Univariate Linear Regression

June 17, 2019

1 Linear Regression Programming Exercise

In [1]:
```
# %load ../../standard_import.txt
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from sklearn.linear_model import LinearRegression
from mpl_toolkits.mplot3d import axes3d

pd.set_option('display.notebook_repr_html', False)
pd.set_option('display.max_columns', None)
pd.set_option('display.max_rows', 150)
pd.set_option('display.max_seq_items', None)

#%config InlineBackend.figure_formats = {'pdf',}
%matplotlib inline

import seaborn as sns
sns.set_context('notebook')
sns.set_style('white')
```

1.1 warmUpExercise

In [2]:
```
def warmUpExercise():
    return(np.identity(5))
```

In [3]:
```
warmUpExercise()
```

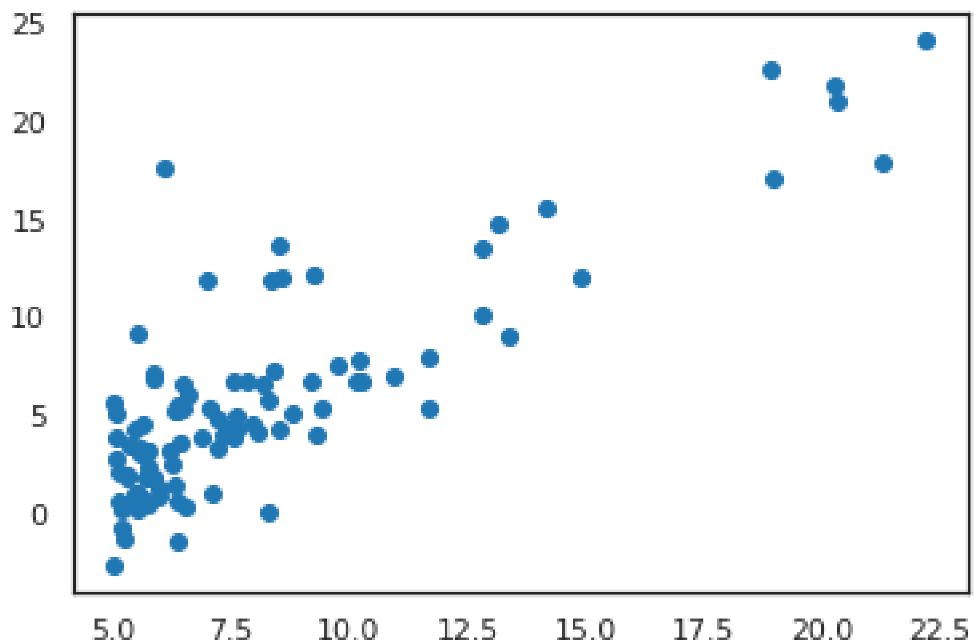
Out[3]:
```
array([[1., 0., 0., 0., 0.],
       [0., 1., 0., 0., 0.],
       [0., 0., 1., 0., 0.],
       [0., 0., 0., 1., 0.],
       [0., 0., 0., 0., 1.]])
```

2 Linear regression with one variable

```
In [4]: data = np.loadtxt('data/ex1data1.txt', delimiter=',')
        print(data.shape)

        X = np.c_[np.ones(data.shape[0]), data[:,0]]
        Y = np.c_[data[:,1]]
        plt.scatter(X[:,1],Y)
```

```
(97, 2)
```

```
Out[4]: <matplotlib.collections.PathCollection at 0x7f2c54ca92e8>
```



2.0.1 We have a training set and we want to find the following hypothesis function:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

We need the theta values which best fit the training data; what we do is to solve this optimization problem:

$$min_{\theta_0,\theta_1} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

where the argument is called (Squared Error) Cost Function

$$J(\theta_0, \theta_1)$$

Vectorized version:

$$J(\theta) = \frac{1}{2m}(X\theta - y)^T(X\theta - y)$$

In [5]:
```
def costFunction(X, y, theta=[[0],[0]]):
    m=y.size
    J=0
    h=X.dot(theta)
    J= 1/(2*m)*(h-y).T.dot(h-y)
    return J
```

