Economics G5410 - Mathematical Methods for Economists - Fall 2015
Math Camp Exam

Date: 11:55 am - 12:55 pm, September 8, 2015
Instructions:
You may use any results covered in class directly without proofs. Please attempt to do all questions.

1. Diagnalize the following matrix

$$A = \begin{bmatrix} 2 & -1.5 \\ 1 & -0.5 \end{bmatrix}$$

   i.e. find a matrix $P_{2\times 2}$ s.t. $P^{-1}AP = D$, where D is a diagnal matrix.

2. Let $f : (1,2) \to \mathbb{R}$ be defined as

$$f(x) := \int_{x^2}^{6} \left(e^t \ln t\right) dt$$

   for any $x \in (1,2)$. Give $f'(x)$.

3. Log-linearize the equation

$$K^\alpha L^{1-\alpha} = C + I$$

   around the steady state $(A^*, K^*, L^*, C^*, I^*)$. ($\alpha$ is a fixed parameter.)

4. In metric space (S,d), the sequences $\{x_n\}_{n=1}^{+\infty}$ converges to x, and the sequence $\{y_n\}_{n=1}^{+\infty}$ converges to y. Show that $\{d(x_n, y_n)\}_{n=1}^{\infty}$ converges to $d(x,y)$ in $(\mathbb{R}, d_{\mathbb{R}})$, where $d_{\mathbb{R}}(s,t) := |t-s|$ for any $s,t \in \mathbb{R}$.

5. The functions $f_1 : \mathbb{R}^k \to \mathbb{R}$ and $f_2 : \mathbb{R}^k \to \mathbb{R}$ are quasi-concave. Define $f : \mathbb{R}^k \to \mathbb{R}$ as

$$f(x) := \min\{f_1(x), f_2(x)\}$$

   for any $x \in \mathbb{R}^k$. Show that f is also quasi-concave.

6. Consider the following maximization problem:

$$\max_{(x_1,x_2)\in\mathbb{R}^2} -x_2$$

   s.t.

$$x_1^2 - x_2^3 = 0$$

   Find all maximizers of this problem, if there is any. Do the maximizers you find satisfy FOC? Explain.