A Scalable Distributed RRT for Motion Planning

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Abstract

Rapidly-exploring Random Tree (RRT), like other sampling-based motion planning methods, has been very successful in solving motion planning problems. This success, in part, has led to a renewed interest in exploring parallel processing in attempts to solve more challenging problems within an acceptable timeframe. However, common to all existing work in parallelizing RRT is the global computation and communication overhead of nearest neighbor search. This overhead is a critical drawback as it limits the scalability of previous algorithms. We present two parallel algorithms to address this problem. The first algorithm extends existing work by introducing a parameter that adjusts how much local computation is done before a global update. The second algorithm radially subdivides the configuration space into regions, constructs in parallel a portion of the tree in each region, and connects the subtrees, removing cycles if they exist. By subdividing the space, we increase computation locality enabling a scalable result. We show that our approaches are scalable. We present performance results on different parallel architectures for different motion planning problems.

1 Introduction

Research in robotic motion planning spans over three decades. Within this timeframe, researchers have proposed and studied different types of sequential and parallel algorithms for motion planning [11,15]. The renewed interest in parallel motion planning algorithms is in part encouraged by the progress made in sequential algorithms, the ubiquity of parallel and distributed machines, and the demand for more efficiency in solving complex, high dimensional problems such as those arising in manipulation and reconfigurable robotics [1,27], computational biology and drug design [3,32,33], as well as virtual prototyping and computer-aided design [2,10]. These new application areas test the limit and capability of existing sequential motion planners [29]. Thus, scalable parallelism has a key role to play, both in supporting existing work and in exploring new algorithms needed to solve complex, high dimensional motion planning problems.

In this work, we present scalable parallel algorithms for computing Rapidly-exploring Random Trees (RRT) [21]. There are two main sampling-based motion planning approaches: RRT and Probabilistic Roadmap Method (PRM) [19]. RRT, PRM, and their variants [5,11,12,16,18,20,35–37] are widely considered as state-of-the-art methods for solving motion planning problems. They are efficient and have been highly successful at solving many unsolved problems. RRT in particular is well suited for non-holonomic and kinodynamic motion planning problems [8, 22, 23].

The focus of this paper is to explore scalable parallel RRT processing as a complimentary research endeavor to speedup computation time, solve large and more difficult problems, and improve quality of solution. In particular, we present two parallel algorithms: (1) an algorithm that extends [13] by introducing a parameter that controls how much local and concurrent computation is done before global update and inter-processor communication and (2) an algorithm with a novel radial subdivision of the high dimensional planning space that concurrently builds a subtree in each region which are later connected to form a single tree. By controlling how the subtrees explore the space, we minimize the communication overhead — a well known bottleneck in parallel processing. While these parallel approaches employ the standard RRT expansion techniques, we note that they both result with trees that are structurally different than one that would be constructed using a sequential method. All approaches were implemented using the Standard Template Adaptive Parallel Library (STAPL), a framework for developing parallel C++ code [7,34].

Contribution. Key contributions of this work include:

- We extend previous work by introducing a new parameter to control the local expansion of RRT and minimize global communication which results in scalable performance.
- A novel radial subdivision of the motion planning space such that RRT computation can be distributed to yield scalability.
- A generic and efficient implementation of nearest neighbor search based on a nested map reduce parallel computation pattern.

We show that our algorithms achieve better and more scalable performance on different problems and different parallel machines.

Outline. The rest of the paper is organized as follows. We provide a background overview to motion planning and discuss some important related work in Section 2. In Section 3, we describe our strategy for parallelizing RRT. We focus on implementation details in Section 4 and experimental details and results in Section 5. Finally, we conclude the paper in Section 6.

2 Preliminaries and Related Work

2.1 Sampling-based Motion Planning

The motion planning problem is to find a valid path (e.g., collision-free and satisfies any joint limit and/or loop closure constraints) for a movable object starting from a specified start configuration to a goal con-

figuration in an environment [11]. A single configuration is specified in terms of the movable object's d independent parameters or degrees of freedom (DOF). The set of all possible configurations (both feasible and infeasible) is configuration space (\mathbb{C}_{space}). \mathbb{C}_{space} is partitioned into two sets: \mathbb{C}_{free} (feasible) and $\mathbb{C}_{obstacle}$ (infeasible). Motion planning then becomes finding a continuous sequence of points in \mathbb{C}_{free} that connects the start and the goal configuration.

