Lezione 31/03/2020

Meet Streaming

Inizio Lezione 14:15

La Lezione sarà videoregistrata !!

Calcolo Forze Aerodinamiche:

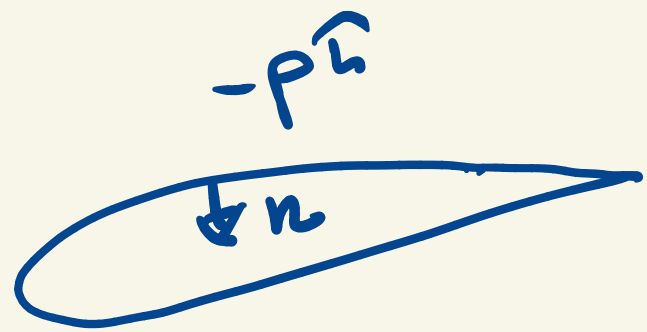
$$v_n\big|_{\partial B} = -U_\infty \cdot n \qquad U_n = 0$$

$$v_\tau\big|_{\partial B} = \frac{\partial \psi}{\partial n} \leftarrow \text{incognita} \qquad U_\tau = v_\tau + U_\infty \cdot \tau$$

$$\tfrac{1}{2}\rho_0 U^2 + p = \tfrac{1}{2}\rho_0 U_\infty^2 + p_\infty$$

$$p - p_\infty = \frac{1}{2}\rho_\infty \left(U_\infty^2 - u^2\right) = \frac{1}{2}\rho_\infty U_\infty^2 \left(1 - \frac{u^2}{U_\infty^2}\right)$$

$$c_p = \frac{p - p_\infty}{1/2\, \rho_\infty U_\infty^2} = 1 - \left(u/U_\infty\right)^2$$



$$F_A = \oint_{\partial B} p\,\hat{n}\, dl =$$

$$= \frac{1}{2}\rho_0 U_\infty^2 \left[\oint_{\partial B} c_p\, \hat{n}\, dl + \cancel{\oint p_\infty \hat{n}\, dl}\right]$$

$$\oint_{\partial B} p_\infty \hat{n}\, dl = \int_B \nabla p_\infty \, dA \equiv 0$$

$$F_A = \frac{1}{2}\rho_\infty U_\infty^2 \oint_{\partial B} c_p \hat{n}\, dl$$

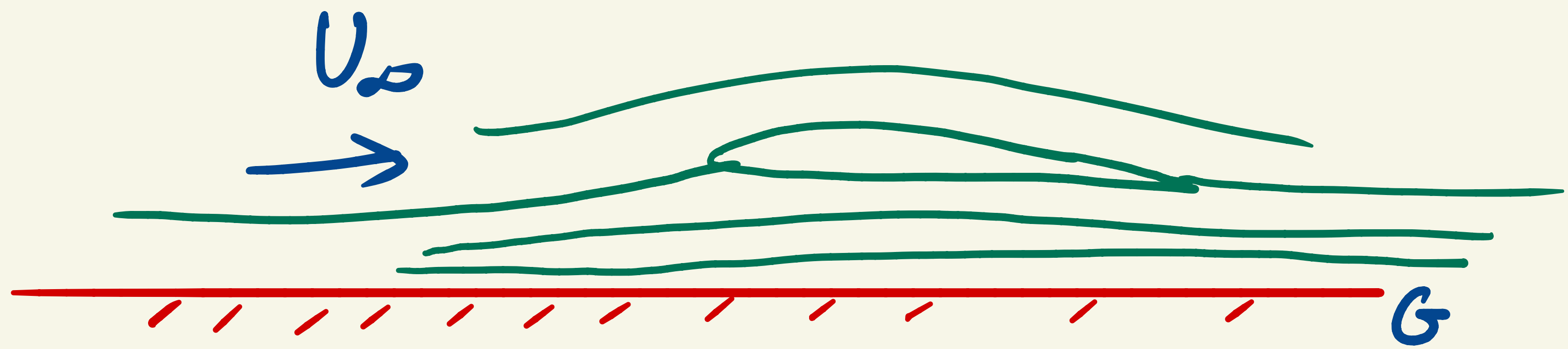
$$\xrightarrow[U_\infty]{\qquad\qquad} \qquad F_A = D\hat{e}_x + L\hat{e}_y$$

