Time Series Econometrics Project

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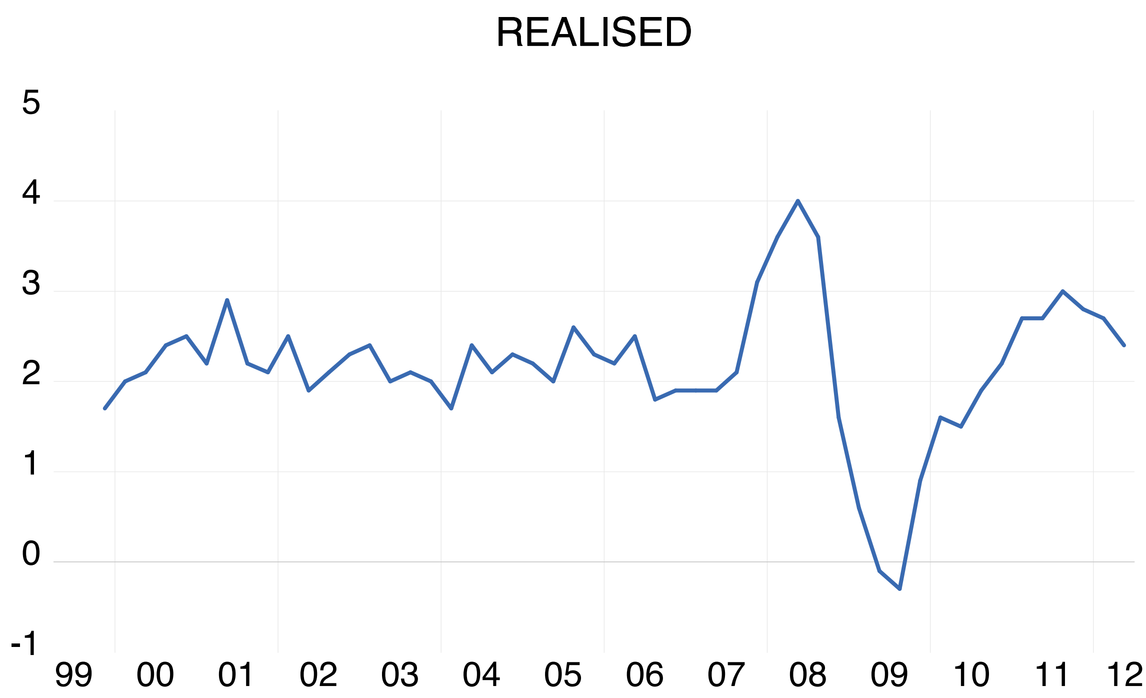
Introduction

Series in ECB forecasts are the forecasts of the professional forecasters and the Realizations of inflation for the euro area.

We are going to use observations 2 to 54 (1999Q2-2012Q2) to produce a model to forecast inflation (for 2012Q3-2020Q1).

In the following graph we can see how realizations (true values) of inflation go in the sample 1999Q2-2012Q2:



We can see how inflation stays more or less at 2% level. In 2008 there is a shock in which it steeply increases up to about 4% and then decreases to a 0% before adjusting from 2010 on, again to that "fair" level of 2%.

1. Model Selection and Estimation

Before estimating the equations with the parameters and the attached likelihood and information criterion, we are going to make a preliminary investigation on the eventual presence of a unit root.

Null Hypothesis: REALISED has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.218593	0.0016
Test critical values:	1% level	-3.574446	
	5% level	-2.923780	
	10% level	-2.599925	

*MacKinnon (1996) one-sided p-values.

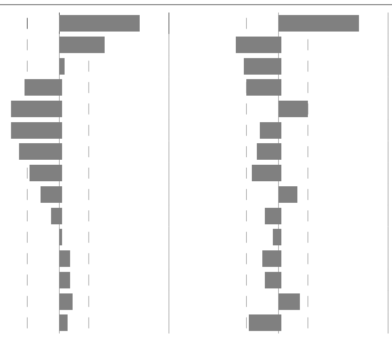
From the Augmented Dickey-Fuller Test we can see that the process has NOT a unit root, by rejecting the null hypothesis (intercept and no time trend). Here the process seems to be almost “mean-reverting” (not fully) because there is a “fair” level of inflation of 2%. (Mean = 2.15).

