Lezione 7/04/2020

(Metodo Streaming)

La Lezione inizia @ 14:15

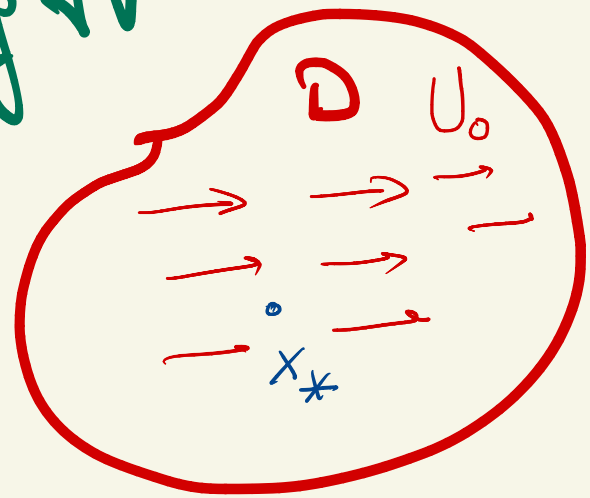
Le lezione sarà Video Registrata !

14:20 5 min ritardo!

$$U_* = -\nabla_* \oint_{\partial D} u \cdot n \, g \, dS - \nabla_* \times \oint_{\partial D} u \times n \, g \, dS$$

$$- \nabla_* \times \int_D \zeta \, g \, dV$$

$$\zeta = \nabla \times u$$

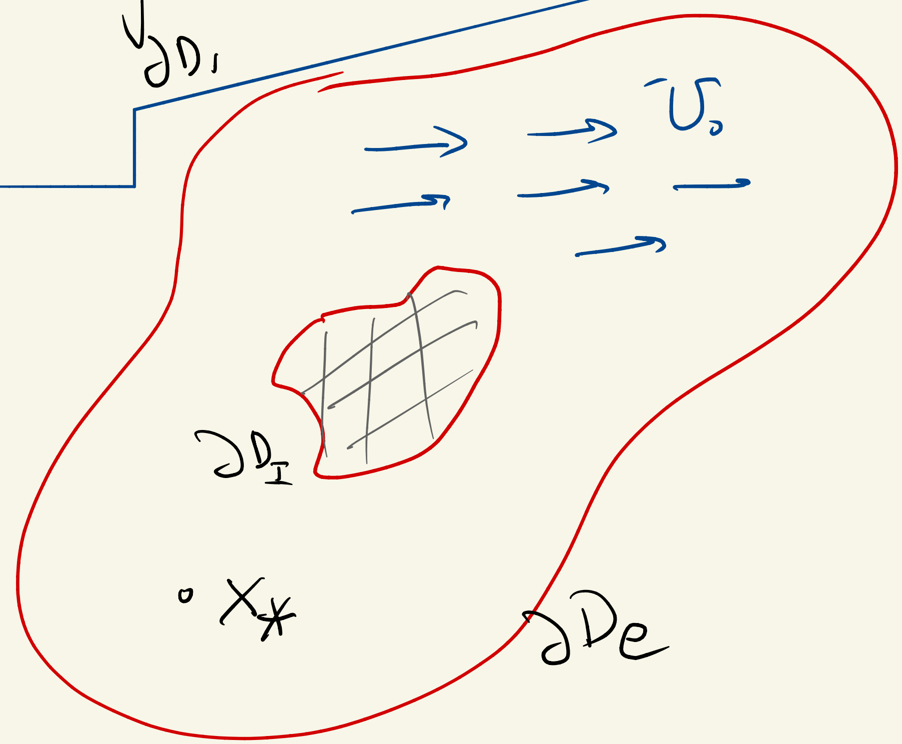


$$\nabla \times U_0 \equiv$$

$$U_0 = -\nabla_* \oint_{\partial D} (U_0 \cdot n) \, g \, dS - \nabla_* \times \oint_{\partial D} U_0 \times n \, g \, dS$$

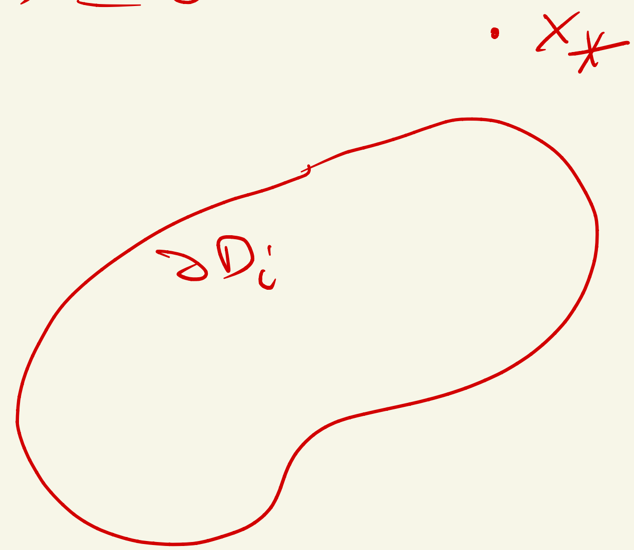
$$\cancel{v_0} = \boxed{-\nabla_* \oint_{\partial D_e} \underset{\cancel{v_0}}{v_0 \cdot n\, g}\, dS - \nabla_* \times \oint_{\partial D_e} v_0 \times n\, g\, dS}$$

