THÈSE

En vue de l'obtention du

DOCTORAT DE L'UNIVERSITÉ DE TOUTOUSE

Délivré par : *l'Institut Supérieur de l'Aéronautique et de l'Espace (ISAE)*

Présentée et soutenue le *25 Avril 2019* **par :**

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Validation expérimentale d'un système de pointage de grande précision

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Experimental validation of a high accuracy pointing system

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Abstract

Satellite imagery provides contributions to land surveying, cartography, forestry and water management. In crisis management scenarios imagery can help to carry out rescue operations and to mitigate the results of natural catastrophes. Modern Space observation missions demand stringent pointing requirements that motivated a significant amount of research on the topic of microvibration isolation and line-of-sight stabilization systems. Microvibrations are a particular class of structural vibrations of small amplitude (micrometer) and with a large spectral content. They propagate from the sources to the flexible elements of the spacecraft and they cause the degradation of the gathered images and the loss of part of the vital information. There are many sources of micro-vibrations in the spacecraft. Reaction wheels are generally the main ones. Also other components, such as solar array drive mechanisms, antenna stepper motors, antenna trimming mechanisms or chemical thrusters, can have a huge impact. In order to have better pointing performances, the disturbances can be reduced by mounting some of the noisy equipment on various isolation platforms. However these solutions result ineffective at low frequency. Residual vibrations can still propagate through the spacecraft and be amplified by the flexible structure of the satellite. The present PhD thesis relies on an architecture based on an actuator that directly acts at payload level and on an additional rate sensor integrated in the line-of-sight. The conventional LOS control architectures use low frequency sensors. This work proposes to integrate an additional rate sensor on the most sensitive structural elements in the optical path to raise the bandwidth of the active control system. The first contribution relates to the flexible structure modeling. The multi-body modeling methodology called “Two-Input Two-Output Port (TITOP) approach” is extended to the case of flexible structures with N ports (N-Input N-Output Port: NINOP). This methodology is applied to Space systems to develop models of deployable solar arrays, reaction wheels and solar array drive mechanisms. The versatility of this framework allows the co-design of a structure/control problem by simple connection of elementary models associated to each sub-structure. All possible parametric uncertainties can be easily taken into account in order to perform robust performance analysis with the modern structured singular value techniques. These models permit the analysis of the pointing performances under the effect of microvibrations and their interactions with the gyroscopic effects of the reaction wheels and the spacecraft structural modes that vary with the solar panels rotation. The second contribution of this work is about the control laws synthesis. Following a comprehensive model identification and uncertainty quantification part, the robust control strategy is designed in the structured-$\mathcal{H}_\infty$ framework to account for plant uncertainty and provide formal worst-case performance guarantees. The last contribution is experimental. The comparison of the theoretical predictions and the experimental results obtained on the test bench developed at the European Space Agency permits the validation of the proposed approach. The results of this thesis, both on the modeling and experimental aspect, are generalized to be scaled to different observation missions.

Aknowledgements

When you come to write the acknowledgements it means that maybe the worst is over and it is time to reconcile with everyone. It is the moment of forgiveness and gratitude. Three dense years have passed. You have the feeling of having been hit by a spaceship at full speed and of having miraculously survived. You suddenly find yourself in deep Space after a failed extravehicular activity with the rope broken, like an umbilical cord in the vacuum of unemployment.

You have time to think whilst looking at the Earth that moves away with all its inhabitants. You think back to your first days of your PhD, when the doors of your office opened and phagocytized you.

A new family welcomed you: the PhD candidates (Flavio L.C.R., Pierrick R., Antoine S., Marie L., Vincent L., Nabil K., Anass S., Luca M. *et al.*). These are women and men known for their extreme bravery or the extravagant recklessness with which you can laugh at your common misfortunes or share successes. During afternoon breaks, shrouded in the reassuring smoke of Laurent and Michèle's cigarettes as we sipped coffee together, I improved my French and discovered that in some regions of France you can be "alone with someone else".

And now that I am swimming in the emptiness of Space, I look around to find the corner where paradise is hidden and say hello to my dear grandfather 'Ntino. Leaning on the handle of a hoe, he winks at me from a distance as if to to tell me: "Open your eyes, man!"

From Space you can see an infinite number of stars. There are so many that you lose direction. On Earth stars are few, but sometimes they help you to navigate in a stormy sea. I remember their names. A constellation, called "Family Major", was always the first one to be identified. It was so distant but so bright that you could hear its warm voice on the phone. Where the olive trees flourish in arid land, these stars have taught me the tenacity to believe that I can make great things with a few drops of water, with the humility and silence of a growing tree.

