Motorcycles

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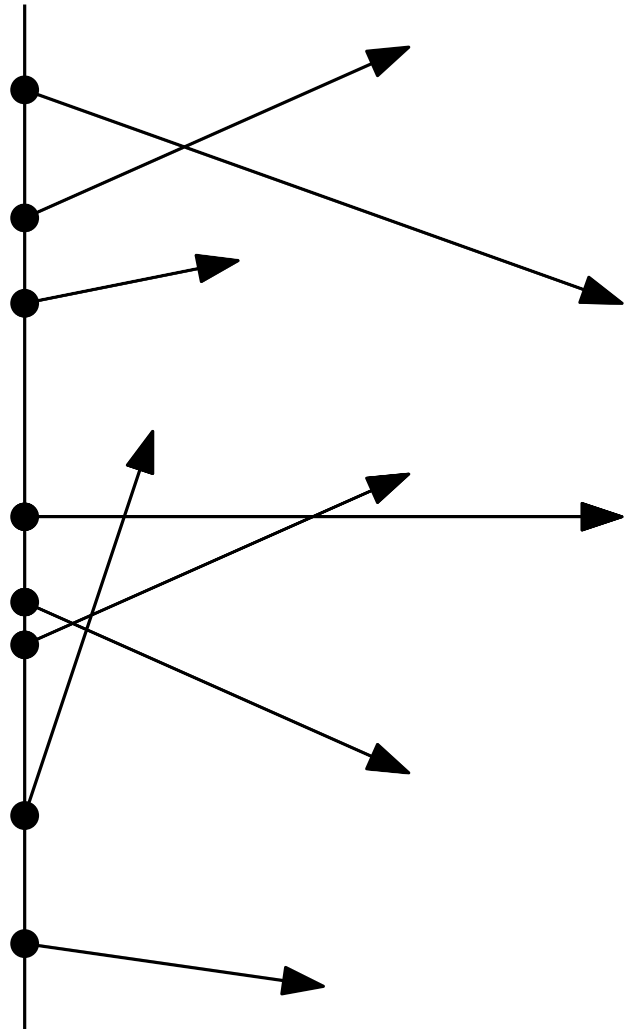

