer [3], Shen [4], and Holt [5], have asserted that the technique is only valid for QPC signals. They each give a simple system such as:

$$y_1(k) = 1 + \cos(f_a k + \theta_a) + \cos(f_b k + \theta_b) + \frac{1}{2}\cos(f_c k + \theta_c) + \frac{1}{2}\cos(f_d k + \theta_d)$$

and apply the constraints $f_a = f_1 + f_2$, $f_b = f_1 - f_2$, $f_c = 2f_1$, $f_d = 2f_2$, and $\theta_a, \theta_b, \theta_c, \theta_d$ are random and independent so that the mixture is *not* QPC.

They go on to state that such a system will result in a zero bispectrum over all the frequency space. However, this assertion is incorrect and inconsistent with example B.

In both example B and the system described above, the phases are set to be random constants. Thus, the signal components are coherent as there is no relative phase drift between them, and the bispectrum will be significant.

In conclusion, we have sought to provide an intuitive guide to the bispectrum using a graphical explanation, and have proceeded to show that previous literature on the subject is confusing and misleading.

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