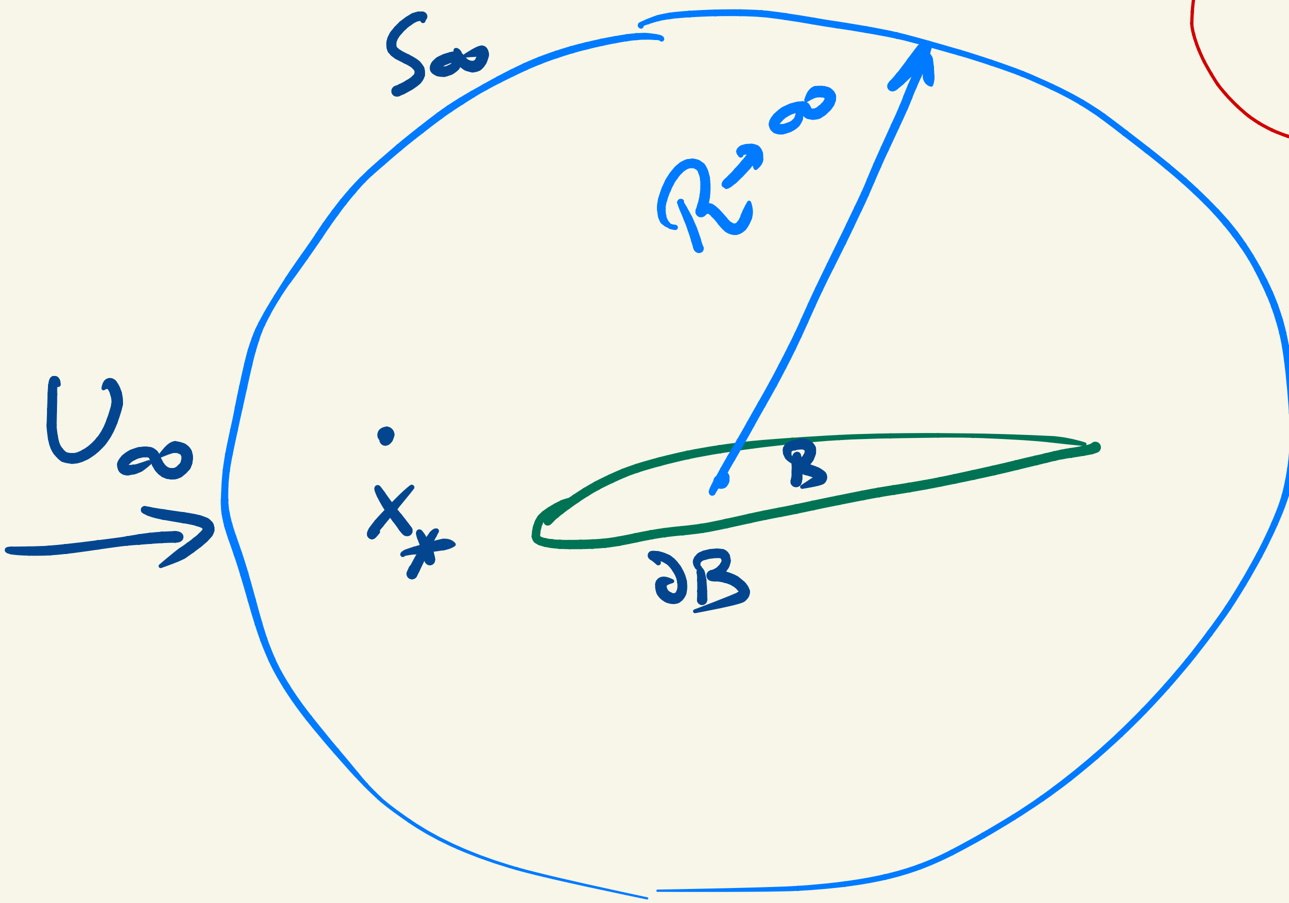
$$\boxed{\underbrace{-\nabla_* \oint_{\partial D_i} v_0 \cdot n\, g\, dS}_{=0} - \nabla_* \times \oint_{\partial D_i} v_0 \times n\, g\, dS}$$



$$\partial D = \partial D_e \cup \partial D_I$$

$$-\nabla_* \oint_{\partial D_i} v_0 \cdot n\, g\, ds - \nabla_* \times \oint_{\partial D_i} v_0 \times n\, g\, ds = 0$$





