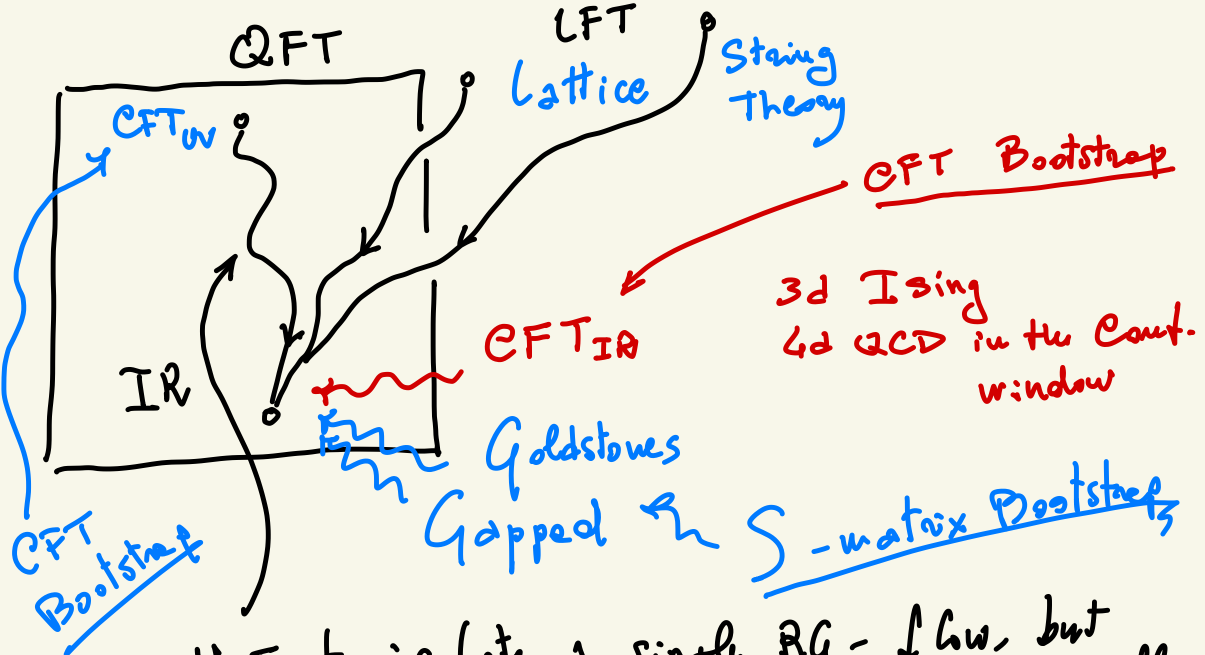
Quantum Gravity S-matrix Bootstrap!

arXiv: 2102.02847

arXiv: 2212.00151

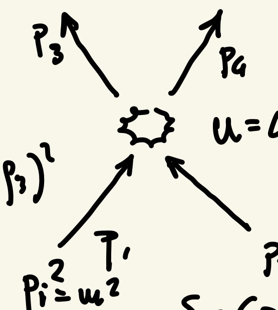
→ maybe more with

Armanini, Häring, Zhiboedov



NOT to isolate a single RG-flow, but constrain the space of RG flows

Example: 1 gapped scalar with mass m



$u = 4m^2 - s - t$

$t = (p_1 - p_3)^2$

$s = (p_1 + p_2)^2$

$\lambda\phi^4 , \; c_2 \partial^4\phi^4 , \ldots \qquad T =$

how do I define them non-pert.

on-shell renorm.

