Debug

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Chapter 1

Foundamental concepts

1.1 What is an RRT algorithm?

Rapidly Random exploring Tree(s), aka RRT(s), is one of the most popular technique adopted for solving planning path problems in robotics. In essence, a planning problem consists of finding a feasible trajectory or path that leads a manipulator, or more in general a dynamical system, from a starting configuration/state to an ending desired one, consistently with a series of constraints. RRTs were firstly proposed in [5]. They are able to explore a state space in an incremental way, building a search tree, even if they may require lots of iterations before terminating. They were proved be capable of always finding at least one solution to a planning problem, if a solution exists, i.e. they are probabilistic complete. RRT were also proved to perform well as kinodynamic planners, designing optimal LQR controllers driving a generic dynamical system to a desired final state, see [9] and [8].
The typical disadvantage of RRTs is that even for medium-complex problems, they require thousands of iterations to get the solution. For this reason, the aim of this library is to provide multi-threaded planners implementing parallel version of RRTs, in order to speed up the planning process.
It is possible to use this library for solving each possible problem tackled by an RRT algorithm. The only necessary thing to do when facing a new class of problem is to derive a specific object describing the problem itself.
Then it is clear that one of the most common problem one may solve with RRT is a standard path planning problem for an articulated arm. What matters in such cases is to have a collision checker, which is not provided by this library. Anyway, the interfaces Tunneled_check_collision and Bubbles_free_ configuration allows you to integrate the collision checker you prefer for solving standard path planning problems (see also Section 2.2.3).

The next Section briefly reviews the basic mechanism of the RRT. The notations and formalisms introduced in the next Section will be also adopted by the other Sections. Therefore, the reader is strongly encouraged to read before the next Section.
Section 1.3 will describe the typical pipeline to consider when using MT_RRT, while some examples of planning problems are reported in Chapter 2. Chapter 3 will describe the possible parallelization strategy that MT_RRT offers you.

1.2 Background on RRT

1.2.1 Standard RRT

RRTs are able to find a series of states connecting two particular ones: a starting state x_o and an ending one x_f. This is done by building a search tree $T(x_o)$ having x_o as root. Each node $x_i \in T$ is connected to its unique father $x_{fi} = Fath(x_f)$ by a trajectory $\tau_{fi \to i}$. The root x_o is the only node not having a father ($Fath(x_o) = \emptyset$). The set $\mathcal{X} \subseteq \mathbb{R}^d$ will contain all the possible states x of the system whose motion must be controlled, while $\underline{\mathcal{X}} \subseteq \mathcal{X}$ is a subset describing the admissible region induced by a series of constraints. The solution we are interested in, consists clearly of a sequence of trajectories τ entirely contained in $\underline{\mathcal{X}}$.
If we consider classical path planning problems, the constraints are represented by the obstacles populating the scene, which must be avoided. However, according to the nature of the problem considered, different kind of constraints might need to be accounted. The basic version of an RRT algorithm is described by Algorithm 1, whose steps are visually represented by Figure 1.2. Essentially, the tree is randomly grown by performing several steering operations. Sometimes, the extension of the tree toward the target state x_f is tried in order to find an edge leading to that state.

Data: x_o, x_f
$T = \{x_o\}$;
for $k = 1 : MAX\_ITERATIONS$ **do**
  sample $r \sim U(0, 1)$;
  if $r < \sigma$ **then**
    $x_{steered}$ =Extend(T, x_f);
    if $x_{steered}$ *is* $VALID$ **then**
      if $\|x_{steered} - x_f\| \leq \epsilon$ **then**
        Return Path_to_root($x_{steered}$)$\cup x_f$;
      end
    end
  end
  else
    sample a $x_R \in \mathcal{X}$;
    Extend(T, x_R);
  end
end

Algorithm 1: Standard RRT. A deterministic bias is introduced for connecting the tree toward the specific target state x_f. The probability σ regulates the frequency adopted for trying the deterministic extension. The Extension procedure is described in algorithm 2.

Data: T, x_R
$x_{Nearest}$ =Nearest_Neighbour(T, x_R);
$x_{steered}$ = Steer($x_{Nearest}, x_R$);
if $x_{steered} \notin \underline{\mathcal{X}}$ **then**
  Mark $x_{steered}$ as $INVALID$;
end
if $x_{steered}$ *is* $VALID$ **then**
  $T = T \cup x_{steered}$;
end
Return $x_{steered}$;

Algorithm 2: The Extend procedure.

Data: T, x_R
Return $\underset{x_i \in T}{\operatorname{argmin}}(C(\tau_{i \to R}))$;

Algorithm 3: The Nearest_Neighbour procedure: the node in T closest to the given state x_R is searched.

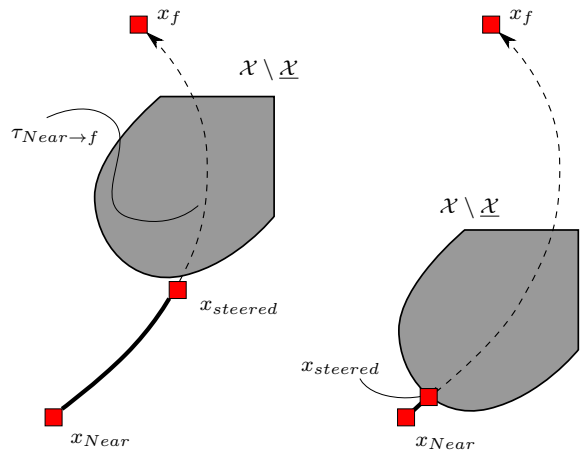


Figure 1.1: The dashed curves in both pictures are the optimal trajectories, agnostic of the constraints, connecting the pair of states x_{Near} and x_f, while the filled areas are regions of X not allowed by constraints. The steering procedure is ideally in charge of searching the furthest state to x_{Near} along $\tau_{Near \to f}$. For the example on the right, the steering is not possible: the furthest state along $\tau_{Near \to f}$ is too much closer to x_{Near}.

The Steer function in algorithm 2 must be problem dependent. Basically, It has the aim to extend a certain state x_i already inserted in the tree, toward another one x_R. To this purpose, an optimal trajectory $\tau_{i \to R}$, agnostic of the constraints, going from x_i to x_R, must be taken into account. Ideally, the steering procedure should find the furthest state from x_i that lies on $\tau_{i \to R}$ and for which the portion of $\tau_{i \to R}$ leading to that state is entirely contained in $\underline{\mathcal{X}}$. However, in real implementations the steered state returned might be not the possible farthest from x_i. Indeed, the aim is just to extend the tree toward x_R. At the same time, in case such the steered state results too closer to x_i, the steering should fails [1].

The Nearest_Neighbour procedure relies on the definition of a cost function $C(\tau)$. Therefore, the closeness of states does not take into account the shape of $\underline{\mathcal{X}}$. Indeed $C(\tau)$ it's just an estimate agnostic of the constraints. Then, the constraints are taken into account when steering the tree. The algorithm terminates when a steered configuration x_s sufficiently close to x_f is found.
The steps involved in the standard RRT are summarized by Figure 1.2.

1.2.2 Bidirectional version of the RRT

The behaviour of the RRT can be modified leading to a bidirectional strategy [6], which expands simultaneously two different trees. Indeed, at every iteration one of the two trees is extended toward a random state. Then, the other tree is extended toward the steered state previously obtained. At the next iteration, the roles of the trees are inverted. The algorithm stops, when the two trees meet each other. The detailed pseudocode is reported in Algorithm 4.

This solution offers several advantages. For instance, the computational times absorbed by the Nearest Neighbour search is reduced since this operation is done separately for the two trees and each tree contains at an average half of the states computed. The steps involved in the bidirectional strategy are depicted in Figure 1.3.

