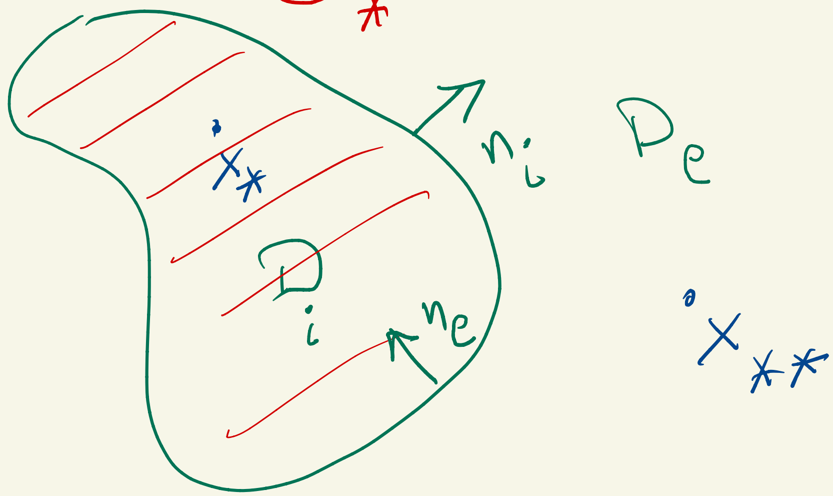
Lezione 10/03/220

$g(x_1, x_2)$ Regular



$$\oint_{\partial D_i} \left(\varphi_i \frac{\partial g}{\partial n_x} - g\,\frac{\partial \varphi_i}{\partial n_x}\right) dS_x = \begin{cases} \varphi_{i*} & x_* \in \mathring{D}_i \\ 1/2\,\varphi_{i*} & x_* \in \partial D_i \\ 0 & x_* \in \mathring{D}_e \end{cases}$$

$$-\oint_{\partial D_i} \left(\varphi_e \frac{\partial g}{\partial n_x} - g\,\frac{\partial \varphi_e}{\partial n_x}\right) dS = \begin{cases} 0 & x_* \in \mathring{D}_i \\ 1/2\,\varphi_{e*} & x_* \in \partial D_i \\ \varphi_{e*} & x_* \in \mathring{D}_e \end{cases}$$

$$\oint_{\partial D_i} \left[\underbrace{(\varphi_i - \varphi_e)}_{=0} \frac{\partial g}{\partial n_x} - \underbrace{\left(\frac{\partial \varphi_i}{\partial n_x} - \frac{\partial \varphi_e}{\partial n_x} \right)}_{=0} g \right] dS = \begin{cases} \underline{\varphi_{i_*}} & x_* \in \mathring{D}_i \\ \frac{1}{2}(\varphi_{i_*} - \varphi_{e_*}) & x_* \in \partial D_i \\ \varphi_{e_*} & x_* \in \mathring{D}_e \end{cases}$$

$$\frac{\partial \varphi_e}{\partial n_x} = \frac{\partial \varphi_i}{\partial n_x}$$

$$\varphi_{i_*} = \oint_{\partial D_i} \tau \frac{\partial g}{\partial n_x} \, dS \qquad \tau := \varphi_i - \varphi_e$$

Rappresentazione indiretta in doppio strato (dipoli, doppietti)

$$\varphi_{i_*} = \oint_{\partial D_i} \eta \frac{\partial g}{\partial n_x} \, dS \qquad x_* \to x_0 \in \partial D_i$$

$$\downarrow$$

$$\varphi_{i_0} = 1/2\, \eta_0 + \cancel{\oint_{\partial D_i}} \eta \frac{\partial g}{\partial n_x} \, dS$$

$$x_* \to x_0 \in \partial D_i$$

$$\varphi_{i_*} = \oint_{\partial D_i} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS \longrightarrow$$

$$\downarrow$$

$$\varphi_{i_0} = 1/2\, \varphi_{i_0} + \cancel{\oint_{\partial D_i}} \left(\varphi \frac{\partial g}{\partial n_x} - g \frac{\partial \varphi}{\partial n_x} \right) dS$$

