MA Math Camp 2022
Columbia University
Syllabus

Andrea Ciccarone

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Course Information

- Instructor: Andrea Ciccarone
- Email: ac4790@columbia.edu
- Graders: TBA
- Course website: https://andreaciccarone.github.io/MA-Columbia-Math-Camp-2022
- Dates: Monday Aug. 15 - Thur Sep. 1
- Time: 9.30am - 12pm
- Place : Hamilton 517 (See a Map of Campus Here) and on Zoom (link and recordings will be shared by email)
- Office Hours : TBA

Course Description

The course will cover the mathematical tools and concepts required for the first year sequence of the Master in Economics. The main goal of the course is to prepare for first year classes by reviewing or introducing fundamental concepts in various domains of mathematics: real analysis, linear algebra, calculus and optimization. While studying these topics, we will refine proof-writing skills and develop familiarity with mathematical rigor. Emphasis will also be put on problem-solving and application of the tools. While the course is largely self-contained, students are expected to have taken courses in elementary analysis and unidimensional calculus, as well as have some familiarity with concepts in linear algebra.

The class will be taught in a hybrid format from Monday August 15th to Thursday September 1st. Lectures will be held in person (Hamilton 517) every weekday from 9.30am to 12pm EST. Lectures will

simultaneously be available on Zoom as well as recorded for asynchronous attendance. If possible, students are strongly encouraged to attend the lectures in real time.

The course is largely self-contained. Lecture notes will be posted on the website. Teaching itself will mostly take place on the blackboard but additional notes or slides might be provided. Some additional notes and textbook references are provided below.

Problem sets will be assigned weekly. These are important practice and will be graded for feedback, although no grade will be given for the class. Problem sets will have to be submitted online (modalities to be specified) and **will have to be typed**, in order to be graded. LaTeX is very strongly encouraged as it is an extremely valuable skill that students should acquire as soon as possible. There will be a final exam, the date and modality of the exam will be announced later.

Course Material

Course Outline and Lecture Notes

Here is a tentative course outline (I will soon publish all the related lecture notes):

1. Preliminaries : Mathematical Logic, Sets, Functions, Numbers
   (a) Introduction to Mathematical Logic
   (b) Sets
   (c) Relations
   (d) Numbers
   (e) Countability and Cardinality
2. Real Analysis
   (a) Metric Spaces
   (b) Basic Topology
   (c) Sequences and Convergence
   (d) Compactness
   (e) Cauchy Sequences and Completeness
   (f) Continuity of Functions
3. Linear Algebra
   (a) Vectors and Vector Spaces
   (b) Matrices
   (c) Systems of Linear Equations
   (d) Eigenvalues, Eigenvectors, and Diagonalization
   (e) Quadratic Forms

4. Multivariate Calculus
    (a) Derivatives
    (b) Mean Value Theorem
    (c) Higher Order Derivatives and Taylor Expansions
    (d) Log-Linearization
    (e) Implicit and Inverse Function Theorems
    (f) Integration
5. Convexity
    (a) Convex Sets, Separation Theorem, Fixed Point Theorems
    (b) Convex and Concave Functions
    (c) Quasi-convex and Quasi-concave functions
6. Optimization
    (a) General Setup
    (b) Result on the set of Maximizers
    (c) Optimization on R^n
    (d) Kuhn-Tucker Theorem
    (e) Introduction to Dynamic Programming
7. Correspondences (if time permits)

Lectures notes are susceptible to being continuously updated (be sure to check the date of last update, which is always mentioned at the top of the pdf). I'm sure there are typos in the notes. If you spot one, please point it out.

Problem Sets and Exam

Problem sets will be posted here. Below is a tentative schedule:

- Problem Set 1 (Logic, Sets, Analysis)
    - Date Posted : Monday August 15th
    - Date Due : Monday August 22nd
- Problem Set 2 (Real Analysis, Linear Algebra)
    - Date Posted : Monday August 22nd
    - Date Due : Monday August 29th
- Problem Set 3 (Multivariate Calculus, Convexity, Optimization)
    - Date Posted : Monday August 29th
    - Date Due : Monday August 5th

References and Textbooks

Two very useful short introductions to mathematical proofs :

- Proving Things, by Stéphane Dupraz
- How to do proofs in Mathematics, by Xingye Wu

Below is a list of useful references and textbooks sorted by theme. Within each theme, references are listed in (approximately) increasing complexity. References marked with a (!) are more advanced and are included either for future references or very motivated students.

- General references
  - Knut Sydsaeter, Peter Hammond, Arne Strom and Andr'es Carvajal. "Essential mathematics for economic analysis.", 5th Edition, (2016), Pearson.
  - Knut Sydsaeter, Peter Hammond, Atle Seierstad and and Arne Strom. "Further mathematics for economic analysis.", 2nd Edition, (2008), Pearson.
- Analysis
  - Walter Rudin. "Principles of Mathematical Analysis" (1976), International Series in Pure & Applied Mathematics, McGraw-Hill.
  - Ok, Efe A. "Real Analysis with Economic Applications" (2007).
  - (!) Walter Rudin, Real and Complex Analysis, Third Edition (1987), McGraw-Hill.
- Linear Algebra
  - Treil, Sergei. "Linear Algebra Done Wrong." (2014) (available online here
  - Lang, Serge. "Linear Algebra", Third Edition (2004), Springer Undergraduate Texts in Mathematics.
- Optimization
  - Rangarajan K. Sundaram, "A First Course in Optimization Theory" (1996), Cambridge University Press.
- Dynamic Programming
  - Klaus Walde, "Applied Intertemporal Optimization" (2020), available online for free here
  - Nancy L. Stokey, Robert E. Lucas Jr, and Edward C. Prescott. "Recursive Methods in Economic Dynamics" (1989), Harvard University Press.

The problem sets will have to be typed and students are encouraged to use LaTeX. LaTeX is a powerful tool for seamless and systematic typesetting that produces clean and readable documents. It is arguably the best practical options to typeset mathematical notations and it is the standard tool in the academic world in Economics. For those that are not familiar with LaTeX, here are a few references to get started:

- The website Overleaf is a great practical way to get started with LaTeX. You can create a free account and work on LaTeX documents without having to install anything on your computer, it is all browser based. Furthermore, Overleaf has some useful templates and a very good guide to getting started with LaTeX (which is useful even you choose to use another editor) and many tutorials.
- If you prefer to install a local LaTeX distribution and editor on your laptop, there are several good options that come "pre-packaged" with everything you need. Notable among those are MikTeX for Windows or MacTeX for MacOS, which includes the editor TeXshop and a number of useful packages.
- For the more adventurous, you can download separately a LaTeX distribution and then pair it with any editor you like (VSCode, Sublime Text, Atom,...).
- If you are strongly averse to writing explicit commands, LyX is an alternative LaTeX-based software that wraps it in a more visual "Word-like" environment.
- Another good guide to LaTeX: The Not So Short Guide to LaTeX
- A useful guide for all the math command that you might need in the AMS package : Short Math Guide for LaTeX
- There are a lot of good LaTeX tutorials out there, don't hesitate to look for them and see if you find one you like. Most importantly, after you grasp the general idea of how LaTeX works, you'll learn the most by just using it and figuring out how to do what you need to do.