Lezione 28/04/2020

Meet Streaming

La lezione inizia @ 14:15

Sara' Video-Registrata

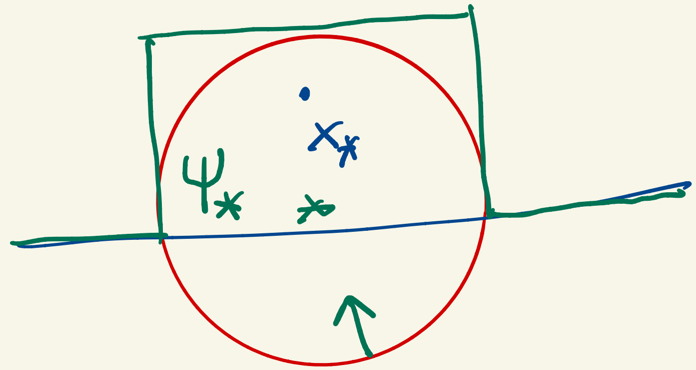
https://meet.google/xkd-szfj-xfr

2D:

$$\varphi_* = \oint_{\partial B} \left(\varphi \frac{\partial g}{\partial n} - g\frac{\partial \varphi}{\partial n}\right) dl$$

$$\psi|_{\partial B} \equiv \varphi|_{\partial B}$$

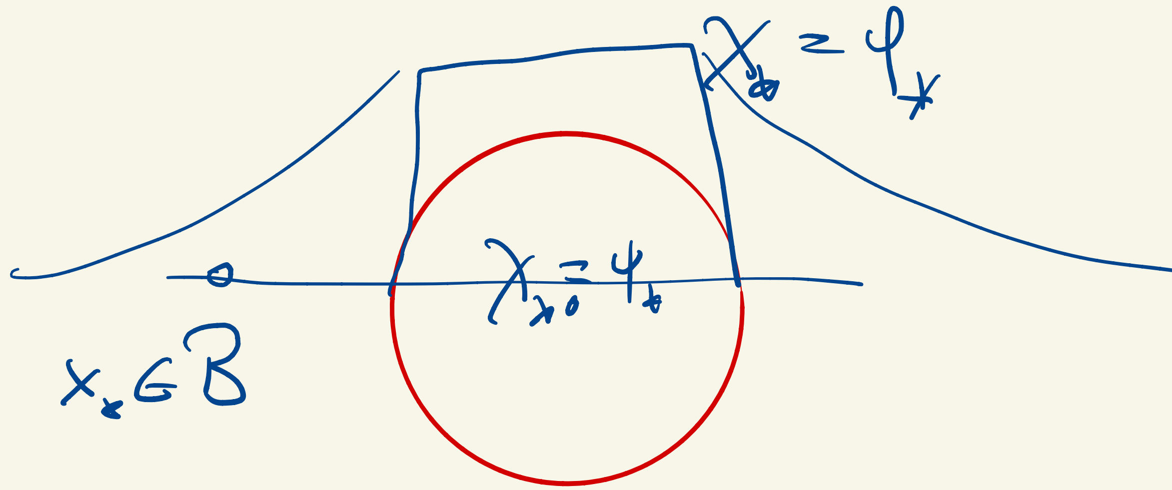
$$\oint_{\partial B} \frac{\partial g}{\partial n}(x,x_*)\,dl = \begin{cases} -1 & x_*\in \mathring{B} \\ 0 & x_*\notin B\end{cases}$$



$$\psi_* = -\oint_{\partial B}\left(\psi \frac{\partial g}{\partial n} - g\frac{\partial\psi}{\partial n}\right)dl$$

$$\chi_* = \oint_{\partial B}\left[\cancel{(\varphi-\psi)}\frac{\partial g}{\partial n} - \left(\frac{\partial \varphi}{\partial n}-\frac{\partial\psi}{\partial n}\right)g\right]dl$$

$$\chi_x = \oint_{\partial B} \sigma \, g \, dl$$



$$\chi\big|_{\partial B} = 1 \to \chi_x \equiv 1 \quad x_x \in B$$

$$\sigma \equiv \text{cost} \qquad \sigma \equiv 1 \qquad \oint_{\partial B} g \, dl = \text{cost} \quad \forall x_0 \in \mathring{B}$$

