Lezione 24/03/2020

(meet streaming)

La Lezione inizia alle 14:15

La lezione sarà Video-Registrata

(dovete accettare la video registrazione per la privacy)

Dalla prossima volta utilizzeremo sempre lo stesso indirizzo Meet:

$\boxed{\text{https://meet.google.com/sit-sjju-yur}}$

Rappresentazione Integrale Diretta

$$\varphi_* = \oint_{\partial B} \left(\varphi \frac{\partial g}{\partial n} - g \frac{\partial \varphi}{\partial n} \right) dS$$

Potenziale

2D:

Dato

$$\boxed{\frac{1}{2} \varphi_* - \oint_{\partial B} \varphi \frac{\partial g}{\partial n} dS} = - \oint_{\partial B} g \frac{\partial \varphi}{\partial n} dS$$

$$\boxed{\oint_{\partial B} g \frac{\partial \psi}{\partial n} dS} = - \frac{1}{2} \psi_* + \oint_{\partial B} \psi \frac{\partial g}{\partial n} dS$$

Dato

Rappresentazioni Indirette

Semplice Strato:

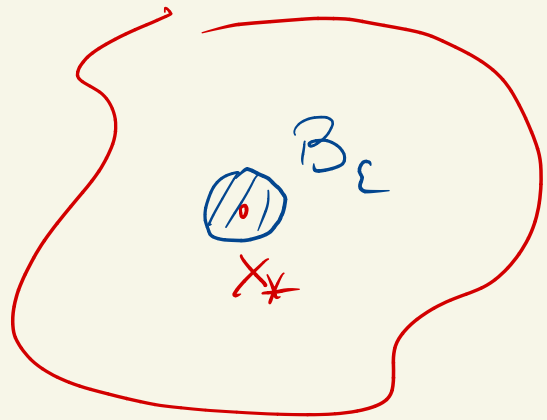
$$\varphi_* = \oint_{\partial B} \sigma g \, dS \rightarrow \frac{\partial \varphi_*}{\partial n_*} = 1/2 \, \sigma_* + \oint_{\partial B} \sigma \frac{\partial g}{\partial n_*} dS$$

$$\nabla^2 g = \delta(x - x_*) \qquad \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}}$$

$$x_* \neq x \rightarrow \nabla^2 g = 0 \qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dg}{dr}\right) = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\rho}{dr}\right)=0 \to \frac{d}{dr}\left(r\frac{d\rho}{dr}\right)=0 \to$$

$$r\frac{d\rho}{dr}=A \to \frac{d\rho}{dr}=\frac{A}{r} \to \rho = A\ln r + \cancel{B}$$



$$1=\int_{B_\varepsilon}\nabla^2\rho\, dA = \oint_{\partial B_\varepsilon}\frac{\partial\rho}{\partial n}\,dl =$$

$$=\int \frac{d\rho}{dr}\,dl \to A=\frac{1}{2\pi}$$

$$\rho_{2D} = \frac{1}{2\pi}\ln r$$

$$g = \frac{1}{2\pi} \ln r \qquad r = |x - x_*|$$

$$\nabla g = \frac{dg}{dr} \nabla r = \frac{1}{2\pi r} \nabla r = \frac{\hat{r}}{2\pi r} \qquad \nabla r = \hat{r}$$

$$\frac{\partial g}{\partial n} = \frac{1}{2\pi} \frac{\hat{n} \cdot \hat{r}}{r} \qquad \frac{\partial g}{\partial n_*} = \hat{n}_* \cdot \nabla_* g = -\hat{n}_* \cdot \nabla g$$

$$\nabla_* g = \frac{dg}{dr} \nabla_* r \qquad r = |x - x_*| = |x_* - x|$$

$$\nabla_* r = -\nabla r$$

$$r = \sqrt{(x_n - x_n^*)(x_n - x_n^*)} \qquad \underset{\sim}{\delta_{ns}}$$

$$\frac{\partial r}{\partial x_s} = \frac{2 \, (x_n - x_n^*)}{2\sqrt{\cdots}} \quad \frac{\partial (x_n - \cancel{x_n^*})}{\partial x_s} = \frac{x_s - x_s^*}{r} = \hat{r}_s$$

