**REVIEW PAPER APM-D-18-02037**

## 1. General comments

The paper presents very interesting and novel results on the Port Hamiltonian formulation of the Mindlin plate using a tensor notation which allows the coordinate free description.

This is an extension of existing work which make clearly appear the intrinsic definition of the Hamiltonian operator uniquely defined by the divergence and gradient operators. From this formulation the port variables and the Stokes-Dirac structure are consistently derived. A major outcome of this formulation (given in the section 4), is the structure preserving (e.g. preserving the port Hamiltonian structure) discretization of the Mindlin plate using a Partioned Finite Element Method. This section is extremely well written , in details and offers the method to incorporate various choices for the input and output boundary variables and will also be easily be adapted to the use of conventional FEM softwares. By the compliance of the method to various choice of input and output variables including mixed boundary conditions, the presented results will also find numerous applications to control.

However the paper could be improved in its structure. A first, more formal but important remark is , that several sections contain only one subsection: this should not happen !

But more important are the following points.

In the section 1. *Recall on port-Hamiltonian systems*, the definition of PHS on Dirac structures as well as the definition of Stokes-Dirac structure (in subsection 1.2. Infinite-dimensional PHs) are missing in order to understand the following sections !

In the section 2. *Mindlin theory for thick plates* , the presentation may be quite improved (see the detailed remarks). It is not clear why the EL formulation is recalled ? I think that the all the geometry, stress and strain variables could be directly introduced without giving the Euler-Lagrange formulation. If however the authors like to recall the Euler-Lagrange formulation, then there should be some reference given about its relation with the Port Hamiltonian formulation (for instance the order of the Euler-Lagrange system is different from the Port Hamiltonian one) which is maybe not so easy.

In the same spirit, why is the vectorial formulation presented ? I think that the reference [16] could be mentioned but then all variables of the model presented and directly expressed in the tensorial formulation ! That would simplify the paper and the understanding. This remark applies to the section 3. *PH formulation of the Mindlin plate* as well as to the section 4. *Discretization of the Mindlin plate using a Partioned Finite Element Method (PFEM)* which is extremely well-written and intersting but I do not think that the section 4.1. *Weak Form for the vectorial formulation* is really nececessary.

The section *Conclusion and Perspectives* should also be improved. The paragraph of lines 51 to 55 should be developed and enriched with more detailes on the result: coordinate free representation, of Hamiltonian operator nd boundary port variables for the definition of the Stokes-Dirac structure. The advantages of the PFEM method should be detailed: natural derivation of the boundary port variables in the discretized system and possibilty of having mixed boundary conditions, the possibility to use classical FEM software ... I am not convinced by the open questions' formulation. If I understand well the first paragraph adresses the questions of convergence but more interstingly, of the interconnection of the plate and the choice of appropriate basis for the discretization. The second paragraph could rather state that this work paves the way for a practical design and implementation of control laws based on methods developed for PHS, like IDA-PBC etc .. with giving references.

Again, the paper presents extremely intersting results as well for the community in modelling, numerics and simulation as well as for control. Therefore, provided the comment sections are taken into account in a revised version, I strongly support the publication of this paper.

## 2. Detailed comments

- page 3, line 42: I would suggest: 1. Reminder on port-Hamiltonian systems
- page 4,
  - line 27: I would suggest: this vector *is called* co-energy variables
  - line 37: I suggest: given by the *skew-symmetric matrix S(x)*
  - line 41: [9] complete with the chapter number or page !
- page 5,
  - lines 21-31: this paragraph should end with some relation with the definition of Dirac structures or be removed. Maybe the authors would have liked to define Port Hamiltonian systems ?
  - line 35-36: again from the text before, it does not appear clearly the relation between Dirac structures and Port Hamiltonian systems. Maybe the defintion of a Port Hamiltonian system should be presented before ? the name distributed port-Hamiltonian systems is given: it should be defined or there should be a precise reference to it.
  - line 51: please give name to the variables : linear, angular momentum etc ..
- page 6,
  - line 14: please give names to the variables: velocity etc ..
  - line 32: the equation (14) mixes two things: the Hamiltonian system equivalent to the equation (10) and the Hamiltonian operator J: what is the interconnection structure: is it not a defined by a Dirac structure ?
- page 7
  - line 47, give a name to $\sigma$: stress field
- page 8
  - line 3: the constant E is not defined.
  - lines 17-18: It seems that the correction factor $k$ could simply be integrated into $E$, as all coefficients of the matrices $D_b$ and $\overline{D}_s$ are proportional to E.
  - line 34: the Euler Lagrange is derived below: maybe a subsection 2.2 is convenient here ?
- page 9
  - line 32: I suggest the Title: Reminder on the PH vectorial formulation of the Mindlin plate [16]
  - lines 34-35: I prefer: in the sequel, the Mindlin plate model written as a the port-Hamiltonian system.
  - lines 35-38: this sentence does not provide useful information.
  - lines 38-42: the energy variables in (32) and the Hamiltonian functional in (33) should be related to the model presented in section 2.
- page 10
  - line 31-33: it is written: *The port-Hamiltonian system and the skew-adjoint operator relating energies and co-energies are found to be* This should be commented a little more. Indeed the Port Hamiltonian System (35) is quite different from the Euler-Lagrange system: firstly it is defined with respect to a differential Hamiltonian operator (whereas the Hamiltonian system obtained by the Legendre transformation of Euler-Lagrange system would lead toa symplectic Hamiltonian system with constant skew-symmetric matrix) and secondly the order of the Port Hamiltonian system is 8 which is strictly greater than the order of the Euler-Lagrange system which is 6 ! This is worth a little explanation, either by referring to an explanation in [16] (if there is any) or to an analoguous example, the vibrating string as exposed in [7, chapter 4] or maybe another reference ?
  - line 47: it is written *The boundary variables are found by evaluating the time derivative of the Hamiltonian* . Is it not true that the boundary port variables are defined from the Hamiltonian operator and the Stokes-Dirac structure derived from it ? At that point it would be more precise to write: *We shall establish the total energy balance in terms of ...*
  - line 59: maybe it would be good to specify that the vectors $n$ and s are unit vectors (normal and tangent to the boundary and in the plane $(x, y)$) ?
- page 11

  - line 28: why is the reference [25] cited ? I have checked and the Midlin plate is not treated there.
  - line 49: it is written *If instead the differential operator is of order two ...* As it is not the case, the sentence is superfluous !
- page 15
  - lines 57-61: the remark 2 would be very appropriate as an introduction of the section 3 !