When I was recruited for the Space program the captains were Valérie P.B., Daniel A. and Fabrice B.. They showed me the instruction manual for the spaceship and helped me to correct the course when I was at the helm. They provided me with their technical knowledge and experience, but we also had time to discuss our lives on Earth.

During our intergalactic journey I landed for a few months on a planet called "the Bubble", where I met Diego N.T., Giordana B., Ozgun Y. and Marina M.. With them I spent unforgettable moments and a special force developed and kept us connected after we were separated.

Sometimes I returned to the land of olive trees to meet my childhood friends: Gabriele R., Daniele L., Cinzia D., Paolo R., Valentina G., Marco Z. and Paola P.. We talked about

how the world was changing around us and how we tried to use the old laws to interpret it. Indeed, we too have changed with the world and everyone has made his own Copernican Revolution. Nevertheless we continue to recognize ourselves, although with much less hair.

On the "Planet of the Artists" I met a group of delirious creatures, Andrea G., Andrea B., Paolo P., Andrea A. and Mina L.. We understood each other from the first moment without saying a word. Then we uttered some words and they were always extraordinary.

A couple of old friends, Marco S. and Geraldina G., opened the front door of their home whenever I needed refreshment or a smile.

I shared my dormitory during training with Simone U. and Aurora B.. With the former we shared the same hard fate of astronaut. The latter, with seraphic calm, bore all my untidiness.

I learned to climb mountains on the planet where there are no mountains, The Netherlands, with Alberto M., Karine Z., Silvia Z. and Chiara M.. On the same planet I met Robert A., Martina M. and Levin G.. With all of them I shared moments of unforgettable hilarity. At the same time an old astronaut whom I had already met in the past, Valentin P., was staying on the same planet. He explained to me all the secrets of hyperspace and how I could survive the intergalactic journey.

Like all Hollywood cult space films, there is also a love story, the fateful encounter with Soizic G., the Breton-Parisian girl, who disrupted the hero’s life by distracting him from the mission’s goal while making his journey all the greater for it.

Remerciements

Quand on arrive à écrire les remerciements, cela veut dire que, peut-être, le pire est passé et c'est le moment de se réconcilier avec tout le monde. C'est le moment du pardon et de la reconnaissance. Trois années denses comme le reste de la vie ont passé. On a l'impression d'avoir été percutés par un vaisseau spatial à toute vitesse mais d'avoir survécu miraculeusement. On se retrouve tout d'un coup dans l'Espace profond, après une activité extravéhiculaire qui s'est mal passée, avec la corde tranchée comme un cordon ombilical, dans le vide du chômage.

T'as le temps pour réfléchir maintenant en regardant la Terre qui s'éloigne avec tous ses habitants. Tu repenses aux premiers jours de ton doctorat, quand les portes du bureau se sont ouvertes en t'engloutissant. Une nouvelle famille qui t'accueille, les doctorants (Flavio L.C.R., Pierrick R., Antoine S., Marie L., Vincent L., Nabil K., Anass S., Luca M. *et al.*), femmes et hommes de courage extrême ou d'extravagante inconscience, avec qui rire sournoisement des mésaventures communes et avec qui partager les succés. Pendant les pauses de l'aprés-midi, en sirotant le café des machines, entourés par la fumée rassurante des cigarettes de Laurent et Michèle, j'ai perfectionné mon français et découvert qu'en certaines régions de France on peut être «seuls tous les deux».

Et maintenant que je nage dans le vide de l'Espace, je regarde autour de moi pour trouver le coin où se cache le paradis, pour saluer mon cher grand-père 'Ntino, qui penché sur le manche d'une houe me fait un clin d'œil de loin, comme pour dire : «Ouvre les yeux petit!».

Depuis l'Espace on voit un nombre infini d'étoiles, il y en a tellement qu'on perd l'orientation. Sur la Terre il y a peu d'étoiles, mais elles m'ont servi à prendre la bonne route dans une mer parfois orageuse. Je m'en rappelle les noms. Une constellation, appelée « Grande famille», a été toujours la première que j'identifiai, si distante mais si lumineuse qu'on en peut entendre la voix chaude au téléphone. Là où les oliviers poussent luxuriants dans une terre aride, ces étoiles m'ont appris la ténacité de croire qu'on peut faire des choses grandes avec quelques gouttes d'eau, avec l'humilité et le silence d'un arbre qui pousse.