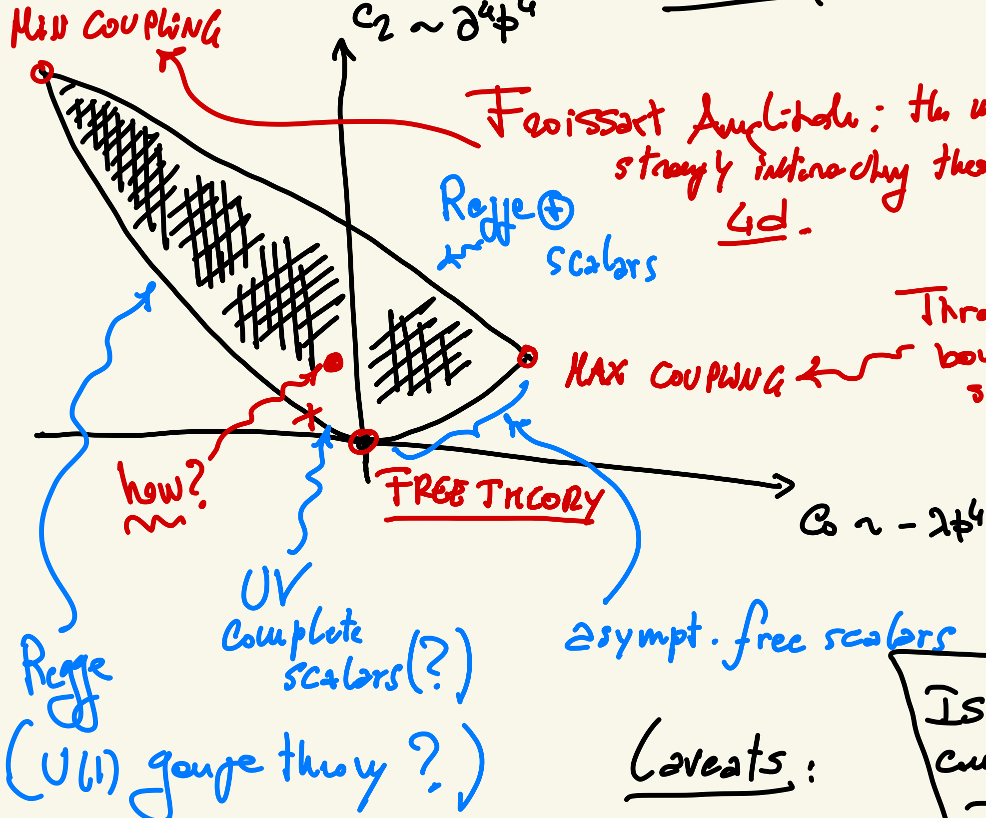
$$\lambda = T(s = t = 4/3\, m^2)$$

$$c_2 = \frac{1}{4} \frac{\partial^2 T}{\partial s^2}(s = t = 4/3\, m^2) \geq 0$$

$$T^{\text{pert. theory}} = \lambda + \lambda^2 \times LOG(s,t) + \ldots$$

what was in $\lambda\phi^4$?

Abrakadabra Bootstrap



Is list complete? Don't know

Caveats:

- Projection of an ∞ dim. space!
- Optimal?

$Prob_{2\to2} \leq 1$

$Prob_{2\to2} + Prob_{2\to4} + Prob_{2\to6} + \dots = 1$

true prob. conservation

Is the cut fully consistent?

Apply this to gravity.

$\underline{D \geq 9}$, $\underline{\text{MAX. SUGRA}}$

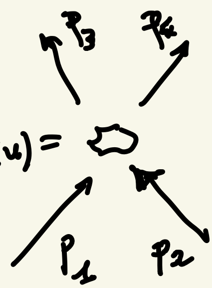
(it restricts the space)

not crucial (kinematics)

we could push to $D = 5$

"D = 4 ?

" don't know, but not just us, nobody "

$T(s,t,u) =$ 

$p_i^2 = 0$

supergravitons

$T(s,t,u)$ obeys causality, unitarity, crossing

space of functions...

gravity?

s, t small in units of G_N

$$A_e = \int dt \int d^{D-1}x \sqrt{-g} \left[R + 0R^2 + 0R^3 + \alpha R^4 + DR^{4,5} + \dots \right]$$

first correction

No-Go Result: $\alpha > 0$

non-perturbatively

$$\mathbb{A}(s,t,u) = \underbrace{\mathbb{R}^4}_{\text{Kinematics}} \boxed{A(s,t,u)}$$

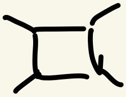
dynamics

$$T_{2\,dil \to 2\,dil}(s,t,u) = \left(s^4+t^4+u^4\right) A(s,t,u)$$

$$T_{2\,axid \to 2\,axid}(s,t,u) = s^4 \underbrace{A(s,t,u)}$$

Not crossing symm.

$= 64\pi^7$ $d=10$

$$A(s,t,u) = \frac{8\pi G_N}{stu} + \alpha + $$  $$+ \cdots$$

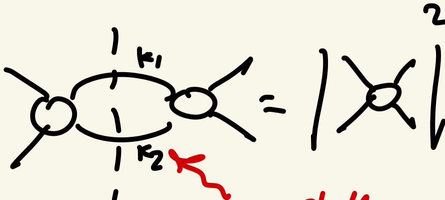
(positive transcendental)

crossing!

Unitarity!