$$g = \frac{1}{2\pi} \ln r \equiv \frac{1}{2\pi} \ln R$$

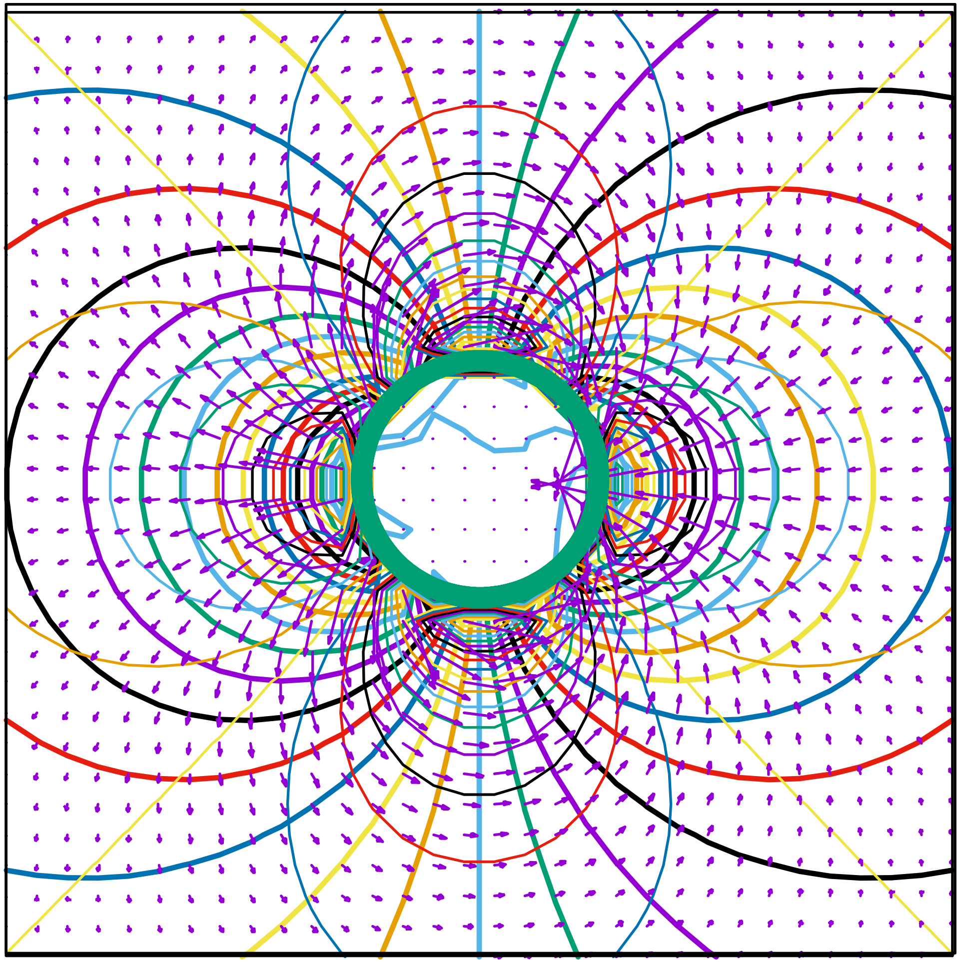
$$\oint g \, dl = \frac{2\pi R \ln R}{2\pi} = R \ln R$$

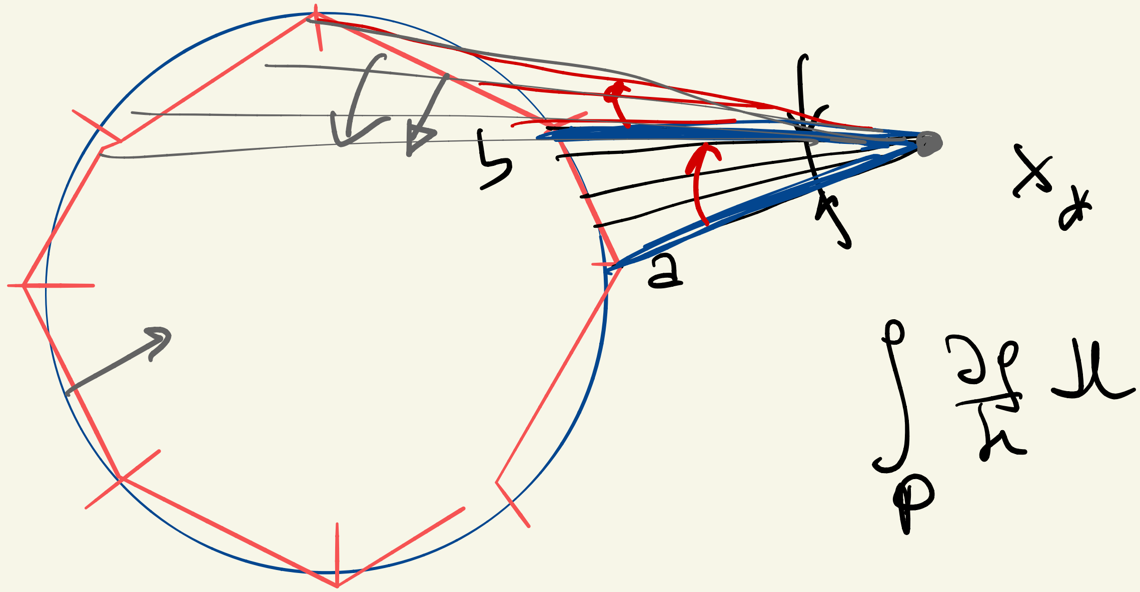
$$A = \int_{P} \frac{\partial g}{\partial n} \, dl \qquad B = \int_{P} g \, dl$$