$$x = x_p + \alpha x^{\#}$$

$$A x = b$$

$$A x^{\#} = 0$$

$$\frac{1}{2} \varphi_0 + \oint_{\partial D_i} \varphi \frac{\partial \varphi}{\partial n_x} dS = \varphi_{i0}$$

$$\frac{1}{2} \varphi_0 - \oint_{\partial D_i} \varphi \frac{\partial \varphi}{\partial n_x} dS = - \oint_{\partial D_j} \rho \frac{\partial \varphi}{\partial n_x} dS$$

$$\nabla^2 \varphi = 0 \qquad x \in \mathring{D}_i$$

$$0 = \int_{D_i} \nabla^2 \varphi \, dV = \oint_{\partial D} \frac{\partial \varphi}{\partial n} dS$$

1) $\frac{\partial \varphi}{\partial n_x} = f(x) \qquad x \in \partial D_i$ (Neumann)

2) $\varphi = h(x) \qquad x \in \partial D_i$ (Dirichlet)

$$\varphi_i^H : \quad \tfrac{1}{2}\varphi^H_{i_0} - \oint_{\partial D_i} \varphi^H \frac{\partial}{\partial n_x} \equiv 0 \qquad \varphi^H_{i_0} \neq 0$$

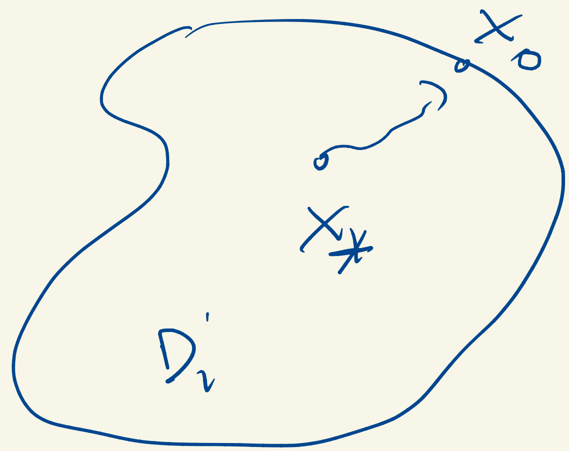
$$\varphi_i = \varphi_i^p + \alpha \varphi_i^H$$

$$\varphi_{i_*} = \oint_{\partial D_i} \sigma g \, dS$$

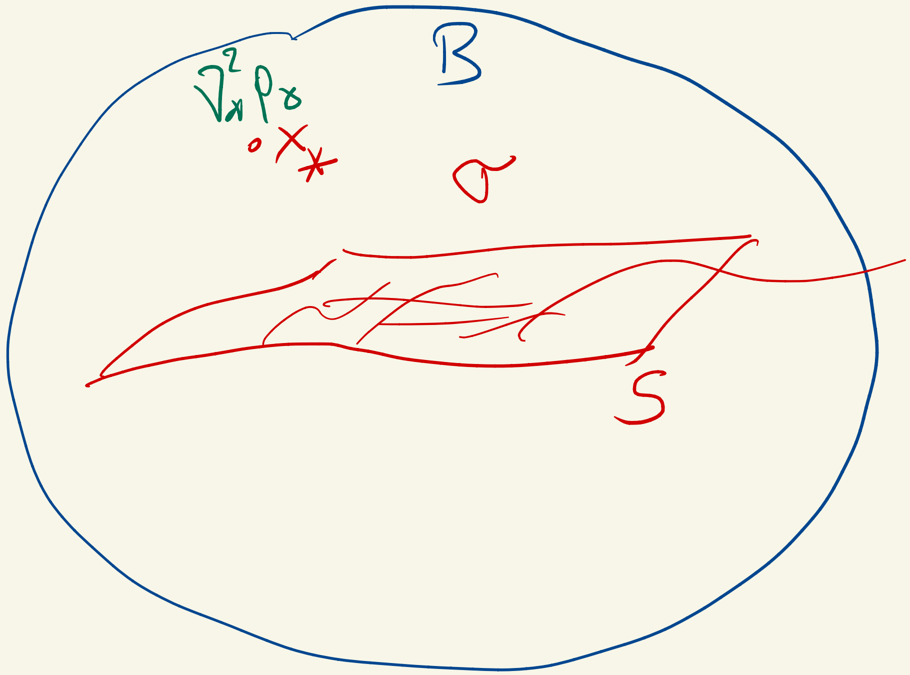
Rapp. Indiretta in semplice strato (sorgenti)