Quand je fus recruté pour le programme spatial les capitaines étaient Valérie P.B., Daniel A. et Fabrice B.. Ils me montrèrent le livret d'instructions du vaisseau et m'aidèrent à corriger la route quand je fus au timon. Ils me transmirent leurs connaissances et leur expérience dans le domaine technique, mais nous eûmes aussi le temps de discuter de nos vies sur Terre.

Pendant le voyage intergalactique j'atterris pour quelque mois sur une planète appelée «The bubble», oû je rencontrai Diego N.T., Giordana B., Ozgun Y. et Marina M.. Avec eux je passai des moments inoubliables et une force spéciale s'établit en nous laissant connectés après notre séparation.

Parfois je rentrais à la terre des oliviers pour retrouver les amis d'enfance : Gabriele R., Daniele L., Cinzia D., Paolo R., Valentina G., Marco Z. et Paola P.. On parlait du monde qui changeait autour de nous, alors qu'on cherchait à utiliser les vieilles lois pour l'interpréter.

En fait nous avons aussi changé avec le monde et chacun a fait sa révolution Copernicienne. Malgré tout on continue encore à se reconnaitre même avec beaucoup moins de cheveux.

Sur la «planète des artistes» je rencontrai un groupe de créatures délirantes, Andrea G., Andrea B., Paolo P., Andrea A. et Mina L.. Avec eux on se comprit depuis le premier instant sans prononcer un mot. Après on en prononça et ils furent toujours extraordinaires.

Une couple de vieux amis, Marco S. et Geraldina G., ouvrirent la porte de leur maison toutes les fois que j'avais besoin d'un rafraichissement ou d'un sourire.

Je partageai le dortoir pendant la formation avec Simone U. et Aurora B.. Avec le premier on partagea le même sort amer de cosmonaute. La deuxième, avec un calme séraphique, toléra tout mon désordre.

J'appris à escalader les montagnes sur la planète où il n'y a pas de montagnes, les Pays-Bas, avec Alberto M., Karine Z., Silvia Z. et Chiara M.. Sur la même planète je rencontrai Robert A., Martina M. et Levin G.. Avec eux on partagea des moments d'inoubliable hilarité. A cette période vivait sur la même planète un vieux cosmonaute, que j'avais déjà rencontré dans le passé, Valentin P.. Il m'expliqua les secrets de l'hyperespace et comment survivre au voyage intergalactique.

Comme dans tous les films cultes hollywoodiens sur l'Espace il y a aussi l'histoire d'amour, la rencontre fatidique avec Soizic G., la breto-parisienne, qui bouleversa la vie du héros en le distrayant de l'objectif de sa mission, mais en rendant agréable le voyage.

Ringraziamenti

Quando si arriva a scrivere i ringraziamenti vuol dire che, forse, il peggio è passato ed è ora di riconciliarsi con tutti. È il momento del perdono e della riconoscenza. Tre anni densi come il resto della vita sono passati. Si ha l'impressione di essere stati investiti da una navicella spaziale a tutta velocità, ma di essere miracolosamente sopravvissuti. Ci si ritrova ad un tratto nello Spazio profondo, dopo un'attività extraveicolare andata male, con la fune spezzata come un cordone ombelicale, nel vuoto della disoccupazione.

Hai tempo per riflettere, ora, guardando la Terra che si allontana con tutti i suoi abitanti. Ripensi ai primi giorni del tuo dottorato, quando le porte dell'ufficio si sono aperte fagocitandoti. Una nuova famiglia che ti accoglie, i dottorandi (Flavio L.C.R., Pierrick R., Antoine S., Marie L., Vincent L., Nabil K., Anass S., Luca M. *et al.*), donne e uomini di estremo coraggio o di stravagante incoscienza, con cui ridere sornioni delle comuni sventure e con cui condividere i successi. Durante le pause pomeridiane, sorseggiando insieme il caffé delle macchinette, avvolti dal fumo rassicurante delle sigarette di Laurent e Michèle, ho perfezionato il francese e scoperto che in alcune regioni di Francia si può essere "soli tutti e due".

E ora che nuoto nel vuoto dello Spazio, mi guardo intorno per cercare l'angolo in cui è nascosto il paradiso, per fare un saluto al mio caro nonno 'Ntino, che appoggiato sul manico di una zappa mi fa da lontano un occhiolino, come per dirmi: "Apri gli occhi vagnò!".