$+ \cdots$

$$2\,\mathrm{disc}_s \mathbb{A}_{ab\to cd} = \sum_X \int d\ell_{IPS}\; \mathbb{A}_{ab\to X} \times \mathbb{A}^*_{cd\to X}$$



on-shell

$s \geq 0$
$(+i\epsilon)$

$$2\, \cancel{\mathbb{R}^4_{ab\to cd}}\, \mathrm{disc}_s A = \int d\ell_{IPS} \sum_X \underbrace{R^4_{ab\to X}\, R^4_{cd\to X}}_{A \times A^*}$$

s^4

duplication formula

$$T = \underset{\sim}{s^4 A}$$

$$\underline{\underline{s^4}}\ \cancel{\mathbb{R}^4_{ab\to cd}} \qquad \underline{\underline{s^4}}$$

$$2\,\mathrm{disc}\,T = \int d\ell_{IPS}\, T \times T^*$$ Unitarity

diagonalize in "effective spin"

?

review a bit of SUGRA

$D=10$ $H_{\mu\nu}$, $C^{[\mu\nu\rho]}$

IIA ϕ dilaton totally

G antisymm.

IIB axidilaton $|NS \times NS|$

$D=11$ graviton $H_{\mu\nu}$

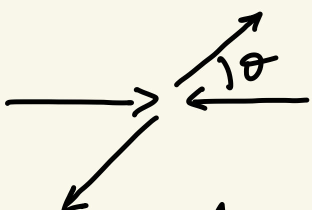
11D $C^{[\mu\nu\rho]}$

(*) how does axi-dilaton avoids the No Global symmetries in QG?

$$S_\ell(s) = 1 + i\, s^{\frac{D-4}{2}} f_\ell(s) =$$

$$= 1 + i\, s^{\frac{D-4}{2}} N_d \int_{-1}^{1} d\cos\theta\, \frac{C_\ell^{(D-3/2)}(\cos\theta)}{C_\ell^{(D-3/2)}(1)} \times$$

$$\times\, T\left(s, t = -\tfrac{s}{2}(1-\cos\theta)\right)$$



$$|S_\ell(s)|^2 \leq 1$$ prob. conservation

$$\forall \ell = 0 \ldots \infty \quad \forall s \geq 0$$

Causality + unit. $\Rightarrow$ analyticity + poly. boundedness

even in Gravity!

Regge boundedness

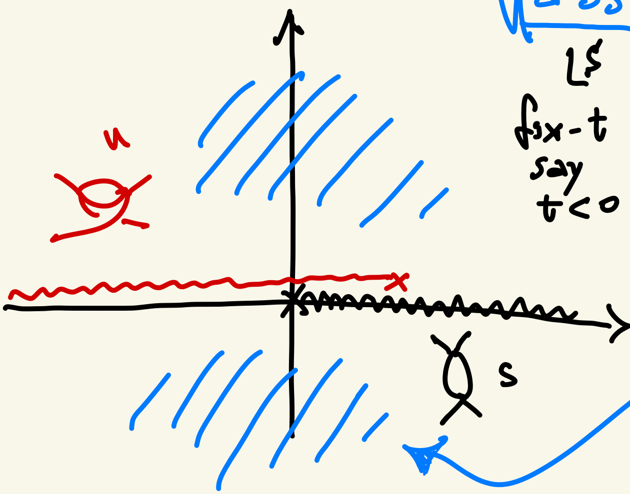
$$\lim_{s\to\infty} \frac{\mathcal{T}(s, t\leq 0)}{|s|^2} = 0$$

fix-t

assumption [no reason to believe is incorrect!]

fix-t
say
$t<0$

$u = -s - t$



analytic outside

Convention $16\pi G_N = (2\pi)^{D-3} \ell_p^{D-2}$

Bootstrap

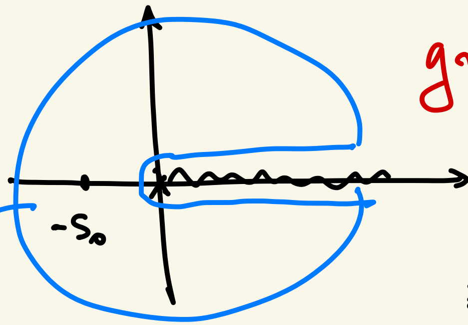
$$A = \sum_{a+b+c\le N} \alpha_{abc}\,\rho_s^a\,\rho_t^b\,\rho_u^c + \frac{8\pi G_N}{stu}$$