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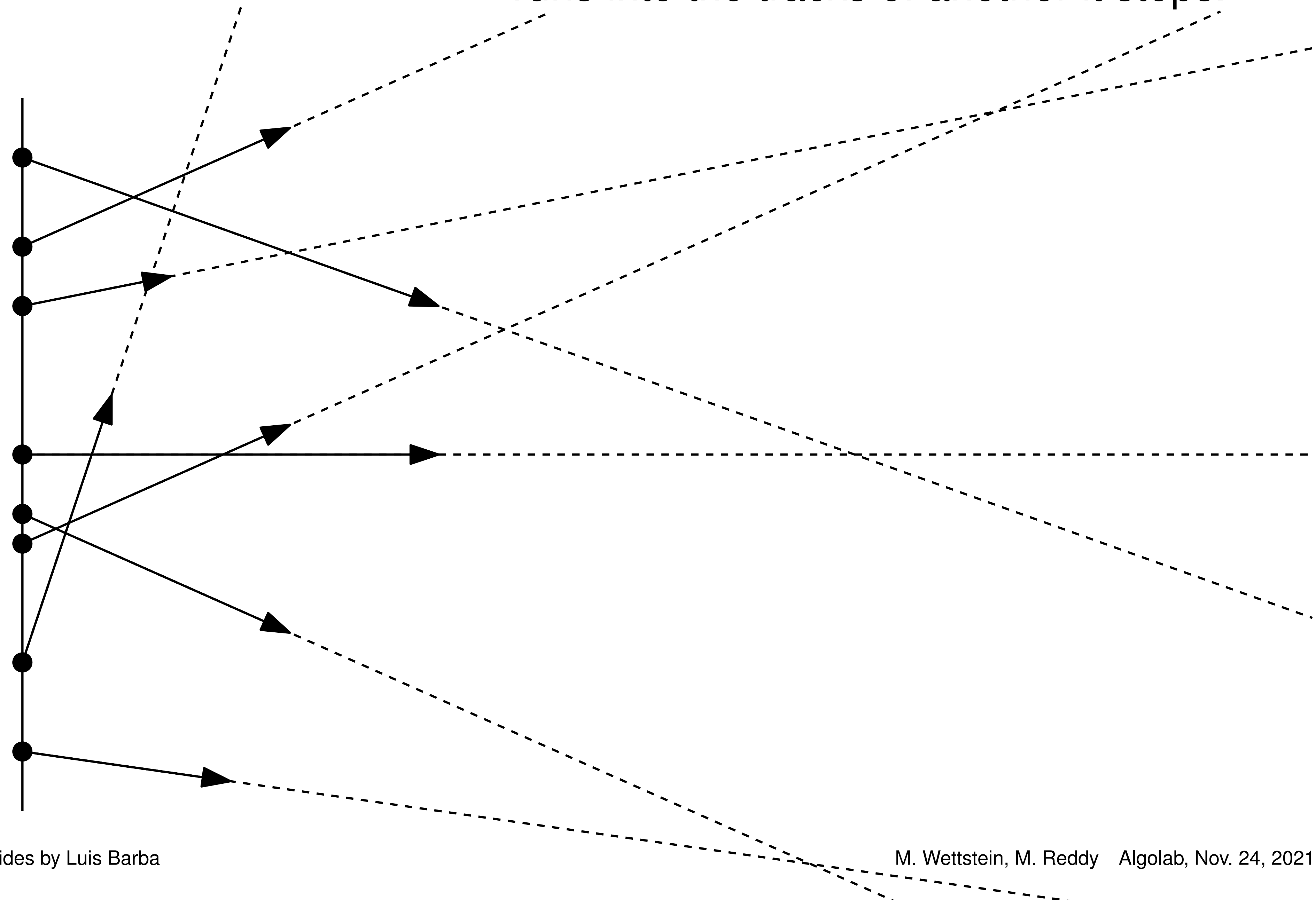

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