REVIEW PAPER APM-D-18-02038

1. General comments

This paper is a very interesting follower of the companion paper on the Port Hamiltonian formulation of the Mindlin plate. Theorem 2 (Stokes-Dirac structure of the Kirchhoff Plate), presented in the section 3, is very nice and interesting and the proof proves the definition of the boundary port variables very nicely. The section 4 about a structure-preserving discretization is also very interesting and will attract a lot of interest.

However the paper should be improved in the following aspects. I find it necessary, to add the definition of the Stokes-Dirac structure at the end of the section 3; it would be a natural to present it as the extension of the operator defined in equation (54). The proof would not be necessary as it would be very similar to that of the section 2 in vector form. The conclusion is very poorly written but can easily been improved. Detailed suggestions are also presented in the next section.

Having read now the two companion paper I would also suggest that in the first one (as indicated in the review) include the precise definition of Dirac and Stokes-Dirac structure as extension of Hamiltonian operators and that the second paper refers to it.

These are however minor changes, concerning essentially the presentation of the results. therefore I recommend the publication of the paper provided the minor revisions suggested here and in the detailed comments, are completed.

2. Detailed comments

- page 1, line 48: mispelled: *first*
- page 2
  - line 20: mispelled: *through*
  - lines 44-47: the sentence *The reader can refer to [17] ...as PH Bernoulli beams.* It is not clear why it is written here; I think that it should rather be inserted at the end of the section as reference to the use of this model for simualtion and control purposes.
- page 5, line37-38: the rigidity is defined in two ways as a constant and as a position depend coefficient: give the properties of this coeffcient and give a justification of your choice !
- page 6
  - line 43: the sentence *The spatial derivatives of the acceleration have been neglected.* is cryptic ! That means that equation (19) does not stems from hamilton's principle ?
  - line 50: please precise: in the sentence: *In the sequel it is assumed* that the load density $p = 0$.
- page 7 :
  - line 7-8 it is written: *Since the Kirchhoff plate represents the 2D extension of the Euler-Bernoulli beam, it is natural to select as energy variables ....* Maybe it would be better not to argue with "common sense in Mechanics" and be more clearly justify the choice of the energy variables from the expression of the kinetic and the potential energy in page 6, lines 18-22.

  - lines 23-24: it is written: *The port-Hamiltonian system and the formally skew-adjoint operator relating energy and co-energy variables are found ...* How do you find it ? there is nowhere written the equations of motion or a reference to it ?
- page 9:
  - line 8: it is written *It is well known that variables v and $\frac{\partial v}{\partial s}$ are kinematically related.* Actually what is used below is the fact that they are differentially dependent (with respect to derivation along s the curvilinear abcsissa of the boundary domain) and this is immediately seen. So what is actually achieved is to express the energy balance equation with respect to independent variables.
  - line 33: it is written *The space of boundary conditions* but the authors mean *The space of boundary variables* as it is written line 43
- page 12
  - line 31: I guess that the authors mean *In fact* instead of *Indeed* ?
  - lines 40-41: the sentence .. *will correspond to the mixed derivatives of the vertical displacement instead of its double*, is not very clear. Rather write (differing by 1/2 from the definition page ...)
  - page 13
    - line 5: it is written: *The port-Hamiltonian system ...* what Hamiltonian system do you mean ? give a precise reference to the eqution you mean.
    - line 49: it is written *We try to identify A^* ..* rather write *We shall compute / identify ..*
- page 14
  - lines 4-6: I would reverse the 2 sentences; first: A classical result is the fact that the adjoint of the vector divergence is $div^* = grad$ and give a reference there. and then. *This may be generalized to* the adjoint of the tensor divergence ..
  - line 33: it is written *Again the boundary values*, whereas it should be *boundary port variables*
- page 15
  - lines 28-34: these lines describing the discretization method are too vague. I suggest to write preciely which of the equations are then integrated by parts and write the choices of input variables to which this corresponds.
- page 20
  - line 10, you write: *shares the same properties*, but it is not clear which properties ? Rather state *share the Port Hamiltonian structure.*
  - line 13: it is written *Furthermore, this method is easily implementable by standard finite element libraries.* This is not justified; which libraries ? why ? I doubt that standard libraries encompass the Port Hamiltonian systems N!
  - the paragraph starting line 16 is very poorly written, although the subject should be mentioned. Please recall where $\phi_1$isdefined and to which physical variable it corresponds and do the same with theremaining variables; F-giev references to the choices of discretization basis that you mention. The sentence *These elements are however difficult ... level.* is not understandable.

Rather give references where ist is used and why they are difficult to implement. What are *the less regular elements*, five examples, references and eventually references to standar/open FEM packages ?

– lines 35 and following. The conclusion has been too hastily written and is quite weak. The results should be summarized precisely in 1 paragraph. Concerning the open questions, they should be formulated in such a way to give a precise idea on the path to follow. The statement: *First of all the functional spaces in which the variables live needs to be specified precisely.* is weak. I'd rather stated that the presented formulation should be completed with amore precise analysis of the well-posedness of the system in the input-output sense, generalizing to a second-order operator the results of Kurula and Zwart in the UT Memorandum 1994 *The duality between the gradient and divergence operators on bounded Lipschitz domains* (see also the reference [9] about the trace operators and Port Hamiltonian systems) for 2-D systems.

The statement *Then it would be of great interest to interconnect ...* is also not precise enough ! State for instance that the results presneted in this paper enables the interconnection in the sense of Dirac structures and their composition on Hilbert spaces Mikael Kurula et al., J. . Math. Anal. Appl. 372 (2010) 402–422, that the structure presernving deiscretization enables to adapt easily commercial software etc ..

- References:
  – please cite as much as possible journal papers and chapters from books instead of conference papers
  – please write Hamiltonian with capital H in the titles of the papers: this is achieved by writing {H}amiltonian in the bib file !
  – for the Port Hamiltonian formulation on jet bundles of the Mindlin plate rather cite Schöberl M., Schlacher K. (2017) Variational Principles for Different Representations of Lagrangian and Hamiltonian Systems. In: Irschik H., Belyaev A., Krommer M. (eds) Dynamics and Control of Advanced Structures and Machines. Springer, First Online 12 November 2016 DOI https://doi.org/10.1007/978-3-319-43080-5_7