UNIVERSITY OF TWENTE.

Mixed finite elements for port-Hamiltonian models of von Kármán beams

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Overview

1. Von-Karman theory of thin beams/plates

Outline

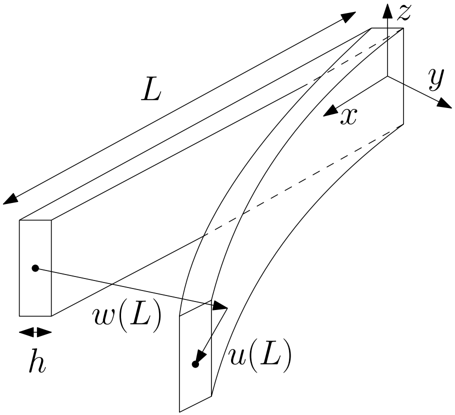
Linear vs Von-Kàrmàn plate theory

Non linearity allows to describe bifurcations, i.e. buckling.

The von-Kàrmàn assumption

Basic geometric assumption

Out of plane deflection comparable compared to the thickness: $w/h = \mathcal{O}(1)$.



Aspect ratio: $\delta = h/L$.
The following terms are kept in the expansion:

$$
\begin{aligned}
w/L &= \mathcal{O}(\delta), \\
u/L &= \mathcal{O}(\delta^2),
\end{aligned}
$$

Stresses and Strains

Decomposition strain field

$$\varepsilon_{xx} = \overbrace{\boxed{\frac{\partial u}{\partial x}} + \boxed{\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2}}^{\varepsilon_{xx}^m} - \overbrace{\boxed{z\frac{\partial^2 w}{\partial x^2}}}^{\kappa_{xx}}$$

Linear membrane def.

Non-linear membrane def.

Linear bending def.

Membrane-bending problem for thin beams

For small deformations the membrane and bending behavior are uncoupled:

$$\frac{\partial}{\partial t}\begin{pmatrix}\alpha_u \\ \alpha_\varepsilon \\ \alpha_w \\ \alpha_\kappa\end{pmatrix} = \begin{bmatrix}0 & \partial_x & 0 & 0 \\ \partial_x & 0 & 0 & 0 \\ 0 & 0 & 0 & -\partial_{xx} \\ 0 & 0 & \partial_{xx} & 0\end{bmatrix}\begin{pmatrix}e_u \\ e_\varepsilon \\ e_w \\ e_\kappa\end{pmatrix}$$

If the material is isotropic

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