REVIEWS

of APM-D-18-02037R1

Review

The authors have rewritten the paper by restructuring it, deleting the section about the vectorial formulation, including properly the definition of the Stokes-Dirac structure and adding a section about numerical results, taking account of the most important remarks.

Apart of some minor comments which I leave to the authors to account for, I recommend the present paper for publication.

Minor remarks

- Introduction
  - page 1: *formally skew-symmetric Hamiltonian differential operator* is a pleonasm as a Hamiltonian operator should be skew-symmetric and obey the Jacobi identities [1].. Maybe you may just keep: Hamiltonian operator ? .
  - the authors might refer to for the relation between Lagrangian and Hamiltonian formulations including port variables and Dirac structures [2, 3]
- Reminder on port Hamiltonian systems
  - page 4: I do not like *orthogonal complement* which reminds of a metric structure but would rather prefer: *isotropic and coisotropic*.
  - page 6 Remark 1: Rather then *inner product* which reminds of a metric, I would write *pairing* (which just means the bilinearity)
- 5. Numerical studies
  - page 20 : The sentence *Anyway, the Lagrange multipliers are defined only over the boundary.* is cryptic.
  - page 20: Remark 4. The sentence *This choice does not correspond to the optimal one given by* $D(\mathcal{J})$. is not understable. Please recall the equation where the operator $\mathcal{J}$ is defined ! Is it not $\mathcal{H}$? What means *optimal* : the projection space does not belong to the domain ?
  - page 21: It is written: *The symplectic Störmer-Verlet time integrator is employed, so that when no solicitation is applied to the system, the Hamiltonian is preserved*. In theory the symplectic scheme should preserve the symplectic structure not the Hamiltonian which is expected not to be conserved by to oscillate around a mean value ? Furthermore how do you aply the scheme to a model which is defined with respect to a skew-symmetric matrix which is not in canonical coordinates ?

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general comment

The authors have rewritten the paper taking account of the most important remarks. The section about the numerics is precise, well presented and very intersting. The paper is satisfactory in the present form.

Apart of some minor comments which I leave to the authors to account for, I recommend the present paper for publication.

Minor remarks

- page 6. After the sentence: *This theorem states that, for smooth functions, higher order partial derivative commute.,* remove all detailed equations. This statement is enough as it corresponds to define the jet bundle over the displacements.
- page 22: I have the same remarks as for the first part. : It is written: *The symplectic Störmer-Verlet time integrator is employed, so that when no solicitation is applied to the system, the Hamiltonian is preserved*. In theory the symplectic scheme should preserve the symplectic structure not the Hamiltonian which is expected not to be conserved by to oscillate around a mean value ? Furthermore how do you aply the scheme to a model which is defined with respect to a skew-symmetric matrix which is not in canonical coordinates ?

References

[1] P.J. Olver. *Applications of Lie Groups to Differential Equations*, volume 107 of *Graduate texts in mathematics*. Springer, New-York, ii edition, 1993. ISBN 0-387-94007-3.

[2] H. Yoshimura and J.E. Marsden. Dirac structures in Lagrangian mechanics part i: Implicit Lagrangian systems. *Journal of Geometry and Physics*, 57(1):133–156, Dec. 2006.

[3] H. Yoshimura and J.E. Marsden. Dirac structures in Lagrangian mechanics part ii: Variational structures. *Journal of Geometry and Physics*, 57(1):209–250, Dec. 2006.