2.0.2 Let's introduce an algorithm for minimizing the cost function, the Gradient Descent.

2.0.3 We start up with an inital guess about the theta parameters then we iterate and we keep changing theta parameters in order to reduce cost function until we reach the local minimum

$$\theta_j = \theta_j - \alpha \frac{\partial J(\theta)}{\partial \theta_j}$$

with

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

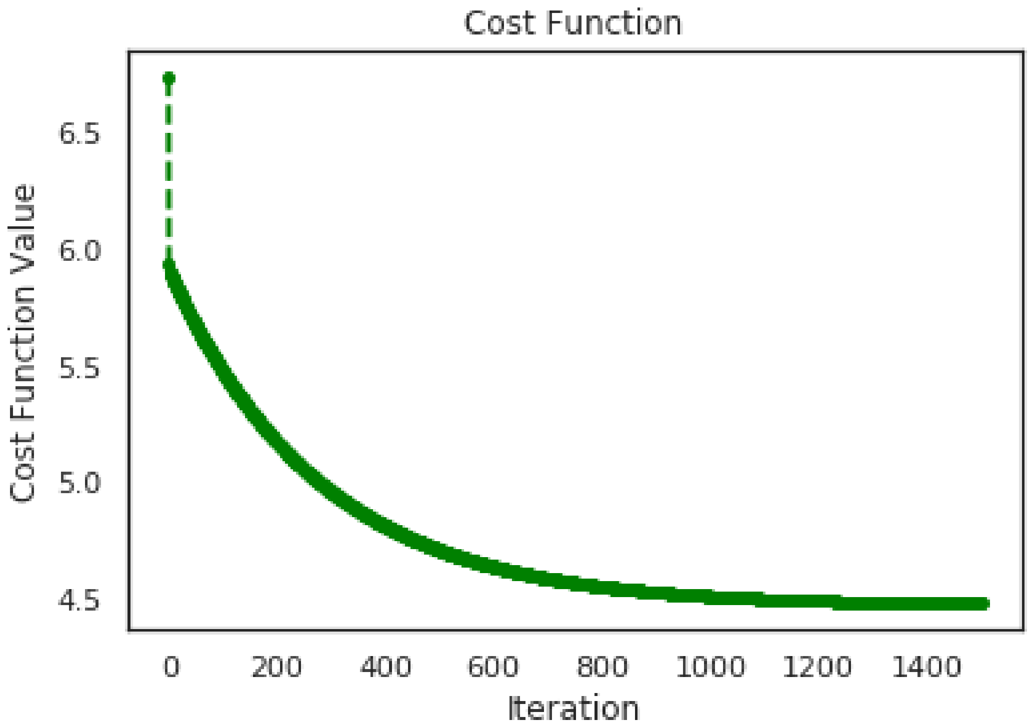
where alpha is called the learning rate and it represents the aggressiveness of the algorithm ### Vectorized:

$$\theta = \theta - \frac{\alpha}{m}((X\theta - y)^T X)^T = \theta - \frac{\alpha}{m}(X^T(X\theta - y))$$