$$F_A \cdot \hat{U}_\infty = D \qquad\qquad \hat{k} \times \hat{U}_\infty \cdot F_A = L$$

$$L = F_A \cdot \hat{k} \times \hat{U}_\infty = \frac{1}{2} \rho_\infty U_\infty^2 \underbrace{\hat{k} \times \hat{U}_\infty \cdot \oint_{\partial B} c_p \, \hat{n} \, dl}_{c_L \, l}$$

$$D = F_A \cdot \hat{U}_\infty = 1/2 \, \rho_\infty U_\infty^2 \underbrace{\hat{k} \times \hat{U}_\infty \cdot \oint_{\partial B} c_p \, \hat{n} \, dl}_{c_D \, l}$$

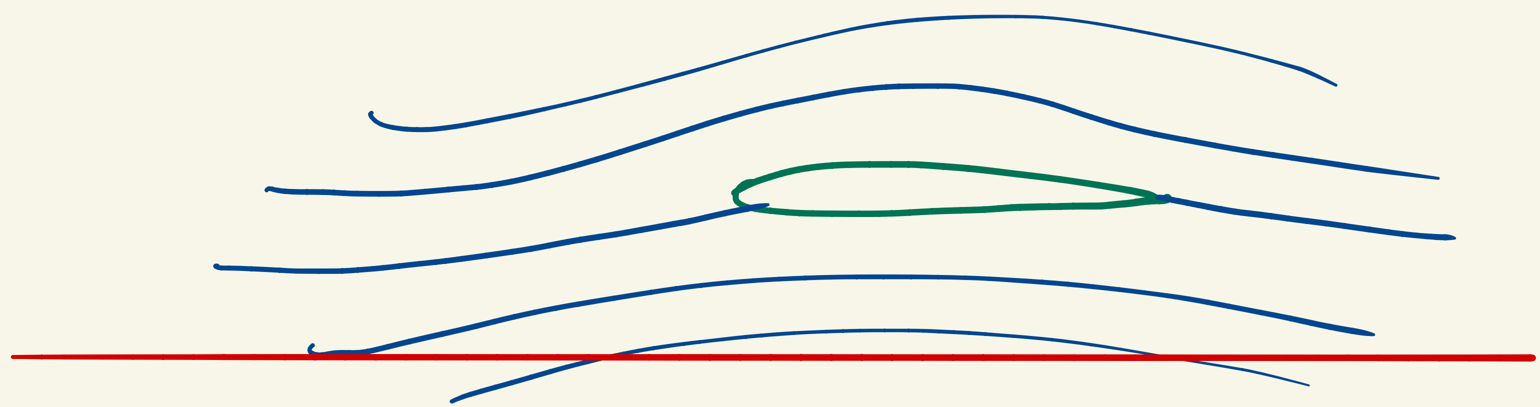
Effetto Suolo:



