# Task-Space Inverse Dynamics: Implementation

Quadratic-Programming based Control for Legged Robots

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This document explains the implementation of the control framework Task-Space Inverse Dynamics (TSID).

 $<sup>^{1} {\</sup>sf https://github.com/stack-of\text{-}tasks/tsid}$ 

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To simplify the job we rely on the open-source C++ library  $\mathsf{TSID}^1$ .

TSID (currently) relies on:

- Eigen for linear algebra
- Pinocchio for multi-body dynamics computations
- Eiquadprog for solving Quadratic Programs

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# Main features: Pros & Cons

#### **CONS**

- Not mature (Feb 2017)
- Many missing features you may need for your application
  - Hierarchy
  - Fixed-base robots
  - Joint pos-vel limits
  - Actuation limits
  - Bilateral contacts
  - Line contacts
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#### **PROS**

- Efficient (<0.6 ms for humanoid)
- Tested in simulation & on HRP-2
- Open source
- Modular design
  - ullet ightarrow easy to extend
- Python bindings
- No alternative (AFAIK)

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# **HQP Solver**

solves a HQP

# **Other Concepts**

# **Constraint**

- affine function
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- contains robot model
- provides utility functions to compute robot quantities
- e.g., mass matrix, Jacobians

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# **Trajectory**

- maps time to vector values
- pos, vel, acc
- position and velocity can have different sizes (Lie groups)

# **Details**

# Robot Wrapper 1/2

Interface for computing robot-related quantities:

```
RobotWrapper(string filename, vector<string> package_dirs,
             JointModelVariant rootJoint);
int nq(); // size of configuration vector q
int nv(); // size of velocity vector v
Model & model(); // reference to robot model (Pinocchio)
// Compute all quantities and store them into data
void computeAllTerms(Data &data, Vector q, Vector v);
```

# Robot Wrapper 2/2

```
Vector rotor_inertias();
Vector gear_ratios();
Vector3 com(Data data):
Vector3 com_vel(Data data);
Vector3 com_acc(Data data);
Matrix3x Jcom(Data data);
Matrix mass(Data data);
Vector nonLinearEffects(Data data);
SE3 position(Data data, JointIndex index);
Motion velocity(Data data, JointIndex index);
Motion acceleration(Data data, JointIndex index);
Matrix6x jacobianWorld(Data data, JointIndex index);
Matrix6x jacobianLocal(Data data, JointIndex index);
```

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• Equalities, represented by matrix A and vector a:

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• Inequalities, represented by matrix A and vectors *lb* and *ub*:

$$lb \le Ax \le ub$$

• Bounds, represented by vectors *lb* and *ub*:

$$lb \le x \le ub$$

```
ConstraintBase(string name, int rows, int cols);
bool isEquality();
bool isInequality();
bool isBound():
Matrix matrix();
Vector vector();
Vector lowerBound():
Vector upperBound();
bool setMatrix(Matrix A);
bool setVector(Vector b);
bool setLowerBound(Vector lb);
bool setUpperBound(Vector ub);
bool checkConstraint(Vector x);
```

```
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TaskBase(string name, Model model);
Constraint compute(double t, Vector q, Vector v, Data data);
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- TaskMotion: linear function of robot accelerations
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# Tasks can compute either:

- equality constraints, e.g., TaskComEquality, TaskJointPosture, TaskSE3Equality
- bounds, e.g., TaskJointBounds (not implemented yet)
- inequality constraints, e.g., friction cones

Interface of ContactBase:

```
ContactBase(name, Kp, Kd, bodyName, regWeight);
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- represents inequality constraints acting on contact forces
- e.g., friction cone constraints
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- e.g., keep them close to friction cone center

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# Force-Generator matrix T:

- maps force variables to motion constraint representation
- Dynamic:  $M\dot{v}_q + h = S^{\top}\tau + J^{\top}Tf$
- Motion constraint:  $J\dot{v}_q = -\dot{J}v_q$
- Friction cones:  $Af \leq a$

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- but 12d representation for force variable  $f \in \mathbb{R}^{12}$
- force-generator matrix  $T \in \mathbb{R}^{6 \times 12}$  defines mapping between two representations:  $\tau_{contact} = J^{\top} T f$