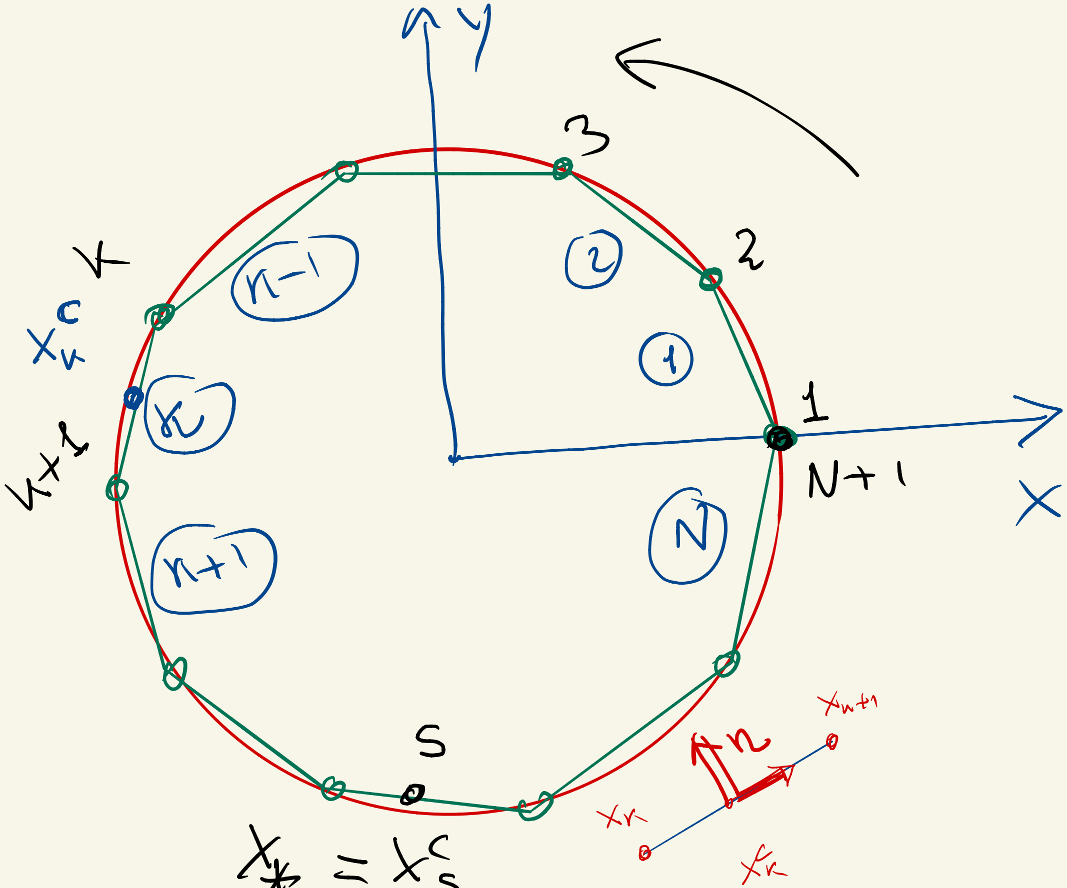
$$g\,; \quad \nabla g = \hat{\tau}\,\frac{\partial g}{\partial \tau} + \hat{n}\,\frac{\partial g}{\partial n}$$

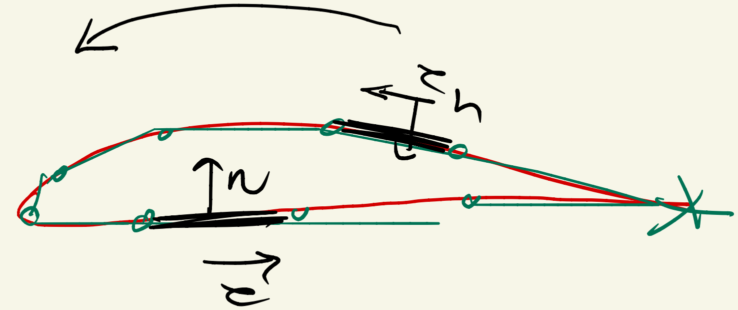
$$g\,; \quad \frac{\partial g}{\partial \tau}\,; \quad \frac{\partial g}{\partial n}$$