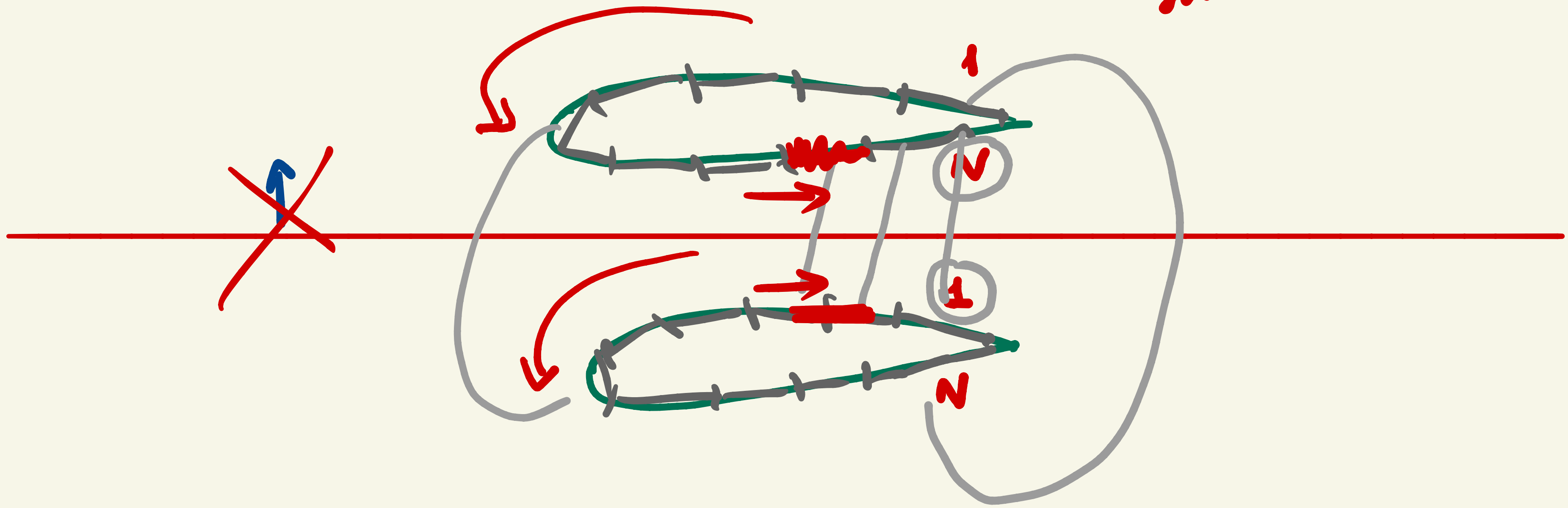
$$\nabla^2 \psi = 0$$

$$\psi\big|_{\partial B} = -U_\infty \, y\big|_{\partial B}$$

