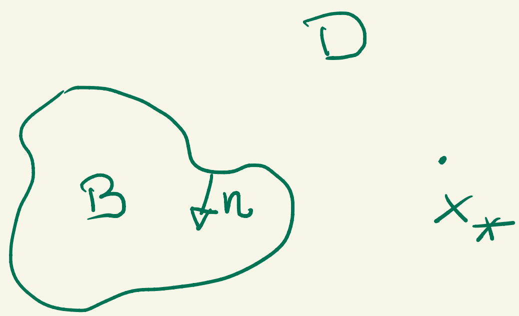
Lezione 17/03/2019

Inizio ore 14:15

$$\nabla^2 \varphi = 0 \qquad\qquad U = U_\infty + \nabla\varphi$$

$$\frac{\partial \varphi}{\partial n} = -U_\infty \cdot n$$



$$\left.\begin{array}{ll} x_* \in \mathring{D} & \varphi_* \\ x_* \in \partial B & \tfrac{1}{2}\varphi_* \\ x_* \in \mathring{B} & 0 \end{array}\right\} = \oint_{\partial B} \left(\varphi \frac{\partial G}{\partial n} - \frac{\partial \varphi}{\partial n} G \right) dS$$

$$\left.\begin{array}{ll} x_* \in \mathring{D} & 0 \\ x_* \in \partial B & \tfrac{1}{2}\varphi_{i*} \\ x_* \in \mathring{B} & \varphi_i^* \end{array}\right\} = -\oint_{\partial B} \left(\varphi_i \frac{\partial G}{\partial n} - \frac{\partial \varphi_i}{\partial n} G \right) dS$$

$$\oint_{\partial B} \left[\underbrace{\overbrace{(\varphi - \varphi_i)}^{\mu}}_{0} \frac{\partial g}{\partial n} - \underbrace{\overbrace{\left(\frac{\partial \varphi}{\partial n} - \frac{\partial \varphi_i}{\partial n}\right)}^{\sigma}}_{= 0} g \right] dS = \begin{cases} \varphi_* & x_* \in \mathring{D} \\ \frac{1}{2}(\varphi_* - \varphi_{i*}) & x_* \in \partial B \\ \varphi_{*i} & x_* \in \mathring{B} \end{cases}$$

(diretti)

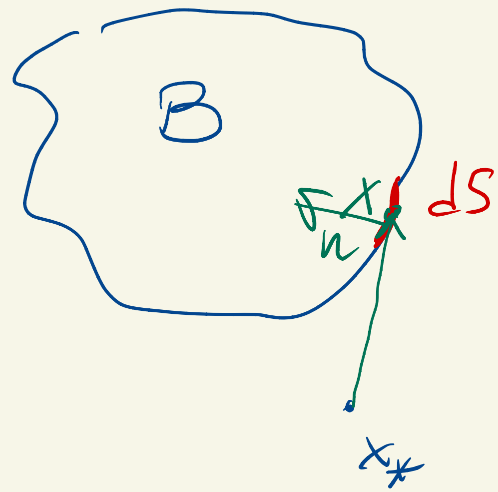
$$\varphi_* = \oint_{\partial B} \mu \frac{\partial g}{\partial n} dS$$ (doppio strato)

$$c\varphi_* = \oint_{\partial B} \left(\varphi \frac{\partial g}{\partial n} - g \frac{\partial \varphi}{\partial n} \right) dS$$

$$\varphi_* = \oint_{\partial B} \sigma g \, dS$$ (semplice strato)

$$0 \equiv \int_{D_i} \nabla^2 \varphi \, dV = \oint_{\partial D_i} \frac{\partial \varphi}{\partial n} \, dS \Longleftarrow \qquad \nabla^2 \varphi = 0$$

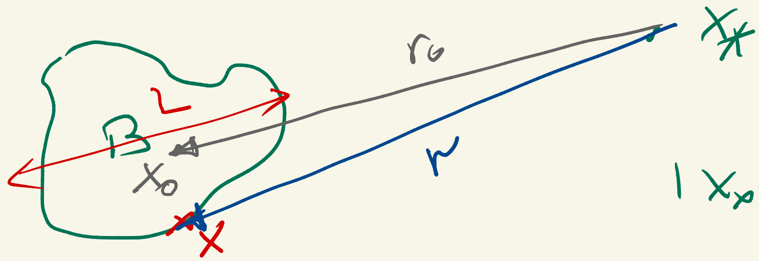
$$g = -\frac{1}{4\pi} \frac{1}{\underbrace{|x - x_*|}_{r}}$$



