NSPDE2021 - Computational exercise 4

This exercises are meant to keep you busy for the next couple of weeks.

(1) Modify your Finite Difference elliptic problems codes to solve general parabolic problems using the θ-method in time and FD in space assuming constant coefficients in time. Note that, in this case, most of the assembly work does not need to be repeated at each time-step, hence it should be computed once and for all before the time-marching loop.

(2) Verify experimentally the theoretical order of convergence in the $l_\infty - l_\infty$ norm seen in the lectures by considering a simple heat-equation problem with known exact solution.

(3) Repeat (1)-(2) for the θ-method in time and FEM in space, this time starting from your FEM codes and considering the $l_\infty - L_2$ norm for the convergence analysis.

(4) Test in practice the theoretical stability conditions seen in the lectures for both FD and FEM.

(5) A question worth considering is that of the linear solvers. Considering the 2D case, compare computational costs using direct and iterative solvers, taking advantage of (incomplete) factorisations and appropriate initial guesses (eg. the solution at the previous time-step). You may also test solution efficiency with respect to the explicit (forward) Euler method with mass lumping.

(6) Implement the second order in time method BDF(2) combined with a 1-step method for the initialisation time-step and compare.