So, we can continue with the estimation of our model without any unit root.

In a preliminary investigation from the correlogram we could attempt to say which model can be fitted, but it will be safer if we choose it with an information criterion.

Date: 12/02/20 Time: 16:01
Sample (adjusted): 1999Q4 2012Q2
Included observations: 51 after adjustments

Autocorrelation Partial Correlation



	AC	PAC	Q-Stat	Prob
1	0.760	0.760	31.206	0.000
2	0.415	-0.384	40.703	0.000
3	0.035	-0.305	40.771	0.000
4	-0.320	-0.290	46.669	0.000
5	-0.427	0.270	57.399	0.000
6	-0.426	-0.160	68.324	0.000
7	-0.358	-0.184	76.204	0.000
8	-0.264	-0.234	80.600	0.000
9	-0.158	0.171	82.204	0.000
10	-0.072	-0.111	82.550	0.000
11	0.012	-0.034	82.560	0.000
12	0.088	-0.133	83.098	0.000
13	0.081	-0.111	83.564	0.000
14	0.115	0.186	84.538	0.000
15	0.067	-0.271	84.877	0.000

In particular we are going to compare the Bayes information criterion (Schwarz criterion) in order to select the model.
I'm going to report only the BIC (Schwarz score) for all the possible ARMA(p, q) models with p<=4 and q<=4.

	iid	MA(1)	MA(2)	MA(3)	MA(4)
iid	2.352063	1.638368	1.641671	1.121882	0.893601
AR(1)	1.569833	1.541158	1.457845	0.530251	1.198534
AR(2)	1.501454	0.952780	1.646536	0.703114	0.926435
AR(3)	1.505486	1.479813	1.456306	1.337207	1.210969
AR(4)	1.508547	1.481536	1.563433	1.463767	1.365355

Our selected model is the **ARMA(1, 3)**

Dependent Variable: REALISED
Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/02/20 Time: 16:06
Sample (adjusted): 2000Q1 2012Q2
Included observations: 50 after adjustments
Failure to improve likelihood (non-zero gradients) after 289 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: OFF (Roots of MA process too large)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.702127	0.193811	13.94210	0.0000
AR(1)	0.732999	0.046979	15.60280	0.0000
MA(1)	0.311345	0.235418	1.322521	0.1927
MA(2)	1.282130	0.256704	4.994587	0.0000
MA(3)	1.275398	0.221636	5.754465	0.0000

R-squared	0.883340	Mean dependent var	2.162000
Adjusted R-squared	0.872970	S.D. dependent var	0.767155
S.E. of regression	0.273423	Akaike info criterion	0.339049
Sum squared resid	3.364215	Schwarz criterion	0.530251
Log likelihood	-3.476217	Hannan-Quinn criter.	0.411860
F-statistic	85.18416	Durbin-Watson stat	2.138918
Prob(F-statistic)	0.000000		

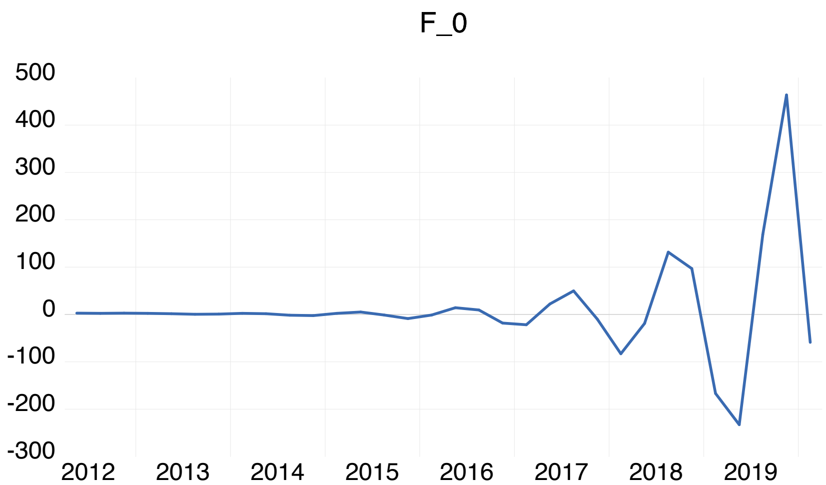
Inverted AR Roots	.73		
Inverted MA Roots	.23+1.26i	.23-1.26i	-.78
	Estimated MA process is noninvertible		

Checking the residuals with Portmanteau test we CANNOT be absolutely sure that they resemble an independent process. The test of independence on the residuals (Portmanteau test) is saying that we can reject the null hypothesis of independence between residuals. To be sure it could be useful to make another test.