$$\frac{\partial g}{\partial n} = n \cdot \nabla g$$

$$\nabla g = \frac{dg}{dr} \nabla r = \frac{1}{4\pi} \frac{1}{r^2} \hat{r} \Longrightarrow \frac{\partial g}{\partial n} = \frac{1}{4\pi} \frac{\hat{r} \cdot n}{r^2}$$

$$\partial g / \partial n = O(1/r^2)$$



$|x_*| \to \infty \qquad r \simeq r_0$

$$\frac{\partial G}{\partial n} = O\left(1/r^2\right) \simeq O\left(1/r_0^2\right)$$

$$\boxed{\varphi_* = \oint_{\partial B} \mu \frac{\partial G}{\partial n}\, dS \sim \left(\int \mu\, dS\right) \frac{1}{r_0^2}}$$

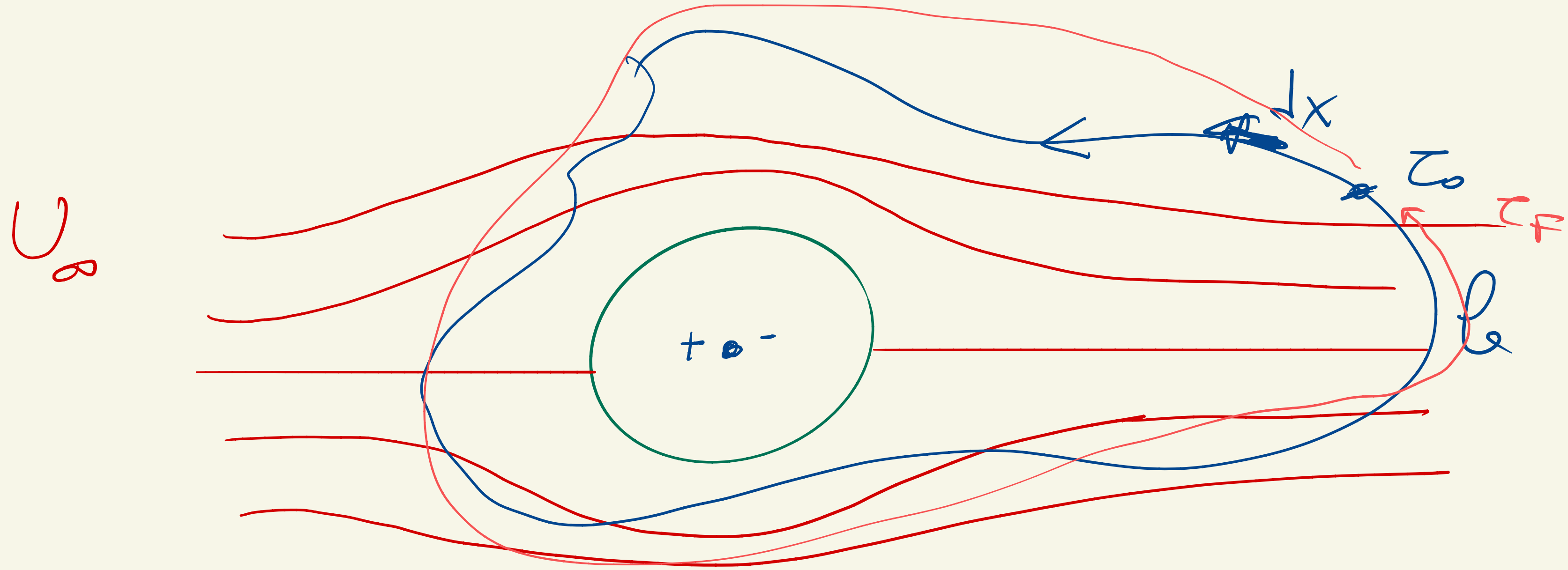
$$\varphi_* = \oint_{\partial B} \Big(\underbrace{\varphi \frac{\partial g}{\partial n}}_{O(1/r_0^2)} - \underbrace{g \frac{\partial \varphi}{\partial n}}_{O\left(\frac{1}{r_0}\right) \oint_{\partial B} \frac{\partial \varphi}{\partial n} dS} \Big) dS \qquad g = -\frac{1}{4\pi}\,\frac{1}{r}$$

$$\varphi_* \sim \oint_{\partial B} \varphi\, dS$$

$$\varphi_0 = 1/2\, \varphi_0 + \oint_{\partial B} \varphi \frac{\partial g}{\partial n} dS$$

