Projet MORPHEUS

Andrea Brugnoli

February 4, 2022

Overview

Modelling multiphysics systems with Hamiltonian systems

A unified discretization framework for port-Hamiltonian systems

Artificial Intelligence for Model Order Reduction (MOR)

Outline

Modelling multiphysics systems with Hamiltonian systems

A unified discretization framework for port-Hamiltonian systems

Artificial Intelligence for Model Order Reduction (MOR)

Modelling framework for the MORPHEUS project

Main objective

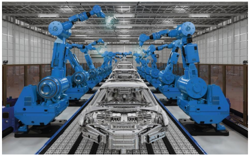
Use a unified port-Hamiltonian (pH) framework to model fluid-structure interactions.

If successful, the approach will:

- be competitive w.r.t. state of the art methods;
- pave the way to other multiphysical problems;
- serve for model reduction and control of complex systems.

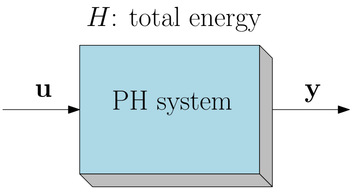
Robotics

Aerospace



Manufacturing

Why port-Hamiltonian systems?



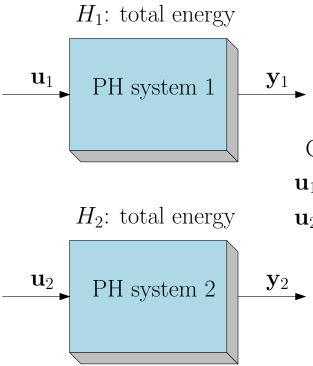
Lossless: $\dot{H} = \mathbf{u}^\top \mathbf{y}$

Passive: $\dot{H} \leq \mathbf{u}^\top \mathbf{y}$

PH systems are:

- Physically motivated;
- Lumped (ODEs) or distributed (PDEs);
- Passivity based control;
- Closed under interconnection (modular multiphysics modelling);

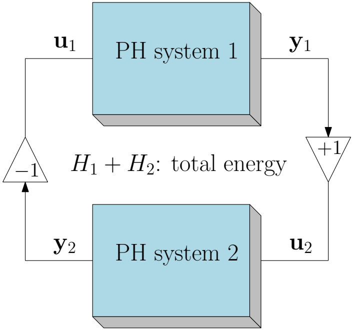
Energy preserving interconnection of pHs



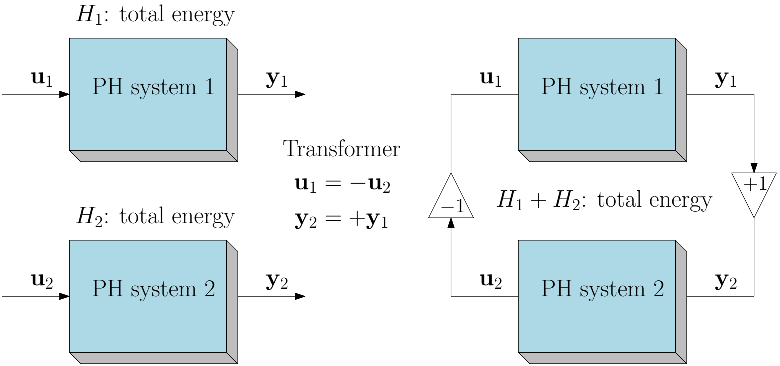
Gyrator

$$\mathbf{u}_1 = -\mathbf{y}_2$$

$$\mathbf{u}_2 = +\mathbf{y}_1$$



Energy preserving interconnection of pHs



Distributed port-Hamiltonian systems

General structure

$H(\boldsymbol{\alpha})$ is the Hamiltonian functional:

$$
\begin{aligned}
\partial_t \boldsymbol{\alpha} &= (\mathcal{J} - \mathcal{R})\, \boldsymbol{e}, & \boldsymbol{\alpha} &\quad \text{State variables,} \\
\boldsymbol{e} &:= \delta_{\boldsymbol{\alpha}} H, & \boldsymbol{e} &\quad \text{Effort variables,} \\
\boldsymbol{u}_\partial &= \mathcal{B}_\partial\, \boldsymbol{e}, & \boldsymbol{u}_\partial &\quad \text{Boundary control input,} \\
\boldsymbol{y}_\partial &= \mathcal{C}_\partial\, \boldsymbol{e}, & \boldsymbol{y}_\partial &\quad \text{Collocated output.}
\end{aligned}
$$

Operators:

- $\mathcal{J}$ Interconnection operator (skew-symmetric);
- $\mathcal{R}$ Dissipation operator (semi-positive definite);
- $\mathcal{B}_\partial$ Boundary input operator;
- $\mathcal{C}_\partial$ Boundary output operator.