1.2.3 Compute the optimal solution: the RRT*

For any planning problem there are infinite $\tau_{o \to f} \subset \underline{\mathcal{X}}$, i.e. infinite trajectories starting from x_o and terminating in x_f which are entirely contained in the admissible region $\underline{\mathcal{X}}$. Among the aforementioned set, we might be interested in finding the trajectory minimizing the cost $C(\tau_{o \to f})$, refer to Figure 1.4. The basic version of the RRT algorithm is proved to find with a probability equal to 1, a suboptimal solution [4]. The optimality is addressed by a variant of the RRT, called RRT* [4], whose pseudocode is contained in Algorithm 5. Essentially, the RRT* after inserting in a tree a steered state, tries to undertake local improvements to the connectivity of the tree, in order to minimize the cost-to-go of the states in the $Near$ set. This approach is proved to converge to the optimal solution after performing an infinite number of iterations [2]. There are no precise stopping criteria for the RRT*: the more iterations are performed, the more the solution found get closer to the optimal one.

[1] This is done to avoid inserting less informative nodes in the tree, reducing the tree size.
[2] In real cases, after a sufficient big number of iterations an optimizing effect can be yet appreciated.

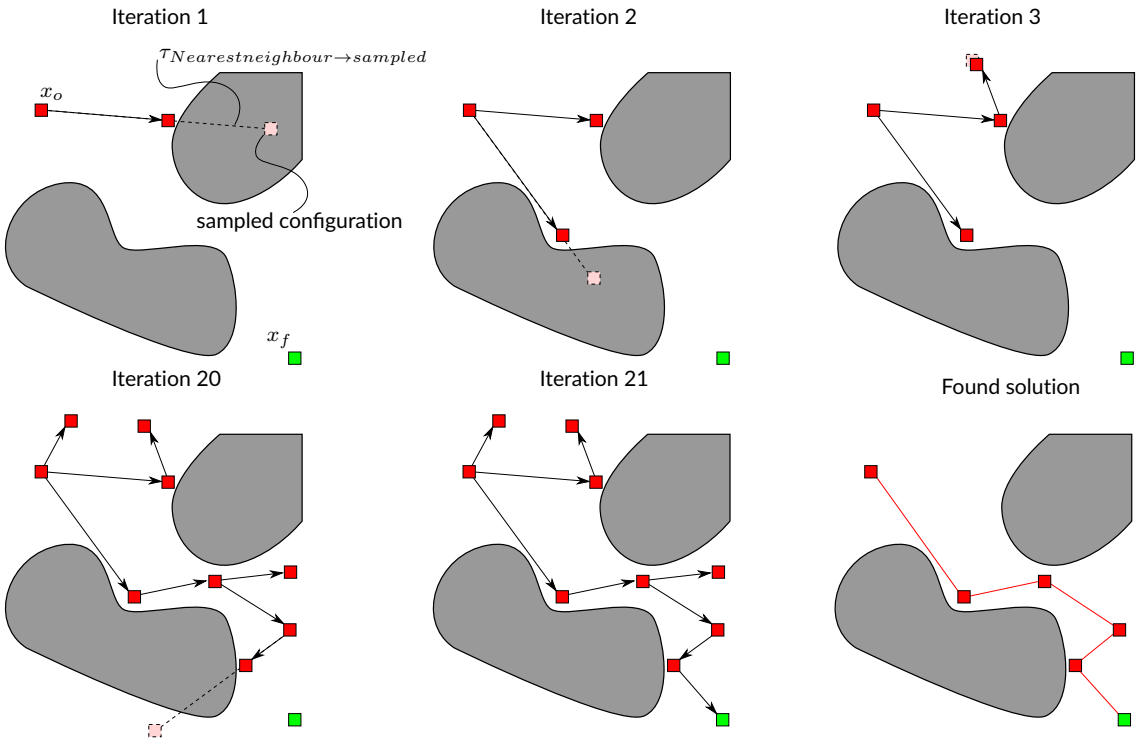


Figure 1.2: Examples of iterations done by an RRT algorithm. The solution found is the one connecting the state in the tree that reached x_f, with the root x_o.

Data: x_o, x_f
$T_A = \{x_o\}$;
$T_B = \{x_f\}$;
$x_{target} =$ root of T_A;
$x_2 =$ root of T_B;
$T_{master} = T_A$;
$T_{slave} = T_B$;
for $k = 1 : MAX\_ITERATIONS$ **do**
  sample $r \sim U(0,1)$;
  if $r < \sigma$ **then**
    $x_{steered} =$Extend(T_{master}, x_{target});
  end
  else
    sample a $x_R \in \mathcal{X}$;
    $x_{steered} =$Extend(T_{master}, x_R);
  end
  if $x_{steered}$ *is* $VALID$ **then**
    $x_{steered2} =$Extend$(T_{slave}, x_{steered})$;
    if $x_{steered2}$ *is* $VALID$ **then**
      if $\|x_{steered} - x_{steered2}\| \leq \epsilon$ **then**
        Return Path_to_root($x_{steered}$) $\cup$ Revert $\left(\text{Path\_to\_root}(x_{steered2})\right)$;
      end
    end
  end
  Swap T_{target} and T_2 ;
  Swap T_{master} and T_{slave} ;
end

Algorithm 4: Bidirectional RRT. A deterministic bias is introduced for accelerating the steps. The probability σ regulates the frequency adopted for trying the deterministic extension. The Revert procedure behaves as exposed in Figure 1.3.

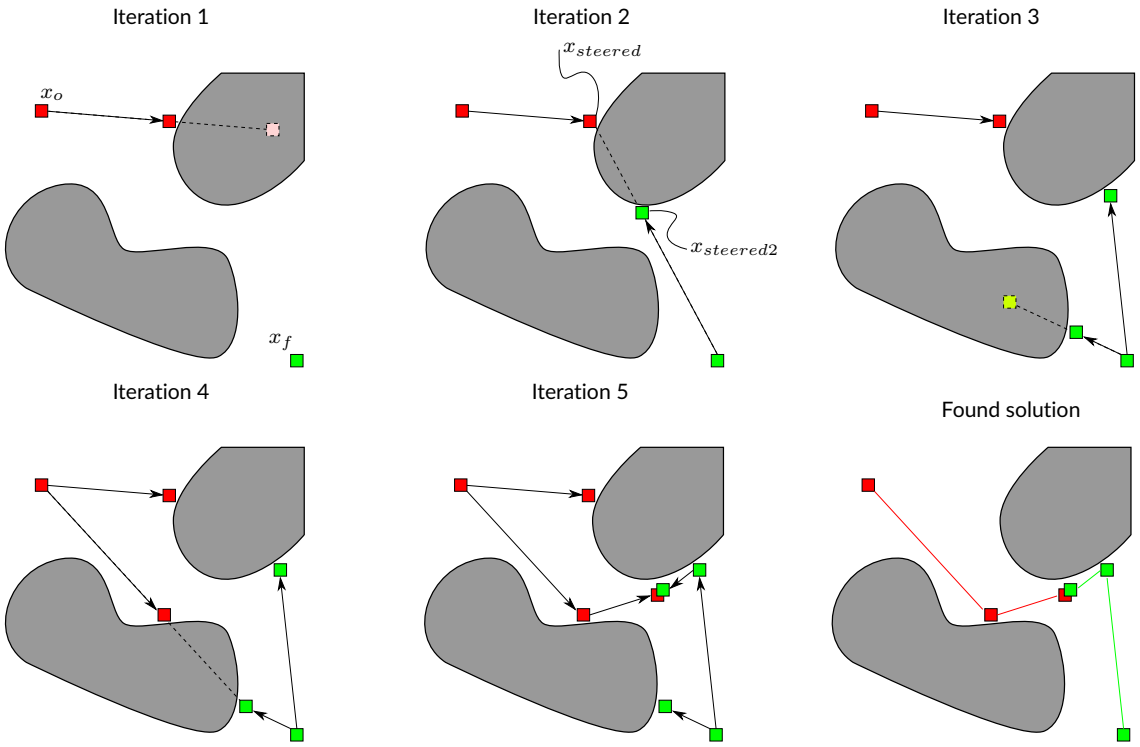


Figure 1.3: Examples of iterations done by the bidirectional version of the RRT. The path in the tree rooted at x_f is reverted to get the second part of the solution.