Ammette alcuno sol. se $\oint \frac{\partial \varphi}{\partial n} dS = 0$

$\boxed{\text{In caso discutiamo la cosa più approfonditamente}}$



$$\varphi \to V = \nabla\varphi$$

$$\oint_{\mathcal{C}} \nabla\varphi \cdot dx = \oint_{\mathcal{C}} \frac{\partial\varphi}{\partial\tau}\, d\tau =$$

$$= \int_{\tau_0}^{\tau_F} \frac{\partial\varphi}{\partial\tau}\, d\tau = \varphi(\tau_F) - \varphi(\tau_0) \equiv 0$$

$$\Gamma = \oint_{\mathcal{C}} V \cdot dx = \oint_{\mathcal{C}} \frac{\partial\varphi}{\partial\tau}\, d\tau \equiv 0$$

2D: $\nabla \cdot v = 0$

$$\nabla \times v = 0 \rightarrow v = \nabla \varphi, \quad \nabla \times v = \nabla \times \nabla \varphi \equiv 0 \rightarrow$$

$$\rightarrow \nabla \cdot v = \nabla \cdot \nabla \varphi = \nabla^2 \varphi = 0$$

$$\nabla \times v = 0$$

$$\nabla \cdot v = 0 \rightarrow v = \hat{k} \times \nabla \psi$$

$\uparrow$ Funzione di corrente

$\hat{i}, \hat{j}, \hat{k}$

$$v = \hat{k} \times \left(\hat{i} \frac{\partial \psi}{\partial x} + \hat{j} \frac{\partial \psi}{\partial y} \right) = \hat{i} \frac{\partial \psi}{\partial y} - \hat{j} \frac{\partial \psi}{\partial x}$$

$$\nabla \cdot v = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} \right) \cdot \left(\hat{i} \frac{\partial \psi}{\partial y} - \hat{j} \frac{\partial \psi}{\partial x} \right) = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} \equiv 0$$

$$\nabla \times V = 0 = \nabla \times (\hat{k} \times \nabla\psi) = \hat{k}\, \nabla^2\psi - \hat{k} \cdot \nabla\, \nabla\psi$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, 0\right) \qquad \hat{k} \cdot \nabla = 0$$

$$\nabla \times V = 0 \rightarrow \nabla^2\psi = 0$$

$$V = \hat{k} \times \nabla\psi \qquad V \cdot \hat{n} = \hat{n} \cdot \hat{k} \times \nabla\psi$$

$$\nabla^2\psi = 0 \qquad \nabla\psi = \hat{\tau}\frac{\partial\psi}{\partial\tau} + \hat{n}\frac{\partial\psi}{\partial n}$$

$$V \cdot \hat{n} = \hat{k} \cdot \nabla\psi \times \hat{n} = \hat{k} \cdot \underbrace{\hat{\tau} \times \hat{n}}_{\hat{k}} \frac{\partial\psi}{\partial\tau} + \cancel{\hat{k} \cdot \hat{n} \times \hat{n} \frac{\partial\psi}{\partial n}}$$

$$v \cdot \hat{n} = \frac{\partial \psi}{\partial \tau} \qquad U = U_\infty + v$$

$$U \cdot \hat{n} = 0 \qquad v \cdot \hat{n} = -U_\infty \cdot n = \frac{\partial \psi}{\partial \tau} \Longleftarrow$$

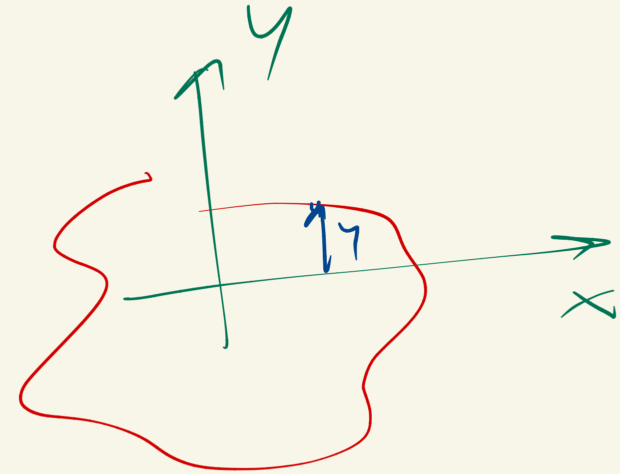
$$\psi = \psi_0 + \int_{\tau_0}^{\tau} \frac{\partial \psi}{\partial \tau} d\tau = \psi_0 + \int_{\tau_0}^{\tau} -U_\infty \cdot n \, d\tau = \psi(\tau)$$