$$\nabla^2 \varphi_i = 0 \qquad x \in \mathring{D}_i$$

$$\frac{\partial \varphi}{\partial n} = f(x) \qquad x \in \partial D_i$$



$$\left.\frac{\partial \varphi}{\partial n_x}\right|_0 = \left.\frac{\partial}{\partial n_x}\right|_0 \oint \sigma g \, dS$$



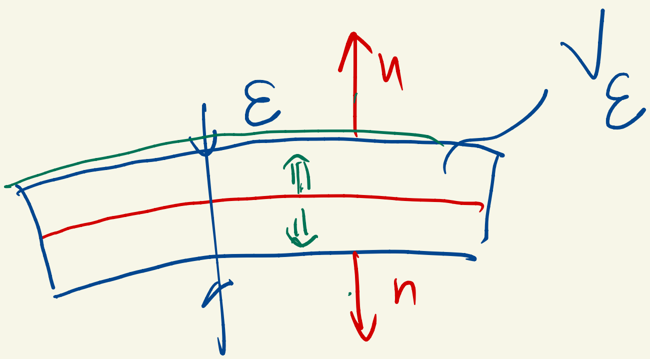
$$\varphi_* = \int_S \sigma \delta \, dS$$

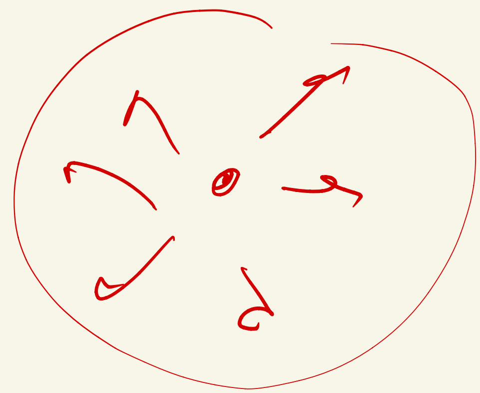
$$\nabla_*^2 \varphi = \nabla_*^2 \int_S \sigma \delta \, dS = \int_S \sigma(x) \overbrace{\nabla_*^2 \delta(x, x_*)}^{\delta(x - x_*)} \, dS$$

$$\int_B \nabla_*^2 \varphi \, dV = \int_B \int_S \sigma(x) \, \delta(x - x_*) \, dS \, dV_* =$$

$$= \int_S \sigma(x) \left(\int_B \underbrace{\delta(x - x_*)}_{\substack{V \\ \Delta}} \, dV_* \right) dS$$

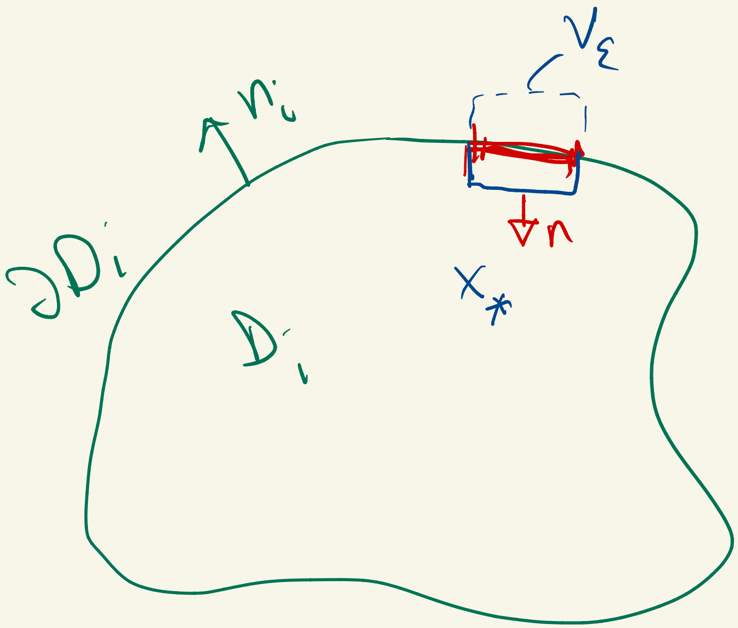
$$\int_B \nabla_y^2 \varphi_x \, dV_y = \int_S \sigma \, dS = \oint_{\partial B} \frac{\partial \varphi}{\partial n} \, dS$$