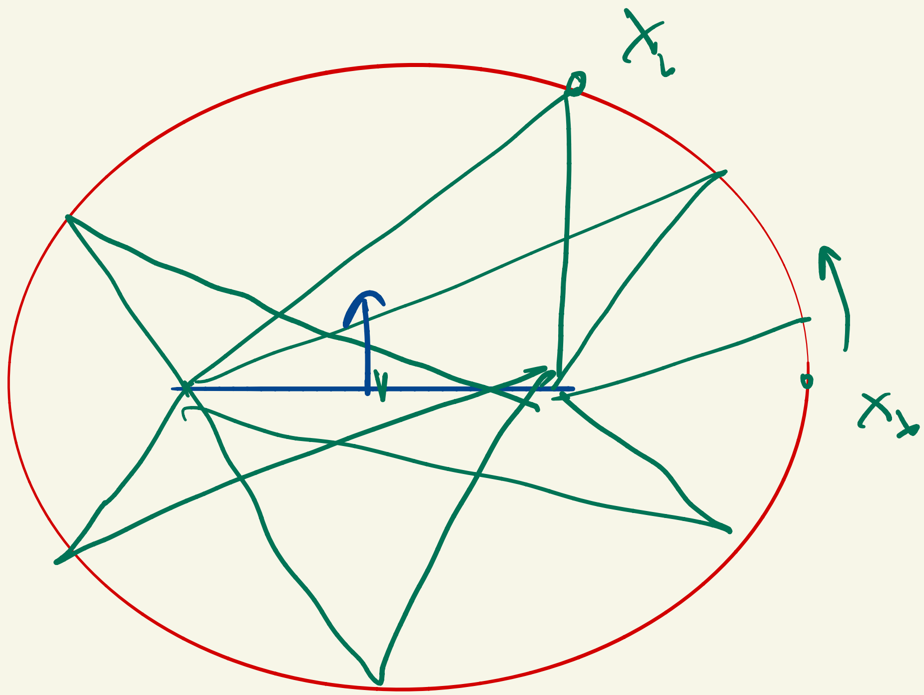
$i = 1, \ , N$

$$\sum_{i=1}^{N} A_i = \begin{cases} -1 & x \in \mathring{B} \\ 0 & x \notin B \end{cases}$$

$$\sum_{i=1}^{N} B_i = \begin{cases} R \ln R & x \in \mathring{B} \\ ? & x \notin B \end{cases}$$




b
a
x*


x₂
x₁

$$\varphi_* = \oint_{\partial B} \left(\varphi \frac{\partial g}{\partial n} - g \frac{\partial \varphi}{\partial n} \right) dl \qquad U_* = \nabla_* g$$

$$U_* = \nabla_* \varphi_* = \nabla_* \oint_{\partial B} \left(\varphi \frac{\partial g}{\partial n} - g \frac{\partial \varphi}{\partial n} \right) dl$$

$$U_* = \sum_{i=1}^{N} \left[\varphi_i \int_{P_i} \nabla_* \frac{\partial g}{\partial n} dl - \left.\frac{\partial \varphi}{\partial n}\right|_i \int_{P_i} \nabla_* g \, dl \right]$$