A complete solution of the motion planning problem is considered computationally intractable and has been shown to be PSPACE-hard with an upper bound that is doubly exponential in movable object's DOF [30]. As an alternative, approximate solutions have been shown to be efficient and practical. Sampling-based methods [11] are the state-of-art practical approach to solving motion planning problems. While not guaranteed to find a solution if one exists, sampling-based methods are known to be probabilistically complete, i.e., the probability of finding a solution given one exists increases with the number of samples generated. Sampling-based methods are broadly classified into two main classes: roadmap or graph-based methods such as the Probabilistic Roadmap Method (PRM) [19] and tree-based methods such as Rapidly-exploring Random Tree (RRT) [21].

2.2 RRT

The basic sequential RRT (shown in Algorithm 1) grows a tree rooted at the start configuration that expands outward into unexplored areas of the \mathbb{C}_{space} . RRT first generates a uniform random sample q_{rand} , valid or not, and identifies the closest node q_{near} in the tree to q_{rand} . q_{near} is extended toward q_{rand} a stepsize Δq . If the extension is successful, q_{new} is added to the tree as node and the pair of q_{near} and q_{new} is added as an edge. To solve a particular query, RRT repeats this process until goal configuration is also connected to the tree. RRT-connect is a variant that grows two trees towards each other: one rooted at the start configuration and the other at the goal configuration [20]. These two trees explore the \mathbb{C}_{space} until they are both connected.

Algorithm 1 Sequential RRT

```
Input: An environment env, a root q_{root}, the number of nodes N
Output: A tree T containing N nodes rooted at q_{root}
 1: T.AddNode(q_{root})
 2: i \leftarrow 0
 3: while i < N do
       q_{rand} \leftarrow \text{GetRandomNode}(env)
 4:
       q_{near} \leftarrow \text{FindNeighbor}(T, q_{rand}, 1)
 5:
 6:
       q_{new} \leftarrow \text{Extend}(q_{near}, q_{rand})
       if !TooSimilar(q_{near}, q_{new}) \wedge IsValid(q_{new}) then
 7:
 8:
          T.AddNode(q_{new})
          T.AddEdge(q_{near}, q_{new})
 9:
          i \leftarrow i + 1
10:
       end if
11:
12: end while
13: \mathbf{return} \ T
```

2.3 Parallel RRT

Early parallel motion planning methods were based on discretization of \mathbb{C}_{space} [9,24]. The discretization as presented limits the algorithm to solving relatively low dimensional problems. However, these methods laid the foundation for subsequent work in parallelizing RRTs.

The OR paradigm [9] was applied to parallelizing RRT computations on shared-memory machines where the computation is replicated on each process [8]. Processes concurrently explore the entire \mathbb{C}_{space} and the

first process to find a solution sends a termination message to other processes. Their work also explored concurrently and cooperatively building a single tree under a shared-memory model. Each process executes their own program and communicates to other processes by exchanging data through the shared memory in a concurrent read exclusive write (CREW) fashion. In addition, they study a hybrid algorithm combining the OR paradigm and the CREW model. The processes are divided into groups and each group cooperatively built its own tree. The first group to find a solution sends a termination message to the others.

Bialkowski et al. parallelize RRT and RRT* by focusing on parallelizing the collision detection phase [4]. They identified nearest neighbor search as the key bottleneck for scalability. The implementation was done in CUDA and GPU. Other work has turned to multicore architecture [13]. They present three algorithms for distributed RRT. The first is a message passing implementation of the OR paradigm. In the second algorithm, each process builds part of tree and globally communicates with the other processes each time a new node and edge is added. The third algorithm adopts a manager-worker approach. Instead of having multiple copies of the tree, only the manager initializes and maintains the tree while the expansion computation is delegated to the worker processes. The drawback with the manager-worker approach is that it does not scale well as it is prone to load imbalance with more workload on master process(es). In this paper, we extend the second algorithm by introducing a user-defined parameter so as to minimize the communication overhead associated with global update.