Dallo Spazio si vede un numero infinito di stelle, ce ne sono talmente tante che si perde l'orientamento. Su Terra le stelle sono poche, ma sono servite a imboccare la giusta rotta in un mare a volte tempestoso. Di esse ne ricordo il nome. Una costellazione, chiamata "Famiglia maggiore", è stata sempre la prima da individuare, così lontana ma così luminosa da poterne ascoltare la calda voce al telefono. Lì dove gli ulivi crescono rigogliosi in un'arida terra, queste stelle mi hanno insegnato la tenacia nel credere di poter fare grandi cose con poche gocce d'acqua, con l'umiltà e il silenzio d'un albero che cresce.

Quando fui recrutato per il programma spaziale i capitani erano Valérie P.B., Daniel A. e Fabrice B.. Mi mostrarono il libretto d'istruzioni della navicella e mi aiutarono a correggere la rotta quando fui al timone. Mi trasmisero le loro conoscenze e la loro esperienza in campo tecnico, ma avemmo tempo anche di discutere delle nostre vite su Terra.

Durante il viaggio intergalattico atterrai per qualche mese su un pianeta, chiamato "The bubble", dove incontrai Diego N.T., Giordana B., Ozgun Y. e Marina M.. Con loro trascorsi momenti indimenticabili ed una forza speciale si istauró tenendoci connessi dopo esserci separati.

Talvolta tornavo nella terra degli ulivi per reincontrare gli amici dell'infanzia: Gabriele R., Daniele L., Cinzia D., Paolo R., Valentina G., Marco Z. e Paola P.. Parlavamo di come il mondo cambiava intorno a noi, mentre cercavamo di usare le vecchi leggi per interpretarlo. Siamo difatti cambiati anche noi col mondo e ognuno ha fatto la sua rivoluzione Copernicana.

Nonostante ció continuiamo a riconoscerci anche se con molti meno capelli.

Sul "pianeta degli artisti" incontrai un gruppo di deliranti creature, Andrea G., Andrea B., Paolo P., Andrea A. e Mina L.. Con loro ci capimmo dal primo istante senza pronunciare parola. Poi ne pronunciammo e furono sempre straordinarie.

Una coppia di vecchi amici, Marco S. e Geraldina G., aprirono la porta della loro casa tutte le volte che avevo bisogno di un ristoro o di un sorriso.

Condivisi il dormitorio durante l'addestramento con Simone U. e Aurora B.. Col primo condividemmo la stessa amara sorte di cosmonauta. La seconda, con serafica calma, sopportò tutto il mio disordine.

Imparai a scalare le montagne sul pianeta in cui non ci sono montagne, l'Olanda, con Alberto M., Karine Z., Silvia Z. e Chiara M.. Sullo stesso pianeta incontrai Robert A., Martina M. e Levin G.. Con tutti loro condividemmo momenti di indimenticabile ilarità. Nello stesso periodo soggiornava sullo stesso pianeta un vecchio cosmonauta, che avevo già incontrato in passato, Valentin P.. Mi spiegò i segreti dell'iperspazio e come avrei potuto sopravvivere al viaggio intergalattico.

Come tutti i cult spaziali holliwodiani c'è anche la storia d'amore, l'incontro fatidico con Soizic G., la breto-parigina, che sconvolse la vita dell'eroe distraendolo dall'obiettivo della missione, ma rendendo gradevole il viaggio.

Ai miei genitori e mio fratello,
che mi hanno insegnato
ciò che è importante

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List of Acronyms

ACS	*Attitude Control System*
APM	*Antenna Pointing Mechanism*
ARS	*Angular Rate Sensor*
AS	*Amplitude Spectrum*
CCS	*Controlled Component Synthesis*
CFSM	*Control Fast Steering Mirror*
CG	*Center of Gravity*
CMG	*Control Momentum Gyro*
CMS	*Component Modes Synthesis*
DFSM	*Disturbance Fast Steering Mirror*
DFT	*Discrete Fourier Transform*
DOF	*Degrees of Freedom*
EMC	*Electromagnetic compatibility*
ESA	*European Space Agency*
FEM	*Finite Element Method*
FE-TM	*Finite Element-Transfer Matrix*
FRF	*Frequency Response Function*
FSM	*Fast Steering Mirror*
HST	*Hubble Space Telescope*
IMU	*Inertial Measurement Unit*
JWST	*James Webb Space Telescope*
LFT	*Linear Fractional Transformation*
LPV	*Linear Parameter-Varying*
LQR	*Linear Quadratic Regulator*
LTI	*Linear Time-Invariant*