Date: 12/02/20 Time: 16:17
Sample (adjusted): 2000Q1 2012Q2
Q-statistic probabilities adjusted for 4 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.094	-0.094	0.4652	
		2	-0.002	-0.011	0.4656	
		3	-0.104	-0.106	1.0610	
		4	-0.343	-0.371	7.7176	
		5	-0.016	-0.119	7.7332	0.005
		6	0.052	0.008	7.8949	0.019
		7	0.024	-0.067	7.9303	0.047
		8	-0.082	-0.281	8.3504	0.080
		9	-0.001	-0.131	8.3504	0.138
		10	-0.018	-0.049	8.3719	0.212
		11	-0.003	-0.111	8.3724	0.301
		12	0.182	0.003	10.646	0.223
		13	-0.188	-0.300	13.135	0.157
		14	0.115	-0.005	14.097	0.169
		15	0.080	0.108	14.570	0.203
		16	-0.161	-0.198	16.551	0.167
		17	0.229	0.051	20.680	0.080
		18	-0.245	-0.246	25.538	0.030
		19	0.003	-0.007	25.539	0.043
		20	0.040	-0.005	25.678	0.059
		21	-0.025	-0.113	25.734	0.079
		22	0.130	0.010	27.298	0.074
		23	-0.005	-0.030	27.301	0.098
		24	-0.072	-0.127	27.825	0.114

Considering the fact that is always preferable the "parsimonious modelling" it seems that the ARMA(1, 3) is overparametrized. The estimated parameters are equally non-sensical (on the non-invertibility region) so we should ignore the options with more than one MA term with these data.
Having a look to the forecast made with ARMA(1, 3) we can easily see that values of inflation explode up to an absurd 500% (below in the graph).



So we can try to make some restriction on our model, selecting an ARMA(1, 1):

Dependent Variable: REALISED
Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/03/20 Time: 16:04
Sample (adjusted): 2000Q1 2012Q2
Included observations: 50 after adjustments
Failure to improve likelihood (non-zero gradients) after 8 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: 1999Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.182847	0.247881	8.806026	0.0000
AR(1)	0.622279	0.139521	4.460105	0.0001
MA(1)	0.383739	0.166146	2.309642	0.0253

R-squared	0.625103	Mean dependent var	2.162000
Adjusted R-squared	0.609150	S.D. dependent var	0.767155
S.E. of regression	0.479610	Akaike info criterion	1.426437
Sum squared resid	10.81120	Schwarz criterion	1.541158
Log likelihood	-32.66092	Hannan-Quinn criter.	1.470123
F-statistic	39.18391	Durbin-Watson stat	1.950766
Prob(F-statistic)	0.000000		

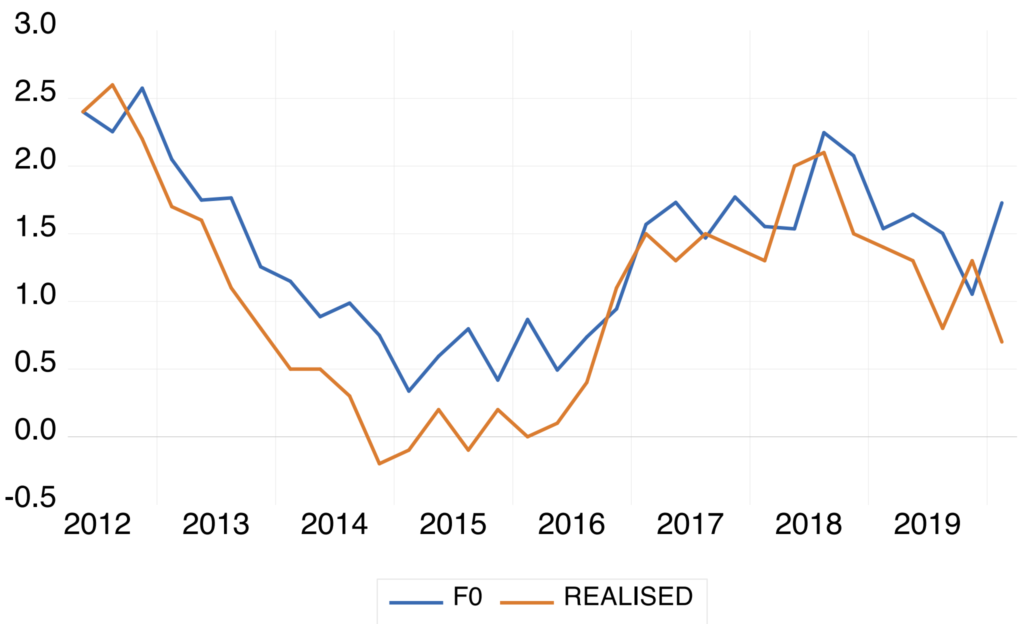
Inverted AR Roots	.62
Inverted MA Roots	-.38