$$\int_{P_i} \nabla_* g \, dl$$

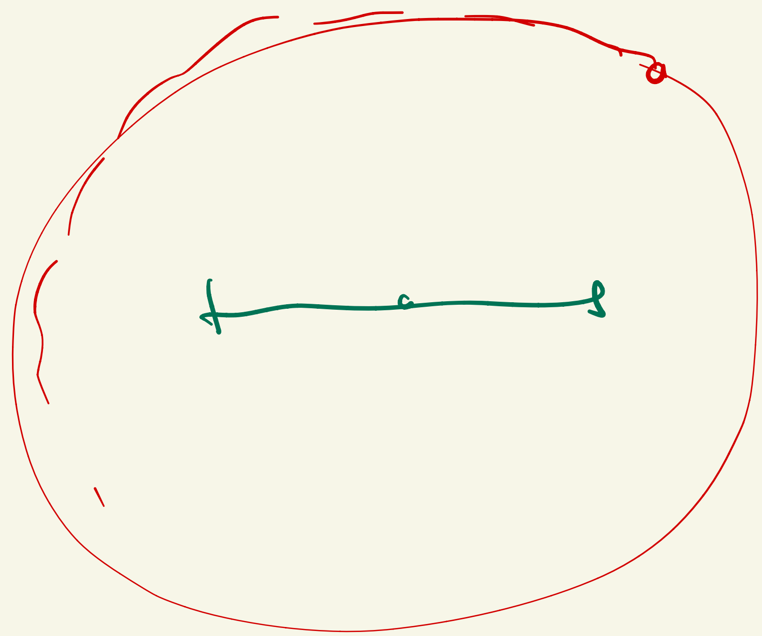
$$g = \frac{1}{2\pi} \ln |x - x_*|$$

$$\nabla_* g = - \nabla g$$

$$\int_{\tau_A}^{\tau_B} \frac{\partial g}{\partial \tau} d\tau = g\Big|_{\tau_B} - g\Big|_{\tau_A}$$

$$\nabla g = \hat{\tau} \frac{\partial g}{\partial \tau} + \hat{n} \frac{\partial g}{\partial n}$$

$$\int_{\tau_A}^{\tau_A} \frac{\partial g}{\partial n} d\tau = A g$$



$$\int_{P_i} \nabla_* \frac{\partial g}{\partial n}\, dl \qquad \nabla_* \frac{\partial g}{\partial n} = \nabla_* \left[n \cdot \nabla g \right] =$$

$$= \frac{\partial}{\partial x_i^*}\left(n_k \frac{\partial g}{\partial x_k} \right) = n_k \frac{\partial}{\partial x_i^*} \frac{\partial g}{\partial x_k} =$$

$$= n_k \frac{\partial}{\partial x_k} \frac{\partial g}{\partial x_i^*} = n \cdot \nabla \nabla_* g = - n \cdot \nabla \nabla g$$

$$= - n \cdot \nabla \otimes \nabla g$$

$$\nabla \otimes \nabla g \qquad \nabla g = \frac{1}{2\pi}\frac{1}{|r|}\nabla r = \frac{1}{2\pi}\frac{r}{|r|^2}$$

$$g = \frac{1}{2\pi}\ln r$$

$$\nabla r = \frac{r}{|r|}$$

$$\frac{\partial}{\partial x_k}\frac{1}{2\pi}\left(\frac{r_i}{r^2}\right) = \frac{1}{2\pi}\left[\frac{\delta_{ik}}{r^2} - 2\frac{r_i}{r^3}\frac{r_k}{r}\right] \qquad r_i = x_i - x_i^*$$

$$\delta_{ik} = \frac{\partial r_i}{\partial x_k} = \begin{cases} 0 & i \neq k \\ 1 & i = k \end{cases}$$

$$\frac{\partial^2 g}{\partial x_k \partial x_i} = \frac{1}{2\pi}\frac{1}{r^2}\left(\delta_{ij} - \frac{r_i r_k}{r^2}\right)$$

$$\nabla \otimes \nabla g = \frac{1}{2\pi}\left(\frac{I}{r^2} - \frac{r \otimes r}{r^4}\right)$$

$$\nabla \otimes \nabla g = \frac{1}{2\pi}\left(\frac{I}{r^2} - \frac{r \otimes r}{r^4}\right)$$

$$\hat{n} \cdot \nabla \otimes \nabla g = \frac{1}{2\pi}\left[\frac{\hat{n} \cdot I}{r^2} - \frac{\hat{n} \cdot r \; r}{r^4}\right] =$$

$$\frac{1}{2\pi}\left[\frac{\hat{n}}{r^2} - \frac{r \cdot n \; r}{r^4}\right] = \frac{1}{2\pi}\hat{n}\left[\frac{1}{r^2} - \frac{(r \cdot \hat{n})^2}{r^4}\right] + \frac{1}{2\pi}\hat{t}\left[-\frac{r \cdot \hat{n} \; r \cdot \hat{t}}{r^4}\right]$$

$$\int_{\tau_A}^{\tau_B} \frac{d\tau}{(\cancel{n}-n_*)^2+(\tau-\tau_*)^2} = \qquad n \equiv 0$$