# Inverse Dynamics Formulation Base

Central class of the whole library

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Methods to add tasks:

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addMotionTask(MotionTask task, double weight, int priority);
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Methods to convert TSID problem into (Hierarchical) QP: HqpData computeProblemData(double time, Vector q, Vector v);

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# Methods to convert TSID problem into (Hierarchical) QP: HgpData computeProblemData(double time, Vector q, Vector v);

### HqpData defined as:

```
#typedef vector<pair<double, ConstraintBase>> ConstraintLevel
#typedef vector<ConstraintLevel> HqpData
```

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HqpOutput is defined as:
class HqpOutput
     QpStatusFlag flag;
     Vector x, lambda;
```

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### Results on HRP-2's computer (very old):

# Python Example

• Code snippets

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- Code snippets
- Biped robot with both feet on the ground (double support)
- Control center of mass (CoM) for balance
- Control joint angles (posture) for whole-body stability
- Good starting point before moving to more complex scenarios

# Create Robot Wrapper

# **Create Inverse Dynamics Formulation**

### **Create Contact**

```
contactRF = Contact6d("contact_rfoot", robot, rf_frame_name,
                       contact_points, contact_normal, mu,
                       fMin, fMax, w_forceReg)
contactRF.setKp(kp_contact * matlib.ones(6).T)
contactRF.setKd(2*sqrt(kp_contact) * matlib.ones(6).T)
rf_joint_id = robot.model().getJointId(rf_frame_name)
H_rf_ref = robot.position(data, rf_joint_id)
contactRF.setReference(H_rf_ref)
invdyn.addRigidContact(contactRF)
# repeat for other contact(s)
```

#### Create Center-of-Mass Task

```
comTask = TaskComEquality("task-com", robot)
comTask.setKp(kp_com * matlib.ones(3).T)
comTask.setKd(2*sqrt(kp_com) * matlib.ones(3).T)
invdyn.addMotionTask(comTask, w_com, 1, 0.0)
```

#### **Create Posture Task**

```
postureTask = TaskJointPosture("task-posture", robot)

postureTask.setKp(kp_posture*matlib.ones(robot.nv-6).T)

postureTask.setKd(2*sqrt(kp_posture)*matlib.ones(robot.nv-6).T)

invdyn.addMotionTask(postureTask, w_posture, 1, 0.0)
```

# **Create Reference Task Trajectories**

```
com_ref = robot.com(data)
trajCom = TrajectoryEuclidianConstant("traj_com", com_ref)

q_ref = q[7:]
trajPosture = TrajectoryEuclidianConstant("traj_joint", q_ref)
```

#### **Create HQP Solver**

```
solver = SolverHQuadProg("qp solver")
solver.resize(invdyn.nVar, invdyn.nEq, invdyn.nIn)
```

# **Control Loop**

```
for i in range(0, N_SIMULATION_STEPS):
    comTask.setReference(trajCom.computeNext())
    postureTask.setReference(trajPosture.computeNext())
    # get current state estimation
    (q, v) = ...
    HQPData = invdyn.computeProblemData(t, q, v)
    sol = solver.solve(HQPData)
    tau = invdyn.getActuatorForces(sol)
    # send desired joint torques (tau) to actuators
```

## Simulation Loop

```
for i in range(0, N_SIMULATION_STEPS):
    . . .
    # assuming perfect torque-acceleration tracking...
    dv = invdyn.getAccelerations(sol)
    # integrate desired accelerations
    q = se3.integrate(robot.model(), q, dt*v)
    v += dt*dv
    # increase time
    t += dt
```

# **Exercises**

 Run provided example (tsid/exercizes/ex\_1.py) and check that the robot does not move

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- Change references of CoM/posture and look what happens
- Change CoM/posture gains and see effect
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- Set reference CoM outside support polygon (e.g., 20 cm to the side), what happens? Why?

 Run provided code (tsid/exercizes/ex\_2.py) and check the sinusoidal reference CoM tracking

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- Increase CoM frequency until tracking gets bad. Why does that happen?
- Set contact feedback gains to zero, what happens? Why?
- Add contact on hand

# **Exercise 3: Taking a Step**

 Extend code to make robot take a step (solution in tsid/demo/demo\_romeo.py)