$$\boxed{g = \frac{1}{2\pi}\ln r}$$

$$\boxed{\frac{\partial g}{\partial \tau} = \frac{\hat{\tau}\cdot\hat{r}}{2\pi r}}$$

$$\boxed{\frac{\partial g}{\partial n} = \frac{\hat{n}\cdot\hat{r}}{2\pi r}}$$





$$x_{N+1} = x_1$$

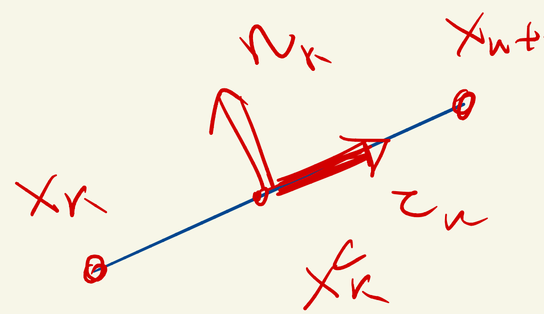
$$\mathbf{x}_k = (x_k, y_k) \quad k = 1, \ldots, N+1$$

$$x_* = x^c_s$$

$$k: \quad x_k, \; x_{k+1}$$

$$x^k_c = \frac{x_k + x_{k+1}}{2}$$

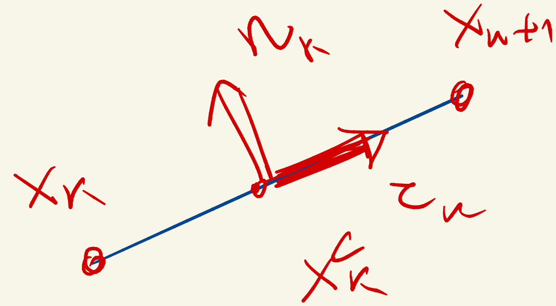
$$\tau_k = \frac{x_{k+1} - x_k}{|x_{k+1} - x_k|}$$



$$\hat{e}_y = \hat{e}_z \times \hat{e}_x \qquad \vec{J} = \vec{r} \times \vec{\tau}$$

$$\hat{n} = \hat{k} \times \hat{\tau} \qquad \hat{\tau} = \left(\tau_x \hat{e}_x + \tau_y \hat{e}_y\right)$$

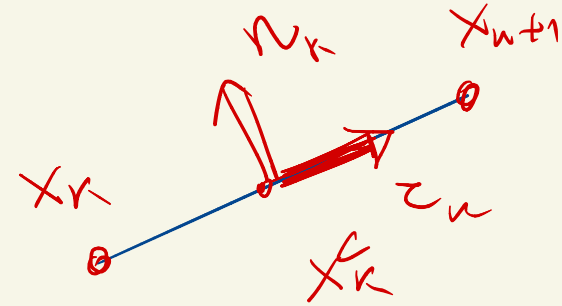
$$\hat{n} = \tau_x \underbrace{\hat{k} \times \hat{e}_x}_{\hat{e}_y} + \tau_y \underbrace{\hat{k} \times \hat{e}_y}_{-\hat{e}_x} = -\tau_y \hat{e}_x + \tau_x \hat{e}_y$$



$$\oint_{\partial B} \varphi \frac{\partial g}{\partial n} ds \simeq \sum_{k=1}^{N} \int_{P_k} \varphi(x) \frac{\partial g(x, x_a)}{\partial n_x} dl_x$$

$$\varphi(x)\Big|_{x \in P_k} \simeq \varphi(x_k^c) = \varphi_k$$

$$\oint_{\partial B} \varphi \frac{\partial g}{\partial n} \, ds \simeq \sum_{k=1}^{N} \varphi_k \int_{P_k} \frac{\partial g(x, x_\gamma)}{\partial n_x} \, dl_x$$



$$x_\gamma = x_s^c$$

x_γ

$$\frac{1}{2} \varphi_s - \sum_{k=1}^{N} \varphi_k \underbrace{\int_{P_k} \frac{\partial g}{\partial n}(x, x_s^c) \, dl_x}_{A_{sk}} =$$

$$= -\sum_k \left.\frac{\partial \varphi}{\partial n}\right|_k \underbrace{\int_{P_k} g(x, x_s^c) \, dl_s}_{B_{sk}}$$

$$\tfrac{1}{2}\varphi_s - \sum_{k=1}^{N} A_{sk}\varphi_k = -\sum_{n=1}^{N} B_{sn}\frac{\partial\varphi}{\partial n}\Big|_k$$