$$\psi_\infty \to U_\infty \qquad \psi_\infty = U_\infty y \qquad \frac{\partial \psi_\infty}{\partial y} = U_\infty$$

$$\Psi_T = \psi_\infty + \psi \to U \cdot \hat{n} = \frac{\partial \Psi_T}{\partial \tau} = 0$$

$$\psi_T = \text{cnst} = \Psi_0 \to \psi_\infty + \psi = \Psi_0$$

$$\psi = \Psi_0 - \psi_\infty = \Psi_0 - U_\infty y$$



$$\nabla^2 \psi = 0$$

$$\psi\big|_{\partial B} = -U_\infty y + \text{cost}$$

$$\psi_* = \oint_{\partial B} \left(\psi \frac{\partial g}{\partial n} - g \frac{\partial \psi}{\partial n} \right) ds \rightarrow$$

$$\underbrace{\tfrac{1}{2}\psi_0 - \oint_{\partial B} \psi \frac{\partial g}{\partial n} ds}_{\text{Dati}} = \underbrace{- \oint_{\partial B} g \frac{\partial \psi}{\partial n} ds}_{\partial\psi/\partial n \text{ é l'incognita}}$$

$$V \cdot \hat{n} = \frac{\partial \psi}{\partial \tau} \quad \rightarrow \quad V \cdot \hat{\tau} = -\frac{\partial \psi}{\partial n}$$

$$\nabla^2 \varphi = 0 \qquad \frac{\partial \varphi}{\partial n} = -U_\infty \cdot n$$

$$\frac{1}{2}\varphi_* - \oint_{\partial B} \varphi \frac{\partial g}{\partial n} dS = -\oint_{\partial B} \frac{\partial \varphi}{\partial n} g \, dS$$

$$\frac{1}{2}\sigma^* - \oint_{\partial B} \sigma \frac{\partial g}{\partial n^*} dS = \left.\frac{\partial \varphi}{\partial n}\right|_*$$

$$\left(\varphi_* = \oint_{\partial B} \sigma g \, dS\right)$$

$$\nabla^2 \psi = 0 \qquad \psi = -U_\infty y + G$$

$$\oint_{\partial B} \left(-\frac{\partial \psi}{\partial n}\right) g \, dS = \frac{1}{2}\psi_* \overset{+C}{} - \oint_{\partial B} \overset{C+}{} \psi \frac{\partial g}{\partial n}$$

$$\int \sigma g \, dS = \psi_* + G$$

$$\forall G \leftarrow \sigma$$

Elettrostatica:

Problem interno

$$\varphi_{x_i} = \oint_{\partial B} \sigma g \, dS \Rightarrow$$

$$\frac{\partial \varphi_*}{\partial n} = \frac{1}{2}\sigma_* \,?\, + \oint_{\partial B} \sigma \frac{\partial g}{\partial n_*} \equiv 0 \qquad \varphi^{(i)}_*\Big|_{\partial B} = \oint_{\partial B} \sigma g \, dS = C$$

$$\varphi^{(e)}_* = \oint_{\partial B} \sigma g \, dS \longrightarrow \frac{\partial \varphi}{\partial n_e}\Big|_b = \frac{1}{2}\sigma_* - \oint_{\partial B} \sigma \frac{\partial g}{\partial n_*} dS$$

$$\frac{\partial \psi}{\partial n} = v_\tau \longrightarrow v_\tau = v_\tau^{(p)} + \alpha v_\tau'$$

$$C + \psi_* = \oint_{\partial B} \sigma g \, ds \rightarrow \frac{\partial \psi_*}{\partial n} = 1/2 \, \sigma_* - \oint_{\partial B} \sigma \frac{\partial g}{\partial n} ds$$