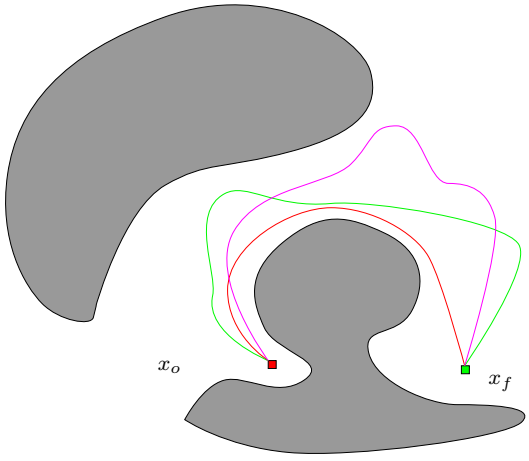


Figure 1.4: Different trajectories connecting x_o with x_f, entirely contained in $\underline{\mathcal{X}}$. If we assume ad cost the length of a path, the red solution is the optimal one.

Data: x_o, x_f
$T = \{x_o\}$;
$Solutions = \emptyset$;
for $k = 1 : MAX\_ITERATIONS$ **do**
  sample $r \sim U(0,1)$;
  if $r < \sigma$ **then**
    $x_{steered}$ =Extend_Star(T, x_f);
    if $x_{steered}$ *is* $VALID$ **then**
      if $\|x_{steered} - x_f\| \leq \epsilon$ **then**
        $Solutions = Solutions \cup x_{steered}$;
      end
    end
  end
  else
    sample a $x_R \in \mathcal{X}$;
    Extend_Star(T, x_R);
  end
end
$x_{best} = \underset{x_S \in Solutions}{\operatorname{argmin}}$ (Cost_to_root(x_S));
Return Path_to_root$(x_{best}) \cup x_f$;

Algorithm 5: RRT*. The Extend_Star, Rewird and Cost_to_root procedures are explained in, respectively, algorithm 6, 7 and 8.

Data: T, x_R
$x_{steered}$ = Extend(T, x_R);
if $x_{steered}$ *is* $VALID$ **then**
  $Near = \left\{ x_i \in T \middle| C(\tau_{i \to steered}) \leq \gamma \left(\frac{log(|T|)}{|T|}\right)^{\frac{1}{d}} \right\}$;
  Rewird$(Near, x_{steered})$;
end
Return $x_{steered}$;

Algorithm 6: The Extend_Star procedure. d is the cardinality of $\mathcal{X}$.

Data: $Near, x_s$
$x_{bestfather} = Fath(x_s)$;
$C_{min} = C(\tau_{bestfather \to s})$;
for $x_n \in Near$ **do**
  if $\tau_{n \to s} \subset \underline{\mathcal{X}}$ *AND* $C(\tau_{n \to s}) < C_{min}$ **then**
    $C_{min} = C(\tau_{n \to s})$;
    $x_{bestfath} = x_n$;
  end
end
$Fath(x_s) = x_{bestfath}$;
C_s =Cost_to_root(x_s);
$Near = Near \setminus x_{bestfath}$;
for $x_n \in Near$ **do**
  if $\tau_{s \to n} \subset \underline{\mathcal{X}}$ **then**
    $C_n = C(\tau_{s \to n}) + C_s$;
    if $C_n <$ *Cost_to_root*(x_n) **then**
      $Fath(x_n) = x_s$;
    end
  end
end

Algorithm 7: The Rewird procedure.

Data: x_n
if $Fath(x_n) = \emptyset$ **then**
  Return 0;
end
else
  Return $C(\tau_{Fath(n)\to n})+$ Cost_to_root($Fath(x_n)$) ;
end

Algorithm 8: The Cost_to_root procedure computing the cost spent to go from the root of the tree to the passed node.

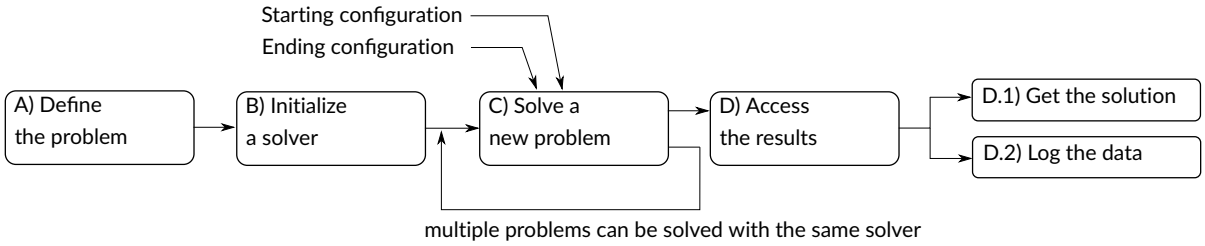


Figure 1.5: Pipeline to follow for using MT_RRT.

1.3 MT_RRT pipeline

When solving a planning problem with MT_RRT, the pipeline of Figure 1.5 should be followed.

A) Define the problem

First of all, you need to derive a class that tells MT_RRT how your problem is made, i.e. you need to defined how to compute the optimal trajectories τ, the shape of the constraints admitted region $\underline{\mathcal{X}}$, how to perform the steering of a node in the tree, etc.. This must be done by deriving a specific object from the interface called Node::I_Node_factory. The object derived is able to describe the kind of problem, without the need to know the particular starting and ending configurations that you need to join with RRT. Therefore, you can recycle this object for resolving different RRT planning, addressing the same kind of problem. Chapter 2 reports some examples of planning problems, describing how to derive the corresponding Node::I_Node_factory object.

B) Initialize a solver

After having defined the problem, you need to build a solver, to be used to solve later a planning problem. The solver can be: a serial standard solver or a multi-threaded solver. In the first case, you must use Planner_canonical and you are basically using the standard RRT versions described at the beginning in Section 1.2.1. In the second case you can chose one of the approach described in Chapter 3, exploiting multi-threading to reduce the computation times.

C) Solve the problem with a certain strategy

One single solver can be used to solve multiple problems, using one of the strategies discussed in Sections 1, 1.2.2 and 5. This can be done by calling RRT_basic, RRT_bidirectional or RRT_star on the built solver. Results are internally stored into the solver and can be later accessed as explained in the next paragraph. Clearly, only the results concerning the last found solution are saved, i.e. data are overwritten when calling multiple times RRT_basic (and the other two methods) for solving different problems of the same category, but with different starting and ending configurations.

D) Access the results

To access the results computed by a solver there are two possible ways.

D.1. The first way is to get the waypoints representing the solution, i.e. a series of states $x_{1,2,3,\dots,M}$ that must be visited to get from the starting configuration to the ending one, by traversing the trajectories $\tau_{1\to 2}, \tau_{2\to 3}, \dots, \tau_{M-1\to M} \subset \underline{\mathcal{X}}$. Such series of waypoints can be obtained by calling I_Planner::Get_solution, which returns a list of configurations. In case a solution was not found, an empty list is externally returned.