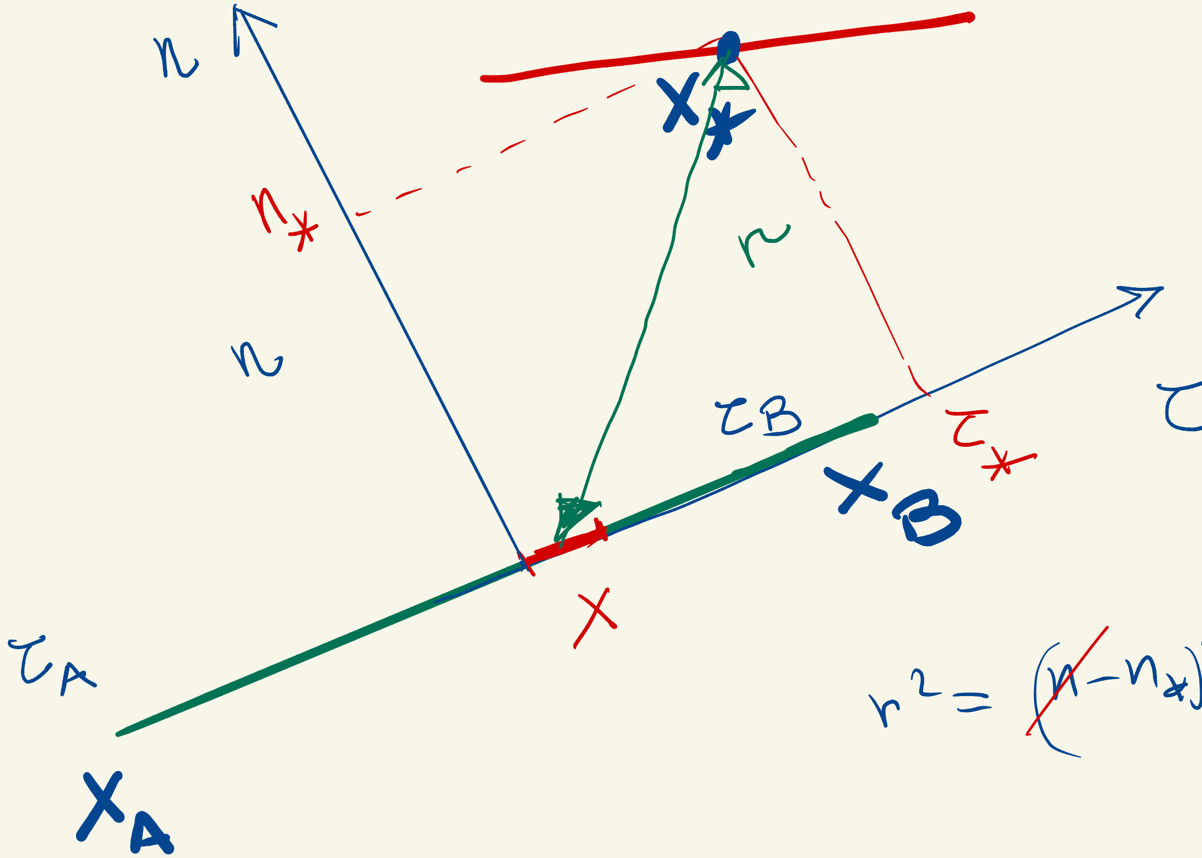
N equazioni in N incognite $\qquad s = 1, \dots, N$