$$U_* = -\nabla_* \oint_{\partial B} u \cdot n \, g \, dS - \nabla_* \times \oint_{\partial B} u \times n \, g \, dS$$

$$- \nabla_* \oint_{S_\infty} \overset{U_\infty \cdot n}{u \cdot n} \, g \, dS - \nabla_* \times \oint_{S_\infty} \overset{U_\infty \times n}{u \times n} \, g \, dS$$

$$- \nabla_* \times \int_D \zeta \, g \, dS$$

$R \to \infty \qquad u \to U_\infty$

$$\lim_{R\to\infty}\left[-\nabla_* \oint_{S(R)} v\cdot n\, g\, dS - \nabla_* \times \oint_{S(R)} v\times n\, g\, dS\right] = U_\infty$$

$u\times n = U_\infty \times n + v\times n$

$u\cdot n = U_\infty\cdot n + v\cdot n$

$$\underbrace{u_* - U_\infty}_{v_*} = -\nabla_* \oint_{\partial B} u\cdot n\, g\, dS - \nabla_* \times \oint_{\partial B} u\times n\, g\, dS$$

$$-\nabla_* \times \int_D \zeta\, g\, dV$$

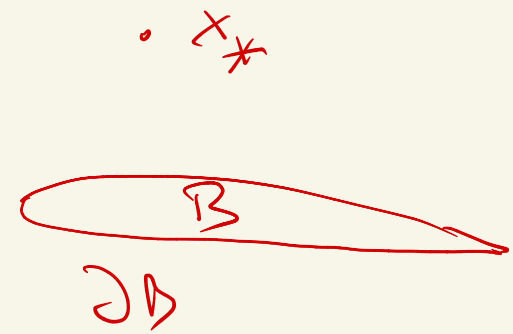
$u = U_\infty + v$

$$
v_* = -\nabla_* \oint_{\partial B} v \cdot n \, \rho \, dS - \nabla_* \times \oint_{\partial B} v \times n \, \rho \, dS \; +
$$

$$
\cancel{- \nabla_* \oint_{\partial B} v_\infty \cdot n \, \rho \, dS} \;\; \cancel{- \nabla_* \times \oint_{\partial B} v_\infty \times n \, \rho \, dS}
$$

$$
- \nabla_* \times \int_{D} \zeta \, \rho \, dV
$$

$$V_* = -\nabla_* \oint_{\partial B} v \cdot n \, g \, dS - \nabla_* \times \oint_{\partial B} v \times n \, g \, dS - \nabla_* \times \int_D \zeta g \, dV$$

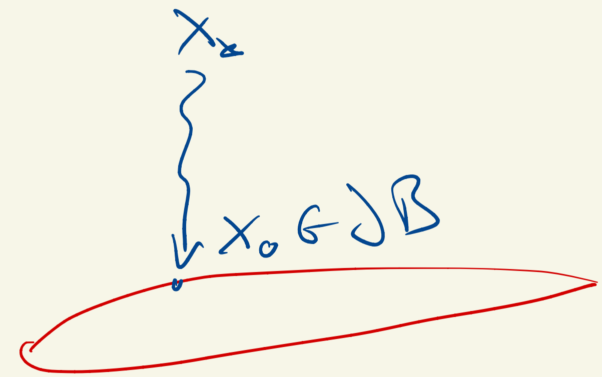


Soluzione Esterna U

in flusso <u>non</u>-separato

$$U \cdot n \big|_{\partial B} = 0 \qquad \boxed{v \cdot n \big|_{\partial B} = -U_\infty \cdot n}$$

$$V_* = +\nabla_* \int_{\partial B} v_\infty \cdot n \, \rho \, dS - \nabla_* \times \oint_{\partial B} v \times n \, \rho \, dS - \nabla_* \times \int_{D} \zeta \rho \, dV$$



$$\lim_{x_* \to x_0 \in \partial B} V_* = V_0 = \frac{1}{2} V_0 + \Big[\qquad \Big]$$

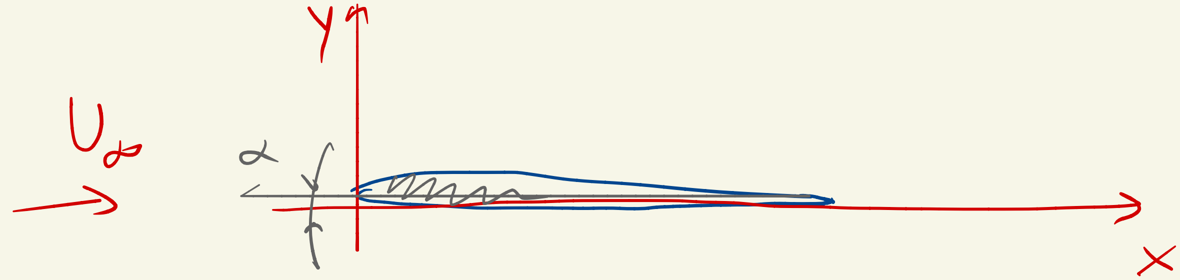
$$= \lim_{x_* \to x_0 \in \partial B} \left[\nabla_* \int_{\partial B} v_\infty \cdot n \, \rho \, dS - \nabla_* \times \oint_{\partial B} v \times n \, \rho \, dS - \nabla_* \times \int_{D} \zeta \rho \, dV \right]$$