Date: 12/03/20 Time: 16:05
Sample (adjusted): 2000Q1 2012Q2
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.023	0.023	0.0277	
		2	0.157	0.156	1.3579	
		3	0.083	0.078	1.7389	0.187
		4	-0.448	-0.490	13.103	0.001
		5	-0.108	-0.147	13.773	0.003
		6	-0.142	0.047	14.964	0.005
		7	-0.092	0.064	15.472	0.009
		8	-0.089	-0.359	15.960	0.014
		9	0.034	-0.102	16.032	0.025
		10	-0.037	0.005	16.120	0.041

Just take in mind that still Portmanteau test from the 4th lag seems not to validate our model.
I continue with my ARMA(1, 1).

Here the more reasonable forecast made with ARMA(1, 1) and the realized values:

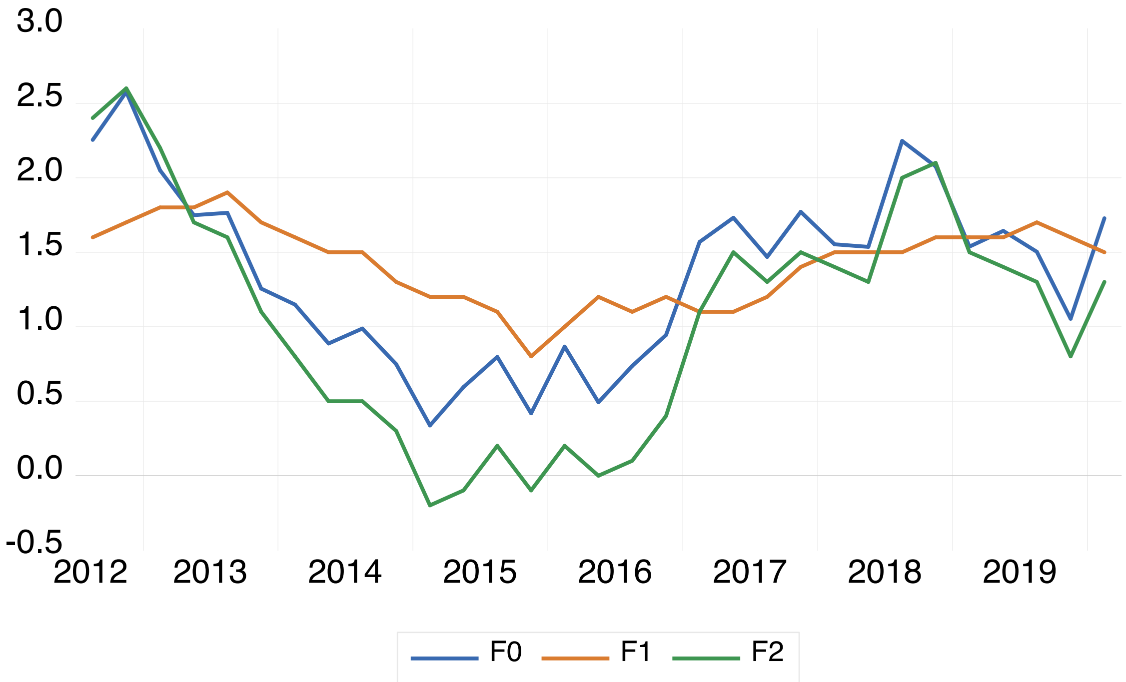


2. Forecast Comparison

Now let's compare the forecast requested:

1. f_0 → The one with the model selected, [ARMA(1, 1)];
2. f_1 → Professionals' forecast (ECB forecast);
3. f_2 → Naive forecast made by taking the last observed value.