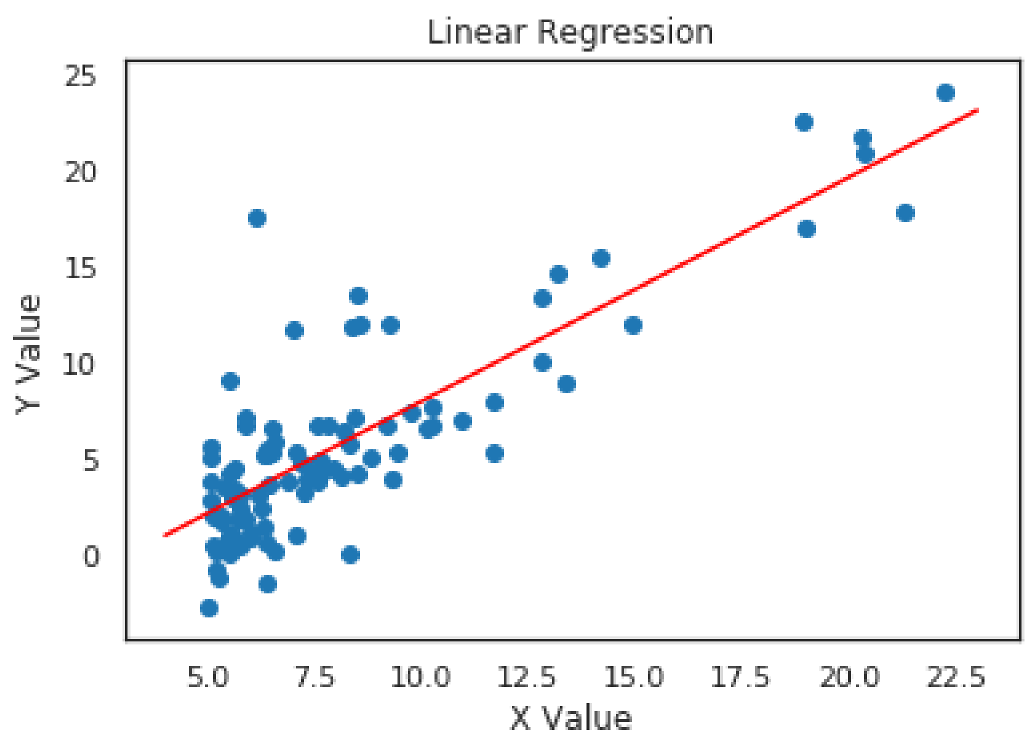
In [6]:
```
def gradientDescent(X, y, theta=[[0],[0]], alpha=0.01, n_iter=1500):
    m = y.size
    J = np.zeros(n_iter)
    for i in range(0, n_iter):
        h = X.dot(theta)
        theta = theta - (alpha/m) * ((X.T).dot(h-y))
        J[i] = costFunction(X,y, theta)

    return (theta,J)
```

In [7]:
```
def plot(X, y, theta=[[0],[0]], alpha=0.01, n_iter=1500):
    theta, cost_J = gradientDescent(X, y, theta, alpha, n_iter)
    plt.plot(cost_J, 'go--', linewidth=2, markersize=4)
    plt.title('Cost Function')
    plt.xlabel('Iteration')
    plt.ylabel('Cost Function Value')
    plt.show()

    a = np.linspace(4, 23, 100)
    b = theta[0] + theta[1] * a
    plt.plot(a, b, 'r')
    plt.scatter(X[:, 1], y)
    plt.title('Linear Regression')
    plt.xlabel('X Value')
    plt.ylabel('Y Value')
    plt.show()
```

```
In [8]: plot(X,Y)
```



Cost Function
Cost Function Value
6.5
6.0
5.5
5.0
4.5
0
200
400
600
800
1000
1200
1400
Iteration



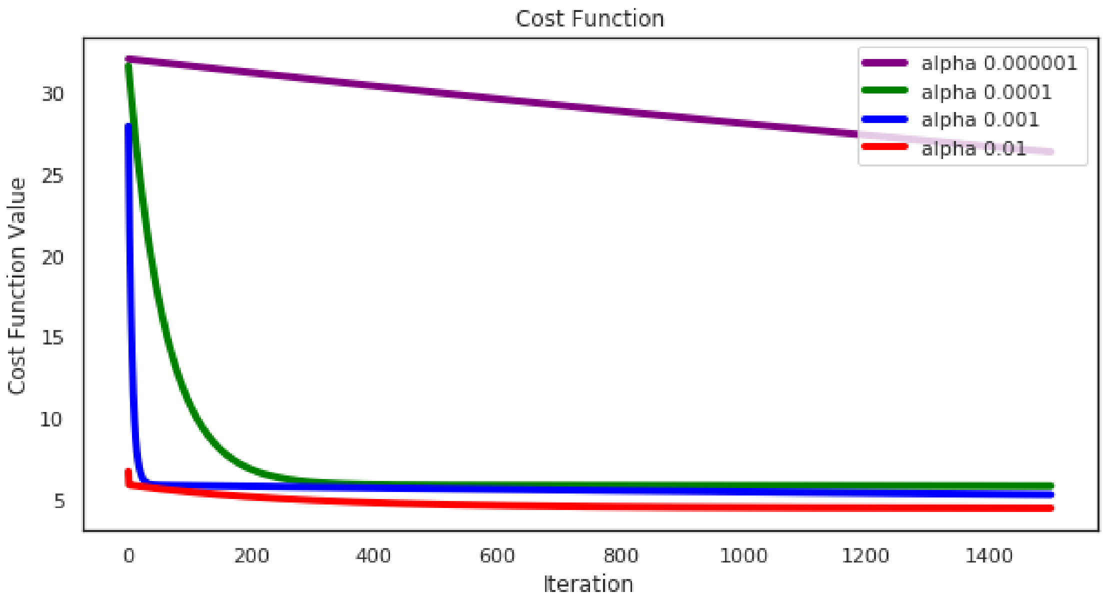
Linear Regression
Y Value
25
20
15
10
5
0
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
X Value