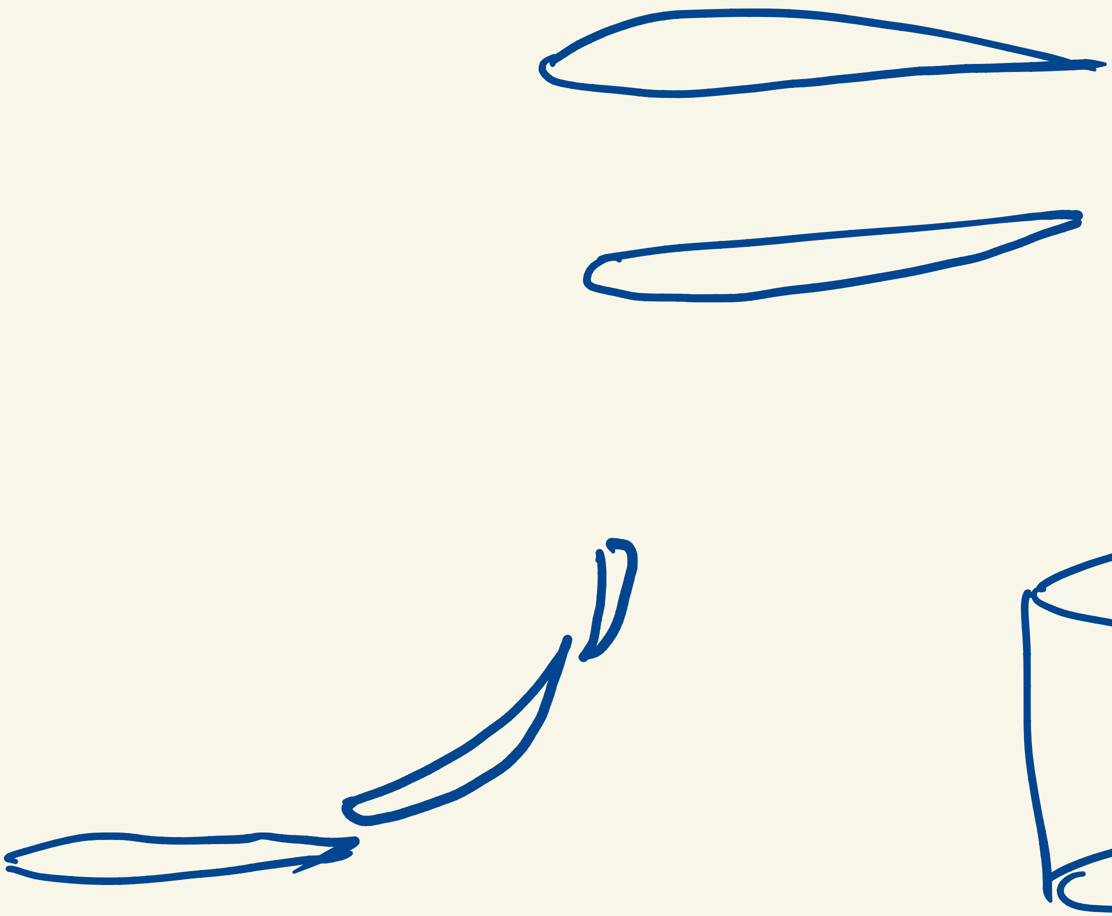
$$\psi\big|_G = 0$$



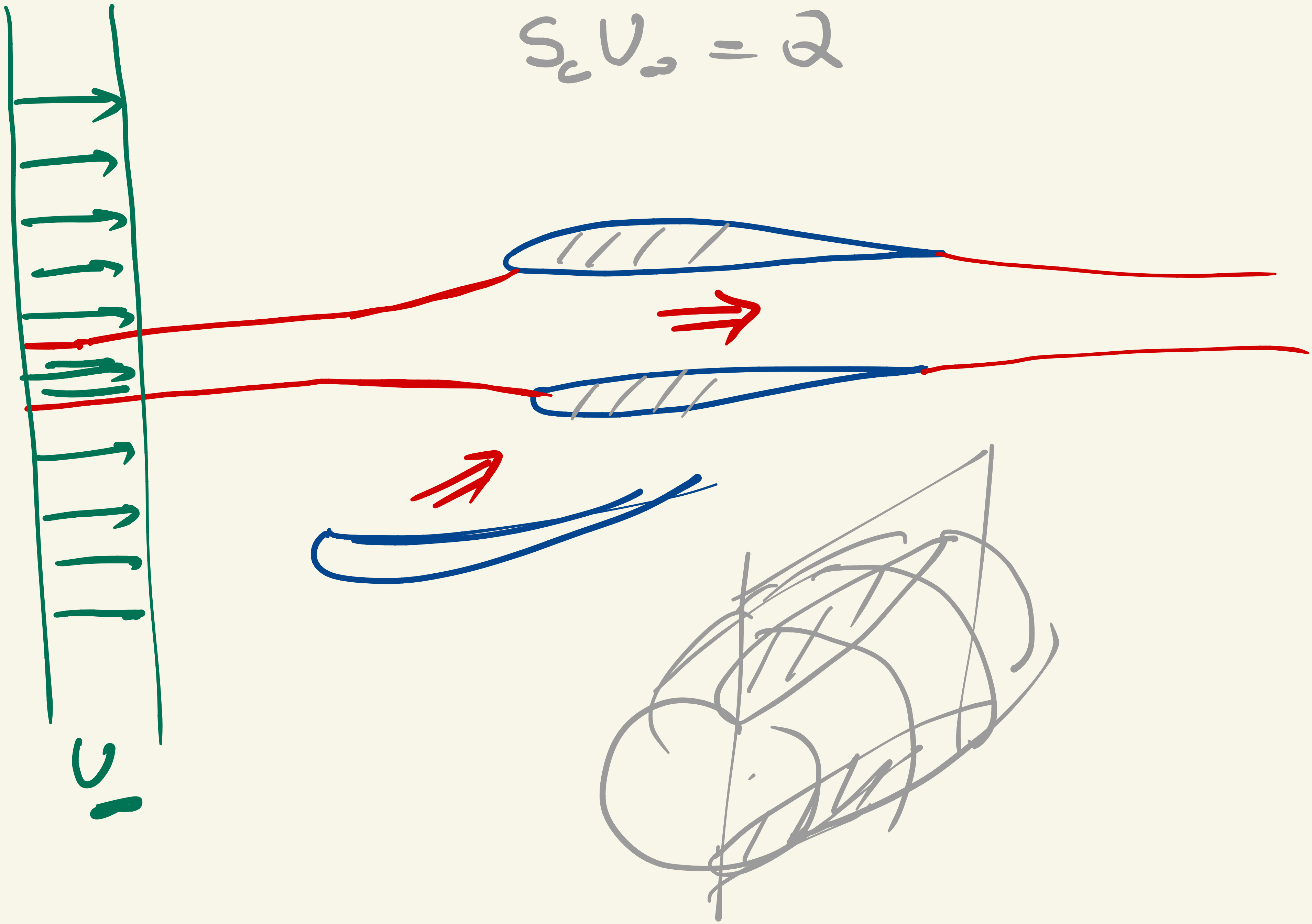
$$\frac{\partial \psi}{\partial n} = v_\tau$$