$$= \int_{\tau_A}^{\tau_B} \frac{d(\tau-\tau_*)}{n_*^2+(\tau-\tau_*)^2} = \frac{\cancel{n_*}}{n_*^{\cancel{2}}} \int_{\tau_A}^{\tau_B} \frac{d\left(\frac{\tau-\tau_*}{n_*}\right)}{1+\left(\frac{\tau-\tau_*}{n_*}\right)^2} =$$

$$= \frac{1}{n_*} \int_{\frac{\tau_A-\tau_*}{n_*}}^{\frac{\tau_B-\tau_*}{n_*}} \frac{dx}{1+x^2} = \frac{1}{n_*} \tan^{-1}\left(\frac{\tau-\tau_*}{n_*}\right)\Bigg|_{\tau=\tau_A}^{\tau=\tau_B}$$

$$\int_{\tau_A}^{\tau_B} \frac{(\cancel{n}-n_*)^2 \, d\tau}{\left[(\cancel{n}-n_*)^2 + (\tau-\tau_0)^2\right]^2} = \frac{n_*^{\cancel{2}}}{n_*^{\cancel{4}\,2}} \int_{\tau_A}^{\tau_B} \frac{d\tau}{\left[1+\left(\frac{\tau-\tau_*}{n_*}\right)^2\right]^2} =$$

$$= \frac{1}{n_*}\int_{\tau_A}^{\tau_B} \frac{d(\tau-\tau_*)/n_*}{\left[1+\left(\frac{\tau-\tau_*}{n_*}\right)^2\right]^2} =$$

$$= \frac{1}{n_*}\int_{\frac{\tau_A-\tau_*}{n_*}}^{\frac{\tau_B-\tau_*}{n_*}} \frac{dx}{(1+x^2)^2} = \frac{1}{n_*}\left[\frac{(\tau-\tau_*)/n_*}{1+\left(\frac{\tau-\tau_*}{n_*}\right)^2} + \tan^{-1}\left(\frac{\tau-\tau_*}{n_*}\right)\right]_{\tau_A}^{\tau_B}$$

$$\int \frac{dx}{(1+x^2)^2} \qquad \frac{d}{dx}\left[\frac{x}{(1+x^2)}\right] = \frac{1}{1+x^2} - \frac{2x^2}{(1+x^2)^2} =$$

$$= \frac{1}{1+x^2} - 2\,\frac{1+x^2}{(1+x^2)^2} + \frac{2}{(1+x^2)^2} = \cancel{\frac{1}{1+x^2}} - \frac{\cancel{2}}{1+x^2} + \frac{2}{(1+x^2)^2}$$

$$\frac{1}{(1+x^2)^2} = \frac{1}{2}\frac{d}{dx}\left(\frac{x}{1+x^2}\right) + \frac{1}{2}\frac{1}{1+x^2} =$$

$$= \frac{1}{2}\frac{d}{dx}\left[\frac{x}{1+x^2} + \tan^{-1}(x)\right]$$

$$\int \frac{dx}{(1+x^2)^2} = \frac{1}{2}\left[\frac{x}{1+x} + \tan^{-1}(x\right]$$

$$\int_{z_a}^{z_b} \frac{r \cdot n \quad n z}{r^4}\, dz = -n_x \int_{z_a}^{z_b} \frac{(z-z_x)\, d(z-z_x)}{\left[n_x^2 + (z-z_x)^2\right]^2} =$$

$$= -\frac{\cancel{n_x^3}}{\cancel{n_x^4}} \int_{z_a}^{z_b} \frac{\frac{z-z_x}{n_x}\; d\left(\frac{z-z_x}{n_x}\right)}{\left[1 + \left(\frac{z-z_x}{n_x}\right)^2\right]^2}$$

$$= -\frac{1}{n_x} \int_{\frac{z_a - z_x}{n_x}}^{\frac{z_b - z_x}{n_x}} \frac{x\, dx}{[1+x^2]^2}$$

$$\int \frac{x\,dx}{(1+x^2)^2} = \frac{1}{2}\int \frac{dx^2}{(1+x^2)^2} = \frac{1}{2}\int \frac{d\xi}{(1+\xi)^2} =$$

$$= -\frac{1}{2}\int \frac{d}{d\xi}\left[\frac{1}{(1+\xi)}\right] d\xi = -\frac{1}{2}\,\frac{1}{1+x^2}$$

$$-\frac{1}{n_x}\int_{\frac{\tau_a-\tau_x}{n_x}}^{\frac{\tau_b-\tau_x}{n_x}} \frac{x\,dx}{[1+x^2]^2} = +\frac{1}{2n_x}\,\frac{1}{1+\left(\frac{\tau-\tau_x}{n_x}\right)^2}\Bigg|_{\tau_a}^{\tau_b}$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \qquad i, j = 1, \ldots, n$$

$$a_{11} x_1 + \sum_{j=2}^{n} a_{1j} x_j = b_1 \rightarrow x_1 = \frac{b_1}{a_{11}} - \sum_{j=2}^{n} \frac{a_{1j}}{a_{11}} x_j$$