$$M_{sk} = \tfrac{1}{2}\delta_{sk} - A_{sk}$$

$$t_s = -\sum_{n=1}^{N} B_{sn}\frac{\partial\varphi}{\partial n}\Big|_k$$

$$\sum_{k=1}^{N} M_{sk}\varphi_k = t_s \qquad M\{\varphi\} = \{t\}$$

$$\int_P \frac{\partial g}{\partial n}\, dl = \int_{-\tau_A}^{\tau_B} \frac{\partial g}{\partial n}\, d\tau$$



$$r^2 = (\cancel{n} - n_*)^2 + (\tau - \tau_*)^2$$

$$\frac{\partial g}{\partial n} = \frac{\hat{n} \cdot \hat{r}}{2\pi\, r} = \frac{\hat{n} \cdot \vec{r}}{2\pi\, r^2} = \frac{n - n_*}{2\pi\ n_*^2 + (\tau - \tau_*)^2} = - \frac{n_*}{2\pi\ n_*^2 + (\tau - \tau_*)^2}$$

$$-\frac{1}{2\pi}\int_{-L/2}^{L/2}\frac{n_*}{n_*^2+(z-z_*)^2}\,dz = -\frac{1}{2\pi}\,n_*^2\int_{-L/2\,\frac{1}{n_*}}^{L/2\,1/n_*}\frac{1}{n_*^2+(z-z_*)^2}\,d(z/n_*)$$

$$+\left(\frac{L}{2}-z_*\right)/n_* = \xi_B$$

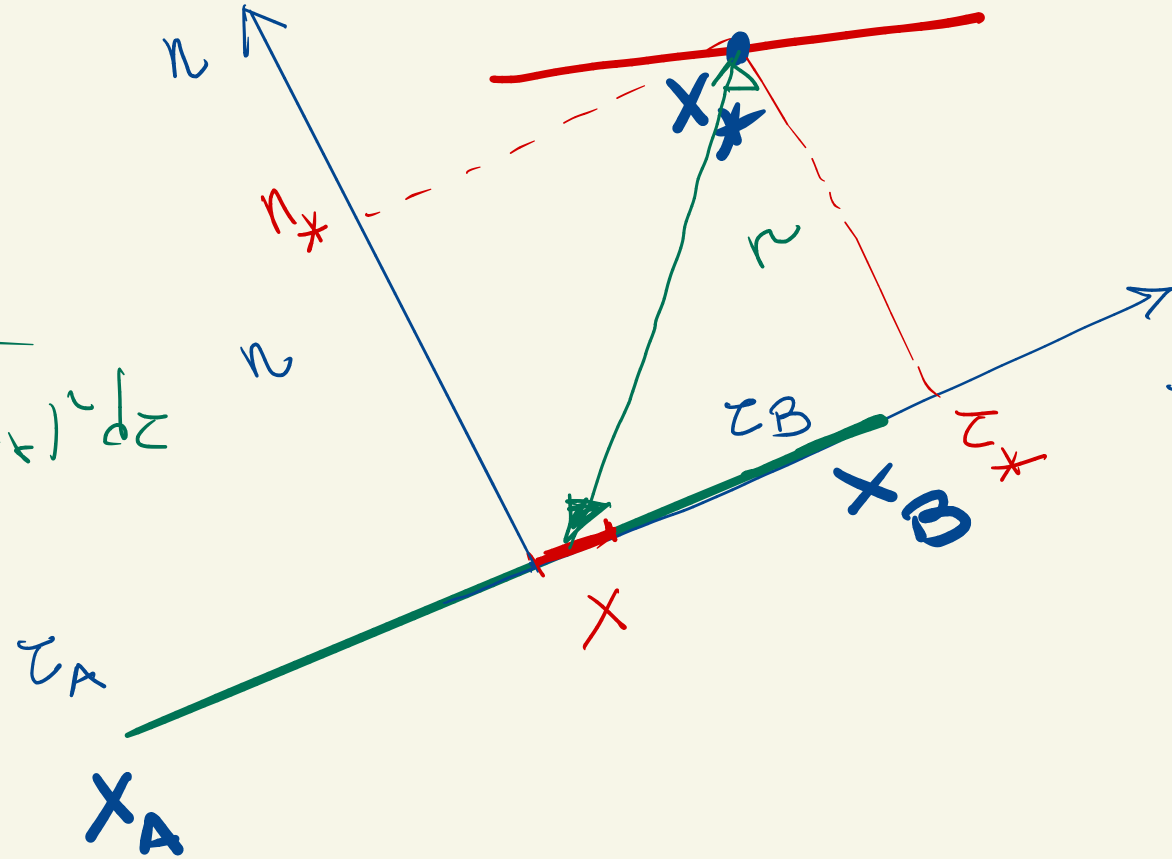
$$= -\frac{1}{2\pi}\frac{n_*^2}{n_*^2}\int\frac{1}{1+\left(\frac{z-z_*}{n_*}\right)^2}\,d\left(\frac{z-z_*}{n_*}\right) =$$