$$\boxed{\sigma = \sigma_p + C \sigma_H}$$

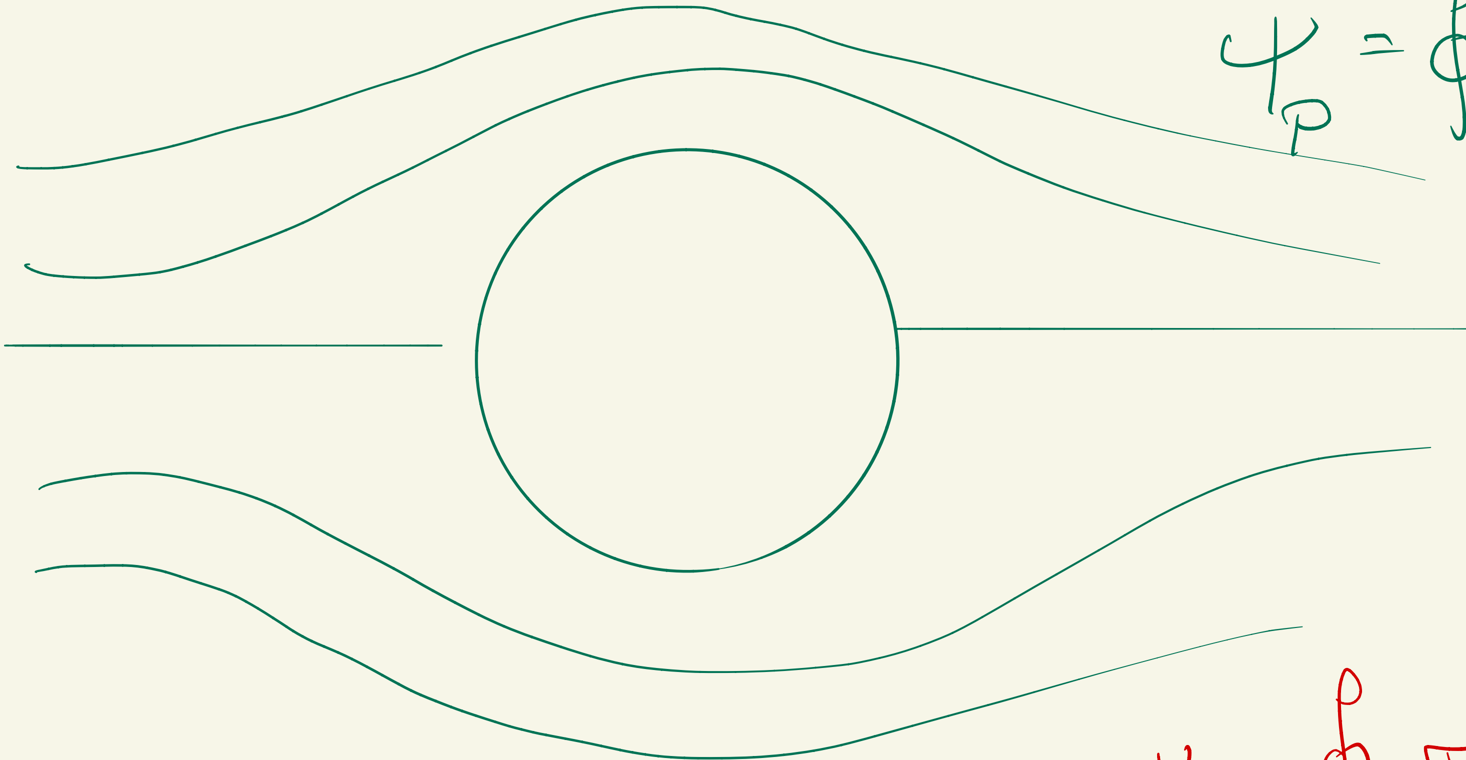
$\sigma_p \leftarrow C = 0 \quad \psi_* = \text{dado}$

$\sigma_H \leftarrow C = 1 \quad \psi_* = 0$

$$\frac{\partial \psi_*}{\partial n} = 1/2 \, \sigma_p^* - \oint_{\partial B} \sigma_p \frac{\partial g}{\partial n} ds + C \left[1/2 \, \sigma_H^* - \oint_{\partial B} \sigma_H \frac{\partial g}{\partial n} ds \right]$$