MHD	*Magneto-hydrodynamic*
NINOP	*N-Input N-Output Port*
PMA	*Proof-Mass Actuator*
PSD	*Power Spectral Density*
PZT	*Lead Zirconate Titanate piezoelectric actuator*
RW	*Reaction Wheel*
RWA	*Reaction Wheel Assembly*
SADM	*Solar Array Drive Mechanism*
SGS	*Strain Gauge Sensor*
STR	*Star Tracker*
TITOP	*Two-Input Two-Output Port*

Appendix A

TITOP theory

A.1 Inversion of TITOP models

Let us consider a square (same number of inputs and outputs) linear system G(s) with order n and m channels (i.e., m inputs and m outputs). A state-space realization of the system $\mathbf{G}(s)$ is a set of four matrices $\mathbf{A}_{n\times n}$, $\mathbf{B}_{n\times m}$, $\mathbf{C}_{m\times n}$, and $\mathbf{D}_{m\times m}$ such that

$$\mathbf{G}(s) = \mathbf{D} + \mathbf{C}\left(s\mathbf{I}_n - \mathbf{A}\right)^{-1}\mathbf{B} \tag{A.1}$$

The matrix $\mathbf{M}(\mathbf{I},\mathbf{J})$ (corresponding to the subsystem $\mathbf{G}_{\mathbf{I},\mathbf{J}}(s)$) denotes the system $\mathbf{G}(s)$ restricted to the rows ordered in vector $\mathbf{I}$ and the columns ordered in vector $\mathbf{J}$.

Single-channel inversion The system corresponding to the inversion of the i-th channel of the system $\mathbf{G}(s)$ ($i \in [1, m]$) is denoted $G^{-1i}(s)$. Let $\mathbf{J}$ be the vector of indexes from 1 to m without i: $\mathbf{J} = [1, \ldots, i-1, i+1, \ldots, m]$. If $\mathbf{D}\,(i,i) \neq 0$, let define $f_i = \mathbf{D}^{-1}\,(1,1)$. The inversion of the channel i is made in two steps [CDGA17]:

1. Compute the intermediate inverse system

$$\tilde{G}^{-1i}(s) = \left[\begin{array}{c|c} \mathbf{A} - f_i\mathbf{B}(:,i)\mathbf{C}(i,:) & \begin{bmatrix} \mathbf{B}(:,\mathbf{J}) - f_i\mathbf{B}(:,\mathbf{i})\mathbf{D}(i,\mathbf{J}) & f_i\mathbf{B}(:,i) \end{bmatrix} \\ \hline \begin{bmatrix} \mathbf{C}(\mathbf{J},:) - f_i\mathbf{D}(\mathbf{J},i)\mathbf{C}(i,:) \\ -f_i\mathbf{C}(i,:) \end{bmatrix} & \begin{bmatrix} \mathbf{D}(\mathbf{J},\mathbf{J}) - f_i\mathbf{D}(\mathbf{J},i)\mathbf{D}(i,\mathbf{J}) & f_i\mathbf{D}(\mathbf{J},i) \\ -f_i\mathbf{D}(i,\mathbf{J}) & f_i \end{bmatrix} \end{array}\right]. \tag{A.2}$$

   Note that in $\tilde{G}^{-1i}(s)$ the i-th inverted channel appears on the last channel.

2. Reorder the channels using the vector of indexes $\mathbf{K} = [1, \ldots, i-1, m, i+1, \ldots, m-1]$, and then

$$\mathbf{G}^{-1i}(s) = \tilde{\mathbf{G}}^{-1i}_{\mathbf{K},\mathbf{K}}(s) \tag{A.3}$$

A.2 Mass and kinetic matrices for the uniform beam element

$$\mathbf{M}_e = \frac{\rho S L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}, \tag{A.4}$$

$$\mathbf{K}_e = \frac{EJ}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}, \tag{A.5}$$

with ρ density, S cross-sectional area, L length, E Young Modulus and J second moment of area.