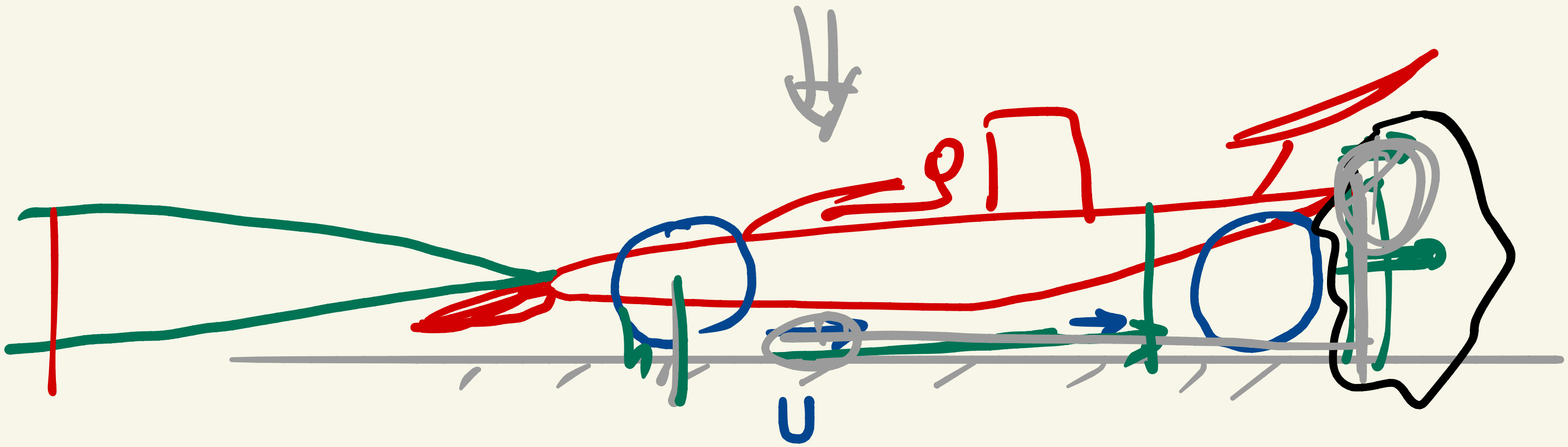


$$\Phi_j \int_{P_j} \frac{\partial \varphi}{\partial n}(x, x_s)\, ds$$



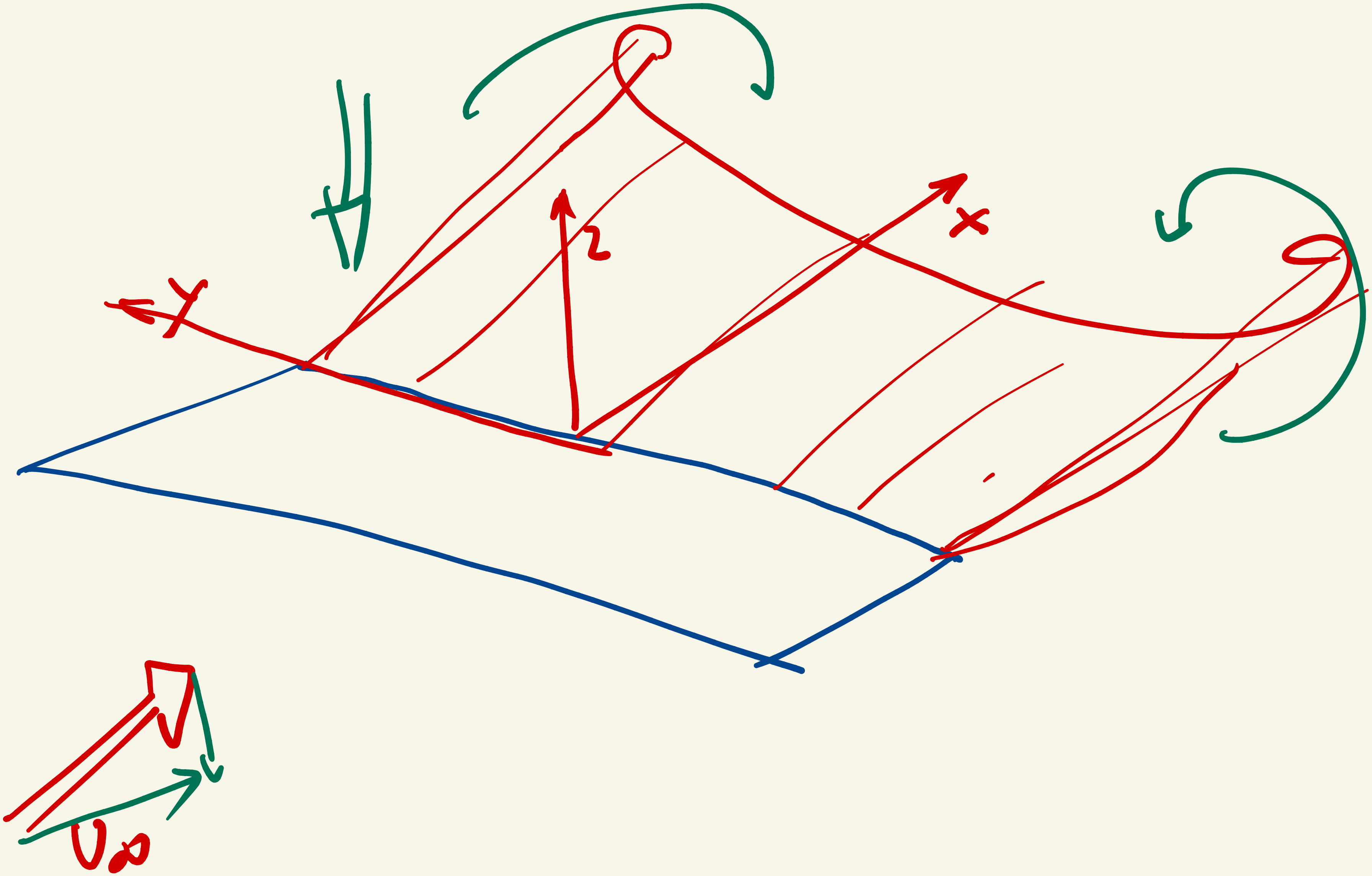
$$S_c V_\infty = 2$$





$$U_1 h_1 = U_2 h_2$$

$$\tfrac{1}{2}\rho U_1^2 + p_1 = \tfrac{1}{2}\rho U_2^2 + p_2$$


y
z
x
V_∞

$$U = U(x) \qquad \nabla \cdot U \equiv 0 \qquad \nabla \times U = \zeta \neq 0$$

$$\nabla^2 B = U \qquad B_* = \int g_{3D}(x, x_*)\, U(x_*)\, dV$$

$$+ \oint_{\partial D} \left(\cancel{B \frac{\partial g}{\partial n}} - g \frac{\partial B}{\partial n} \right) dS$$

$$\nabla_*^2 B_* = \nabla_*^2 \int g_{3D}(x, x_*)\, U(x)\, dV =$$

$$= \int \underbrace{\nabla_*^2 g_{3D}(x, x_*)}_{\delta(x - x_*)}\, U(x)\, dV = U(x_*)$$

$$\nabla_*^2 B_* = U_* \qquad B_* = \int g(x, x_*)\, U(x)\, dV$$

$$\nabla \times (\nabla \times B) = \nabla(\nabla \cdot B) - \nabla^2 B$$