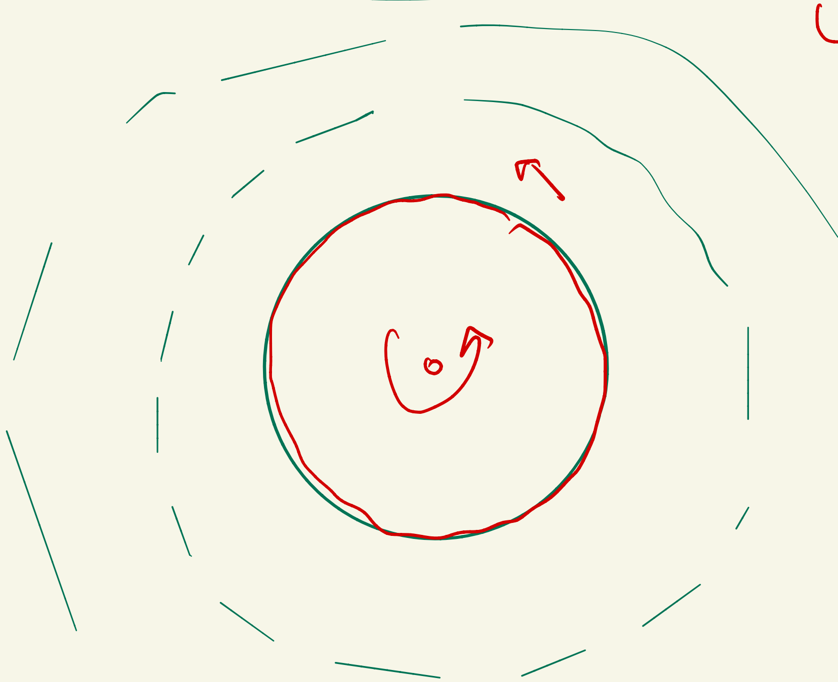
$$\hookrightarrow U_z^* = -U_z^{P*} - C \, U_z^{H*} \rightarrow \boxed{U_z = U_z^P + C \, U_z^H}$$

$$\psi_p = \oint_{\partial B} \sigma_p g \, dS$$



$$\psi_H = \oint_{\partial B} \sigma_H g \, dS = ch$$

$$\Gamma = \Gamma_0$$



ψ_{tP}

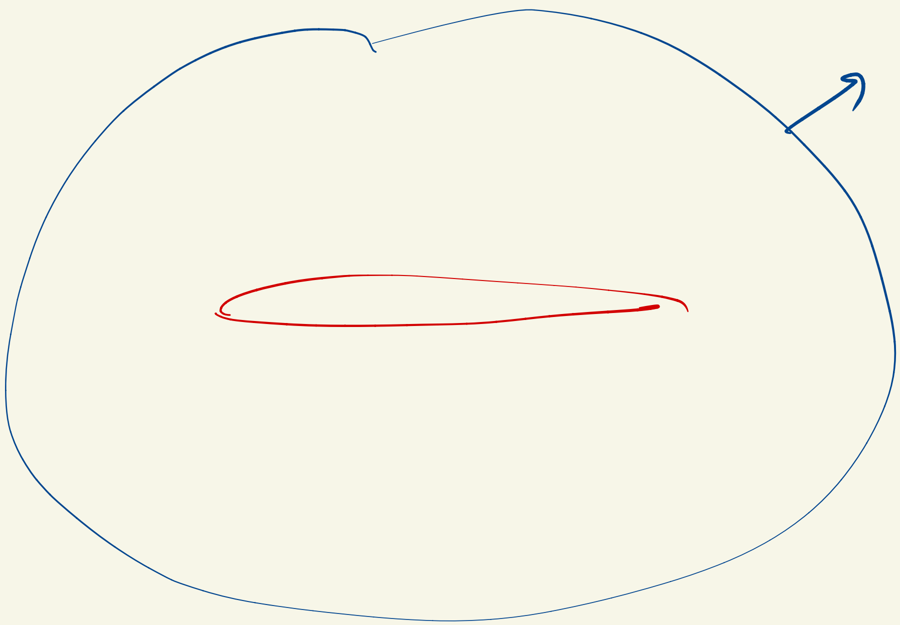
ψ_H

$\psi'_P + C\,\psi_H$

2D:

$$\psi_* = \oint_{\partial B} \eta \frac{\partial \varrho}{\partial n} \, dS \rightarrow \psi_* \sim 1/r_0 \rightarrow v \sim 1/r_0^2$$

$$v \sim 1/r_0 \rightarrow \Gamma \neq 0 = \oint_{C} v_\tau \, d\tau \sim O(1/r_0)$$



$$g_{3D} = -\frac{1}{4\pi}\frac{1}{|x-x_*|} \longrightarrow \nabla g_{3D} \sim O\left(1/r_0^2\right)$$

$$g_{2D} = 2\pi \ln |x-x_*| \longrightarrow \nabla g_{2D} \sim O\left(1/r_0\right)$$