$$-\frac{L/2-z_*}{n_*} = \xi_A$$

$$= -\frac{1}{2\pi}\int_{\xi_A}^{\xi_B}\frac{1}{1+\xi^2}\,d\xi = -\frac{1}{2\pi}\tan^{-1}(\xi)\Big|_{\xi_A}^{\xi_B}$$

$$\int_B g(x, x_*)\, dl_x =$$

$$= \frac{1}{2\pi}\int_{\tau_A}^{\tau_B} \ln\sqrt{(\cancel{n}-n_*)^2+(\tau-\tau_*)^2}\, d\tau$$



$$\ln\sqrt{n_*^2+(\tau-\tau_*)^2} = \frac{1}{2}\ln\left[n_*^2+(\tau-\tau_*)^2\right] =$$

$$= 1/2\,\ln\left\{n_*^2\left[1+\left(\frac{\tau-\tau_*}{n_*}\right)^2\right]\right\} = \frac{1}{2}\ln n_*^2 + \frac{1}{2}\ln\left[1+\left(\frac{\tau-\tau_*}{n_*}\right)^2\right]$$

$$B = \frac{1}{4\pi}\int_{\tau_A}^{\tau_B}\left\{\ln n_*^2 + \ln\left[1+\left(\frac{\tau-\tau_*}{n_*}\right)^2\right]\right\} d\tau =$$

$$= \frac{1}{4\pi}\ln n_*^2\left(\tau_B-\tau_A\right) + \frac{n_*}{4\pi}\int_{\frac{\tau_A-\tau_*}{n_*}}^{\frac{\tau_B-\tau_*}{n_*}} \ln\left[1+\left(\frac{\tau-\tau_*}{n_*}\right)^2\right] d\,\frac{\tau-\tau_*}{n_*}$$

$$\xi = \frac{\tau-\tau_*}{n_*}$$

$$= \frac{1}{4\pi}\ln n_*^2\left(\tau_B-\tau_A\right) + \frac{n_*}{4\pi}\int_{\frac{\tau_A-\tau_*}{n_*}}^{\frac{\tau_B-\tau_*}{n_*}} \ln\left(1+\xi^2\right) d\xi$$

$$\int \ln(1+\xi^2)\, d\xi$$

$$\frac{d}{d\xi}\left[\xi \ln(1+\xi^2)\right] = \ln(1+\xi^2) + \frac{2\xi^2}{1+\xi^2} =$$

$$= \ln(1+\xi^2) + 2\left[\frac{1+\xi^2}{1+\xi^2} - \frac{1}{1+\xi^2}\right] =$$

$$= \ln(1+\xi^2) + 2\left(1 - \frac{1}{1+\xi^2}\right) =$$

$$= \ln(1+\xi^2) + 2\frac{d}{d\xi}\left(\xi - \tan^{-1}\xi\right)$$

$$\ln(1+\xi^2) = \frac{d}{d\xi}\left[\xi \ln(1+\xi^2) - 2\xi + 2\tan^{-1}\xi\right]$$

$$\int_{\frac{\tau_A-\tau_*}{n_*}}^{\frac{\tau_B-\tau_*}{n_*}} \ln(1+\xi^2) = \frac{n_*}{4\pi}\left[\xi \ln(1+\xi^2) - 2\xi + 2\tan^{-1}\xi\right]_{\frac{\tau_A-\tau_*}{n_*}}^{\frac{\tau_B-\tau_*}{n_*}}$$