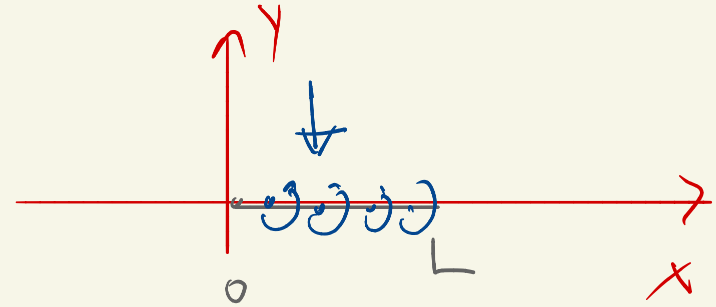
$$\frac{1}{2} V_0 = \nabla_0 \oint_{\partial B} \overbrace{U_\infty \cdot n}^{-v \cdot n}\, \rho \, dS - \nabla_0 \times \oint_{\partial B} v \times n \, \rho \, dS - \nabla_0 \times \int_D \zeta \rho \, dV$$

$$\overbrace{\frac{1}{2} V_0 \cdot n_0}^{-U_\infty \cdot n} = \frac{\partial}{\partial n_0} \oint_{\partial B} U_\infty \cdot n \, \rho \, dS - n_0 \cdot \nabla_0 \times \oint_{\partial B} \overbrace{v \times n}^{\gamma} \, \rho \, dS$$

