Logistics 2020/2021
Posizionamento Parcheggi

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July 14, 2021

1 Description of problem

The aim of this project is to help a mayor to place new parking lots on both sides of a road. The request is to positioning them in order to balance the amount of space occupied by each side of the road as much as possible.
There are tree types of parking:

	Parking Type	Length
1	Car	2.3 m
2	Motorcycle	1.4 m
3	Tourist buses	15 m

2 Mathematical formulation

As first task we formulate in terms of PLI the problem of determining the position of the parking lots on both sides of the road in order to optimize the criterion specified above, then we implement the proposed model using the AMPL modeling language, and solve it using CPLEX.

2.1 Integer Linear Programming Model

Here we present our ILP model for the problem, with the sets of vehicles I and sides of road J so formed:

$$
\begin{aligned}
& min \quad w \\
& \sum_{j \in J} x_{1j} = 7 && && (1) \\
& \sum_{j \in J} x_{2j} = 11 && && (2) \\
& \sum_{j \in J} x_{3j} = 3 && && (3) \\
& \sum_{i \in I} x_{ij} \cdot C_i \leq w && \forall j \in J && (4) \\
& x_{ij} \geq 0 \quad \text{integer} && \forall i \in I, \forall j \in J && (5)
\end{aligned}
$$

Model 1.1

2.2 Description

In this problem we are dealing with two sets, vehicle types and sides of the road, in our model we defined them in the following way:

- I = set of **vehicle types**, $i \in I = \{1, 2, 3\}$

  Each value of the set i correspond to one parking type, respectively: 1 for car, 2 for motorcycle and 3 for tourist buses.

- J = set of **road sides**, $j \in J = \{1, 2\}$

  Each value of the set j correspond to one side of the road: 1 for the left side and 2 for the right side

We represented the parking length of each vehicle type as:

- C_i, **parking length** of type vehicle i

  Our input parking lengths are the following: $C_1 = 2.3$ for cars, $C_2 = 1.4$ for motorcycles and $C_3 = 15$ for tourist buses

In order to keep track of the number of vehicles in each side of the road we defined:

- x_{ij}, the **quantity** of vehicle type i in the road side j ;

The stated linear constraints, in our **Model 1.1**, have the following meaning:

1, 2 and 3 are **allocation constraints** used to impose the number of parking lots to set. In these tree constraints we specify that the sum on both road sides must be equal to the number of parking lots to be located for each type of vehicle;
4 is a **linking constraint** used to link each parking slot to its respectively side of the row. Here we defined w as an upper bound of the sum of the parking length for each type for both the side of the road;
5 is defined our variable as an integer greater than 0.

Our aim is to balance the total length in meters on both road sides as much as we can. In order to impose this we stated our objective function as $min \quad w$. We previously defined w as an upper bound of the sum of the parking lots length for each type in both our road sides. For the purpose of balancing the total length we want to minimise this upper bound since the only way to do it is to try to split the total sum of meters length occupied by all the parking lots as close as possible to its half.
The optimal case of our objective function is represented as: $w \approx \frac{\text{TotalParkingLotsLength}}{2}$.
Our mathematical model will try to state a w as close as possible to the optimal solution.

2.3 CPLEX Results

The optimal integer solution found with AMPL, using CPLEX as solver, has an objective function value of **38.3**. It is achieved by locating the parking lots as shown in the table below.

	Quantity Located	
	Left Side	Right Side
Type 1	4	3
Type 2	10	1
Type 3	1	2
Total length	38.2	38.3

3 Additional Constraint

The problem introduces some additional constraints. In this new scenario we have to locate all parking of type 3 in only one side of the road, knowing that the maximum length on the left size of the road that can be exploited is 30 meters. Our aim is still to balance the amount of space occupied by each side of the road as much as possible, but now some new constraints must be added. We defined a new variable:

$$y_j = \begin{cases} 1, & \text{if parking type 3 is in j} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

This y_j variable is added with the aim to keep track on which side of the road we decided to place all parking lots of type 3.
The **new model** can be represented as follows:

Model 1.1

$$+$$

$$\sum_{i \in I} x_{i1} \cdot C_i \leq 30 \qquad (6)$$

$$\sum_{j \in J} y_j = 1 \qquad (7)$$

$$x_{3j} \leq 3 \cdot y_j \qquad \forall j \in J \qquad (8)$$

$$y_j = \{0, 1\} \qquad \forall j \in J \qquad (9)$$

We added to our **Model 1.1** some new constraints. They have the following meaning:

(6) force the total sum of all parking lots on the left side of our road must be at most *30 meters*
(7) is used to set equal to 1 our new variable only once, so as to place all type 3 parking lots in only one road side
(8) here is defined x_{3j}, so as to impose that all type 3 parking lots must be located in only one side of the road
(9) we specify that the new variable is binary and it state in which side of the road type 3 parking lots are positioned

3.1 CPLEX Results

The optimal solution found for the model with additional constraints is **47.3**. Indeed the parking lots of Type 3 must be located in the same side, but the left side can only contain 2 out of 3 lots of Type 3 due to its maximum length constraint. So, as intuition, right side is chosen for locating lots of Type 3.
In the table below are shown the quantity located in each side.

	Quantity Located	
	Left Side	Right Side
Type 1	6	1
Type 2	11	0
Type 3	0	3
Total length	29.2	47.3