$$B = \frac{1}{4\pi}(\tau_B-\tau_A)\ln n_*^2 +$$

$$\frac{n_*}{4\pi}\left\{\frac{\tau_B-\tau_*}{n_*}\ln\left[1+\left(\frac{\tau_B-\tau_*}{n_*}\right)^2\right] - 2\,\frac{\tau_B-\tau_*}{n_*} + 2\tan^{-1}\left(\frac{\tau_B-\tau_*}{n_*}\right)\right.$$

$$\left. - \frac{\tau_A-\tau_*}{n_*}\ln\left[1+\left(\frac{\tau_A-\tau_*}{n_*}\right)^2\right] - 2\,\frac{\tau_A-\tau_*}{n_*} + 2\tan^{-1}\left(\frac{\tau_A-\tau_*}{n_*}\right)\right\}$$

$$C = \int_{\tau_1}^{\tau_2} \frac{\partial \rho}{\partial \tau} d\tau = \rho \Big|_{\tau_1}^{\tau_2} =$$

$$= \frac{\ell}{4\pi} \left\{ \ln\left[n_x^2 + (\tau_2 - \tau_x)^2\right] - \ln\left[n_x^2 + (\tau_1 - \tau_x)^2\right] \right\}$$

$$\rho, \frac{\partial \rho}{\partial m}, \frac{\partial \rho}{\partial \tau} \longrightarrow \rho, \frac{\partial \rho}{\partial n}, \frac{\partial \rho}{\partial n_x}$$

$$\frac{\partial \rho}{\partial m_x} = -n_x \cdot \nabla \rho = -n_x \cdot \tau \frac{\partial \rho}{\partial \tau} - n_x \cdot n \frac{\partial \rho}{\partial n}$$

$$\tfrac{1}{2}\varphi_s - \sum_{k=1}^{N} A_{sk}\varphi_k = -\sum_{k=1}^{N} B_{sk}\left.\frac{\partial\varphi}{\partial n}\right|_k$$

$$\sum_{k=1}^{N} B_{sk}\left.\frac{\partial\psi}{\partial n}\right|_k = -\tfrac{1}{2}\psi_s + \sum_{k=1}^{N} A_{sk}\psi_k$$

ψ: $\quad \Psi = \Psi_\infty + \psi \qquad \Psi_\infty = U_\infty Y$

$$\left. U\cdot n\right|_{\partial B} = 0 \rightarrow \frac{\partial\Psi}{\partial\tau} = 0 \rightarrow \Psi = \text{cost} = \alpha$$

$$\left.\psi\right|_{\partial B} = \alpha - U_\infty Y$$

$$\sum_{n=1}^{N} B_{sk}\, U_\tau\big|_k = -\frac{1}{2}\left(\alpha - U_\infty Y_s\right) + \sum_{n=1}^{N} A_{sk}\left(\alpha - U_\infty Y_k\right)$$