$$- n_0 \cdot \nabla_0 \times \int_D \zeta \rho \, dV$$

Forma molto più generale dell'equazione di Prandtl



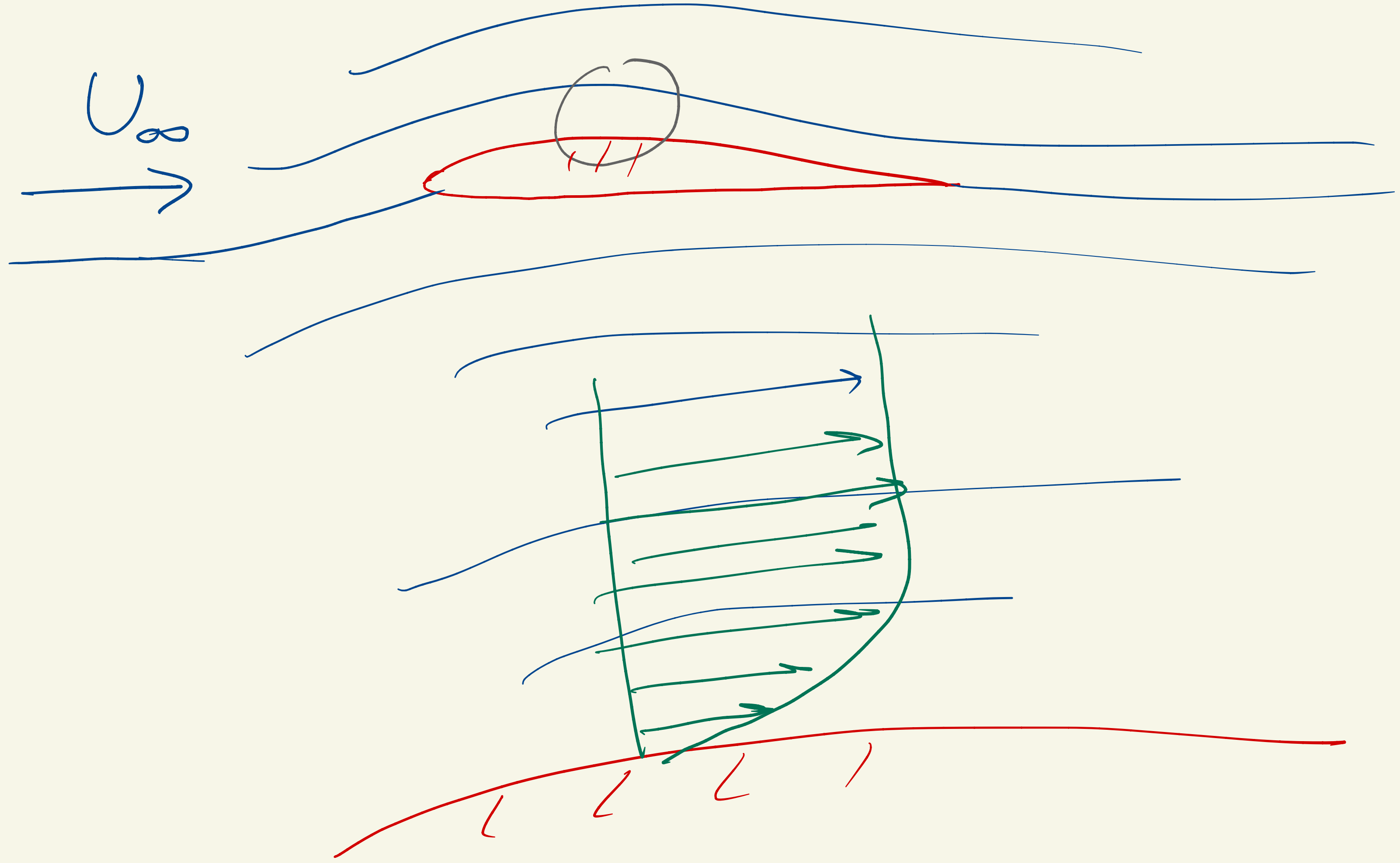


$$\frac{1}{2} V_0 \times n_0 = - n_0 \times \nabla_0 \oint_{\partial B} U_\infty \cdot n \, g \, dS + n_0 \times \left(\nabla_0 \times \oint_{\partial B} v \times n \, g \, dS \right)$$

$$+ \, n_0 \times \nabla_0 \times \int_D \zeta g \, dV$$

$$\frac{1}{2} V_0 \times n_0 - n_0 \times \left(\nabla_0 \times \oint_{\partial B} v \times n \, g \, dS \right) = t_0$$

3D



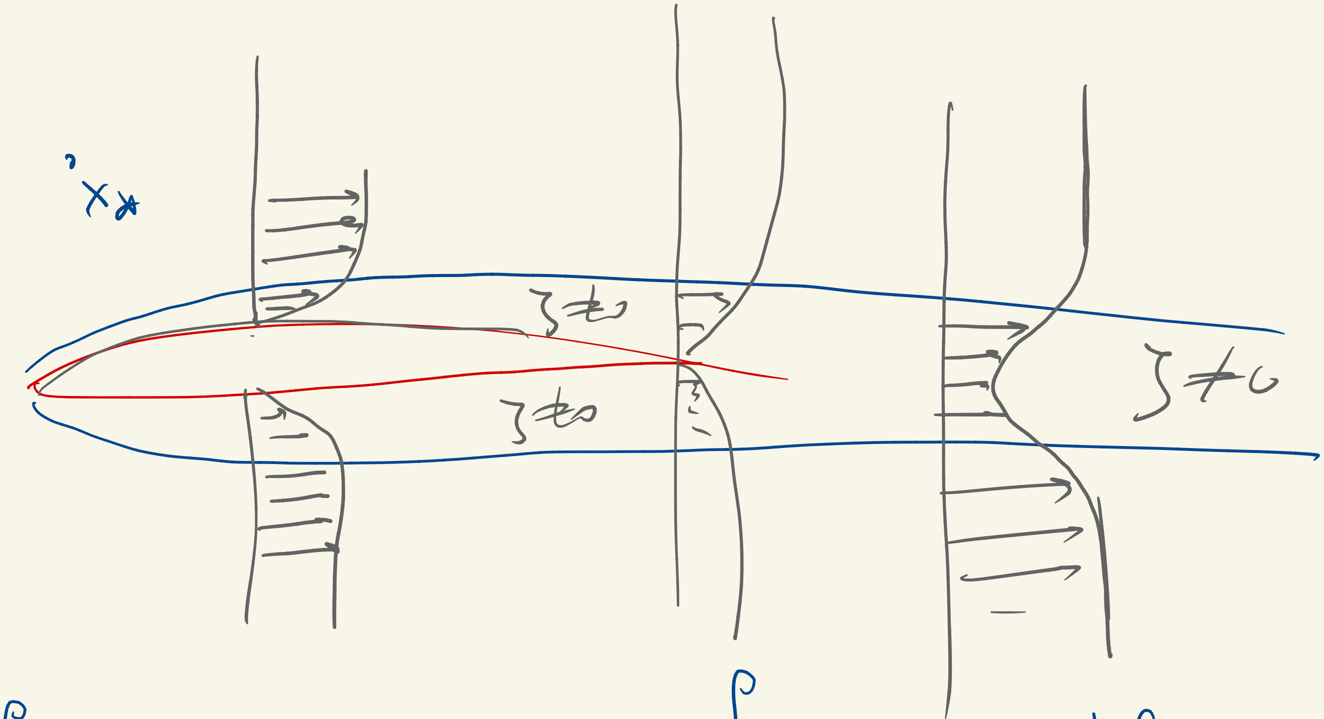
$$U = V + U_\infty$$

$$U\big|_{\partial B} \equiv 0$$

U_∞

$\longrightarrow$

$$V^{NS}\big|_{\partial B} = -U_\infty$$



$$V_*^{NS} = -\nabla_* \oint_{\partial B} V^{NS}\cdot n \, g \, dS - \nabla_* \times \oint_{\partial D} V^{NS} \times n \, g \, dS$$