A.3 Mass and kinetic matrices for the uniform beam super-element

$$\mathbf{M}_e = \frac{\rho S L}{55440}\begin{bmatrix} 21720 & 3732L & 281L^2 & 6000 & -1812L & 181L^2 \\ 3732L & 832L^2 & 69L^3 & 1812L & -532L^2 & 52L^3 \\ 281L^2 & 69L^3 & 6L^4 & 181L^2 & -52L^3 & 5L^4 \\ 6000 & 1812L & 181L^2 & 21720 & -3732L & 281L^2 \\ -1812L & -532L^2 & -52L^3 & -3732L & 832L^2 & -69L^3 \\ 181L^2 & 52L^3 & 5L^4 & 281L^2 & -69L^3 & 6L^4 \end{bmatrix}, \tag{A.6}$$

$$\mathbf{K}_e = \frac{EJ}{70L^3}\begin{bmatrix} 1200 & 600L & 30L^2 & -1200 & 600L & -30L^2 \\ 600L & 384L^2 & 22L^3 & -600L & 216L^2 & -8L^3 \\ 30L^2 & 22L^3 & 6L^4 & -30L^2 & 8L^3 & L^4 \\ -1200 & -600L & -30L^2 & 1200 & -600L & 30L^2 \\ 600L & 216L^2 & 8L^3 & -600L & 384L^2 & -22L^3 \\ -30L^2 & -8L^3 & L^4 & 30L^2 & -22L^3 & 6L^4 \end{bmatrix}, \tag{A.7}$$

with ρ density, S cross-sectional area, L length, E Young Modulus and J second moment of area.

A.4 Mass and kinetic matrices for the Kirchhoff element

```
%
%  ^
% y|                          Input:
%  |                          rho: density
%  |                          E :  Young Modulus [i.e.: Pa]
%  4 ------- 3                ni:  Poisson Ratio [/]
%  |         |                lx:  x-element length [i.e.: m]
%  |         |                ly:  y-element length [i.e.: m]
%  1 --------2 -----> x       t :  plate thickness  [i.e.: m]
%

syms rho E ni lx ly t x y;

% Coordinates
px = [0;lx;lx;0];
py = [0;0;ly;ly];

% Flexural Rigidity
D = E*t^3/12/(1-ni^2);
DD = [D ni*D 0; ...
      ni*D D 0; ...
      0 0 (1-ni)*D/2];
P = sym([]);
P = [1 x y x^2 x*y y^2 x^3 x^2*y x*y^2 y^3 x^3*y x*y^3; ...
     0 0 1 0 x 2*y 0 x^2 2*x*y 3*y^2 x^3 3*x*y^2; ...
     0 -1 0 -2*x -y 0 -3*x^2 -2*x*y -y^2 0 -3*x^2*y -y^3];

C = [];
for i = 1:4
   C = [C; [1 px(i) py(i) px(i)^2 px(i)*py(i) py(i)^2 px(i)^3 px(i)^2*py(i) px(i)*py(i)^2 py(i)^3 px(i)^3*py(i) px(i)*py(i)^3; ...
     0 0 1 0 px(i) 2*py(i) 0 px(i)^2 2*px(i)*py(i) 3*py(i)^2 px(i)^3 3*px(i)*py(i)^2; ...
     0 -1 0 -2*px(i) -py(i) 0 -3*px(i)^2 -2*px(i)*py(i) -py(i)^2 0 -3*px(i)^2*py(i) -py(i)^3]];
end

% Shape function matrix
N = sym([]);
N = P*inv(C);
Q = sym([]);
Q = [0 0 0 -2 0 0 -6*x -2*y 0 0 -6*x*y 0; ...
     0 0 0 0 0 -2 0 0 -2*x -6*y 0 -6*x*y; ...
     0 0 0 0 -2 0 0 -4*x  -4*y 0 -6*x^2 -6*y^2];
B = sym([]);
B = Q*inv(C);

% Stiffness Matrix
Kel = int((int(B.'*DD*B,x,0,lx)),y,0,ly);

% Mass Matrix
J = rho.*diag([t,t^3/12,t^3/12]);
Mel = int(int(N.'*J*N,x,0,lx),y,0,ly);
```