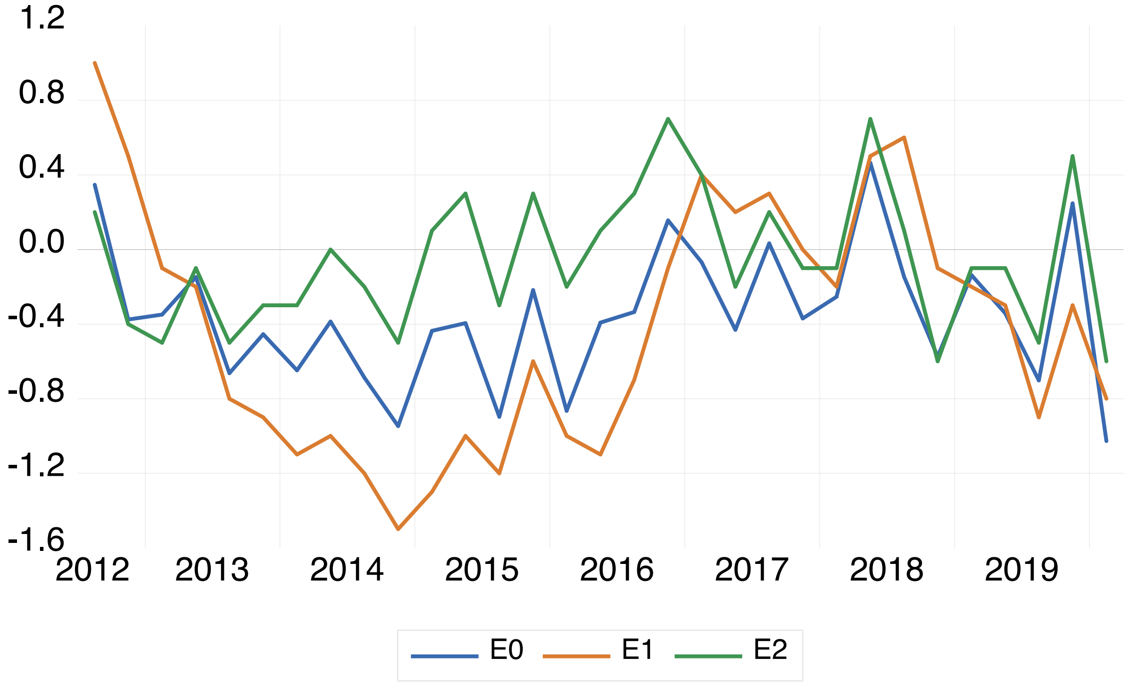
Below I report the plot of the forecast f_1, f_2 and f_3.



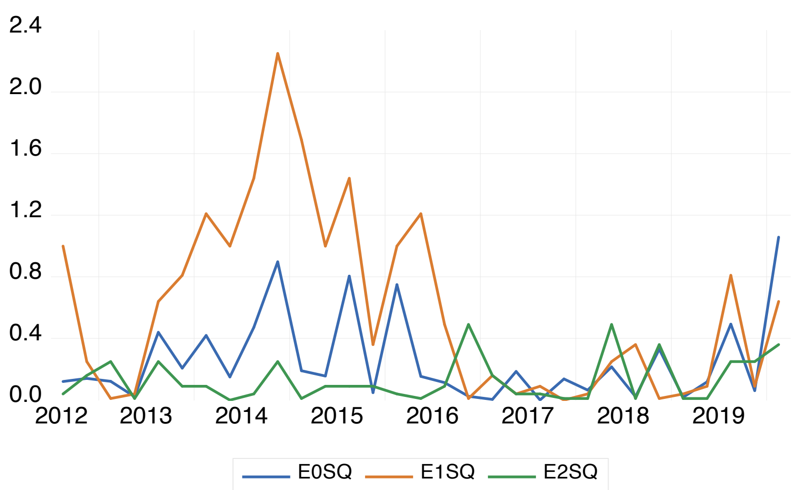
To better see how they are performing we have to take into account the error that each forecast makes. So, I generate series for the errors of each forecast and then I put it to the power of 2 (squared).