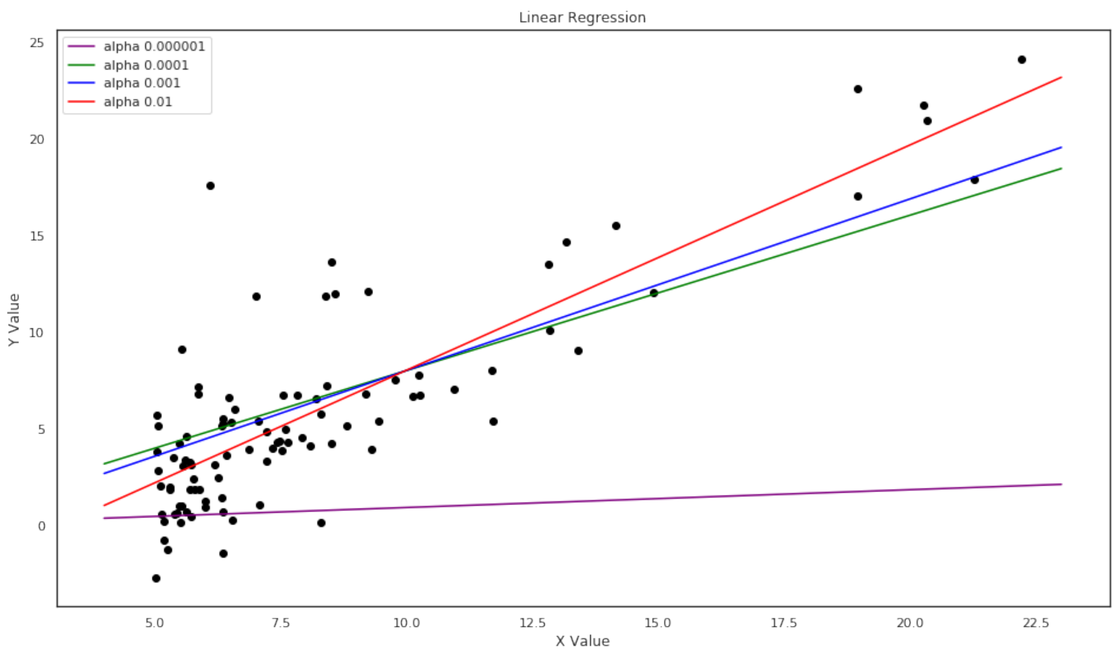
2.1 Thanks to the Gradient Descent algorithm, at each iteration we can see the decrease of the cost function, as we expect. The second plot shows the result achieved after 1500 iteration, a model which can be used to predict new values.

3 Now let's try to plot the cost function to the change of one of the most important parameters, the learning rate (alpha).