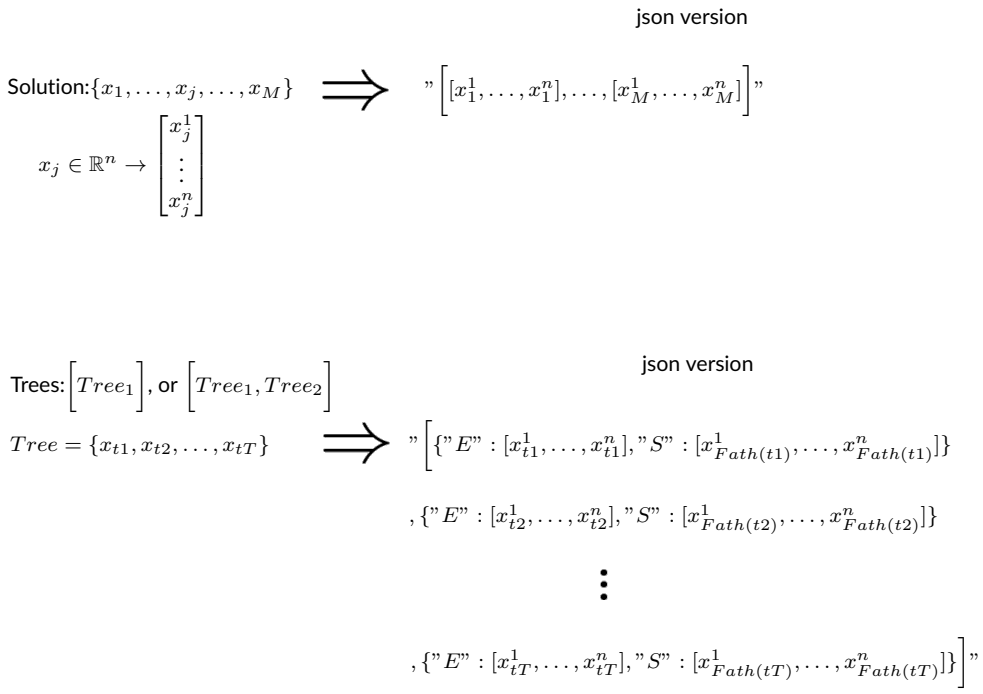


Figure 1.6: Format of the json file returning the results.

D.2. The second way to get the results is to log them into a json string. This can be done by using two possible methods. I_Planner::Get_Solution_as_JSON is similar to I_Planner::Get_solution, but returns the waypoints representing the solution as a json, i.e. an array of arrays refer to top of Figure 1.6. On the opposite, I_Planner::Get_Trees_as_JSON returns a json structure that describes the tree(s) [3] computed by the solver in order to get the solution [4]. The bottom part of Figure 1.6 shows the structure of the json representing a single searching tree.

[3] A single tree is addressed when using RRT_basic or RRT_star, while two trees are computed when considering RRT_bidirectional.
[4] In case of the solver described in Sections 3.0.3 and 3.0.4 the tree owned by the main thread is returned.

Chapter 2

Customize your own planning problem

MT_RRT can be deployed to solve each possible problems for which RRT can be used. The only thing to do is to derive a specific object from the interface called Node::I_Node_factory to have an object describing your particular problem. The methods contained in this object are in charge of sampling new random states in $\mathcal{X}$ or computing the optimal trajectories τ, which is the pre-requisite for performing steering operations (Figure 1.1). Such functions are problem-specific and for this reason they must be implemented every time a new kind of problem must be solved. In order to help the user in understanding how to implement a derivation to Node::I_Node_factory, three main kind of examples are part of the library. In the following Sections, they will be briefly reviewed.

2.1 Planar maze problem

The state space characterizing this problem is a two dimensional one, having $x_{1,2}$ as coordinates. The aim is to connect two 2D coordinates while avoiding the rectangular obstacles depicted in Figure 2.1. The state space is bounded by two corners describing the maximum and minimum possible x_1 and x_2, see Figure 2.1.

2.1.1 Sampling

A sampled state x_R lies in the square delimited by the spatial bounds, i.e.:

$$x_R = \begin{bmatrix} x_{R1} \sim U(x_{1min}, x_{1max}) \\ x_{R2} \sim U(x_{2min}, x_{2max}) \end{bmatrix} \tag{2.1}$$

2.1.2 Optimal trajectory and constraints

The optimal trajectory $\tau_{i \to k}$ between two states in $\mathcal{X}$ is simply the segment connecting that states. The cost $C(\tau_{i \to k})$ is assumed to be the length of such segment:

$$C(\tau_{i \to k}) = \|x_i - x_k\| \tag{2.2}$$

The admissible region $\underline{\mathcal{X}}$ is obtained subtracting the points pertaining to the obstacles. In other words, the segment connecting the states in the tree should not traverse any rectangular obstacle, refer to the right part of Figure 2.1.

2.1.3 Steer procedure

The steering procedure is done as similarly described in Section 2.2.3, checking a close state $x_{steered}$ that lies on the segment departing from the state to steer.

2.2 Articulated arm problem

This is for sure one of the most common problem that can be solved using MT_RRT. Consider a cell having a group of articulated serial robots. Q^i will denote the vector describing the configuration of the i^{th} robot, i.e. the positional values assumed by each of its joint. A generic state x_i is characterized by the series of poses assumed by all the robots in the cell:

$$x_i = Q_i = \begin{bmatrix} (Q_i^1)^T & \dots & (Q_i^n)^T \end{bmatrix}^T \tag{2.3}$$

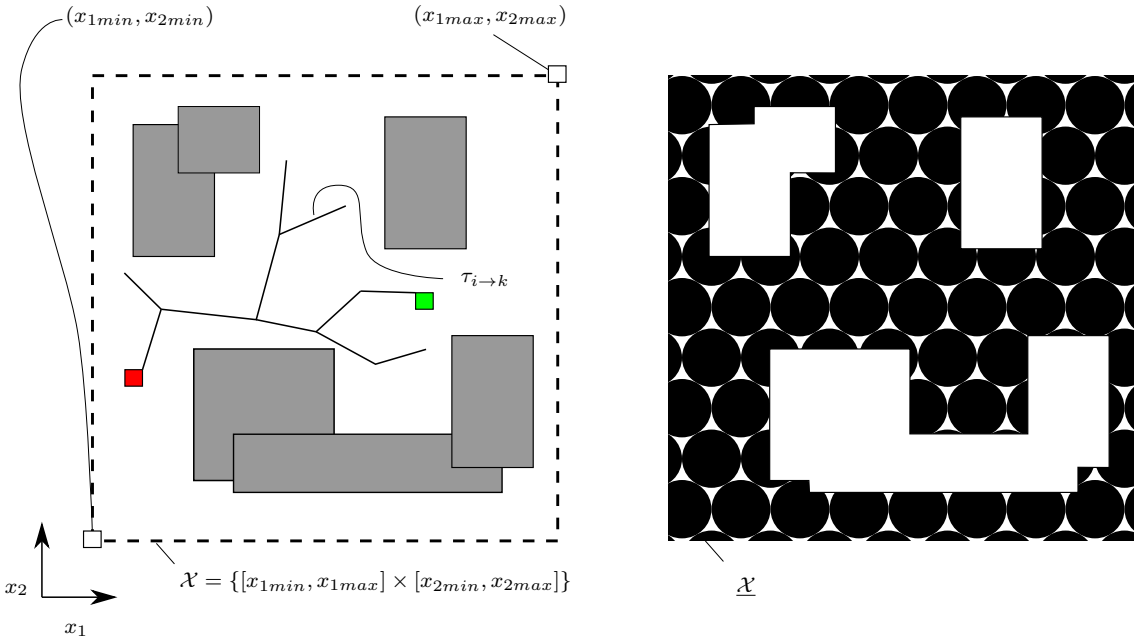


Figure 2.1: Example of maze problem.

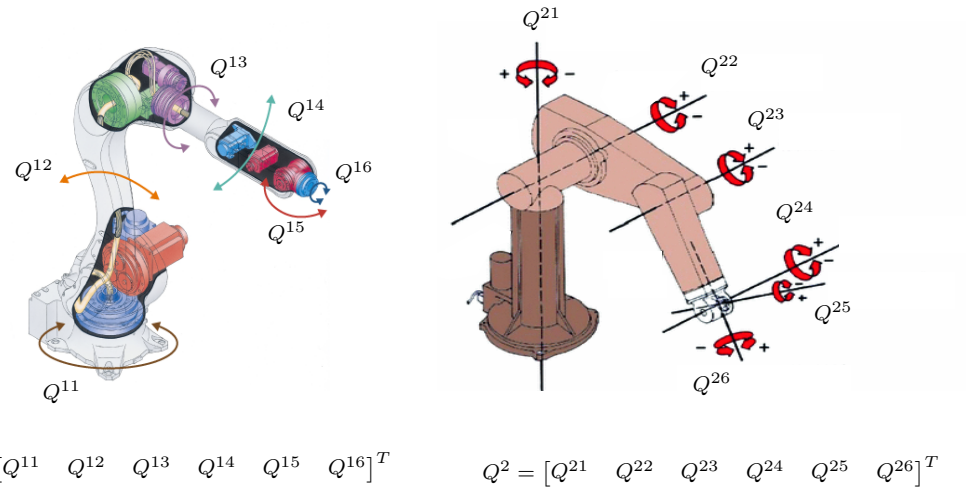


Figure 2.2: Rotating joints of two articulated manipulators.

refer also to Figure 2.2.

