RELAZIONI

Def: Il prodotto cartesiano di $A_1, \dots, A_n$ é così definito:

$$A_1 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) \text{ con } a_i \in A_i \text{ dove } i = 1, \dots, n\}$$

Es: $A = \{1,2\}$ $B = \{0, \square\}$ $\Rightarrow A \times B = \{(1,0); (1,\square); (2,0); (2,\square)\}$

Def: Una relazione n-aria sugli insiemi $A_1, \dots, A_n$ é un sottoinsieme:

$$R \subseteq A_1 \times \dots \times A_n$$

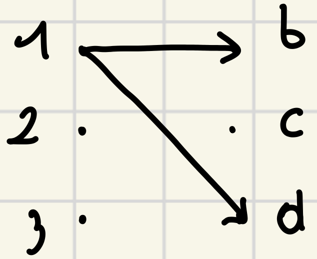
- 1-ARIA $R \subseteq A_1$
- 2-ARIA o BINARIA $R \subseteq \underset{\text{dominio}}{A_1} \times \overset{\text{codominio}}{A_2}$

· METODO DI RAPPRESENTAZIONE (relazione R binaria con A_1 e A_2 finiti)

- ELENCARE ELEMENTI: $R = \{(a_1, b_1) ; (a_2, b_2)\}$

- GRAFO DI ADIACENZA: considero vertici gli elementi di A_1 e A_2 poi disegno una freccia se sono in relazione

ES: $A_1 = \{1, 2, 3\}$ $A_2 = \{b, c, d\}$ $R = \{(1, b) ; (1, d)\}$



- MATRICE DI ADIACENZA: $A_1 = \{a_1, \dots, a_m\}$ $A_2 = \{b_1, \dots, b_n\}$, $m, n \in \mathbb{N}$

allora $M_{R_{ij}} = \begin{cases} 1 \text{ se } (a_i, b_j) \in R \\ 0 \text{ altrimenti} \end{cases}$

ES: nella relazione di prima si ha che:

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

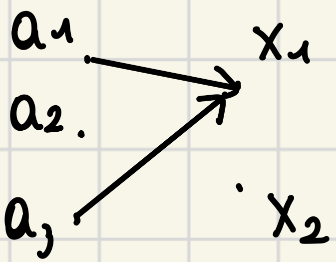
- **OPERAZIONI** :
  - UNIONE : $R, T \subseteq A_1 \times A_2 \Rightarrow R \cup T = \{ (a,b) \mid (a,b) \in R \vee \in T \}$
  - INTERSEZIONE : $R, T \subseteq A_1 \times A_2 \Rightarrow R \cap T = \{ (a,b) \mid (a,b) \in R \wedge \in T \}$
  - PRODOTTO DI RELAZIONI : $R \subseteq A_1 \times A_2 \quad T \subseteq A_2 \times A_3 \quad R \cdot T \subseteq A_1 \times A_3$

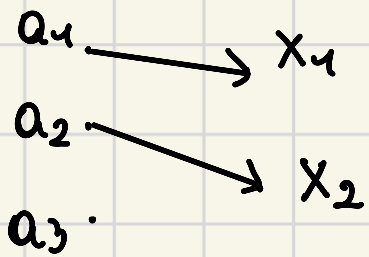
$$R \cdot T = \{ (a,c) \in A_1 \times A_3 \mid \exists b \in A_2 \mid (a,b) \in R \wedge (b,c) \in T \}$$

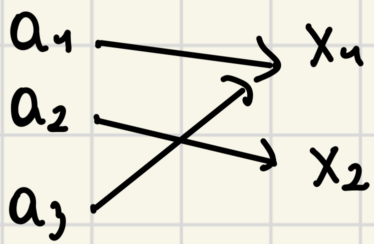
  - INVERSA DI RELAZIONE : $R \subseteq A_1 \times A_2 \Rightarrow R^{-1} \subseteq A_2 \times A_1$

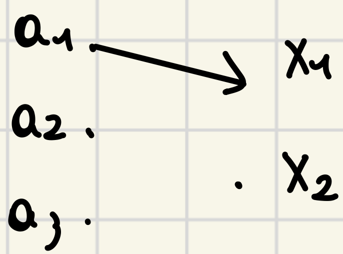
$$R^{-1} = \{ (b,a) \in A_2 \times A_1 \mid (a,b) \in R \}$$

· RAPPRESENTAZIONE DELLE OPERAZIONI

- $\cup, \cap$: $R =$ 

$T =$ 

$\Rightarrow R \cup T =$ 

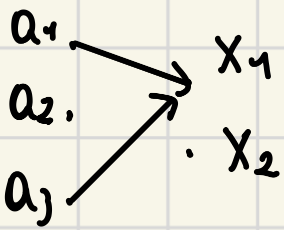
$R \cap T =$ 

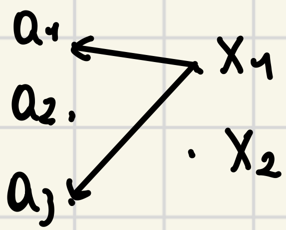
$$M_R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad M_T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

somma booleana

$$M_{(R \cup T)} = M_R \vee M_T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

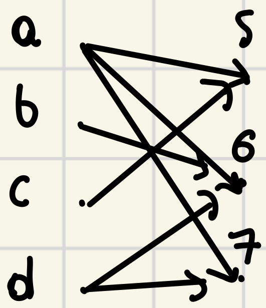
$$M_{(R \cap T)} = M_R \wedge M_T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

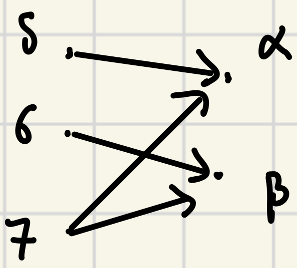
- R^{-1} : $R=$ 

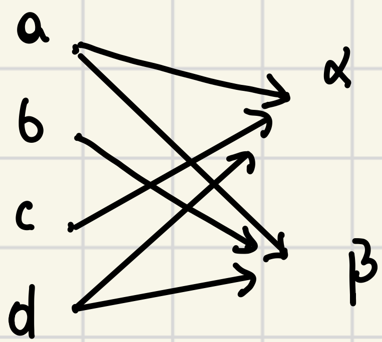
$R^{-1}=$ 

$$M_R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$M_{R^{-1}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = M_R^T$$

- $\cdot$: $R=$ 

$T=$ 

$R \cdot T=$ 

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad M_T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$M_{(R \cdot T)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = M_R \cdot M_T$$ infatti:

percorsi possibili

$$M_R \cdot M_T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \text{``}=\text{''} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$