$i = 2, n$

$$a_{i1} x_1 + \sum_{j=2}^{n} a_{ij} x_j = b_i$$

$$\frac{b_1 a_{i1}}{a_{11}} - \sum_{j=2}^{n} \frac{a_{i1} a_{1j}}{a_{11}} x_j + \sum_{j=2}^{n} a_{ij} x_j = b_i$$

$$\sum_{j=2}^{n} \frac{a_{11} a_{ij} - a_{i1} a_{1j}}{a_{11}} x_j = \frac{a_{11} b_i - b_1 a_{i1}}{a_{11}}$$

$$n \times n \rightarrow x_1 = F(x_2, \ldots x_n)$$

$$(n-1, n-1)$$

$$\sum_{j=2}^{n} \frac{a_{11} a_{ij} - a_{i1} a_{1j}}{a_{11}} x_j = \frac{a_{11} b_i - b_1 a_{i1}}{a_{11}}$$

$$\hat{a}_{ij} = \frac{a_{11} a_{ij} - a_{i1} a_{1j}}{a_{11}} \qquad \hat{b}_i = \frac{a_{11} b_i - b_1 a_{i1}}{a_{11}}$$

$$\sum_{j=2}^{n} \hat{a}_{ij} x_j = \hat{b}_i \qquad i = 2, \ldots, n$$

$$x_2$$

$$x_2 = \frac{\hat{f}_2}{\hat{a}_{22}} - \sum_{j=3}^{n} \frac{\hat{a}_{2j}}{\hat{a}_{22}} x_j \qquad \hat{*} \to *$$

$$\hat{a}_{ij} = \frac{a_{22} a_{ij} - a_{i2} a_{2j}}{a_{i2}} \qquad \hat{f}_i = \frac{a_{22} f_i - f_2 a_{i2}}{a_{22}}$$

$$x_2 = \frac{f_2}{a_{22}} - \sum_{j=3}^{n} \frac{a_{2j}}{a_{22}} x_j$$

$$k: \quad \hat{a}_{ij} = \frac{a_{kk} a_{ij} - a_{ik} a_{kj}}{a_{kk}} \qquad \hat{f}_i = \frac{a_{kk} f_i - f_k a_{ik}}{a_{kk}}$$

$$a_{n-1\,n-1}\, x_{n-1} + a_{n-1\,n}\, x_n = r_{n-1}$$

$$Ax = r$$

$$a_{n\,n-1}\, x_{n-1} + a_{n\,n}\, x_n = r_n$$

$$Ax^p = r$$

$$x_{n-1} = \frac{r_{n-1}}{a_{n-1\,n-1}} - \frac{a_{n-1\,n}\, x_n}{a_{n-1\,n-1}}$$