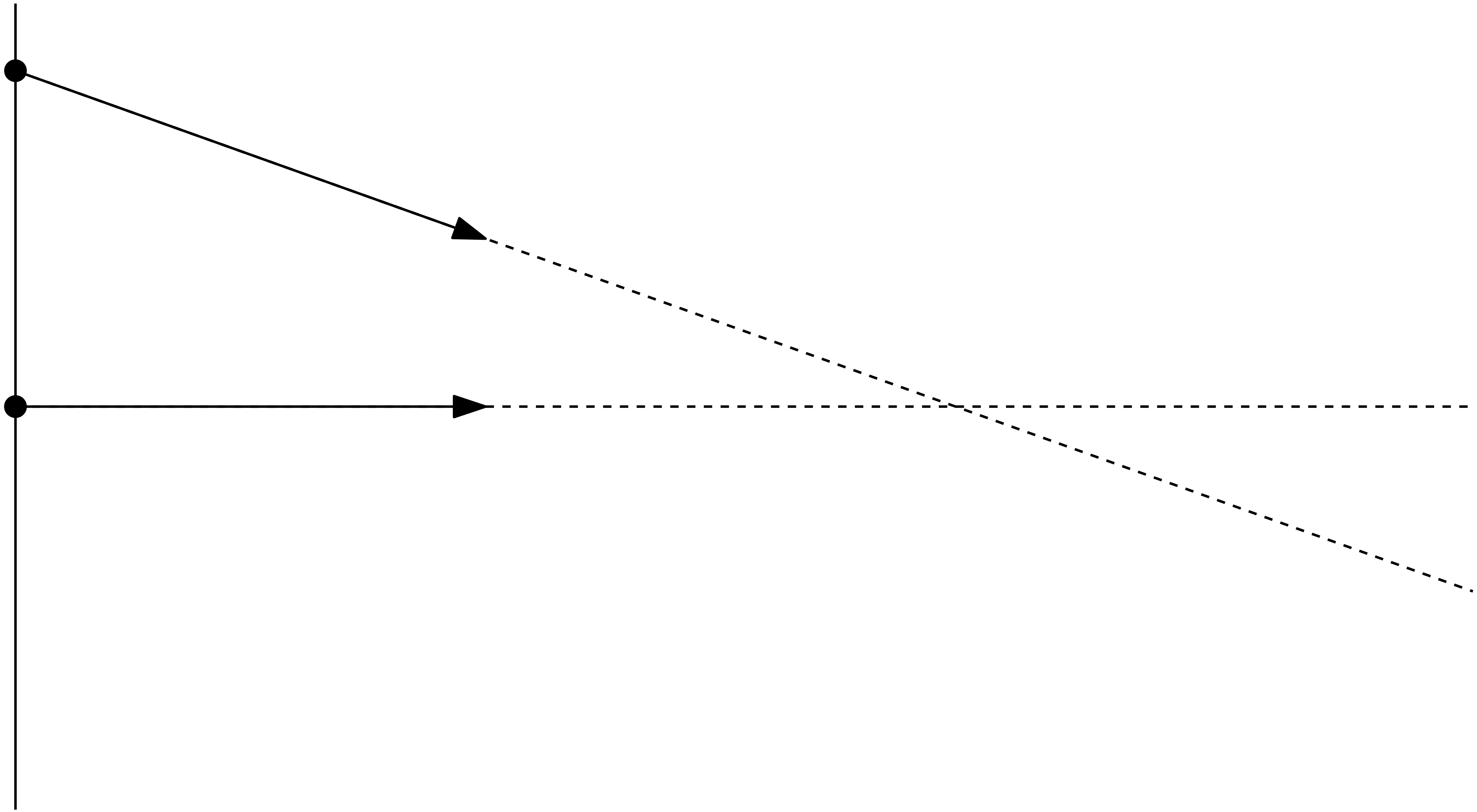

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