2.4 C-space Subdivision

 \mathbb{C}_{space} subdivision has been very useful in solving sequential motion planning problems. Early work in \mathbb{C}_{space} subdivision computes the exact representation of \mathbb{C}_{space} by uniformly dividing it into cells [6]. Each cell is then classified as empty, full, or mixed depending on the obstacle position in the cell. An A* search algorithm is then used to compute a path through the purely empty or mixed cells.

Sampling-based motion planning approaches also employ \mathbb{C}_{space} subdivision. In [25, 26], \mathbb{C}_{space} is subdivided by randomly selecting splitting points from randomly selected positional DOF. These splitting points define an orthogonal boundary in the selected DOF. This splitting process is recursively repeated until homogeneous but overlapping sets of regions are obtained, where homogeneity is defined according to a set of features measured for each region. A similar algorithm of interest is the region-sensitive adaptive motion planning (RESAMPL) [31]. RESAMPL subdivides the \mathbb{C}_{space} into local regions using an initial set of samples, valid or not. Some of these initial samples are randomly selected as the representative samples for the local regions. The distance of the representative sample to its k-closest neighbors determines the region's size. Approximate Cell Decomposition (ACD) subdivides \mathbb{C}_{space} into rectangular cells [38]. Similar to [6], each cell is labeled as empty, full, or mixed. PRM is combined with ACD to compute localized roadmaps by generating samples within these cells. The connectivity graph for adjacent cells in ACD is augmented with pseudo-free edges that are computed based on localized roadmaps.

In our previous work [17], we present \mathbb{C}_{space} subdivision-based parallel methods for randomized motion planning algorithms, particularly PRMs. We demonstrate that by subdividing the space and restricting the locality of connection attempts, scalable performance can be achieved. However, the regular subdivision method as presented is not well suited for RRT. Here, we design a novel radial subdivision technique for parallelizing RRT.

3 Parallelizing RRT

In this section, we present two different parallel algorithms for RRT computation. The first is the bulk synchronous version of the distributed RRT algorithm presented in [13]. We extend the algorithm by introducing a user-defined parameter that controls how much local expansion is made before a global broadcast. The second algorithm subdivides \mathbb{C}_{space} into regions radially and distributes RRT computation in each region to available processes. Regional branches in adjacent regions are later connected to form one single tree. In both cases, although the algorithms use the familiar RRT expansion operations, the trees that are

constructed are structurally different from a tree that would be constructed using a sequential method. We describe each approach in detail below.

3.1 Bulk Synchronous Distributed RRT

The distributed RRT algorithm in [13] incurs global broadcasts each time a new node and edge are added to the tree. However, this is not scalable. We extend this work but differ in two key ways. First, in order to optimize the use of space and memory, each process does not maintain a copy of the tree. Instead, they all have a shared access to the tree which is stored in a global distributed data structure. Requests to access an element on another process are sent and received through the global identifier (GID) assigned to the element. Second, we regulate inter-processor communication (a critical bottleneck to parallel processing) by introducing a variable m that controls how much expansion will done before a global update and broadcast. Setting m = 1 gives the same computational pattern as in [13].

Algorithm 2 describes the high-level implementation of bulk synchronous distributed RRT. We first initialize the tree T with the root node q_{root} . Subsequently, each process locally (in parallel) samples m nodes and finds its nearest node q_{near} from the tree. If the expansion q_{near} toward q_{rand} is successful, then the new node q_{new} and q_{near} are added to a temporary container N_m . After m steps, the global tree is updated. This process is continues until the termination condition is met. A simple illustration of bulk synchronous distributed RRT computation in which p=2, m=2 and N=8 is shown in Figure 1.