These kind of problems consist in finding a path in the configurational space that leads the set of robots from an initial state Q_o to and ending one Q_f, while avoiding the obstacles populating the scene, i.e. avoid collisions between any object in the cell and any part of the robots as well as cross-collision between all the robot parts. Here the term path, refer to a series of intermediate waypoints $Q_{1,\dots,m}$ to traverse to lead the robot from Q_o to Q_f.

2.2.1 Sampling

The i^{th} joint of the k^{th} robot, denoted as Q^{ki}, is subjected to some kinematic limitations prescribing that its positional value must remain always within a compact interval $Q^{ki} \in [Q^{ki}_{min}, Q^{ki}_{max}]$. Therefore, the sampling of a random

configuration Q_R is done as follows:

$$Q_R = \begin{bmatrix} Q_R^{11} \sim U(Q_{min}^{11}, Q_{max}^{11}) \\ Q_R^{12} \sim U(Q_{min}^{11}, Q_{max}^{11}) \\ \vdots \\ Q_R^{21} \sim U(Q_{min}^{21}, Q_{max}^{21}) \\ Q_R^{22} \sim U(Q_{min}^{21}, Q_{max}^{21}) \\ \vdots \\ Q_R^{n1} \sim U(Q_{min}^{n1}, Q_{max}^{n1}) \\ Q_R^{n2} \sim U(Q_{min}^{n1}, Q_{max}^{n1}) \\ \vdots \end{bmatrix} \tag{2.4}$$

2.2.2 Optimal trajectory and constraints

Similarly to the problem described in Section 2.1.2, $\tau_{i\to k}$ is assumed to be a segment in the configurational space and the cost C is the Euclidean distance of a pair of states. The admissible region $\underline{X}$ is made by all the configurations Q for which a collision is not present.

2.2.3 Steer procedure

The trajectory going from Q_i to Q_k can be parametrized in order to characterize all the possible configurations pertaining to $\tau_{i\to k}$:

$$Q(s) = \tau_{i\to k}(s) = Q_i + s\Big(Q_k - Q_i\Big) \tag{2.5}$$

s is a parameter spanning $\tau_{i\to k}$ and can assume a value inside $[0, 1]$. Ideally, the steer process has the aim of determine that state $Q(s_{steered})$ that is furthest from Q_i and at the same time contained in $\underline{X}$ (Figure 1.1). Anyway, determine the exact value of $s_{steered}$ would be too much computationally demanding. Therefore, in real situations, two main approaches are adopted: a tunneled check collision or the bubble of free configuration.

Tunneled check collision

This approach consider as steered state $Q_{steered}$ the following quantity:

$$Q_{steered} = \begin{cases} \textit{if}(\|Q_k - Q_i\| \le \epsilon) \Rightarrow Q_k \\ \textit{else} \Rightarrow Q_i + s_\Delta(Q_k - Q_i) \textit{ s.t. } s_\Delta \|Q_k - Q_i\| = \epsilon \end{cases} \tag{2.6}$$

with ϵ in the order of few degrees. $Q_{steered}$ is checked to be or not in $\underline{X}$ and is consequently marked as $VALID$ or $INVALID$. The class Tunneled_check_collision is in charge of implementing such an extension strategy. It absorbs an object of type I_Collision_checker to check whether for a certain state are present or not collisions. I_Collision_checker is just an interface: you can integrate your own collision checker (using for example [1] or [2]) by deriving an object from this interface.
Clearly, multiple tunneled check, starting from Q_i, can be done in order to get as close as possible to Q_k. This process can be arrested when reaching Q_k or an intermediate state for which a collision check is not passed. This behaviour can be obtained by using Node_factory_multiple_steer, absorbing a Tunneled_check_collision object.
Figure 2.3 summarizes the above considerations.

Bubble of free configuration

This approach was first proposed in [7] and is based on the definition of a so called bubble of free configuration $\mathcal{B}$. Such a bubble is a region of the configurational space that is built around a state Q_i. More formally, $\mathcal{B}(Q_i)$ is defined as follows:

$$\mathcal{B}(\bar{Q} = \begin{bmatrix} \bar{Q}^{1T} & \dots & \bar{Q}^{nT} \end{bmatrix}^T) = \Bigg\{ \bar{Q} \Bigg| \quad \forall \quad j \in \{1, \dots, n\} \sum_i r^{ji} |Q^{ji} - \bar{Q}^{ji}| \le d_{min}^j \textit{ and}$$
$$\forall \quad j, k \in \{1, \dots, n\} \sum_i r^{ji} |Q^{ji} - \bar{Q}^{ji}| + \sum_i r^{ki} |Q^{ki} - \bar{Q}^{ki}| \le d_{min}^{jk} \Bigg\} \tag{2.7}$$

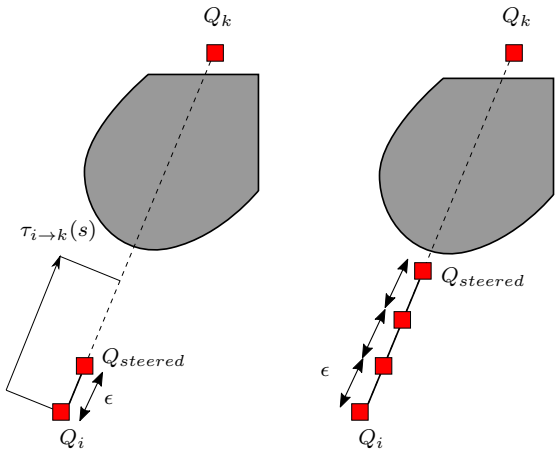


Figure 2.3: Steer extension along the segment connecting two states in the configuration space. On the left a single steer approach, on the right a multiple one.

where d^j_{min} is the minimum distance between the i^{th} robot and all the obstacles in the scene, while d^{jk}_{min} is the minimum distance between the i^{th} and the k^{th} robot. r^{ki} is the distance of the furthest point of the shape of the k^{th} robot to its i^{th} axis of rotation. Refer also to Figure 2.4.
Each configuration $Q \in \mathcal{B}$ is guaranteed to be inside the admitted region $\underline{X}$. This fact can be exploited for performing steering operation. Indeed, we can take as $Q_{steered}$ the pose at the border of $\mathcal{B}(Q_i)$ along the segment connecting Q_i to Q_k. It is not difficult to prove that such a state is equal to:

$$\begin{aligned} Q_{steered} &= \begin{bmatrix} Q^{1T}_{steered} & \dots & Q^{nT}_{steered} \end{bmatrix}^T = Q_i + s_{steered}(Q_k - Q_i) \\ s_{steered} &= min\left\{ s_A, s_B \right\} \\ s_A &= min_{j\in\{1,\dots,n\},q}\left\{ \frac{d^j_{min}}{r^{jq}|Q^{jq}_i - Q^{jq}_k|} \right\} \\ s_B &= min_{j,k\in\{1,\dots,n\},q,q_2}\left\{ \frac{d^{jk}_{min}}{r^{jq}|Q^{jq}_i - Q^{jq}_k| + r^{kq2}|Q^{kq_2}_i - Q^{kq_2}_k|} \right\} \end{aligned} \tag{2.8}$$

Also in this case a multiple steer approach is possible for this strategy, by using Node_factory_multiple_steer, absorbing a Bubbles_free_configuration object, refer also to Figure 2.5.
Bubbles_free_configuration contains the functionalities for performing steering operations using the bubble of free configuration. Then, you have to deploy your own geometric engine in order to compute the distances d^j_{min}, d^{jk}_{min} as well as the radii r^{ki}, deriving an object from I_Proximity_calculator. Robots_info stored in these kind of objects is a structure containing the distance d^j_{min}, as well as the radii r^{ki} (with an order that goes from the base to the end effector), while Robot_distance_pairs is a buffer of distances storing all the possible d^{jk}_{min}, with the order indicated in Figure 2.6.