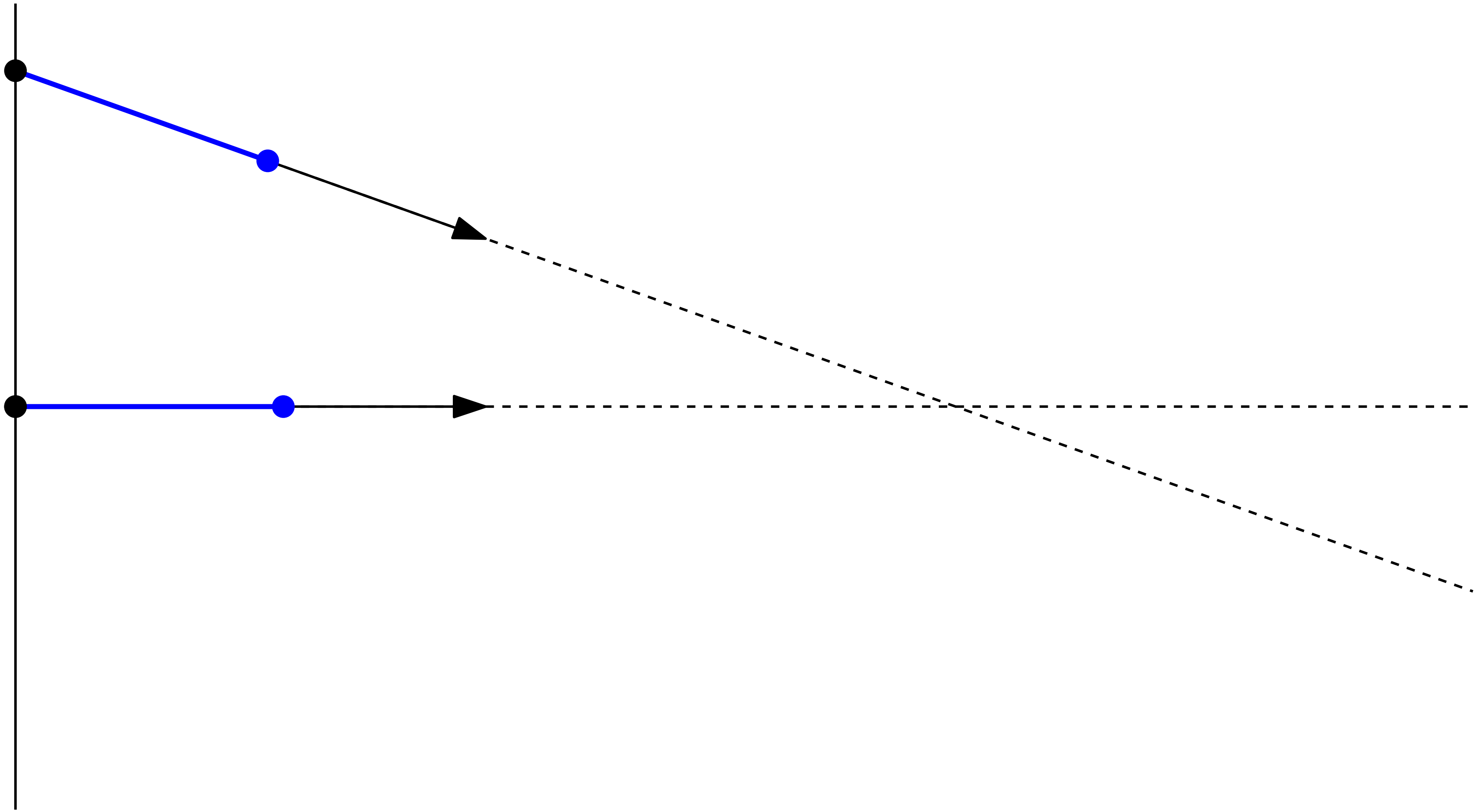

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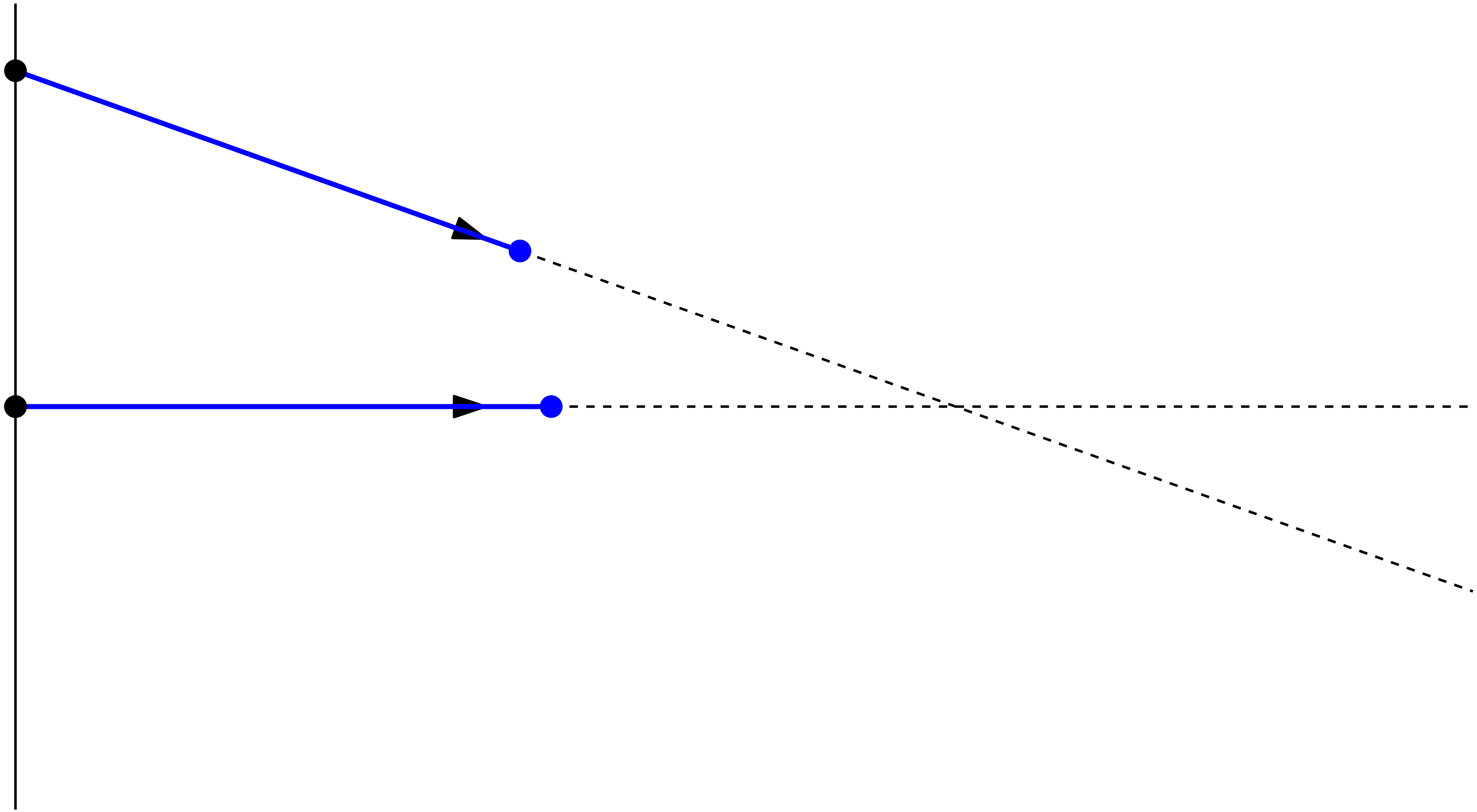