Algorithm 2 Bulk Synchronous Distributed RRT

Input: An environment env, a root q_{root} , the number of nodes N, the number of processes p, the number of local expansion steps m

```
Output: A tree T containing N nodes rooted at q_{root}
 1: T.AddNode(q_{root})
 2: for all proc p \in P par do
       i \leftarrow 0
 3:
       while i < N/p do
 4:
          localContainer N_m
 5:
          for j = 1 \dots m do
 6:
             q_{rand} \leftarrow \text{GetRandomNode}(env)
 7:
             q_{near} \leftarrow \text{FindNeighbor}(T, q_{rand}, 1)
 8:
             q_{new} \leftarrow \text{Extend}(q_{near}, q_{rand})
 9:
             if !TooSimilar(q_{near}, q_{new}) \wedge IsValid(q_{new}) then
10:
                N_m.Insert(q_{near}, q_{new})
11:
12:
             end if
          end for
13:
          for all node pair n \in N_m do
14:
             T.AddNode(n.q_{new})
15:
             T.AddEdge(n.q_{near}, n.q_{new})
16:
             i \leftarrow i + 1
17:
          end for
18:
       end while
19:
20: end for
```

3.2 Radial Subdivision Distributed RRT

21: return T

We also present a novel radial \mathbb{C}_{space} subdivision for parallelization especially suited for RRTs. Starting from the root q_{root} , we subdivide \mathbb{C}_{space} into conical regions and build part of the tree (branches) in each

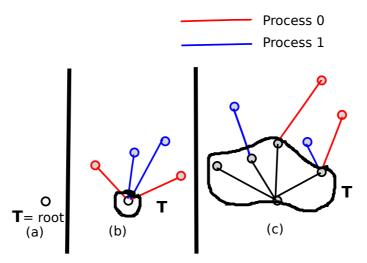


Figure 1: Bulk Synchronous Distributed RRT. (a) T is initialized to root. (b) The first iteration with m=2. (c) The second iteration where globally communicated data is shown in black.

region. These branches are later connected in a manner such that no cycle exists after region connection. We exploit locality by only attempting to connect branches that reside in neighboring regions. Figure 2 shows an example for a two dimensional \mathbb{C}_{space} . Each process builds a branch (shown in different colors) starting at the root that is biased toward their region of \mathbb{C}_{space} .

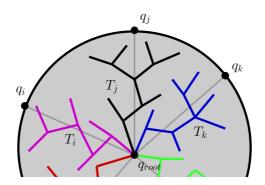


Figure 2: Example of radial subdivision for a 2D \mathbb{C}_{space} . Each process concurrently builds a branch (using sequential RRT) rooted at q_r and biased toward a target q_i (e.g., q_n for the black process).

Algorithm 3 describes the \mathbb{C}_{space} subdivision-based RRT computation in detail. Region construction first creates a hypersphere in d-dimensional \mathbb{C}_{space} S^d centered at $q_{root} \in R^d$ with radius r. We randomly sample N_r points $q_i \in R^d$ on the surface of S^d . Each point defines a conical region centered around the ray $\overline{q_{root}q_i}$. We construct a region graph G(V,E) where each vertex v_i represents a region defined by q_i and an edge (v_i,v_j) is added if q_j is in the k closest neighbors of q_i . Thus, the edges in the region graph encode the adjacency information between regions.

After region graph construction, we independently (in parallel) run sequential RRT in each region. The RRT construction is done in a way that the tree is biased toward the region candidate q_i . Each region is centered around the random ray $\overrightarrow{qroot}, \overrightarrow{q_i}$. Some overlap between regions is allowed so branches can explore part of the space in adjacent regions.

Using the adjacency information provided by the region graph, we make connection attempts between each region branch and its adjacent neighbors. We check if any edge connection at this point creates a cycle.