$$- \nabla_* \times \int_D \zeta^{NS} g \, dV$$

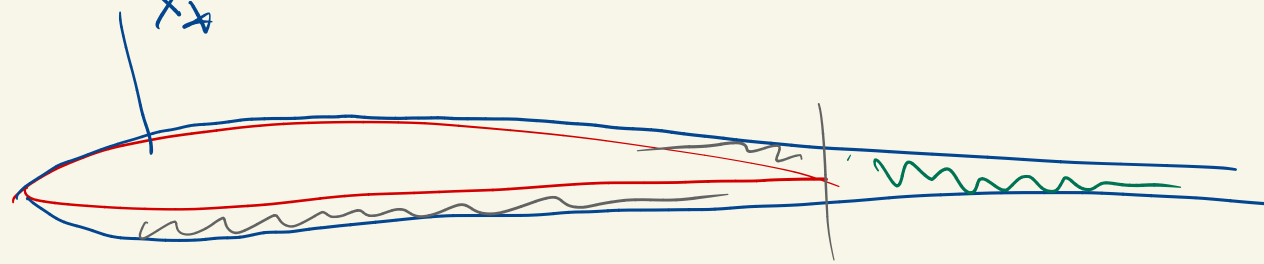
$$V_*^{NS} = + \boxed{\cancel{\nabla_* \oint_{\partial B} U_\infty \cdot n \, \rho \, dS + \nabla_* \times \oint_{\partial B} U_\infty \times n \, \rho \, dS}}$$

$$- \nabla_* \times \int_D \zeta^{NS} \rho \, dV$$

$$V_*^{NS} = - \nabla_* \times \int_D \zeta^{NS} \rho \, dV$$

$Re \to \infty$

$$\lim V^{NS}(x_*) = V^{EX}(x_*)$$



$Re \to \infty$

$x_* = \text{ext}$

$$V^{EX}(x_*) = \nabla_* \oint_{\partial B} U_\infty \cdot n\, g \, dS - \nabla_* \times \oint_{\partial B} v^{EX} \times n\, g \, dS - \nabla_* \times \int_D \zeta^{EX} g \, dV$$

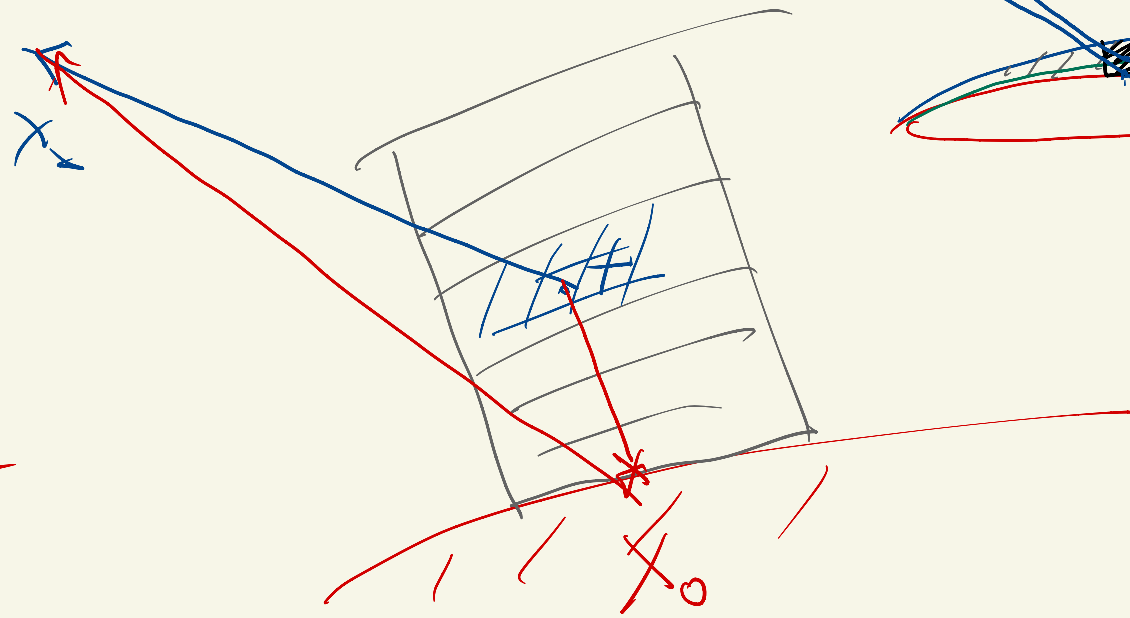
$$V^{NS}(x_*) = -\nabla_* \times \int_{BL} \zeta_{BL}\, g \, dV - \nabla_* \times \int_{W} \zeta_{W}\, g \, dV$$