In [18]:
```
theta0=[[0],[0]]
theta1=[[0],[0]]
theta2=[[0],[0]]
theta3=[[0],[0]]
alpha0=0.000001
alpha1=0.0001
alpha2=0.001
alpha3=0.01
n_iter=1500
theta0, cost_J0 = gradientDescent(X, Y, theta0, alpha0, n_iter)
theta1, cost_J1 = gradientDescent(X, Y, theta1, alpha1, n_iter)
theta2, cost_J2 = gradientDescent(X, Y, theta2, alpha2, n_iter)
theta3, cost_J3 = gradientDescent(X, Y, theta3, alpha3, n_iter)
plt.figure(figsize=(10,5))
plt.plot(cost_J0, 'purple', linewidth=4, markersize=4, label='alpha 0.000001')
plt.plot(cost_J1, 'g', linewidth=4, markersize=4, label='alpha 0.0001')
plt.plot(cost_J2, 'b', linewidth=4, markersize=4, label='alpha 0.001')
plt.plot(cost_J3, 'r', linewidth=4, markersize=4, label='alpha 0.01')
plt.title('Cost Function')
plt.xlabel('Iteration')
plt.ylabel('Cost Function Value')
plt.legend(loc=1)
plt.show()

a = np.linspace(4, 23, 100)
b = theta0[0] + theta0[1] * a
c = theta1[0] + theta1[1] * a
d = theta2[0] + theta2[1] * a
e = theta3[0] + theta3[1] * a
plt.figure(figsize=(16,9))
plt.plot(a, b, 'purple', label='alpha 0.000001')
plt.plot(a, c, 'g', label='alpha 0.0001')
plt.plot(a, d, 'b', label='alpha 0.001')
plt.plot(a, e, 'r', label='alpha 0.01')
plt.scatter(X[:, 1], Y, color='black')
plt.title('Linear Regression')
plt.legend(loc=2)
```

```
plt.xlabel('X Value')
plt.ylabel('Y Value')
plt.show()
```