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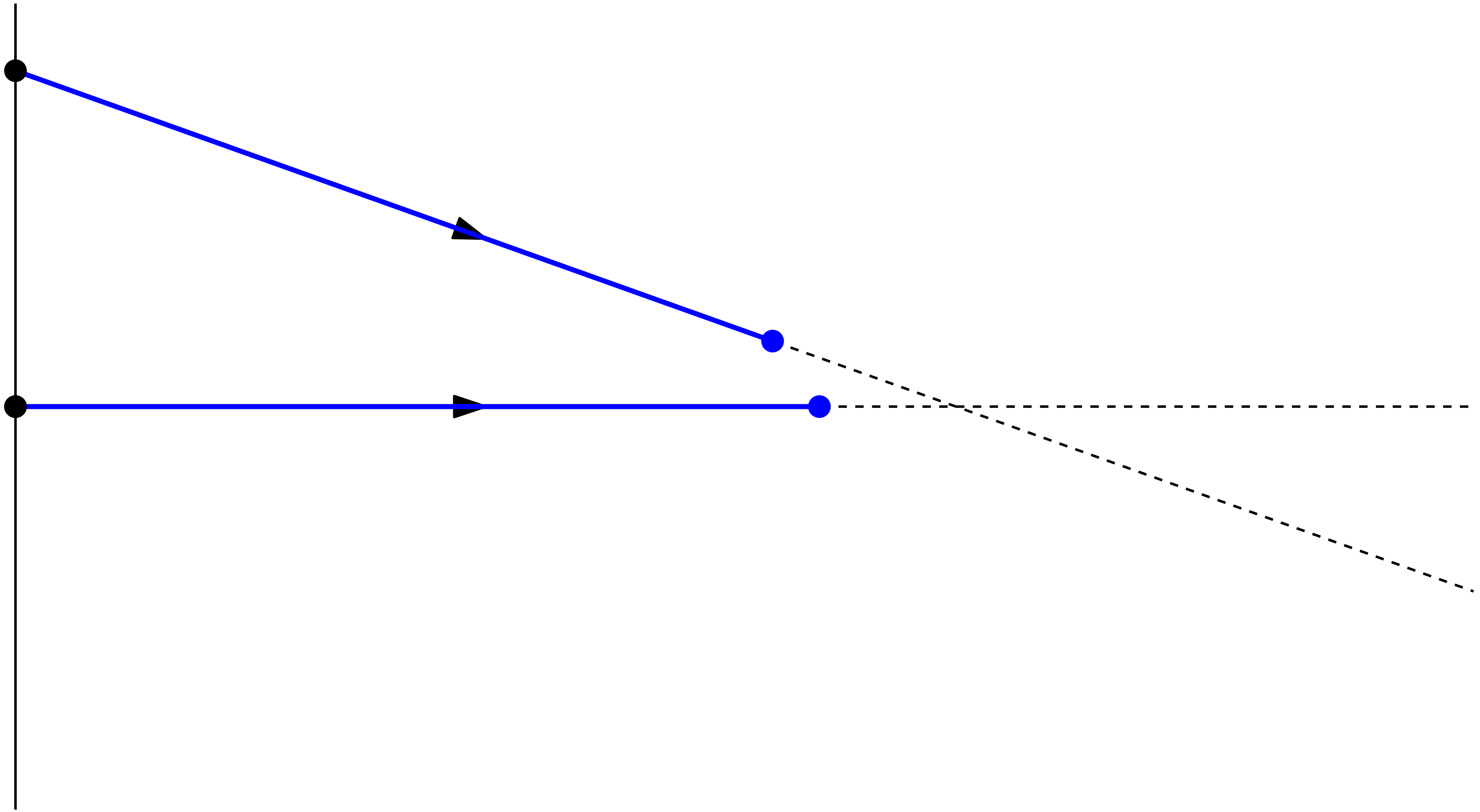