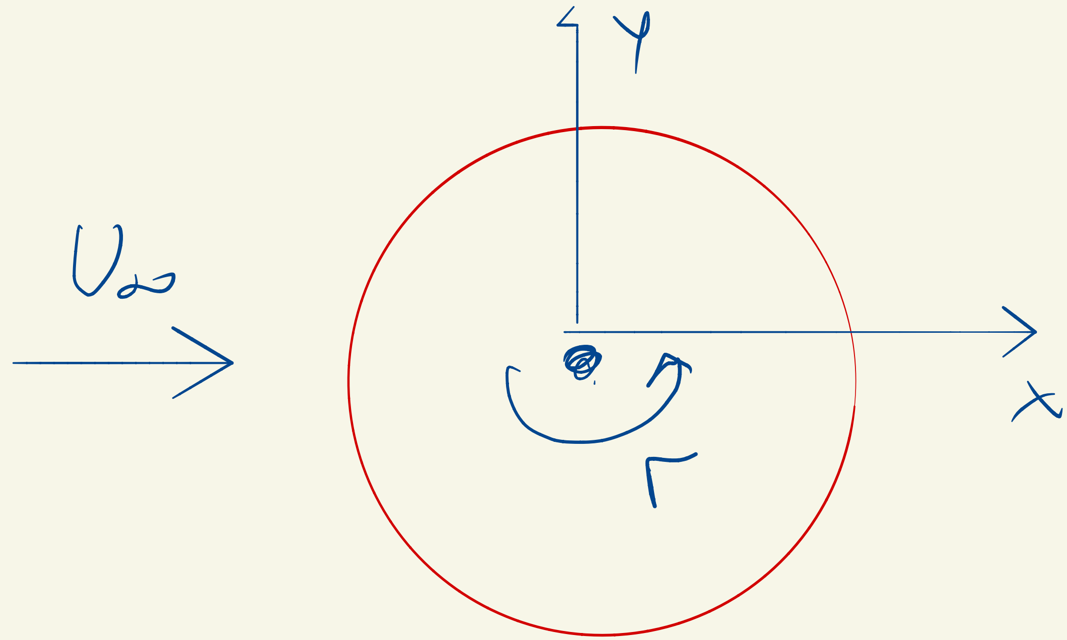
$$= \left(-1/2 + \sum_{k=1}^{N} A_{sk}\right)\alpha + U_\infty\left[1/2\, Y_s - \sum_{k=1}^{N} A_{sk} Y_k\right]$$

$$\boxed{\frac{\partial \psi}{\partial n} = U_\tau}$$

$$U_\tau\big|_k = U_\infty U_\tau^P\big|_k + \alpha\, U_\tau^H\big|_k$$

$$\sum_{k=1}^{N} B_{sk}\, U_\tau^P\big|_n = \left[1/2\, Y_s - \sum_{n=1}^{N} A_{sn} Y_n\right]$$

$$\sum_{n=1}^{N} B_{sn}\, U_\tau^H\big|_k = -1/2 + \sum_{n=1}^{N} A_{sk}$$



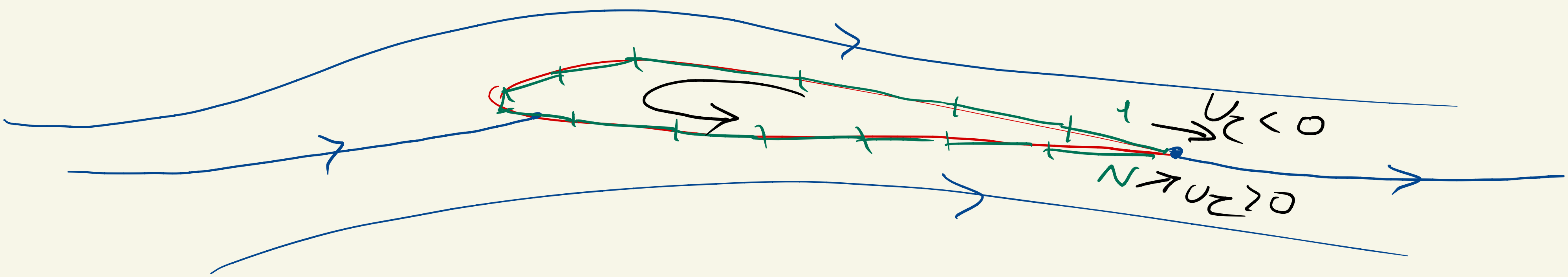
$$U_\tau = U_\infty u_\tau^P + \alpha u_\tau^H$$

$$\oint_{\partial B} u_\tau \, dl = \Gamma$$

$$\sum_{n=1}^{N} u_\tau\big|_n L_k = \Gamma$$

$$\sum_{n=1}^{N} U_\infty u_\tau^P\big|_n L_k + \alpha \sum_{n=1}^{N} u_\tau^H\big|_n = \Gamma$$

Condizione di Kutta



$$v_\tau|_1 = - v_\tau|_N \rightarrow$$

$$U_\infty v_\tau^p|_1 + \alpha v_\tau^H|_1 = - U_\infty v_\tau^w|_N + \alpha v_\tau^H|_N$$

$$\rightarrow \alpha !$$