Algorithm 3 Radial Subdivision Distributed RRT

Input: An environment env, a root q_{root} , the number of nodes N, the number of processes p, the number of regions N_r , a region radius r, the number of adjacent regions k

```
Output: A tree T containing N nodes rooted at q_{root} 1: Construct a sphere S^d rooted at q_{root} with radius r
 2: Q_{N_r} \leftarrow \text{sample } N_r \text{ random points on the surface of } S^d
 3: Initialize region graph G(V, E) with V \leftarrow Q_{N_r} and E \leftarrow \emptyset
 4: for all q_i \in Q_{N_r} par do
       neighbors \leftarrow \text{FindNeighbors}(G, q_i, k)
 6:
       for all n \in neighbors do
           G.AddEdge(q_i, n)
 7:
        end for
 8:
 9: end for
     for all v_i \in V par do
10:
        T \leftarrow \text{ConstructBiasedRRT}(N/p, env, q_{root}, q_i)
    end for
12:
     for all (v_i, v_i) \in E par do
13:
        ConnectTree(T, v_i, v_i)
14:
       if Cycle(T) then
15:
           Prune(T)
16:
        end if
17:
18: end for
    return T
19:
```

If a cycle exists, we prune the tree so as to remove any cycles. In the results presented here, tree pruning is performed by running a graph search algorithm. A simple pictorial illustration for tree pruning is shown in Figure 3.

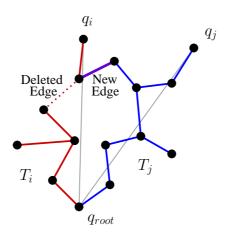


Figure 3: Tree pruning example. The new edge (purple) between the red and blue branches causes a cycle in the red branch (dashed edge). The dashed edge is identified for removal.

4 Implementation Details

4.1 STAPL Framework

Our code was written in C++ using the Standard Template Library (STL) and the Standard Template Adaptive Parallel Library (STAPL) as supporting libraries. STAPL [7,34] is a platform independent superset of STL that provides a collection of building blocks for writing parallel programs. These building blocks (as shown in Figure 4) include a collection of parallel algorithms (pAlgorithms), parallel and distributed containers (pContainers), a general mechanism to access the data of a pContainer similar to STL iterators called pViews, an abstraction of the computation task graph (PARAGRAPH), and an Adaptive Runtime System (ARMI) that includes a communication library, scheduler, and performance monitor.

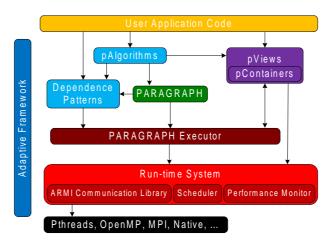


Figure 4: STAPL software architecture.

In this work, we made use of the STAPL Parallel Graph Library, one of the STAPL pContainers, as the parallel data structure for representing both the region graph and the RRT tree. We implemented bulk synchronous distributed RRT and radial subdivision distributed RRT as STAPL pAlgorithms. These algorithms were written in terms of pViews. The pViews give a shared object view of a distributed parallel container and facilitate parallel processing by supporting independent random access to a portion (or range) of the distributed container's elements. The tree pruning process described in Section 3.2 was implemented using the STAPL parallel breadth first search (BFS) algorithm.

4.2 Parallelizing Nearest Neighbor Search

There is a clear need for fine-grained parallelism in sampling based motion planning [4, 13, 14, 28, 28]. The nearest neighbor search is considered a key bottleneck to scalable performance because of its seemingly inherently sequential nature. In this work, we implement and incorporate a nested and fine-grained parallel computation of nearest neighbor search within the two parallel RRT algorithms described in Section 3. Our implementation is a map reduce parallel computation pattern. We implement the map reduce parallel computation pattern in STAPL as a palgorithm. It is illustrated in Figure 5.

Algorithm 4 describes the approach in the context of a distributed RRT. To compute the nearest point q_{near} to a query point q_{rand} , each processing element sends q_{rand} to the other processing elements by calling MapReduce(). The mapping function (Algorithm 5) receives the query point q_{rand} and locally computes its nearest neighbor in T_p based on a given distance metric. The reduce function (Algorithm 6) takes the two inputs returned by the mapping function and computes the nearest neighbor to q_{rand} from the two inputs based on the same distance metric. This computation scheme is general in that it can handle any arbitrary distance metric and any number of neighbors to find.