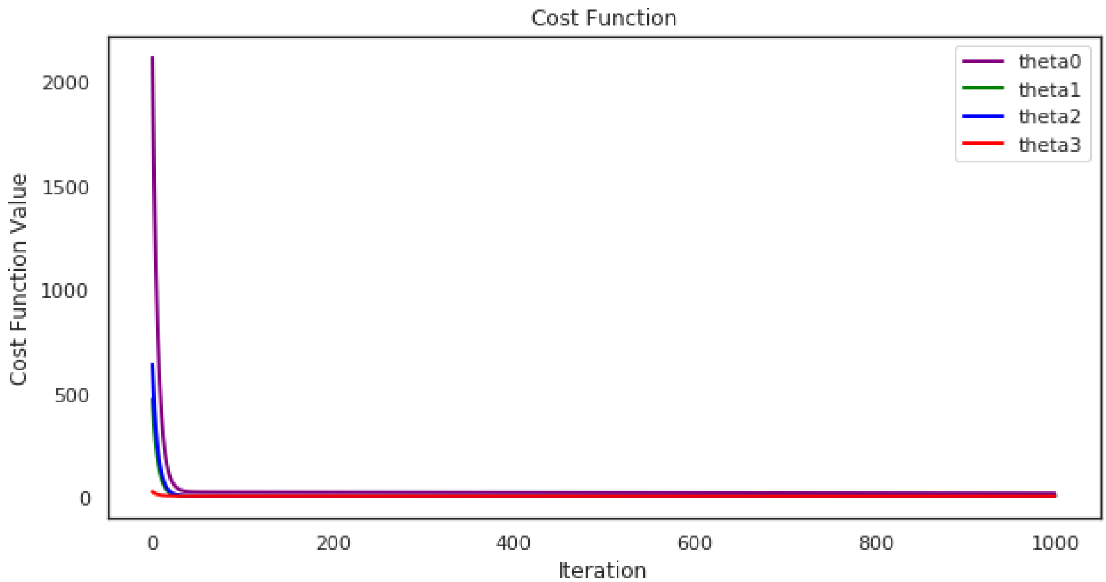


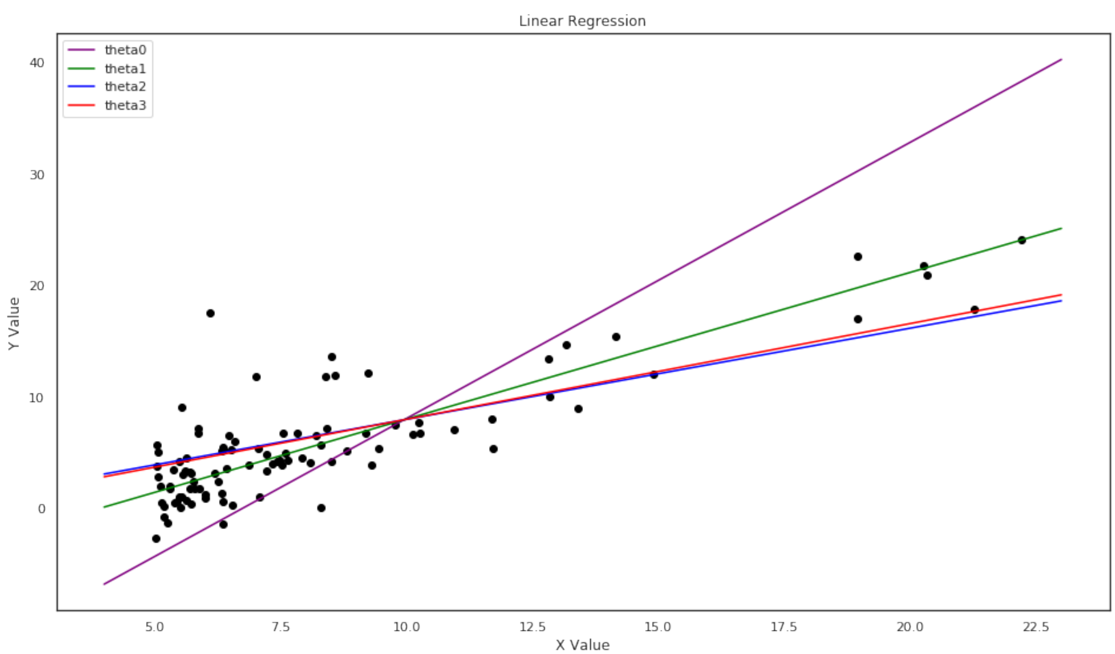

```
[[-3.63029144]
 [ 1.16636235]]
```

3.1 These plots shows the importance of this parameter, in fact alpha represents the learning level in our model; smaller alpha leads to less learning, then we need more iterations in order to converge with the iterative algorithm, the gradient descent.

4 Now let's observe the behavior of our model with different initial guesses and a fixed learning rate and number of iterations

In [28]:
```
theta0=[[-20],[-5]]
theta1=[[-5],[5]]
theta2=[[1],[5]]
theta3=[[0],[0]]
alpha=0.001
n_iter=1000
theta0, cost_J0 = gradientDescent(X, Y, theta0, alpha, n_iter)
theta1, cost_J1 = gradientDescent(X, Y, theta1, alpha, n_iter)
theta2, cost_J2 = gradientDescent(X, Y, theta2, alpha, n_iter)
theta3, cost_J3 = gradientDescent(X, Y, theta3, alpha, n_iter)
plt.figure(figsize=(10,5))
plt.plot(cost_J0, 'purple', linewidth=2, markersize=4, label='theta0')
plt.plot(cost_J1, 'g', linewidth=2, markersize=4, label='theta1')
plt.plot(cost_J2, 'b', linewidth=2, markersize=4, label='theta2')
plt.plot(cost_J3, 'r', linewidth=2, markersize=4, label='theta3')
plt.title('Cost Function')
plt.xlabel('Iteration')
plt.ylabel('Cost Function Value')
plt.legend(loc=1)
plt.show()

a = np.linspace(4, 23, 100)
b = theta0[0] + theta0[1] * a
c = theta1[0] + theta1[1] * a
d = theta2[0] + theta2[1] * a
e = theta3[0] + theta3[1] * a
plt.figure(figsize=(16,9))
plt.plot(a, b, 'purple', label='theta0')
plt.plot(a, c, 'g', label='theta1')
plt.plot(a, d, 'b', label='theta2')
plt.plot(a, e, 'r', label='theta3')
plt.scatter(X[:, 1], Y, color='black')
plt.title('Linear Regression')
plt.legend(loc=2)
plt.xlabel('X Value')
plt.ylabel('Y Value')
plt.show()
```