2.3 Navigation problem

This problem is typical when considering autonomous vehicle. We have a 2D map in which a cart must move. In order to simplify the collision check task, a bounding box $\mathcal{L}$ is assumed to contain the entire shape of the vehicle, Figure 2.7. The cart moves at a constant velocity when advancing on a straight line and cannot change instantaneously its cruise direction. Indeed, the cart has a steer, which allows to do a change direction by moving on a portion of a circle, refer to Figure 2.7. In order to simplify the problem, we assume that the steering radius must be constant and equal to a certain value R and the velocity of the cart while performing the steering maneuver is constant too.
Since the cart is a rigid body, its position and orientation in the plane can be completely described using three quantities: the coordinates p_x, p_y of its center of gravity and the absolute angle θ. Therefore, a configuration $x_i \in \mathcal{X}$ is a vector defined as follows: $x_j = \begin{bmatrix} p_{xi} & p_{yi} & \theta_i \end{bmatrix}$. The admitted region $\underline{\mathcal{X}}$ is made by all the configurations x for which the vehicle results to be not in collision with any obstacles populating the scene.

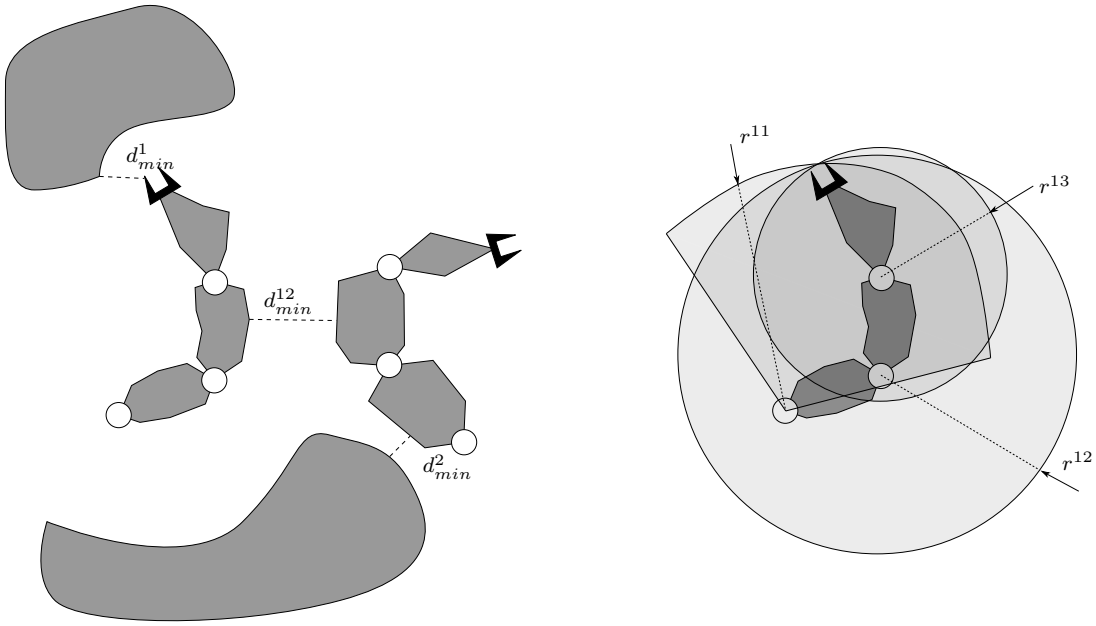


Figure 2.4: The quantities involved in the computation of the bubble $\mathcal{B}$.

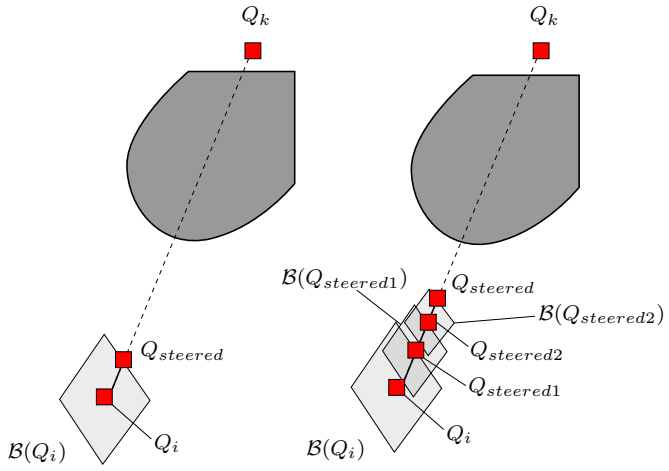
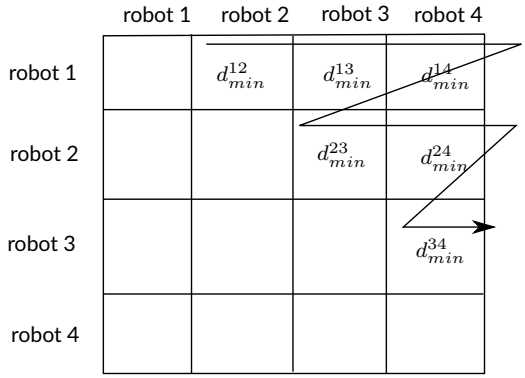


Figure 2.5: Single (left) and multiple (right) steer using the bubbles of free configurations.



Robot_distance_pairs:

d^{12}_{min}	d^{13}_{min}	d^{14}_{min}	d^{23}_{min}	d^{24}_{min}	d^{34}_{min}

Figure 2.6: Values stored in Robot_distance_pairs.

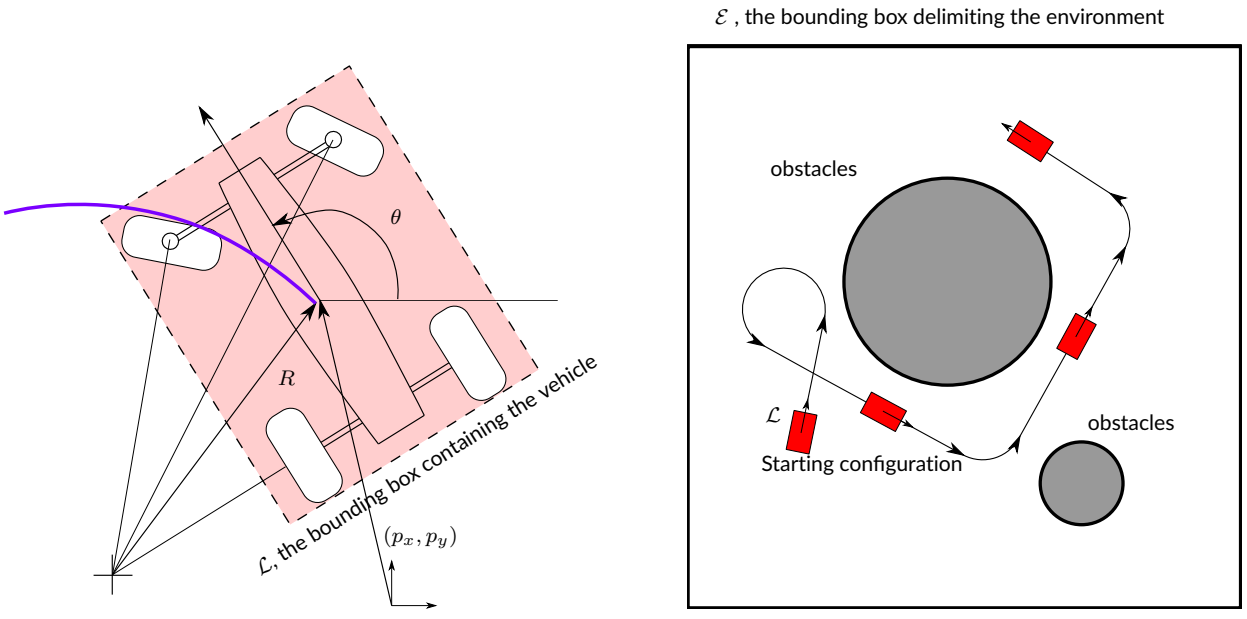


Figure 2.7: Vehicle motion in a planar environment.