$$\int_{V_\varepsilon} \nabla_y^2 \varphi_x \, dV_y = \oint_{\partial V_\varepsilon} \frac{\partial \varphi_y}{\partial n_x} \, dS_x = \int_S \sigma \, dS = 2 \int_{S_\varepsilon^+} \frac{\partial \varphi}{\partial n} \, dS$$

$$\sigma \Delta S = 2 \left. \frac{\partial \varphi}{\partial n} \right|_+ \Delta S \rightarrow \frac{\partial \varphi}{\partial n} = 1/2 \, \sigma$$



$$\frac{\partial \varphi}{\partial n}$$

$$\int_{\partial D_i} \sigma \frac{\partial g}{\partial n_*} \, dS =$$

$$= \oint_{\partial D_i / V_\varepsilon} \sigma \frac{\partial g}{\partial n_*} \, dS - 1/2\, \sigma_*$$

$$\varphi_* = \oint_{\partial D_i} \sigma g \, dS$$

$$\frac{\partial \varphi_*}{\partial n} = -1/2\, \sigma_* + \oint_{\partial D_i} \sigma \frac{\partial g}{\partial n_*} \, dS$$

$$\frac{1}{2}\varphi_0 + \oint_{\partial D_i} \sigma \frac{\partial g}{\partial n}\, dS = \varphi_{i_0}$$

$$-\frac{1}{2}\varphi_{i_x} + \oint_{\partial D_i} \varphi \frac{\partial g}{\partial n}\, dS = + \oint_{\partial D_i} g \frac{\partial \varphi}{\partial n_x})\, dS$$

$$-\frac{1}{2}\sigma_x + \oint_{\partial D_i} \sigma \frac{\partial g}{\partial n_x}\, dS = \left.\frac{\partial \varphi}{\partial n}\right|_x$$

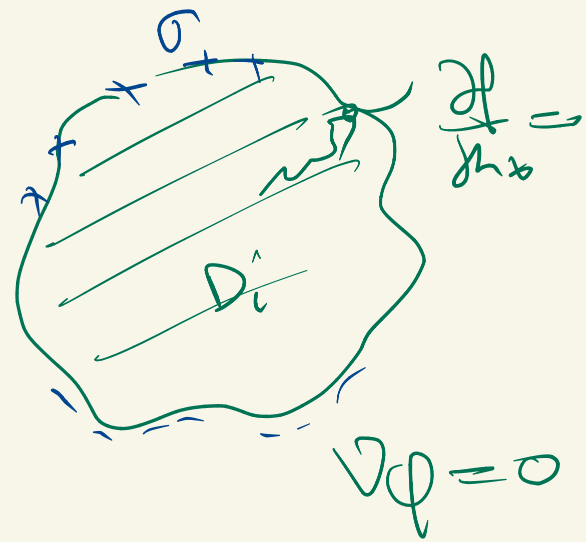
$$Ax = b$$

$$y^T A = b^T$$

$$A^T y = b$$

$$-\tfrac{1}{2}\sigma_x + \oint_{\partial D_i} \sigma \frac{\partial g}{\partial n_x}\, dS = 0 = \frac{\partial \varphi}{\partial n_x} =$$