Cost Function
theta0
theta1
theta2
theta3
2000
1500
1000
500
0
Cost Function Value
0
200
400
600
800
1000
Iteration


Linear Regression
theta0
theta1
theta2
theta3
40
30
20
10
0
Y Value
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
X Value

4.1 The result is that the global minimum is reached under different iterations but the purple model needs more iterations because it starts farther than the other models.

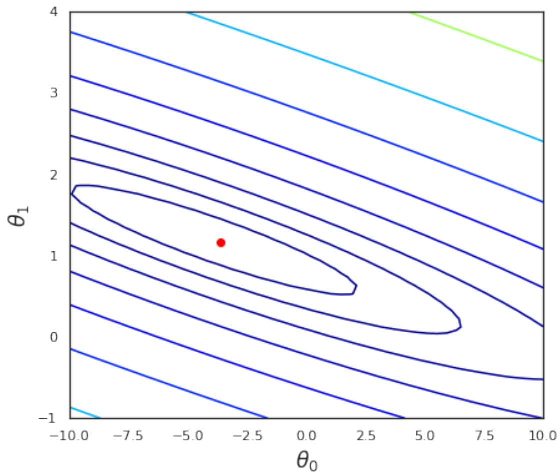
4.2 This behavior is explained by the convexity of the function that leads to a contour plot as a bowl. A bowl-shaped function has only one optimum which is global (Contour plot shown later).

5 New value prediction

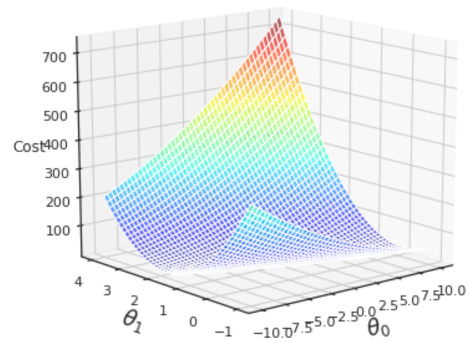
In [26]:
```
theta, cost_J = gradientDescent(X, Y)
print(theta.T.dot([1, 3.5])*10000)
print(theta.T.dot([1, 7])*10000)
```

```
[4519.7678677]
[45342.45012945]
```

In [27]:
```
# Create grid coordinates for plotting
B0 = np.linspace(-10, 10, 50)
B1 = np.linspace(-1, 4, 50)
xx, yy = np.meshgrid(B0, B1, indexing='xy')
Z = np.zeros((B0.size,B1.size))

# Calculate Z-values (Cost) based on grid of coefficients
for (i,j),v in np.ndenumerate(Z):
    Z[i,j] = costFunction(X,Y, theta=[[xx[i,j]], [yy[i,j]]])

fig = plt.figure(figsize=(15,6))
ax1 = fig.add_subplot(121)
ax2 = fig.add_subplot(122, projection='3d')

# Left plot
CS = ax1.contour(xx, yy, Z, np.logspace(-2, 3, 20), cmap=plt.cm.jet)
ax1.scatter(theta[0],theta[1], c='r')

# Right plot
ax2.plot_surface(xx, yy, Z, rstride=1, cstride=1, alpha=0.6, cmap=plt.cm.jet)
ax2.set_zlabel('Cost')
ax2.set_zlim(Z.min(),Z.max())
ax2.view_init(elev=15, azim=230)

# settings common to both plots
for ax in fig.axes:
    ax.set_xlabel(r'$\theta_0$', fontsize=17)
    ax.set_ylabel(r'$\theta_1$', fontsize=17)
```



4
3
2
1
0
−1
θ_1
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
θ_0



Cost
700
600
500
400
300
200
100
θ_1
θ_0