A.5 Data of the test spacecraft

System	Parameter	Description	Value & Uncertainty
Main Hub	m_{MH}	Mass	$800\,\mathrm{kg} \pm 5\%$
	$\begin{bmatrix} J_x^{\mathrm{MH}} & J_{xy}^{\mathrm{MH}} & J_{xz}^{\mathrm{MH}} \\ & J_y^{\mathrm{MH}} & J_{yz}^{\mathrm{MH}} \\ & & J_z^{\mathrm{MH}} \end{bmatrix}$	Inertia in main body frame	$\begin{bmatrix} 381 & -9 & 3 \\ & 800 & -40 \\ & & 94 \end{bmatrix}\,\mathrm{kg\,m^2} \pm 5\%$
	$\mathbf{r}_G$	Spacecraft CG	$[1.4\ 0.01\ 0.03]\,\mathrm{m}$
	$\mathbf{r}_P$	SA attachment point	$[1.25\ -0.6\ 0]\,\mathrm{m} \pm 5\%$
Solar Array	m_{SA}	Mass	$40\,\mathrm{kg} \pm 5\%$
	$\begin{bmatrix} J_x^{\mathrm{SA}} & J_{xy}^{\mathrm{SA}} & J_{xz}^{\mathrm{SA}} \\ & J_y^{\mathrm{SA}} & J_{yz}^{\mathrm{SA}} \\ & & J_z^{\mathrm{SA}} \end{bmatrix}$	Inertia in SA frame	$\begin{bmatrix} 60 & 0.4 & -0.1 \\ & 18 & 21 \\ & & 64 \end{bmatrix}\,\mathrm{kg\,m^2} \pm 5\%$
	$\mathbf{r}_o^{\mathrm{SA}}$	SA CG in SA frame	$[0\ -1.7\ 0.03]\,\mathrm{m}$
	$\left[\omega_1^{\mathrm{SA}}\ \omega_2^{\mathrm{SA}}\ \omega_3^{\mathrm{SA}}\ \omega_4^{\mathrm{SA}}\ \omega_5^{\mathrm{SA}}\right]$	Flexible modes' frequencies	$[0.45\ 2.5\ 12\ 28\ 62]\,\mathrm{Hz} \pm 5\%$
	$\zeta_1^{\mathrm{SA}}, \zeta_2^{\mathrm{SA}}, \zeta_3^{\mathrm{SA}}, \zeta_4^{\mathrm{SA}}, \zeta_5^{\mathrm{SA}}$	Flexible modes' damping	$0.03 \pm 5\%$
	$\mathbf{L}_{\mathrm{SA}}$	Modal participation factors	$\begin{bmatrix} 0 & 5 & 0.3 & 0 & 0 \\ -1.5 & 0 & 0 & -1 & 6 \\ -4.7 & 0 & 0.03 & 2.8 & -1 \\ 14 & 0 & -0.02 & -2 & 3 \\ 0 & 2 & 3 & -0.03 & -0.01 \\ 0 & 14 & -0.2 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$
Reaction Wheel Assembly	m_W	Wheel mass	$2.95\,\mathrm{kg} \pm 1\%$
	J_{W_r}	Wheel radial inertia	$21 \cdot 10^{-3}\,\mathrm{kg\,m^2} \pm 1\%$
	J_{W_z}	Wheel axial inertia	$75 \cdot 10^{-3}\,\mathrm{kg\,m^2} \pm 1\%$
	m_A	Plate mass	$1.01\,\mathrm{kg} \pm 1\%$
	J_A	Plate inertia	$\mathrm{diag}(15.8\ 15.8\ 64.45) \cdot 10^{-3}\,\mathrm{kg\,m^2} \pm 1\%$
	$[d_W\ d_A\ d_{A_1}\ d_{A_2}\ d_I\ d_S]$	see Fig. **??**	$[6\ 0.3\ 3\ 3\ 9.18\ 4.2] \cdot 10^{-2}\,\mathrm{m} \pm 1\%$
	$\mathbf{K}_B$	Bearing stiffness (translations)	$7.861 \cdot 10^6 \mathbf{I}_3\,\mathrm{N/m}$
		Bearing stiffness (rotations)	$3.7 \cdot 10^4 \mathbf{I}_2\,\mathrm{N\,m/rad}$
	$\mathbf{C}_B$	Bearing damping (translations)	$200\mathbf{I}_3\,\mathrm{N\,s/m}$
		Bearing damping (rotations)	$0.942\mathbf{I}_2\,\mathrm{N\,s\,m/rad}$
	$\mathbf{K}_I$	Isolator stiffness (translations)	$1.013 \cdot 10^5 \mathbf{I}_3\,\mathrm{N/m}$
		Isolator stiffness (rotations)	$\begin{bmatrix} 2.026 \cdot 10^3 \mathbf{I}_2 & \\ & 2 \cdot 2.026 \cdot 10^3 \end{bmatrix}\,\mathrm{N\,m/rad}$
	$\mathbf{K}_I$	Isolator damping (translations)	$256\mathbf{I}_3\,\mathrm{N\,s/m}$
		Isolator damping (rotations)	$\begin{bmatrix} 5.12\mathbf{I}_2 & \\ & 2 \cdot 5.12 \end{bmatrix}\,\mathrm{N\,s\,m/rad}$
	$\left[C_{f_1}\ C_{f_2}\ C_{f_3}\ C_{f_4}\right]$	Force coefficients	$[7.160\ 2.721\ 0.709\ 0.945] \cdot 10^{-6}\,\mathrm{N/(rad/s)^2}$
	$[C_{t_1}\ C_{t_2}\ C_{t_3}\ C_{t_4}]$	Torque coefficients	$[2.954\ 0.678\ 0.282\ 0.110] \cdot 10^{-6}\,\mathrm{N\,m/(rad/s)^2}$
Solar Array Drive Mechanism	K_m	Motor torque const	$3\,\mathrm{N\,m/(A\,rad)}$
	K_d	Detent torque const	$1.08\,\mathrm{N\,m/rad}$
	z	Rotor teeth	90
	p	Motor poles	4
	n_μ	μ-step subdivisions	8
	I	Phase current	$0.1\,\mathrm{A}$
	J_r	Rotor inertia	$1.3 \cdot 10^{-4}\,\mathrm{kg\,m^2}$
	C_r	Viscous damping	$15 \cdot 10^{-4}\,\mathrm{N\,m\,s/rad}$
	Ω_{S}	Rotor angular speed	$1 \cdot 10^{-3}\,\mathrm{rad/s}$
Gearbox	K_b	Stiffness	$96000\,\mathrm{N\,m/rad}$
	C_b	Viscous damping	$0.1\,\mathrm{N\,m\,s/rad}$
	N_g	Reduction ratio	60

