

eiquadprog solves problems of the form

$$\begin{aligned} \min_x \quad & \frac{1}{2}x^T Qx + q^T x \\ \text{s.t.} \quad & A_{eq}x + b_{eq} = 0 \\ & A_{ineq}x + b_{ineq} \geq 0 \end{aligned} \quad (55)$$

How to write the QP? we can start by writing the contact force as

$$\begin{aligned} \bar{\lambda} &= \underbrace{\lambda_{avg}}_{3.nactive \times 1} - \frac{1}{\delta t} \underbrace{D}_{3.nactive \times 6.nactive} \underbrace{\int_0^{\delta t} e^{\tau A} d\tau}_{6.nactive \times 6.nactive} \underbrace{\begin{bmatrix} \frac{\delta t}{2} I \\ I \end{bmatrix}}_{6.nactive \times 3.nactive} \underbrace{\dot{p}_0}_{3.nactive \times 1} \\ \bar{\lambda} &= \underbrace{\lambda_{avg}}_{3.nactive \times 1} - \underbrace{\bar{A}}_{3.nactive \times 3.nactive} \underbrace{\dot{p}_0}_{3.nactive \times 1} \end{aligned}$$

where λ_{avg} is defined as

$$\lambda_{avg} = \frac{1}{\delta t} D \int_0^{\delta t} x(\tau) d\tau + K p_0 \quad (56)$$

To approximate the friction cone by an inner 4-sided pyramid, let $\tilde{\mu} = \mu/\sqrt{2}$ where μ is the coefficient of friction, then the constraints can be written as

$$\bar{\lambda}_i \cdot \hat{n}_i \geq 0 \quad (57)$$

$$-\tilde{\mu} (\bar{\lambda}_i \cdot \hat{n}_i) \leq \bar{\lambda}_i \cdot \hat{t}_{ai} \leq \tilde{\mu} (\bar{\lambda}_i \cdot \hat{n}_i) \quad (58)$$

$$-\tilde{\mu} (\bar{\lambda}_i \cdot \hat{n}_i) \leq \bar{\lambda}_i \cdot \hat{t}_{bi} \leq \tilde{\mu} (\bar{\lambda}_i \cdot \hat{n}_i) \quad (59)$$

let \hat{n} is matrix containing all the normal vectors to all the active contacts and written as

$$\hat{n}^T = \underbrace{\begin{bmatrix} \hat{n}_1^T & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{n}_{nactive}^T \end{bmatrix}}_{nactive \times 3.nactive} \quad (60)$$

Similarly define the matrices \tilde{t}_a^T and \tilde{t}_b^T . Then, rewrite the inequalities to match the solver notation

$$\hat{n}^T \bar{\lambda} \geq 0 \quad (61)$$

$$\hat{t}_a^T \bar{\lambda} + \tilde{\mu} \hat{n}^T \bar{\lambda} \geq 0 \quad (62)$$

$$-\hat{t}_a^T \bar{\lambda} + \tilde{\mu} \hat{n}^T \bar{\lambda} \geq 0 \quad (63)$$

$$\hat{t}_b^T \bar{\lambda} + \tilde{\mu} \hat{n}^T \bar{\lambda} \geq 0 \quad (64)$$

$$-\hat{t}_b^T \bar{\lambda} + \tilde{\mu} \hat{n}^T \bar{\lambda} \geq 0 \quad (65)$$

We have to write the constraints in terms of $\dot{\bar{p}}_0$. Expand and regroup the constraints above to get

$$-\hat{n}^T \bar{A} \dot{\bar{p}}_0 + \hat{n}^T \lambda_{avg} \geq 0 \quad (66)$$

$$-(\tilde{\mu} \hat{n}^T + \hat{t}_a^T) \bar{A} \dot{\bar{p}}_0 + (\tilde{\mu} \hat{n}^T + \hat{t}_a^T) \lambda_{avg} \geq 0 \quad (67)$$

$$-(\tilde{\mu} \hat{n}^T - \hat{t}_a^T) \bar{A} \dot{\bar{p}}_0 + (\tilde{\mu} \hat{n}^T - \hat{t}_a^T) \lambda_{avg} \geq 0 \quad (68)$$

$$-(\tilde{\mu} \hat{n}^T + \hat{t}_b^T) \bar{A} \dot{\bar{p}}_0 + (\tilde{\mu} \hat{n}^T + \hat{t}_b^T) \lambda_{avg} \geq 0 \quad (69)$$

$$-(\tilde{\mu} \hat{n}^T - \hat{t}_b^T) \bar{A} \dot{\bar{p}}_0 + (\tilde{\mu} \hat{n}^T - \hat{t}_b^T) \lambda_{avg} \geq 0 \quad (70)$$

then we can construct the inequality matrix and vector A_{ineq} and b_{ineq} as follows

$$A_{ineq} = \begin{bmatrix} -\hat{n}^T \\ -(\tilde{\mu} \hat{n}^T + \hat{t}_a^T) \\ -(\tilde{\mu} \hat{n}^T - \hat{t}_a^T) \\ -(\tilde{\mu} \hat{n}^T + \hat{t}_b^T) \\ -(\tilde{\mu} \hat{n}^T - \hat{t}_b^T) \end{bmatrix} \bar{A} \quad (71)$$

$$b_{ineq} = \begin{bmatrix} \hat{n}^T \\ (\tilde{\mu} \hat{n}^T + \hat{t}_a^T) \\ (\tilde{\mu} \hat{n}^T - \hat{t}_a^T) \\ (\tilde{\mu} \hat{n}^T + \hat{t}_b^T) \\ (\tilde{\mu} \hat{n}^T - \hat{t}_b^T) \end{bmatrix} \lambda_{avg} \quad (72)$$