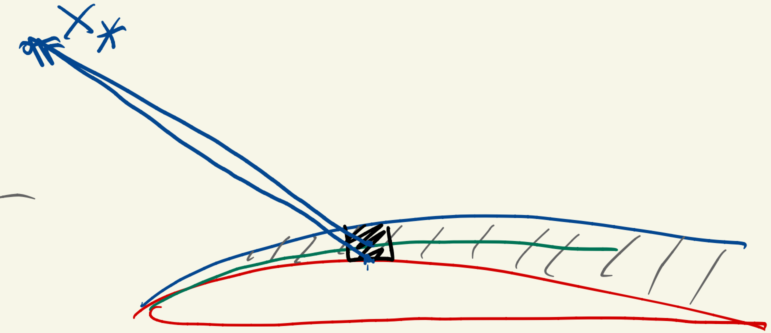
$$-\nabla_x \times \int_{BL} \zeta_{BL}\, \rho\, dV \overset{?}{=} \underbrace{-\nabla_x \times \oint_{\partial B} U_\infty \cdot n\, \rho\, dS}_{-\nabla_x \times \oint_{\partial B} U_\infty \times n\, \rho\, dS} - \nabla_x \times \oint_{\partial B} V^{ex} \times n\, \rho\, dS =$$

$$= -\nabla_x \times \oint_{\partial B} \left(U_\infty + V^{ex}\right) \times n\, \rho\, dS = -\nabla_x \times \oint_{\partial B} u \times n\, \rho\, dS$$

$$\int_{BL} \zeta_{BL}\, \rho\, dV \to \oint_{\partial B} u \times n\, \rho\, dS$$

$$\int_{BL} \zeta_{BL}\, g\, dV$$



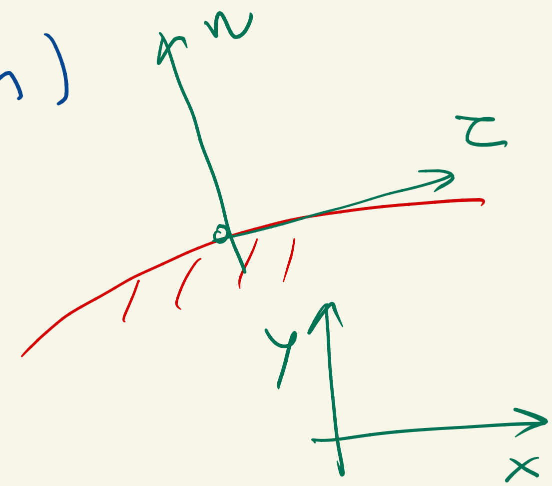


$$g = -\frac{1}{4\pi}\,\frac{1}{r}$$

$$\int_{BL} \zeta_{BL}\, g\, dV \cong \int_S dS \int_0^{\delta_{BL}} dn\, \zeta_{BL}(x_0, n)\; g(r_0)$$

$$\int_{BL} \zeta_{BL}\, g\, dV \cong \oint_{\partial B} dS \int_0^{\delta_{BL}} dn\, \zeta_{BL}(x_0, n)\; g(r_0) =$$

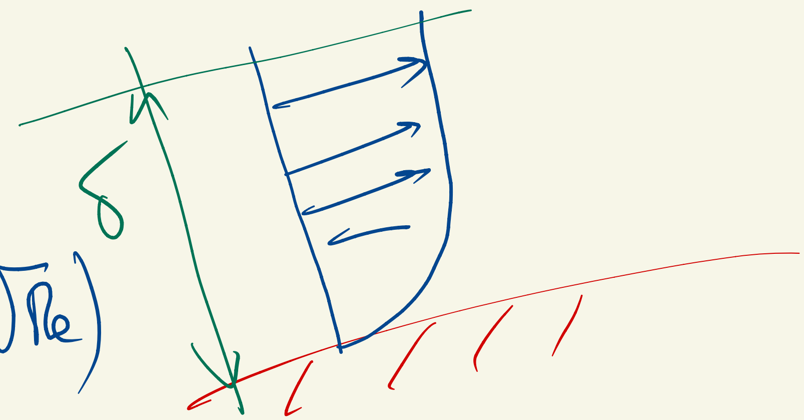
$$= \oint_{\partial B} dS\; g(r_0) \int_0^{\delta_{BL}} dn\, \zeta_{BL}(x_0, n)$$