$$Ax^h = 0$$

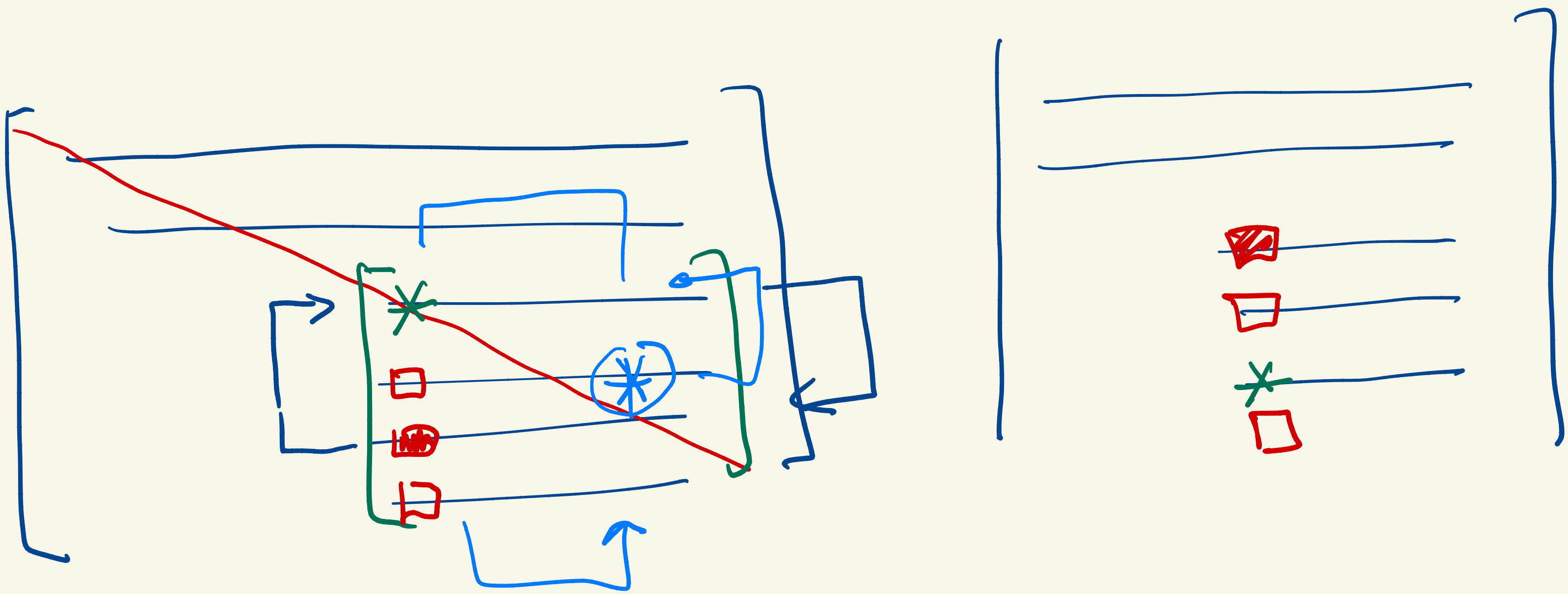
$$x = x^p + \alpha x^h$$

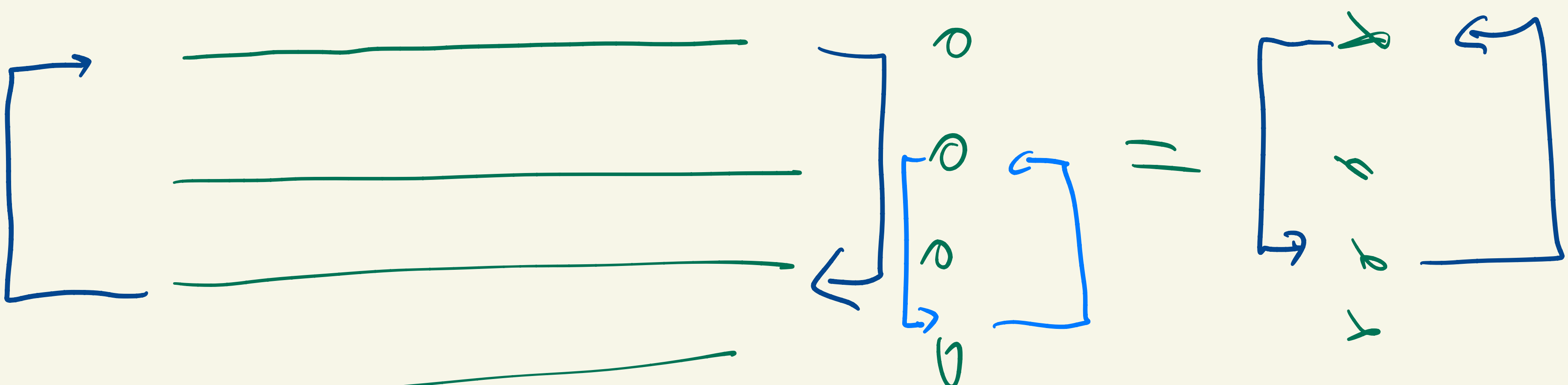
$$\left[a_{nn} - \frac{a_{n\,n-1}\, a_{n-1\,n}}{a_{n-1\,n-1}} \right) x_n = r_n - \frac{a_{n\,n-1}}{a_{n-1}\,a_{n-1}}\, r_{n-1}$$

$$x_n = 0 \longrightarrow x^p_n = 0,\; x^p_{n-1} \neq 0 \;\ldots\; x^p_1 \neq 0$$

$$x_n = 1 \,,\; r_1 = \ldots = r_n = 0 \rightarrow x^h_n,\; x^h_{n-1} \ldots x^h_1$$

$$X = x^{P} + \alpha x^{h_1} + \beta x^{h_2}$$





$$\nabla^2 \phi = 0$$

$$\left.\frac{\partial \phi}{\partial n}\right|_{\partial B} = \boxed{-\, U_\infty \cdot n}$$

$$\phi_* = \oint_{\partial B} \left(\phi \frac{\partial g}{\partial n} - g \boxed{\frac{\partial \phi}{\partial n}} \right) dl$$

$$U = \nabla \phi$$

$$U = \hat{k} \times \nabla \psi = \nabla^{\perp} \psi$$

$$U = \hat{e}_3 \times \left(\frac{\partial \psi}{\partial x} \hat{e}_1 + \frac{\partial \psi}{\partial y} \hat{e}_2 \right)$$

$$= \hat{e}_2 \frac{\partial \psi}{\partial x} - \hat{e}_1 \frac{\partial \psi}{\partial y}$$