$$U \times (V \times W) = U \cdot W\, V - U \cdot V\, W$$

$$\nabla^2 B = \nabla(\nabla \cdot B) - \nabla \times (\nabla \times B)$$

$$\boxed{U_* = \nabla_*(\nabla_* \cdot B_*) - \nabla_* \times (\nabla_* \times B_*)}$$

$$U_* = \nabla_* (\nabla_* \cdot B_*) - \nabla_* \times (\nabla_* \times B_*)$$

$$B_* = \int_D g(x, x_*)\, U(x)\, dV$$

$$\nabla_* \cdot B_* = \nabla_* \cdot \int_D g(x, x_*)\, U(x)\, dV =$$

$$= \int_D \nabla_* \cdot \left[g(x, x_*)\, U(x) \right] dV = \int_D \nabla_* g \cdot U\, dV$$

$$\nabla_* \cdot B_* = \int_D \nabla_* \rho \cdot u \, dV = -\int_D \nabla \rho \cdot u \, dV =$$

$$= -\int_D \left[\nabla \cdot (\rho u) - \rho \, \nabla \cdot u \right] dV$$

$$\nabla_* r = -\nabla r$$

$$\rho(x, x_*) = \rho(\overbrace{|x - x_*|}^{r}) \rightarrow \nabla_* \rho = \frac{d\rho}{dr} \nabla_* r$$