Bibliography

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Résumé — L'imagerie satellitaire aide à la surveillance des terrains agricoles, à la cartographie, à la sylviculture et à la surveillance des eaux. Dans des situations de crise, l'imagerie peut contribuer à la conduite d'opérations de sauvetage et à l'atténuation des effets des catastrophes naturelles. Les missions d'observation de la Terre actuelles requièrent de hautes performances en pointage fin qui justifient les nombreuses recherches sur l'atténuation des micro-vibrations et les systèmes de stabilisation de la ligne de visée. Les micro-vibrations sont une classe particulière de vibrations structurelles de petite amplitude (micrométrique) et avec un large contenu spectral. Elles se propagent depuis les sources aux éléments flexibles du satellite et provoquent la dégradation des images collectées et la perte d'une partie de l'information vitale. Dans les plateformes satellitaires, elles sont causées par différentes sources. Parmi les plus importantes se trouvent les roues à réaction utilisées pour le contrôle d'attitude et les moteurs d'entrainement des panneaux solaires. Ce travail de thèse présente une méthodologie complète pour la modélisation d'un satellite flexible avec ses sources principales de micro-vibrations et la synthèse d'une nouvelle stratégie de contrôle actif pour réduire la dégradation de la ligne de visée. Un banc d'essai expérimental développé à l'Agence Spatiale Européenne a permis de valider l'architecture de contrôle proposée.

Mots clés : Micro-vibrations, dynamique flexible, contrôle rôbuste, fusion senseurs.

Abstract — Satellite imagery provides contributions to land surveying, cartography, forestry and water management. In crisis management scenarios imagery can help to carry out rescue operations and to mitigate the results of natural catastrophes. Modern Space observation and Science missions demand stringent pointing requirements that motivate a significant amount of research on the topic of microvibration mitigation and line-of-sight stabilization systems. Microvibrations are a particular class of structural vibrations with small amplitude (micrometer and sub-micrometer) and wide frequency content. They propagate from the sources to the flexible elements of the spacecraft and cause the degradation of the collected images and the loss of a part of the vital information. On spacecraft they are generated by different sources. Among them the most important sources are the reaction wheels used for attitude control and the solar array drive mechanisms used to orient rotating solar panels. The present PhD thesis aims to present an end-to-end methodology to model a flexible spacecraft with its main sources of microvibrations and to implement a novel active control strategy to mitigate the line-of-sight degradation. An experimental test bench developed at the European Space Agency has enabled the validation of the proposed control architecture.

Keywords: Microvibrations, flexible dynamics, robust control, hybrid sensing.

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