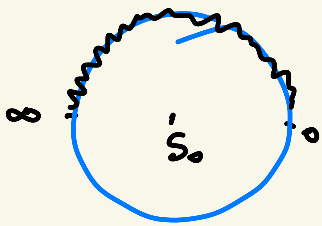
any UV completion

when $N\to\infty$ spans any analytic function

gravity

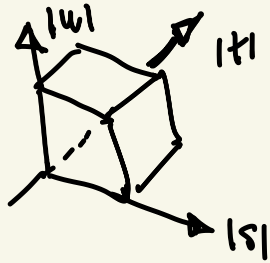
$$\rho_s = \frac{\sqrt{-s_0}-\sqrt{-s}}{\sqrt{-s_0}+\sqrt{-s}}$$





polydisc

crossing, analytic and poly-bounded



Unitarity numerically

$\ell = 0, 2, 4, \dots, L$

$s \in$ grid ≥ 0

What is α?

$$\alpha = \sum_{a+b+c\le N} \alpha_{abc}$$

max/min α subject to constraints

$$\left[\lim_{N\to\infty}\lim_{L\to\infty}\right]\cancel{\lim_{S_{\text{grid}}\to\infty}} \rightsquigarrow \text{triple extrapolation!}$$

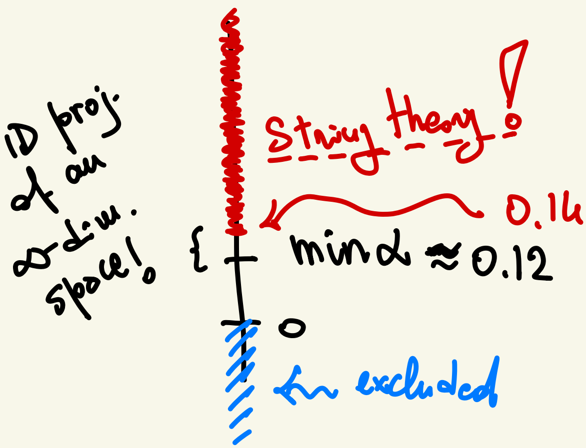
many terms

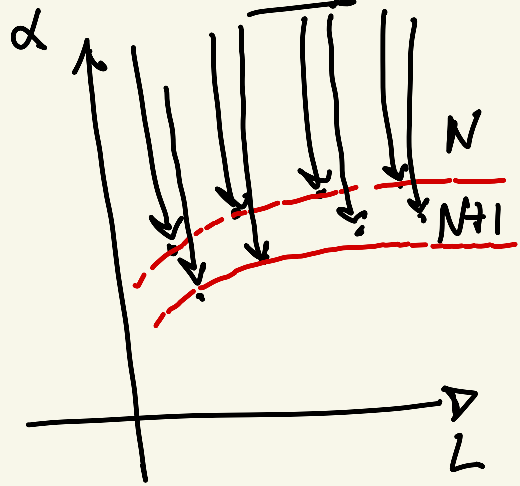
we go to $L \sim 200$ + positivity in the sky

$$\max \alpha = \infty$$

$$\min \alpha = \#$$

in $D = 9, 10, 11$

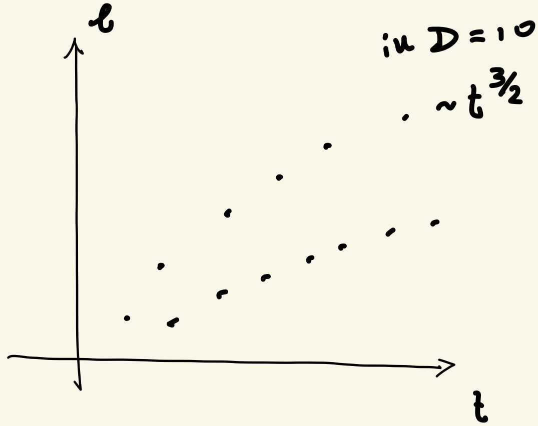




$$\alpha = \frac{2}{\pi}\int_0^\infty \frac{\operatorname{Im} A(s,t=0)}{s}\,ds \geq 0$$