$$r = \sqrt{(x - x_*)^2}$$

$$\nabla \rho = \frac{d\rho}{dr} \nabla r$$

$$\nabla_x \cdot B_x = \int_D \rho \, \nabla \cdot u \, dV - \int_D \nabla \cdot (\rho u) \, dV =$$

$$\boxed{\nabla_x \cdot B_x = \int_D \rho \, \nabla \cdot u \, dV - \oint_{\partial D} \rho \, u \cdot n \, dS}$$

$$\nabla_* \times B_* = \nabla_* \times \int_D \rho\, u\, dV = \int_D \nabla_* \times (\rho u)\, dV =$$

$$= \int_D \nabla_* \rho \times u\, dV = -\int_D \nabla \rho \times u\, dV =$$

$$= -\int_D \left[\nabla \times (\rho u) - \rho\, \nabla \times u \right] dV =$$

$$= \int_D \rho\, \nabla \times u\, dV - \oint_{\partial D} n \times u\, \rho\, dS$$

$$\nabla \times (a\, v_0) = \nabla a \times v_0$$

$$\nabla_x \times B_x = \int_D \rho \, \nabla \times u \, dV - \oint_{\partial D} n \times u \, \rho \, dS =$$

$$\boxed{\nabla_x \times B_x = \int_D \rho \, \nabla \times u \, dV + \oint_{\partial D} u \times n \, \rho \, dS}$$

Green-Gauss:

$$\int_D \frac{\partial}{\partial x_i} Q \, dV = \oint_{\partial D} n_i Q \, dS$$

$Q = u_i$ $\qquad \nabla \times b \to \varepsilon_{ijk} \frac{\partial}{\partial x_j} b_k = \frac{\partial}{\partial x_j}\left(\varepsilon_{ijk} b_k\right)$

$$\int_D \nabla \times b \, dV \to \int_D \frac{\partial}{\partial x_j} \underbrace{\left(\varepsilon_{ijk} b_k\right)}_{Q} dV = \oint_{\partial D} n_j \varepsilon_{ijk} b_k \, dS$$

$$\parallel$$

$$\oint_{\partial D} n \times b \, dS$$

$\varepsilon_{ijk} n_j b_k \to n \times b$

$$U_* = -\nabla_* \oint_{\partial D} u \cdot n \, g \, dS + \nabla_* \int_D \nabla \cdot u \, g \, dV$$

$$- \nabla_* \times \oint_{\partial D} u \times n \, g \, dS - \nabla_* \times \int_D \nabla \times u \, g \, dV$$

Rappresentazione di Poincaré

$$\nabla \cdot u = 0$$

$$U_* = - \nabla_* \oint_{\partial D} u \cdot n \, g \, dS - \nabla_* \times \oint_{\partial D} u \times n \, g \, dS$$

$$- \nabla_* \times \int_D \zeta \, g \, dV$$

$$U_* = \nabla_* \underbrace{(\nabla_* \cdot B_*)}_{\varphi_*} - \nabla_* \times \underbrace{(\nabla_* \times B)}_{-A_*}$$

$$U_* = \nabla \varphi_* + \nabla_* \times A_*$$

Decomposizione di Helmholtz

$$U = U_n n + U_\pi$$

$$U \times n = \cancel{U_n n \times n} + U_\pi \times n$$

$$\nabla \times u \equiv 0 \qquad \nabla \cdot u = 0$$

$$u_* = -\nabla_* \oint_{\partial D} g \, u_n \, dS - \nabla_* \times \oint_{\partial D} g \, u \times n \, dS$$

$$\varphi_* = \oint_{\partial D} \varphi \frac{\partial g}{\partial n} \, dS - \oint_{\partial D} \frac{\partial \varphi}{\partial n} g \, dS$$

$$\frac{\partial \varphi}{\partial n} = u \cdot n$$

$$u_* = \nabla_* \varphi_* = -\nabla_* \oint_{\partial D} \frac{\partial \varphi}{\partial n} g \, dS + \nabla_* \oint_{\partial D} \varphi \frac{\partial g}{\partial n} \, dS$$

$$u_x = -\nabla_x \times \int_D \zeta\, g\, dV$$

Formula di Biot-Savart