Now $$\varphi_* = \int_{\partial D} \sigma g\, dS$$



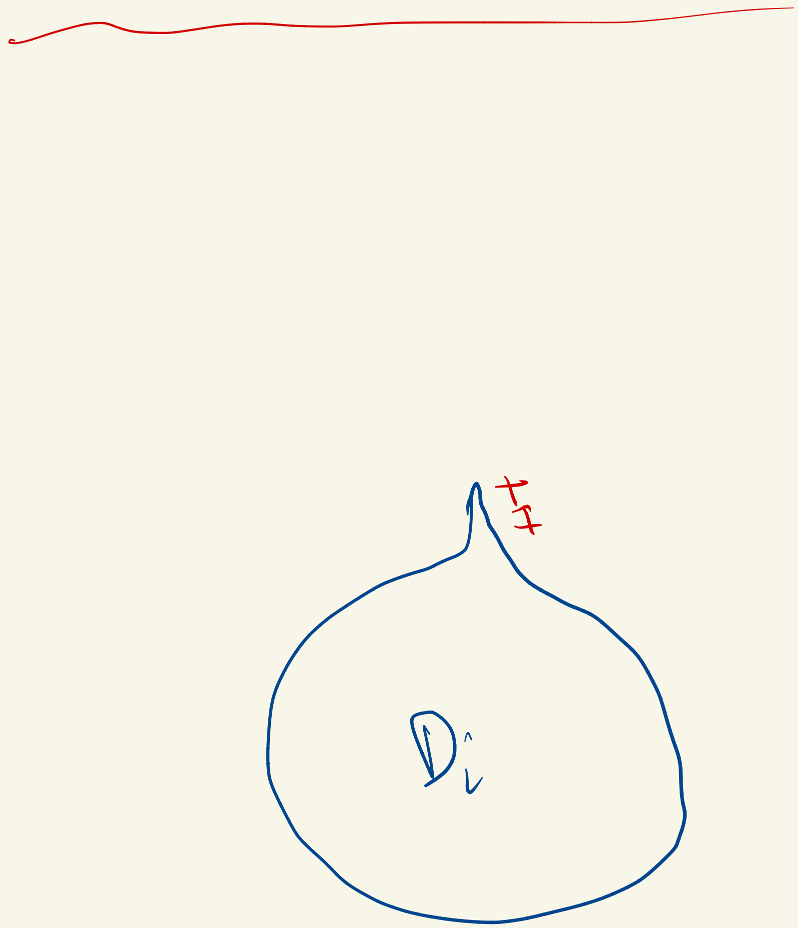
$$\varphi_* = \text{cst} = \oint_{\partial D} \sigma g\, dS$$

