NSPDE2021 - Computational exercise 3

This exercises are meant to keep you busy for the next couple of weeks.

(1) Solve the Poisson problem $-\Delta u = 10$ with Dirichlet boundary conditions on the L-shaped domain $[-1,1]^2$. You may use the provided function `lshape_mesh_generator.m` and `lshape_get_boundary_node.m` to generate a structured triangular mesh and/or the matlab command `[p,e,t] = initmesh('lshapeg','Hmax',0.1)` to generate an unstructured triangular mesh.

(2) Consider the Poisson problem $-\Delta u = 0$ on the L-shaped domain $[-1,1]^2$ with nonhomogeneous Dirichlet boundary conditions compatible with the exact solution $u(x,y) = r^{2/3}$ with $r = \sqrt{x^2+y^2}$. Solve this problem on a sequence of meshes and check the EOC.

(3) Code 1D FEM of order $k = 1, 2$ (or any order if you like!) for reaction-diffusion-convection problems and experiment with convection-dominated and reaction dominated regimes

(4) Modify your code from (3) to implement the artificial-diffusion method for convection-dominated convection-diffusion problems seen in Lecture 10. Verify that the upwind method is first-order in all cases while the Scharfetter-Gummel method with $k = 2$ is second order and nodally exact.

(5) Implement mass-lumping for reaction-dominated reaction-diffusion problems and verify that this yields monotonic solutions if the exact solution is monotonic. Also provide evidence of the EOC of the method.

(6) Combine (2) and (3) appropriately to work in all regimes.

(7) Experiment on the solution of convection-diffusion and reaction diffusion problems in 2D. For instance, you may consider the homogeneous Dirichlet problems $-\varepsilon\Delta u + (1,1)\cdot\nabla u = 1$ and $-\varepsilon\Delta u + u = 1$ varying ε.

(8) Implement the streamline diffusion method and mass lumping methods for linear FEM seen in Lecture 11.