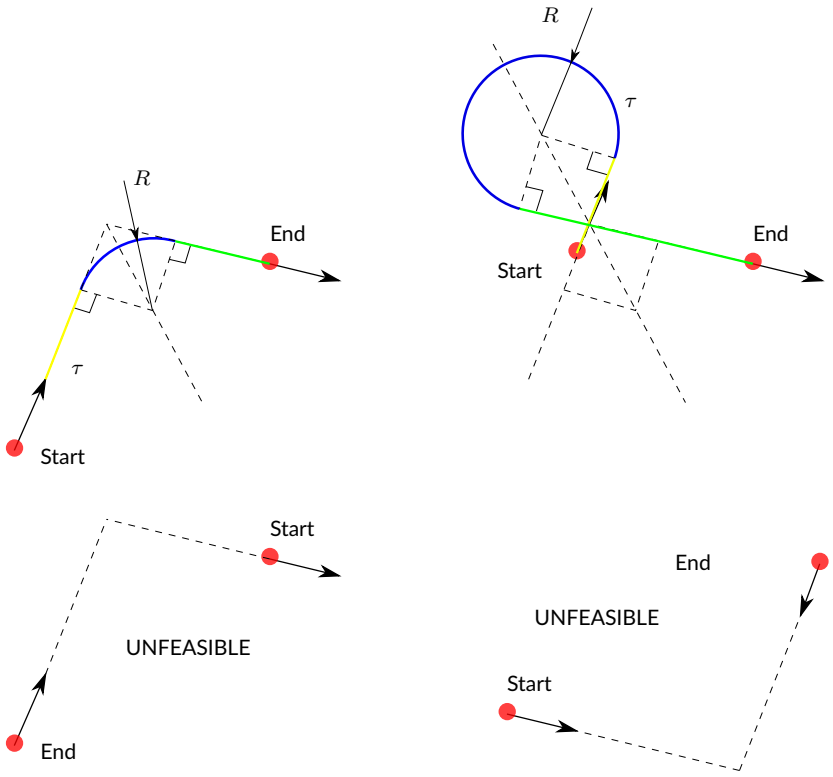


Figure 2.8: Examples of feasible, top, and non feasible trajectories, bottom. The different parts of the feasible trajectories are highlighted with different color.

2.3.1 Sampling

The environment where the vehicle can move is assumed to be finite and equal to a bounding box $\mathcal{E}$ with certain sizes, right portion of Figure 2.7. The sampling of a random configuration for the vehicle is done in this way:

$$x_R = \begin{bmatrix} (p_{xi}, p_{yi}) \sim \mathcal{E} \\ \theta \sim U(-\pi, \pi) \end{bmatrix} \tag{2.9}$$

2.3.2 Optimal trajectory and constraints

The optimal trajectory connecting two configurations x_i, x_j is made of three parts (refer to the examples in the right part of Figure 2.7):

- a straight line starting from x_i
- a circular portion motion used to get from θ_i to θ_j
- a straight line ending in x_j

The cost $C(\tau)$ is assumed to be the total length of τ. It is worthy to remark that not for every pair of configurations exists a trajectory connecting them, refer to Figure 2.8. Therefore, in case the trajectory $\tau_{i\to j}$ does not exists, $C(\tau_{i\to j})$ is assumed equal to $+\infty$.

2.3.3 Steer procedure

The steering from a state x_i toward another x_j is done by moving along the trajectory $\tau_{i\to j}$, advancing every time of a little quantity of space (also when traversing the circular part of the trajectory). The procedure is arrested when a configuration not lying in $\underline{\mathcal{X}}$ is found or x_j is reached. Figure 2.9 summarizes the steering procedure.

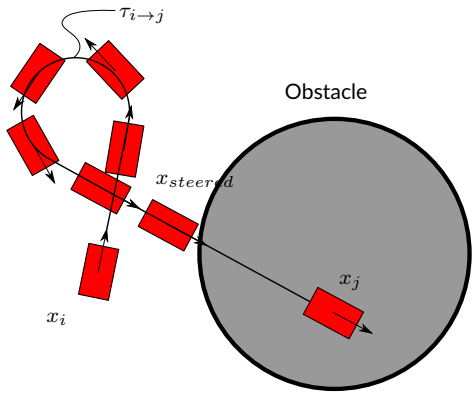


Figure 2.9: Steering procedure for a planar navigation problem.

Chapter 3

Parallel RRT

This Chapter will provide details about the strategies adopted for parallelizing RRT that MT_RRT contains. Further details are contained also in [3], which is the publication were for the first time MT_RRT was presented. In [3] you can find also a comparison in terms of computational times.
Each strategy described in the following Sections is able to parallelize the three RRT versions exposed in Sections 1.2.1, 1.2.2 and 1.2.3. The only exception must be made only for the strategy exposed in Section 3.0.4, for which a bidirectional RRT (Section 1.2.2) cannot be applied.

3.0.1 Parallelization of the query activities

All the RRT versions spend a significant time in performing query operations on the tree, i.e. operations that require to traverse all the tree. Such operations are mainly the nearest neighbour search, algorithm 3, and the determination of the near set, algorithm 6.
The key idea is to perform the above query operations with a parallel for, where at an average all the threads process the same amount of nodes in the tree, computing their distances for determine the nearest neighbour or the near set. The parallel regions are not re-opened and closed every time, but a thread pooling strategy is adopted: all the threads are spawn when a new planning problem must be solved and remain active and ready to perform the parallel for described before. All the operations of the RRT (regardless the version considered) are done by the main thread, which notifies at the proper time when a new query operation must be process collectively by all the threads. Figure 3.1.a summarizes the approach.
The class implementing this approach is Planner_query_parall.

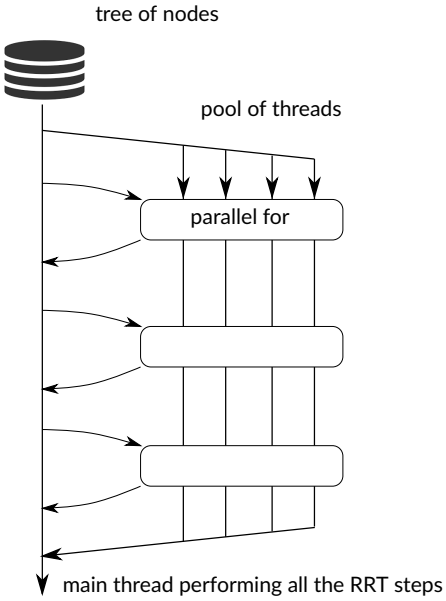
3.0.2 Shared tree critical regions

Another way to obtain a parallelization is to actually do simultaneously, every single step of the RRT versions. Therefore, we can imagine having threads sharing a common tree (or two trees in the case of a bidirectional strategy), executing in parallel every step of the expansion process. Some critical sections must be designed to allow the threads executing the maintenance of the shared tree(s) (inserting new nodes or executing new rewirds) one at a time. More precisely, the steer is done outside and only the insertion of the steered configuration in the tree is performed inside a critical region. Similarly, the extending procedure of the RRT*, algorithm 6, is modified by shifting the determination of the near set and the Rewird procedure in a critical section. Figure 3.1.b summarizes the approach.
The class implementing this approach is Planner_shared_parall.