Algorithm 4 Parallel NNS Distributed RRT

Input: An environment env, a root q_{root} , the number of nodes N, the number of processes p**Output:** A tree T containing N nodes rooted at q_{root} 1: $T.AddNode(q_{root})$ 2: for all proc $p \in P$ par do $i \leftarrow 0$ 3: while i < N/p do 4: subtree $T_p \in T$ 5: $q_{rand} \leftarrow \text{GetRandomNode}(env)$ 6: $q_{near} \leftarrow \text{MapReduce}(Map(T_p, 1, q_{rand}),$ 7: $Reduce(q_{near}, q_{near}))$ $q_{new} \leftarrow \text{Extend}(q_{near}, q_{rand})$ 8: if !TooSimilar $(q_{near}, q_{new}) \wedge \text{IsValid}(q_{new})$ then 9: $T.AddNodeToTree(q_{new})$ 10: T.AddEdgeToTree (q_{near}, q_{new}) 11: end if 12: $i \leftarrow i + 1$ 13: end while 14: 15: end for

Algorithm 5 Map

16: return T

```
Input: A set of points S, a query q
Output: A map of closest point to q and its distance M
1: M_k \leftarrow \text{FindNeighbors}(S, q, 1)
2: return M
```

Algorithm 6 Reduce

```
Input: Two maps M1 and M2 of points and their distances to a query q

Output: The closest point p \in M1 \cup M2

1: if M1.distance \leq M2.distance then

2: p \leftarrow M1

3: else

4: p \leftarrow M2

5: end if

6: return M
```

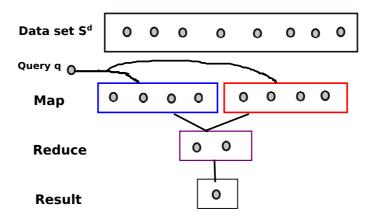


Figure 5: Map reduce computation of nearest neighbor search.

5 Experimental Setup and Results

We investigate the scalability of the two algorithms presented in Section 3: bulk synchronous distributed RRT and radial subdivision distributed RRT. We also examine the effect of robot complexity and machine architecture.

5.1 Experimental Setup

5.1.1 Machine Architecture

Experiments were conducted on two massively parallel computers. The first machine is a major Linux computing cluster at Texas A&M University. It has a total of 300 nodes, 172 of which are made of two quad core Intel Xeon and AMD Opteron processors running at 2.5GHz with 16 to 32GB per node. The 300 nodes have 8TB of memory and a peak performance of 24 Tflops. Each node of the Linux cluster is made of 8 processor cores, thus, for this machine we present results for processor counts in multiples of 8. The second machine is a Cray XE6 petascale machine at Lawrence Berkley National Laboratory. It has 6384 nodes, 217 TB of memory, and a peak performance of 1.288 peta-flops. Each node consists 12 processor cores. This architectural layout influenced our choice of processor counts to be in multiple of 12. Our code was written in C++ and compiled with gcc-4.5.2 on the Linux cluster and gcc-4.6.3 on the Cray XE6 machine. Using STAPL, the same C++ code was used on both architecture types.

5.1.2 Motion Planning Problems

We studied two different kinds of motion planning problems: a 6 DOF cube-like rigid body robot and a 16 DOF articulated linkage. Both robots are placed in the cluttered environment shown in Figure 6. The cluttered environment is $512 \times 512 \times 512$ units. It has 216 obstacles, each of size $2 \times 4 \times 4$ units, randomly scattered across the environment.

5.2 Bulk Synchronous Effect

We first study the effect of the m parameter introduced in Algorithm 2 to tune the amount of local expansion done before a global update. We fixed the sample size at 16,384 and used $m = \{1, 16, 64\}$. Note that m = 1 is the same as the distributed algorithm presented in [13]. Figure 7 shows the running time as a function of the number of processors on the Linux cluster for the rigid body robot up to 256 processors.

Localizing the computation and thus minimizing frequent inter-processor communication by varying m does have impact on the performance of distributed RRT, but this effect is not obvious until higher processor

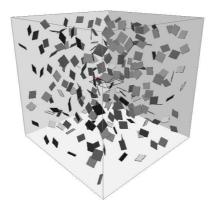


Figure 6: Clutter environment with 216 randomly placed obstacles.

counts, see Figure 7(b). In fact, m = 1 seems to outperform the others until around p = 16. This is quite understandable given that more processors incur more communication.