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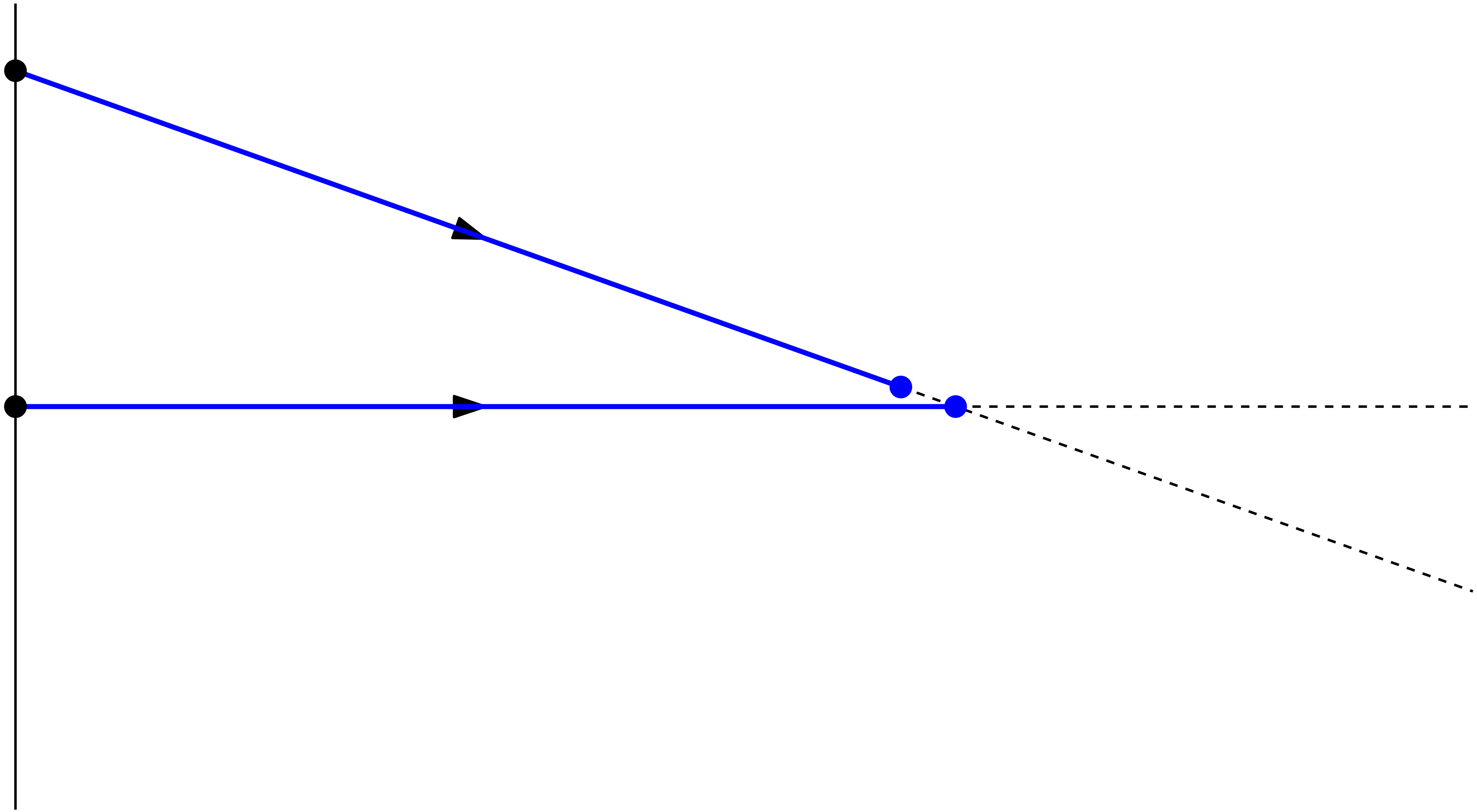

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