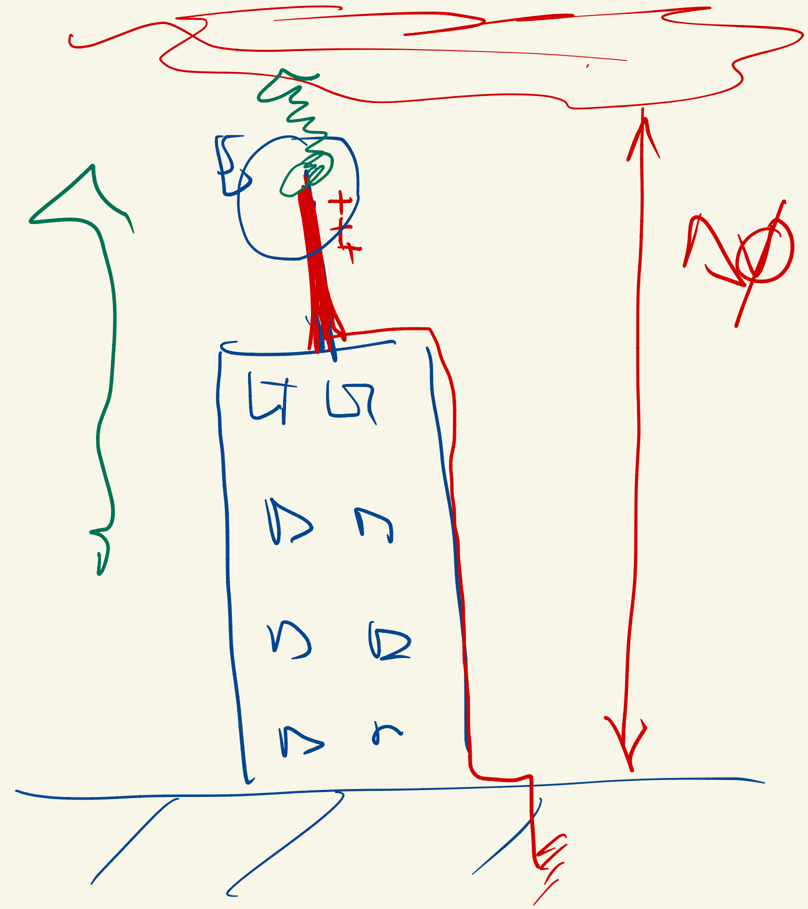
$$E = -\nabla\varphi$$

$$\varphi_* \Big/ \oint \sigma\, dS = C$$

$$E = 0 \to \nabla\varphi = 0 \qquad x \in D_i$$

$$\left.\frac{\partial \varphi}{\partial n}\right|_x = n_x \cdot \nabla\varphi\big|_{x \to x_p}$$




ΔΦ

$$\oint_{\partial\Omega_j} \sigma(x_\ast)\left[-\tfrac{1}{2}\varphi(x_\ast) + \oint_{\partial\Omega_i} \varphi(x)\,\frac{\partial G}{\partial n}(x,x_\ast)\,dS_x\right] dS_{x_\ast} =$$

$$\sigma(x) \qquad \varphi(x_\ast) \qquad \frac{\partial G}{\partial n_\ast}(x_\ast,x)$$

$$= -\tfrac{1}{2}\oint_{\partial\Omega_i} \varphi(x_\ast)\,\sigma(x_\ast)\,dS_{x_\ast} + \oint_{\partial\Omega_i} \sigma(x_\ast)\oint_{\partial\Omega_j} \varphi(x)\,\frac{\partial G}{\partial n}(x,x_\ast)\,dS_x\,dS_{x_\ast} =$$

$$= -\tfrac{1}{2}\oint_{\partial\Omega_i} \varphi(x_\ast)\,\sigma(x_\ast)\,dS_{x_\ast} + \oint_{\partial\Omega_i} dS_{x_\ast}\,\varphi(x_\ast)\oint_{\partial\Omega_i} \sigma(x)\,\frac{\partial G}{\partial n_\ast}\,dS_x =$$

$$= \oint_{\partial\Omega_i} \varphi(x_\ast)\left[-\tfrac{1}{2}\sigma(x_\ast) + \oint_{\partial\Omega_j} \sigma(x)\,\frac{\partial G}{\partial n_\ast}\,dS_x\right]$$

$$y^T A x = x^T A^T y$$

$$Ax = b \qquad y^T A = 0 \rightarrow y_H^T$$

$$0 \equiv y_H^T A x = y_H^T b = 0 \qquad x = x_p + \alpha x_H$$

$$\oint_{\partial D_i} ds_{x_*}\, \sigma(x_*) \oint_{\partial D_i} \frac{\partial \phi}{\partial n}(x)\, g(x, x_*)\, ds_x =$$

$$= \oint_{\partial D} ds_x\, \sigma(x) \oint_{\partial D_i} \frac{\partial \phi}{\partial n_*}(x_*)\, g(x_*, x)\, ds_* =$$

$$= \oint_{\partial D_i} ds_*\, \frac{\partial \phi}{\partial n_*}(x_*) \underbrace{\oint_{\partial D} \sigma(x)\, g(x, x_*)\, ds_x}_{\Phi^* = \mathrm{const}}$$

$$\oint_{\partial D_i} ds_{x_\gamma}\, \sigma(x_\gamma) \oint_{\partial D_i} \frac{\partial f}{\partial n}(x)\, \rho(x, x_\gamma)\, ds_x = \text{const} \oint_{\partial D_i} \left.\frac{\partial f}{\partial n}\right|_b ds_\gamma = 0$$

$$\oint_{\partial D_i} \frac{\partial f}{\partial n_\gamma}\, ds_\gamma = 0$$

$$\nabla^2 \varphi = 0$$

$$\varphi_p + \text{const}$$

$\frac{\partial f}{\partial n} =$ סקמת