Energy balance:

$$
\dot{H} = \langle \boldsymbol{u}_\partial \,|\, \boldsymbol{y}_\partial \rangle - \langle \boldsymbol{e}, \mathcal{R}\boldsymbol{e} \rangle \leq \langle \boldsymbol{u}_\partial \,|\, \boldsymbol{y}_\partial \rangle
$$

Outline

Modelling multiphysics systems with Hamiltonian systems

A unified discretization framework for port-Hamiltonian systems

Artificial Intelligence for Model Order Reduction (MOR)

The MORPHEUS project and its numerical challenges

Numerical methods for multiphysical systems should:

- reproduce the physical properties of the problem (conservation laws, symmetries);
- work in any spatial dimension and coordinate frame;
- distinguish between the dynamics and the constitutive equations;
- adaptable to many physical problems (fluid and solid mechanics, electromagnetism, etc.);
- preserve the exchanges of energy flow for different physical domain;

Methods for computational simulation

- Finite differences (Taylor expansion of derivatives);
- Finite volumes (Integral form of the equations);
- Finite elements (Variational formulation of the PDE);

If the constructions stem from the same ideas, these schemes may be completely equivalent:
James H. Adler et al. (2021). "A Finite-Element Framework for a Mimetic Finite-Difference Discretization of Maxwell's Equations". In: *SIAM Journal on Scientific Computing* 43.4, A2638–A2659

What to choose for port-Hamiltonian systems?

Two important questions:

- Can we rely on established theoretical results to have a clear guideline and unified analysis of different methods?
- Can we rely on existing and robust software packages?

Exterior calculus and discretization of PDEs

A unified discretization framework

Douglas N. Arnold, Richard S. Falk, and Ragnar Winther (2006). "Finite element exterior calculus, homological techniques, and applications". In: *Acta Numerica* 15, pp. 1–155

This framework comes with a periodic table of finite elements for unified analysis[1].

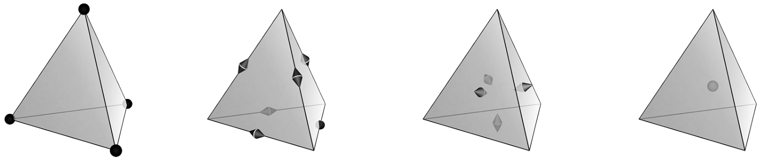


Figure: The $\mathcal{P}_1^- \Lambda^k$ family (Whitney forms 1957)

[1] `https://sinews.siam.org/Details-Page/periodic-table-of-the-finite-elements`

The impact of FEEC

On the importance of FEEC

"Just as the arrangement of the chemical elements in a periodic table led to the discovery of new elements, the periodic table of finite elements has not only clarified existing elements but also highlighted holes in our knowledge and led to new families of finite elements suited for certain purposes."

This theory was employed successfully for:

- Electromagnetism;
- Fluid-Mechanics;
- Solid Mechanics.

Two open source libraries were built to implement the FEEC periodic table[2].

[2] Logg, Mardal, Wells, et al. 2012; Rathgeber et al. 2017.

Outline

Modelling multiphysics systems with Hamiltonian systems

A unified discretization framework for port-Hamiltonian systems

Artificial Intelligence for Model Order Reduction (MOR)

Artificial intelligence for MOR

High dimensional full order system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}), \qquad \mathbf{x} \in \mathbb{R}^n,\ n \gg 1$$

- $\mathbf{x}$ State vector;
- $\mathbf{p}$ Parameters vector (boundary conditions, materials etc.)

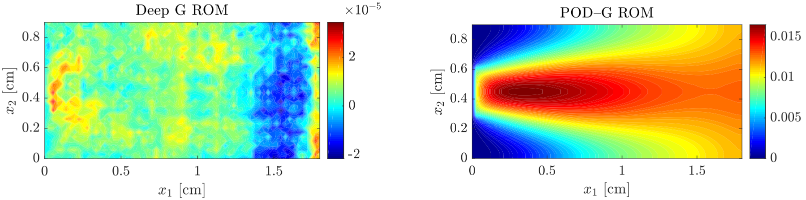
How to obtain a reduced order model

$$\dot{\mathbf{x}}_r = \mathbf{f}_r(\mathbf{x}_r, \mathbf{p}), \qquad \mathbf{x}_r \in \mathbb{R}^r,\ r \ll n$$

valid for the whole parameter space $\mathbf{p}$ using IA and NN?

An IA architecture for Galerkin and Petrov-Galerkin MOR

Kookjin Lee and Kevin T. Carlberg (2020). "Model reduction of dynamical systems on nonlinear manifolds using deep convolutional autoencoders". In: *Journal of Computational Physics* 404. https://doi.org/10.1016/j.jcp.2019.108973, p. 108973



(a) Reduced model with IA.

(b) Reduced model with POD.

Figure: Error of reduced models on the temperature field for a problem of convection-diffusion-reaction. By using a convolutional neural network to generate a nonlinear manifold (left) the error associated with the reduction is drastically reduced, from 10^{-2} to 10^{-5}, compared to the POD method (right).

Extension to port-Hamiltonian systems

- How to incorporate physical structure in the Neural Network architecture?
- What if the system is interconnected to other components or actively controlled?