A port-Hamiltonian formulation for the full Von-Karman plate model

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Summary. In this contribution, a port-Hamiltonian reformulation of the full von-Karman dynamical model for geometrically non-linear plates is detailed. Starting from the canonical equations, a set of variables is chosen so that that make the total energy quadratic. The model, reformulated in these variables, highlights a port-Hamiltonian structure ruled by a state-modulated interconnection operator.

Classical model

The classical full von-Karman dynamical model is detailed in Bilbao et al. [2015]. In dimensionless variables the problem, defined an open connected set $\Omega \subset \mathbb{R}^2$, it reads

$$\begin{aligned} \ddot{\boldsymbol{u}} &= \operatorname{Div} \boldsymbol{N}, & \boldsymbol{N} &= \boldsymbol{\Phi}(\boldsymbol{\varepsilon}), & \boldsymbol{\varepsilon} &= \operatorname{Grad} \boldsymbol{u} + 1/2 \operatorname{grad} w \otimes \operatorname{grad} w, \\ \ddot{w} &= -\operatorname{div} \operatorname{Div} \boldsymbol{M} + \operatorname{div}(\boldsymbol{N} \operatorname{grad} w), & \boldsymbol{M} &= \boldsymbol{\Phi}(\boldsymbol{\kappa}), & \boldsymbol{\kappa} &= \operatorname{Grad} \operatorname{grad} w \end{aligned} \tag{1}$$

where $\boldsymbol{u} \in \mathbb{R}^2$ is the in-plane displacement, w is the vertical displacement, $\boldsymbol{\varepsilon}$ is the in-plane strain tensor, $\boldsymbol{\kappa}$ is the curvature tensor, $\boldsymbol{N}$ is the in-plane stress resultant and $\boldsymbol{M}$ is the bending stress resultant. The notation $\boldsymbol{a} \otimes \boldsymbol{b} = \boldsymbol{a}\boldsymbol{b}^\top$ denotes the outer product of two tensors. The operator div is the divergence of a vector field and grad the gradient of a scalar field. The symmetric part of the gradient operator Grad (i. e. the deformation gradient in continuum mechanics) is given by

$$\operatorname{Grad}(\boldsymbol{u}) := \frac{1}{2}\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^\top\right) \in \mathbb{S} := \mathbb{R}^{2\times 2}_{\text{sym}}.$$

The Hessian operator of a scalar field u is then computed as follows

$$\operatorname{Hess}(u) = \operatorname{Grad}(\operatorname{grad}(u)) \in \mathbb{S}.$$

For a tensor field $\boldsymbol{U} : \Omega \rightarrow \mathbb{M}$, with components U_{ij}, the divergence Div is a vector, defined column-wise as

$$\operatorname{Div}(\boldsymbol{U}) := \sum_{i=1}^{2} \partial_{x_i} U_{ij}, \qquad \forall j = \{1, 2\}.$$

The tensor mapping $\boldsymbol{\Phi}$ is positive and preserves the symmetry

$$\Phi(\boldsymbol{A}) = \nu \operatorname{Tr}(\boldsymbol{A}) \boldsymbol{I}_{2\times 2} + (1-\nu)\boldsymbol{A}, \qquad \boldsymbol{A} = \boldsymbol{A}^\top \implies \boldsymbol{\Phi}(\boldsymbol{A}) = \boldsymbol{\Phi}(\boldsymbol{A})^\top.$$

The total energy of the model is computed

$$H = \frac{1}{2}\int_\Omega \left\{ \|\dot{\boldsymbol{u}}\|^2 + \dot{w}^2 + \boldsymbol{N} : \boldsymbol{\varepsilon} + \boldsymbol{M} : \boldsymbol{\kappa} \right\} \, \mathrm{d}\Omega, \qquad \text{where} \qquad \boldsymbol{A} : \boldsymbol{B} = \operatorname{Tr}(\boldsymbol{A}^\top \boldsymbol{B}). \tag{2}$$

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Figure 1: Greatest figure of all time

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Conclusions

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References

Stefan Bilbao, Olivier Thomas, Cyril Touzé, and Michele Ducceschi. Conservative numerical methods for the full von kármán plate equations. *Numerical Methods for Partial Differential Equations*, 31(6):1948–1970, 2015. doi: 10.1002/num.21974. URL `https://onlinelibrary.wiley.com/doi/abs/10.1002/num.21974`.