2D: $$\zeta_3 = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} =$$

$$= \frac{\partial u_n}{\partial \tau} - \frac{\partial u_\tau}{\partial n}$$

$$\zeta_3 = \frac{\partial u_n}{\partial z} - \frac{\partial u_z}{\partial n}$$



$$\left.\begin{aligned} u_z &= O(1) \\ n &= O\left(1/\sqrt{Re}\right) \end{aligned}\right\} \quad \frac{\partial u_z}{\partial n} = O\left(\sqrt{Re}\right)$$

$$\frac{\partial u_n}{\partial n} + \frac{\partial u_z}{\partial z} = 0$$

$$u_n = O\left(1/\sqrt{Re}\right)$$

$$z = O(1)$$

$$\frac{\partial u_n}{\partial z} = O(1)$$

$$\zeta_3 = -\frac{\partial u_z}{\partial n}\left(1 - \frac{\partial u_n}{\partial z} \Big/ \frac{\partial u_z}{\partial n}\right) = -\frac{\partial u_z}{\partial n}\left[1 - O\left(\frac{1}{\sqrt{Re}}\right)\right]$$

$Re \to \infty$ $\qquad \zeta_3 = -\dfrac{\partial u_\tau}{\partial n}$

$$\int_0^{\delta_{BL}} \zeta_3 \, dn = \int_0^{\delta_{BL}} -\frac{\partial u_\tau}{\partial n} \, dn = -u_\tau(\delta_{BL}) + \cancel{u_\tau(0)}$$

$$\int_0^{d_n} \zeta_3 \, dn = -u_\tau^{\mathrm{EX}}$$

$$\zeta = \nabla \times u =$$

$$u = u_n n + u_\pi$$

$$= \left(n \frac{\partial}{\partial n} + \nabla_\pi\right) \times \left(u_n \hat{n} + u_\pi\right) =$$

$$\nabla = n \frac{\partial}{\partial n} + \nabla_\pi$$

$$= \underbrace{n \times \frac{\partial u_n}{\partial n} \hat{n}}_{O(1)} + \boxed{\underbrace{n \times \frac{\partial u_\pi}{\partial n}}_{O(\sqrt{Re})}} + \underbrace{\nabla_\pi \times u_n \hat{n}}_{O(1/\sqrt{Re})} + \underbrace{\nabla_\pi \times u_\pi}_{O(1)}$$

$$\frac{\partial}{\partial n} = O\left(\sqrt{Re}\right) \qquad u_\pi = O(1) \qquad u_n = O\left(1/\sqrt{Re}\right)$$

$$\nabla_\pi = O(1)$$

$$\zeta \xrightarrow[Re\to\infty]{} n\times\frac{\partial u_\pi}{\partial n}$$

$$\int_0^{\delta_{BL}} dn\, \zeta_{BL}(x_0, n) = \int_0^{\delta_{BL}} dn\; n\times\frac{\partial u_\pi}{\partial n} =$$

$$= n\times\int_0^{\delta_{BL}} dn\,\frac{\partial u_\pi}{\partial n} = n\times u_\pi^{EX} - \cancel{n\times u_\pi(0)}$$

$$\boxed{\int_0^{\delta_{BL}} dn\,\zeta \sim n\times u_\pi^{EX}}$$

$$\int_{BL} \zeta_{BL}\, \rho \, dV \sim \oint_{S} dS\, \rho \; n \times v^{EX}$$

$$\boxed{\nabla_x \times \int_{BL} \zeta_{BL}\, \rho \, dV = \nabla_x \times \oint_{\partial B} \rho \; n \times v^{E}}$$

$$V^{NS}(x_\alpha) = -\nabla_x \times \int_{BL} \zeta_{BL}\, g\, dV - \nabla_x \times \int_{w} \zeta_{w}\, g\, dV \sim$$

$$\sim -\nabla_x \times \oint_{\partial B} n \times v^{EX} g\, dS - \nabla_x \times \int \zeta_w\, g\, dV$$

$$= -\nabla_x \times \oint_{\partial B} n \times v_\infty\, g\, dS - \nabla_x \times \oint_{\partial B} n \times v^{EX} g\, dS$$

$$- \nabla_x \times \int \zeta_n\, dV$$

$$\nabla_x \oint_{\partial B} U_\infty \cdot n \, \rho \, dS + \underbrace{\nabla_x \times \oint_{\partial D} U_\infty \times n \, \rho \, dS}_{-\nabla_x \times \oint_{\partial B} n \times U_\infty \, \rho \, dS} = 0$$

$$V_x^{NS} \sim -\nabla_x \oint_{\partial B} U_\infty \cdot n \, \rho \, dS - \nabla_x \times \oint_{\partial D} n \times v^{B} \times \rho \, dS$$

$$- \nabla_x \times \int_{V_e} \zeta_x \, \rho \, dV$$