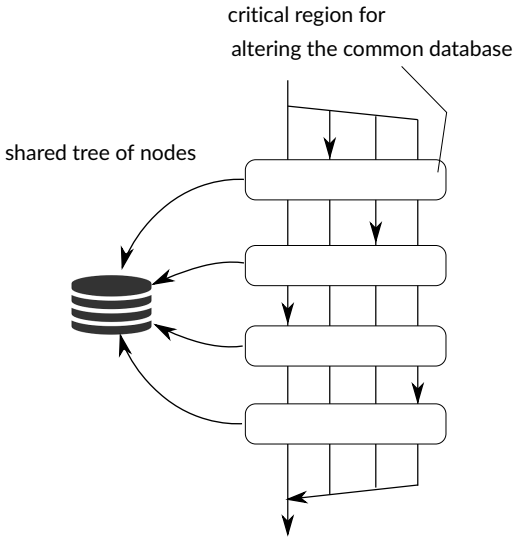
3.0.3 Parallel expansions of copied trees

To limit as much as possible the overheads induced by the presence of critical sections, we can consider a version similar to the one proposed in the previous Section, but for which every thread has a private copy of the search tree. After a new node is added by a thread to its own tree, $P-1$ copies are computed and dispatched [1] to the other threads, were P is the number of working threads. Sporadically, all the threads take into account the list of nodes received from the others and insert them into their private trees. This mechanism is able to avoid the simultaneous modification of a tree by two different threads, avoiding the use of critical sections.
When considering the bidirectional RRT, the mechanism is analogous but introducing for every thread a private copy of both the involved trees.

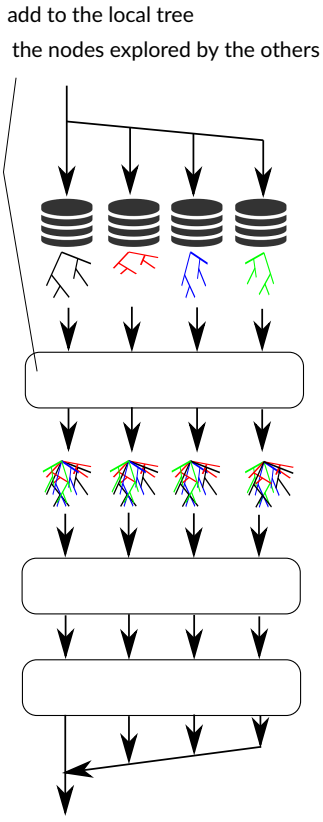
[1] They are dispatched into proper buffer, but not directly inserted in the private copies of the other trees.



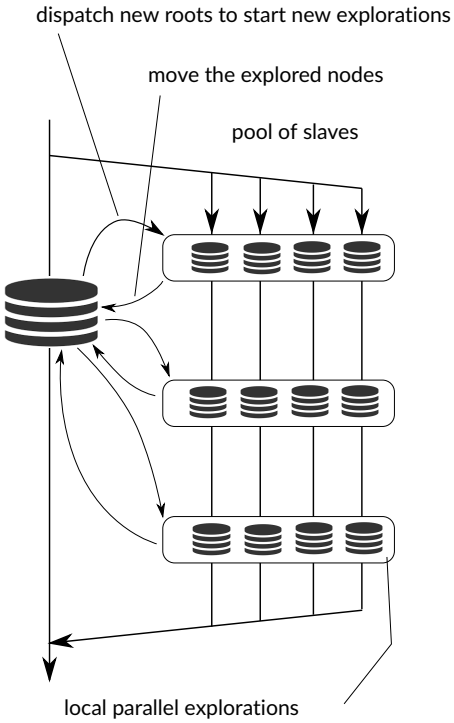
(a) Schematic representation of the parallelization of the query activities approach.



(b) Schematic representation of the parallel extensions of a common tree approach.



(c) Schematic representation of the parallel expansions of copied trees approach.



(d) Schematic representation of the multi agent approach.

Figure 3.1: Approaches adopted for parallelize RRT.

Instead, the RRT* version is slightly modified. Indeed, the rewirds done by a thread on its own tree are not dispatched to the others. At the same time, each thread consider all the nodes produced and added to its own tree when doing their own rewirds. When searching the best solution at the end of all the iterations, the best connections among all the trees in every threads are taken into account. Indeed, the predecessor of a node is assumed to be the parent with the lowest cost to go among the ones associated to each clones. Figure 3.1.c summarizes the approach.
Clearly, the amount memory required by this approach is significantly high, since multiple copies of a node must live in the different threads. This can be a problem to account for.
The class implementing this approach is Planner_copied_parall.

METTERE figura che spiega calcolo path to root

3.0.4 Multi agents approach

The strategy described in this Section aims at exploiting a significant number of threads, with both a reduced synchronizing need and allocation memory requirements. To this purpose, a variant of the RRT was developed for which every exploring thread has not the entire knowledge of the tree, but it is conscious of a small portion of it. Therefore, we can deploy many threads to simultaneously explore the state space $\mathcal{X}$ (ignoring the results found by the other agents) for a certain amount of iterations. After completing this sub-exploration task, all data incoming from the agents are collected and stored in a centralized data base while the agents wait to begin a new explorative batch, completely forgetting the nodes found at the previous iteration. The described behaviour resembles one of many exploring ants, which reports the exploring data to a unique anthill.
Notice that there is no need to physically copy the states computed by the agents when inserting them into the central database, since threads share a common memory: the handler of the node is simply moved.
When considering this approach a bidirectional search is not implementable, while the RRT* can be extended as reported in the following. Essentially, the agents perform a standard non-optimal exploration, implementing the steps of a canonical RRT, Section 1.2.1. Then, at the time of inserting the nodes into the common database, the rewirds are done by the main thread.
The described multi agent approach is clearly a modification of the canonical RRT versions, since the agents start exploring every time from some new roots, ignoring all the previously computed nodes. However, it was empirically found that the global behaviour of the path search is not deteriorated and the optimality properties of the RRT* seems to be preserved.
Before concluding this Section it is worthy to notice that the mean time spent for the querying operations is considerably lower, since such operations are performed by agents considering only their own local reduced size trees.
Figure 3.1.d summarizes the approach. The class implementing this approach is Planner_multi_agents.

Bibliography

[1] *Bullet3*. `https://github.com/bulletphysics/bullet3`.

[2] *ReactPhysics3D*. `https://www.reactphysics3d.com`.

[3] Andrea Casalino, Andrea Maria Zanchettin, and Paolo Rocco. Mt-rrt: a general purpose multithreading library for path planning. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2019)*, pages 1510–1517. IEEE, 2019.

[4] Sertac Karaman and Emilio Frazzoli. Sampling-based algorithms for optimal motion planning. *The international journal of robotics research*, 30(7):846–894, 2011.

[5] James J Kuffner and Steven M LaValle. Rrt-connect: An efficient approach to single-query path planning. In *Robotics and Automation, 2000. Proceedings. ICRA'00. IEEE International Conference on*, volume 2, pages 995–1001. IEEE, 2000.

[6] James J Kuffner and Steven M LaValle. Rrt-connect: An efficient approach to single-query path planning. In *Robotics and Automation, 2000. Proceedings. ICRA'00. IEEE International Conference on*, volume 2, pages 995–1001. IEEE, 2000.

[7] Sean Quinlan. *Real-time modification of collision-free paths*. Number 1537. Stanford University Stanford, 1994.

[8] Adnan Tahirovic and Faris Janjoš. A class of sdre-rrt based kinodynamic motion planners. In *American Control Conference (ACC)*, volume 2018, 2018.

[9] Dustin J Webb and Jur van den Berg. Kinodynamic rrt*: Optimal motion planning for systems with linear differential constraints. 2012.