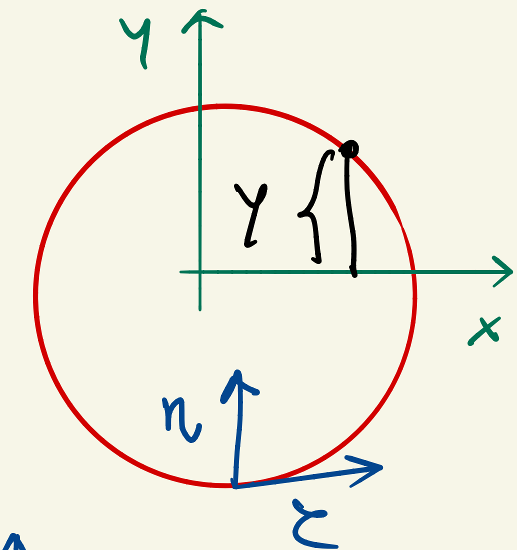
$$U_x = U = -\frac{\partial \psi}{\partial y}$$

$$U_y = V = +\frac{\partial \psi}{\partial x}$$

$$\nabla \cdot U \equiv 0 \qquad \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$\hat{e}_3 \zeta_3 = \nabla \times u = 0 \qquad \nabla = \hat{e}_x \frac{\partial}{\partial x} + \hat{e}_y \frac{\partial}{\partial y}$$

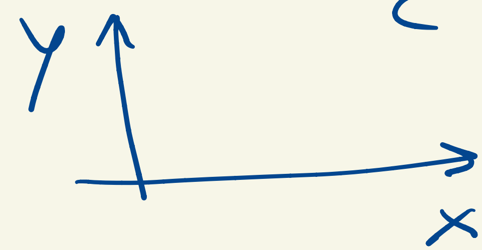
$$\zeta = -\nabla^2 \psi \qquad \zeta = 0 \implies \nabla^2 \psi = 0$$



$$\boxed{\begin{aligned} &\nabla^2 \psi = 0 \\ &\psi\big|_{\partial D} = -\psi_\infty\big|_{\partial B} \end{aligned}}$$

$$u_x = U = -\frac{\partial \psi}{\partial y}$$

$$u_y = V = +\frac{\partial \psi}{\partial x}$$



$$u_x = -\frac{\partial \psi}{\partial y} \qquad u_\tau = -\frac{\partial \psi}{\partial n}$$

$$u_y = \frac{\partial \psi}{\partial x} \qquad u_n = \frac{\partial \psi}{\partial \tau}$$

$$\Psi_T = \Psi_\infty + \psi \qquad \Psi_\infty = -U_\infty y \qquad -\frac{\partial \Psi_\infty}{\partial y} = U_\infty \qquad \frac{\partial \Psi_\infty}{\partial x} = 0$$

$$U_n^T\Big|_{\partial B} = +\frac{\partial \Psi_\infty}{\partial \tau}\Big|_{\partial B} + \frac{\partial \psi}{\partial \tau}\Big|_{\partial B} \equiv 0 \qquad \Psi_\infty + \psi = \psi_0 \equiv 0$$

$$\psi\Big|_{\partial B} = -\Psi_\infty\Big|_{\partial B} = +U_\infty y\Big|_{\partial B}$$

$$\frac{1}{2}\psi_x = \oint_{\partial B}\left(\psi\frac{\partial g}{\partial n} - g\frac{\partial \psi}{\partial n}\right)dl \rightarrow$$

$$\oint_{\partial B} g\underbrace{\frac{\partial \psi}{\partial n}}_{-U_\tau}dl = -\frac{1}{2}\psi_x + \oint_{\partial B}\psi\frac{\partial g}{\partial n}dl$$

$$\sum_{j=1}^{N} - B_{ij} u_{\tau_j} = \sum_{j=1}^{N} \left(-\frac{1}{2} \delta_{ij} + A_{ij} \right) \underbrace{\psi|_j}_{U_\infty y_i}$$