- **RELAZIONI**

DEF: Il prodotto cartesiano di $A_1, .., A_n$ é:

$$A_1 \times A_2 \times \dots \times A_n = \{ (a_1, a_2, .., a_n) \; a_i \in A_i \text{ con } i = 1, .., n \}$$

ES: $A = \{1, 2\}$ $B = \{0, \square\}$ $\Rightarrow A \times B = \{(1,0); (1,\square); (2,0); (2,\square)\}$

DEF: Una relazione n-aria sugli insieme $A_1, ..., A_n$ é un sottoinsieme

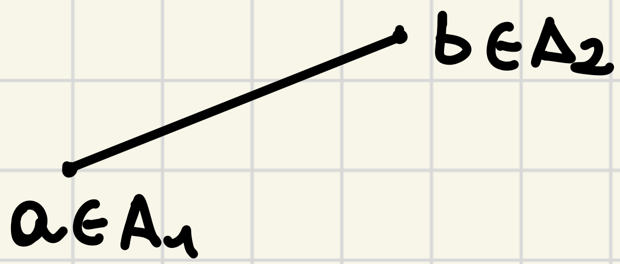
$$\mathbb{R} \subseteq A_1 \times \dots \times A$$

- 1-ARIA $\mathbb{R} \subseteq A_1$
- Binaria $\mathbb{R} \subseteq \underset{\text{dominio}}{A_1} \times \overset{\text{codominio}}{A_2}$

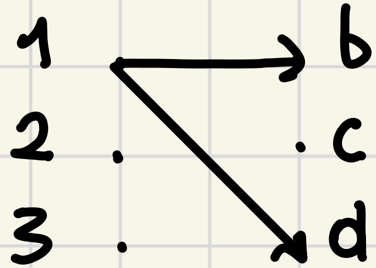
METODO DI RAPPRESENTAZIONE $\mathcal{R} \subseteq A_1 \times A_2$ se A_1, A_2 sono finiti

1. Elencare gli elementi $\mathcal{R} = \{(a_1, b_1), (a_2, b_2)\}$

2. Grafo di adiacenza: i vertici $A_1 \cup A_2$ e disegno una freccia



ES: $A_1 = \{1, 2, 3\}$ $A_2 = \{b, c, d\}$

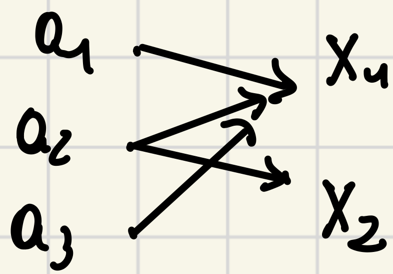


$\mathcal{R} = \{(1, b), (1, d)\}$

3. Matrice d'adiacenza: numero $A_1 = \{a_1, \dots, a_m\}$ $A_2 = \{b_1, \dots, b_n\}$

allora $M_{\mathcal{R}} \in \mathbb{M}_{m,n}(\{0,1\})$ definita $(M_{\mathcal{R}})_{ij} = \begin{cases} 1 & \text{se } (a_i, b_j) \in \mathcal{R} \\ 0 & \text{altrimenti} \end{cases}$

ES: $A = \{a_1, a_2, a_3\}$ $B = \{x_1, x_2\}$



$$M_R = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

OPERAZIONI:

1) UNIONE / INTERSEZIONE: $R, T \subseteq A_1 \times A_2$ $R \cup T = \{(a,b) : (a,b) \in R \text{ o } \in T\}$

$R \cap T = \dots$ e

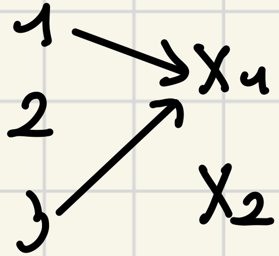
2) PRODOTTO DI RELAZIONI: $R \subseteq A_1 \times A_2$ $T \subseteq A_2 \times A_3$ $R \cdot T \subseteq A_1 \times A_3$

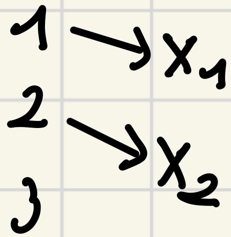
$$R \cdot T = \{(a,c) \in A_1 \times A_3 \mid \exists b \in A_2 : (a,b) \in R \wedge (b,c) \in T\}$$

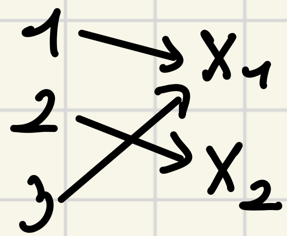
3) INVERSA DI RELAZIONE: $R \subseteq A_1 \times A_2 \Rightarrow R^{-1} \subseteq A_2 \times A_1$

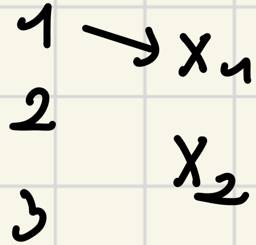
$$R^{-1} = \{(b,a) \in A_2 \times A_1 : (a,b) \in R\}$$

METODI DI RAPPRESENTAZIONE E OPERAZIONI

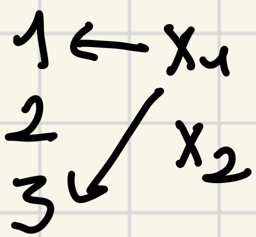
$\cup, \cap$) $R=$ 

$T=$ 

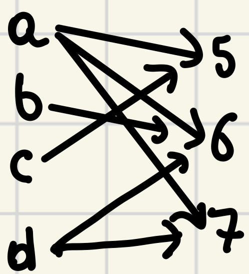
$R \cup T =$ 

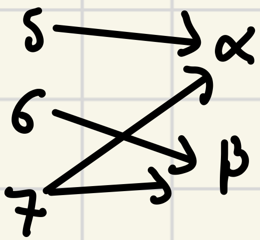
$R \cap T =$ 

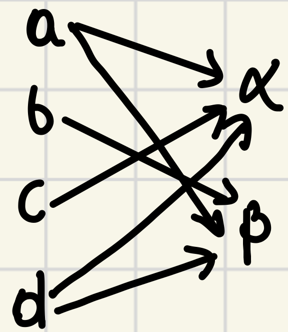
$$M(R\cup T) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad M(R\cap T) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

R^{-1}) $R^{-1} =$ 

$$M(R^{-1}) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = R^T$$

PRODOTTO) R: 

T: 

$R \cdot T$: 

$$M(R) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad M(T) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$M(R \cdot T) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad (M_R \cdot M_T)_{ij} = \sum_{k=1}^{A_2} (M_R)_{ik} (M_T)_{kj}$$

se $(M_R \cdot M_T)_{ij} > 1$ allora sono in relazione ed ho più di un modo

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$