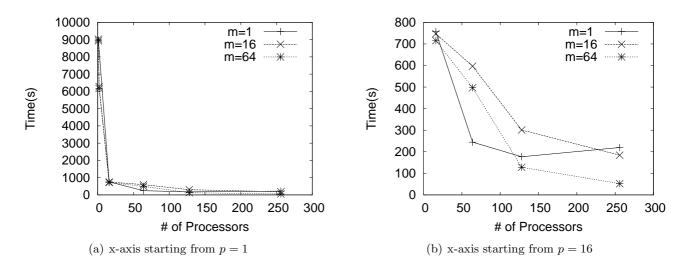


Figure 7: Effect of varying m in the bulk synchronous distributed RRT.

5.3 Radial Subdivision Scalability Study

As seen with the bulk synchronous distributed RRT, localizing computation reduces communication overhead which in turn improves the overall scalability of the algorithm. We now look at the scalability of radial subdivision distributed RRT on the two different robots: the 6 DOF rigid body and the 16 DOF articulated linkage. Figure 8 shows scalability on the Linux cluster up to 64 processors. Radial subdivision distributed RRT was able to achieve almost near linear speedups for both robot types.

5.4 Effect of Machine Architecture

We next study how the machine architecture impacts performance for both the bulk synchronous distributed RRT and the radial subdivision distributed RRT. For the bulk synchronous distributed RRT we use $m = \{1, 25, 50\}$ while keeping the sample size constant. Figure 9 shows the time and scalability for the rigid body robot on the Cray XE6 machine. Radial subdivision distributed RRT scales almost linearly with processor

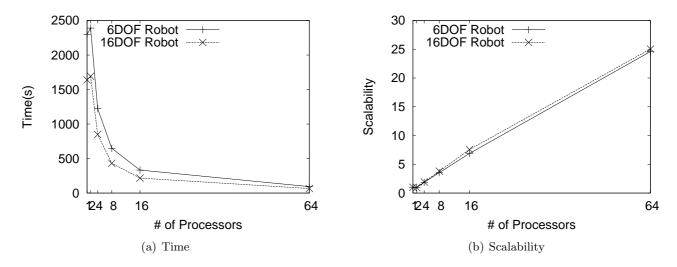


Figure 8: Radial subdivision distributed RRT performance on Linux cluster.

counts, similarly to on the Linux cluster (Figure 8(b)). Scalability of the bulk synchronous distributed RRT depends on the value of m and the number of processors. As in the previous experiments (Figure 7), the impact of increasing m is much felt at higher processor counts at which inter-processor communication become significant. It is at higher processor counts that this bulk synchronous effect is needed.

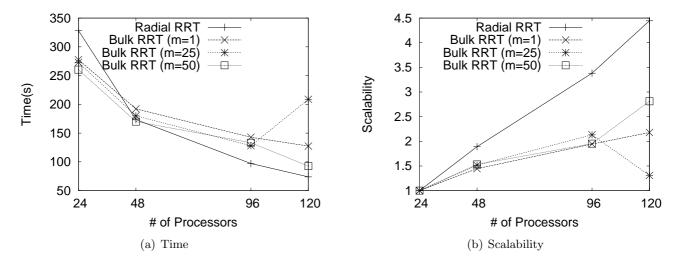


Figure 9: Distributed RRT performance on Cray XE6 machine.

6 Conclusion

In this paper, we present two parallel algorithms for RRT computation. The first algorithm extends existing work by introducing a parameter m that controls local computation of the tree before a global update across processors. The second algorithm radially subdivides the \mathbb{C}_{space} into regions and lets each processor build part of the tree in each region. By controlling local computation and subdividing the \mathbb{C}_{space} , we minimize the overhead associated with inter-processor communication in parallel processing. We present results for both a rigid body robot and an articulated linkages on two different parallel machine architectures.

In future, we will extend our work on parallelizing RRT to more challenging and high-dimensional motion planning problems like protein folding. We will also perform a detailed structural analysis of tree resulting from both \mathbb{C}_{space} subdivision and parallel processing.

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