Then we can see in a graph the comparison between the absolute errors of the forecast f_0, f_1 and f_2:

- e0: is the absolute error made by the forecast of ARMA(1, 1);
- e1: is the absolute error made by the professional forecasters;
- e2: is the absolute error made by the naïve forecast.



In order to see which one of the 4 forecasts is performing better we can compare the mean of the squared errors and pick up the one with the lowest value for it.



Date: 12/03/20 Time: 16:21
Sample: 2012Q3 2020Q1

	E0SQ	E1SQ	E2SQ
Mean	0.256073	0.595806	0.131935
Median	0.149779	0.360000	0.090000
Maximum	1.057302	2.250000	0.490000
Minimum	0.001033	0.000000	0.000000
Std. Dev.	0.281786	0.600049	0.143861
Skewness	1.473578	0.894674	1.168613
Kurtosis	4.185582	3.045011	3.353860
Jarque-Bera	13.03463	4.138230	7.217632
Probability	0.001478	0.126297	0.027084
Sum	7.938253	18.47000	4.090000

So the forecast that is performing best in terms of least mean squared error is the naïve forecast (0.131935).

One last comparison between forecast can be done with Diebold and Mariano test, regressing the difference between the errors squared with a constant.

- Diff0 = difference between the errors squared of f_0 with f_1 (ARMA(1, 1) against professionals' forecast – [namely e0sq-e1sq]);
- Diff1 = difference between the errors squared of f_0 with f_2 (ARMA(1, 1) against naive forecast - [namely e0sq-e2sq]);
- Diff2 = difference between the errors squared of f_1 with f_2 (professional forecasters against naive forecast - [namely e1sq-e2sq]).

From these regressions we can see whether the difference between the errors squared (comparison between 2 models) is statistically different from 0 or not.

If you fail to reject the null hypothesis (p-value>0.05) you can say that there is NOT statistically significative difference between the forecasts in comparison.

Then the output of each single regression between differences of squared errors and a constant c:

Dependent Variable: DIFF0
Method: Least Squares
Date: 12/03/20 Time: 16:35
Sample: 2012Q3 2020Q1
Included observations: 31

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.339734	0.087302	-3.891457	0.0005

R-squared	0.000000	Mean dependent var	-0.339734
Adjusted R-squared	0.000000	S.D. dependent var	0.486080
S.E. of regression	0.486080	Akaike info criterion	1.426837
Sum squared resid	7.088200	Schwarz criterion	1.473095
Log likelihood	-21.11598	Hannan-Quinn criter.	1.441916
Durbin-Watson stat	0.593884		

Professionals' forecasters forecast are statistically superior to the one made by ARMA(1, 1)

Dependent Variable: DIFF1
Method: Least Squares
Date: 12/03/20 Time: 16:35
Sample: 2012Q3 2020Q1
Included observations: 31

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.124137	0.050366	2.464711	0.0197

R-squared	0.000000	Mean dependent var	0.124137
Adjusted R-squared	0.000000	S.D. dependent var	0.280425
S.E. of regression	0.280425	Akaike info criterion	0.326706
Sum squared resid	2.359147	Schwarz criterion	0.372964
Log likelihood	-4.063942	Hannan-Quinn criter.	0.341785
Durbin-Watson stat	1.735905		

Arma(1, 1) forecast are statistically superior to the naïve forecast ones.

Dependent Variable: DIFF2
Method: Least Squares
Date: 12/03/20 Time: 16:36
Sample: 2012Q3 2020Q1
Included observations: 31

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.463871	0.114683	4.044810	0.0003

R-squared	0.000000	Mean dependent var	0.463871
Adjusted R-squared	0.000000	S.D. dependent var	0.638528
S.E. of regression	0.638528	Akaike info criterion	1.972423
Sum squared resid	12.23154	Schwarz criterion	2.018681
Log likelihood	-29.57256	Hannan-Quinn criter.	1.987502
Durbin-Watson stat	0.663318		

Professional's forecasters forecasts are statistically superior to the naive ones. Even though we could have understand it by transitivity from the 2 previous regressions.