optical theorem

Extremal Amplitude is stringy

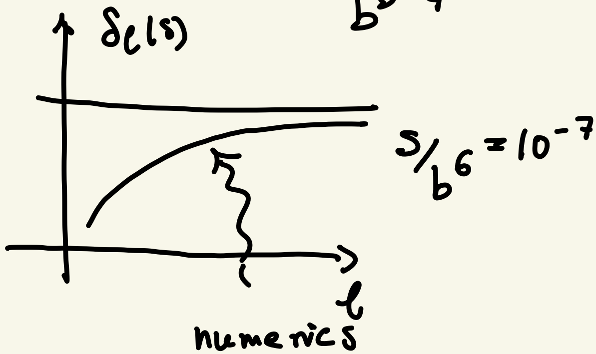


Quantum Regge growth

Clustering

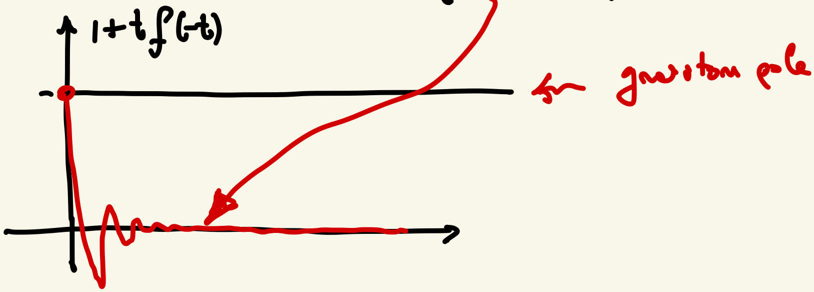
$$\sim \frac{G_N s}{b^{D-4}} \qquad b \to \infty \qquad b \sim s^{1/6}$$

GR dominates



$$\lim_{s\to\infty} \frac{T(s,t)}{|s|^2} = 0 \qquad T^{\text{bot}} = s^2\left(\frac{1}{t} + f(-t)\right)$$

$= 0$ for any t



ST Expectations

$d=11.$ $\quad \alpha = \frac{(2\pi)^2}{3\cdot 2^7} \simeq 0.1028$

$d=10.$ $\quad \alpha^{IIB} = \frac{1}{2^6} E_{3/2}(\tau,\bar\tau) = \frac{1}{2^6} \sum_{\substack{n,m\in\mathbb{Z} \\ \neq(0,0)}} \frac{(\mathrm{Im}\,\tau)^{3/2}}{|m\tau+n|^3}$ $\gtrsim 0.1389$

$$\alpha^{IIA} = \frac{\zeta_3}{32 g_s^{3/2}} + g_s^{1/2}\frac{\pi^2}{96} \gtrsim 0.1403$$

$d=9$ $\quad \alpha = \frac{1}{2^6}\left[\nu^{-3/7} E_{3/2}(\tau,\bar\tau) + \frac{2\pi^2}{3}\nu^{4/7}\right]$

$\gtrsim 0.2417$

Bootstrap.	
	0.223 ± 0.002
	0.124 ± 0.003
	0.101 ± 0.008

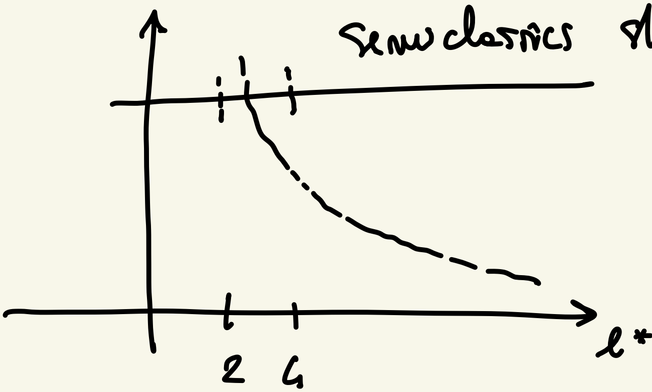
BH production

$$\alpha = \frac{16}{8\pi^4 \ell_p^{16}} \sum_\ell (\ell+1)_6 (2\ell+7) \int_0^\infty ds \, \frac{1 - \mathrm{Re}\, S_\ell(s)}{s^8}$$

$$\alpha \gtrsim \frac{16}{3\pi^4 \ell_p^{16}} (\ell+1)_6 \, (2\ell+7) \int_{s_*(\ell)}^\infty \frac{ds}{s^8}$$

$b = 2\ell/\sqrt{s}$ $\qquad R_s^7 = \frac{105\,\pi^3 \ell_p^8}{2}\sqrt{s}$

$\underline{b = R_s} \longrightarrow \qquad s_*(\ell) \approx \frac{1}{\ell_p^2}\left(\frac{2^8 \ell^7}{105\pi^3}\right)^{1/4}$



semiclassics should break around $\ell = 2, 4$!