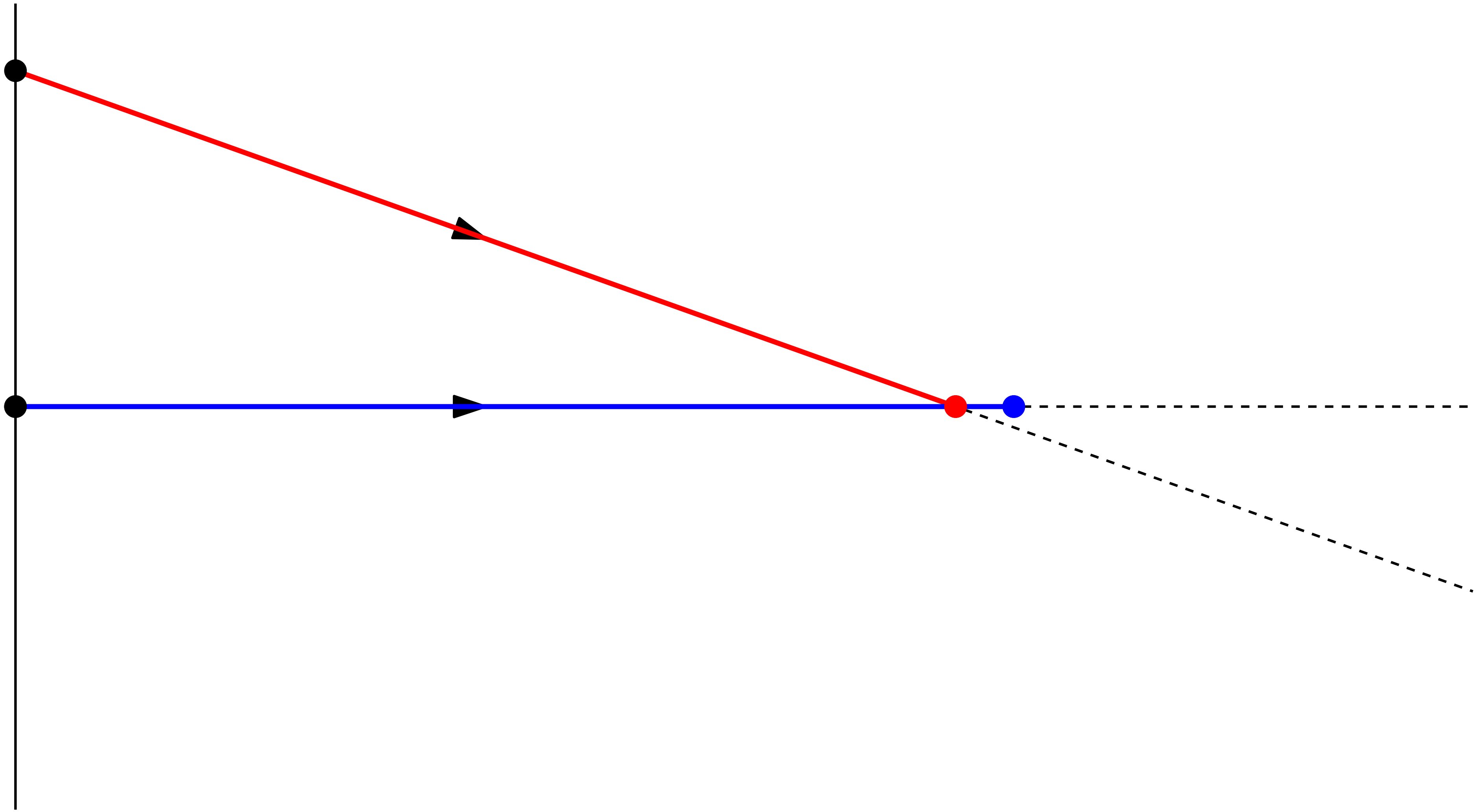

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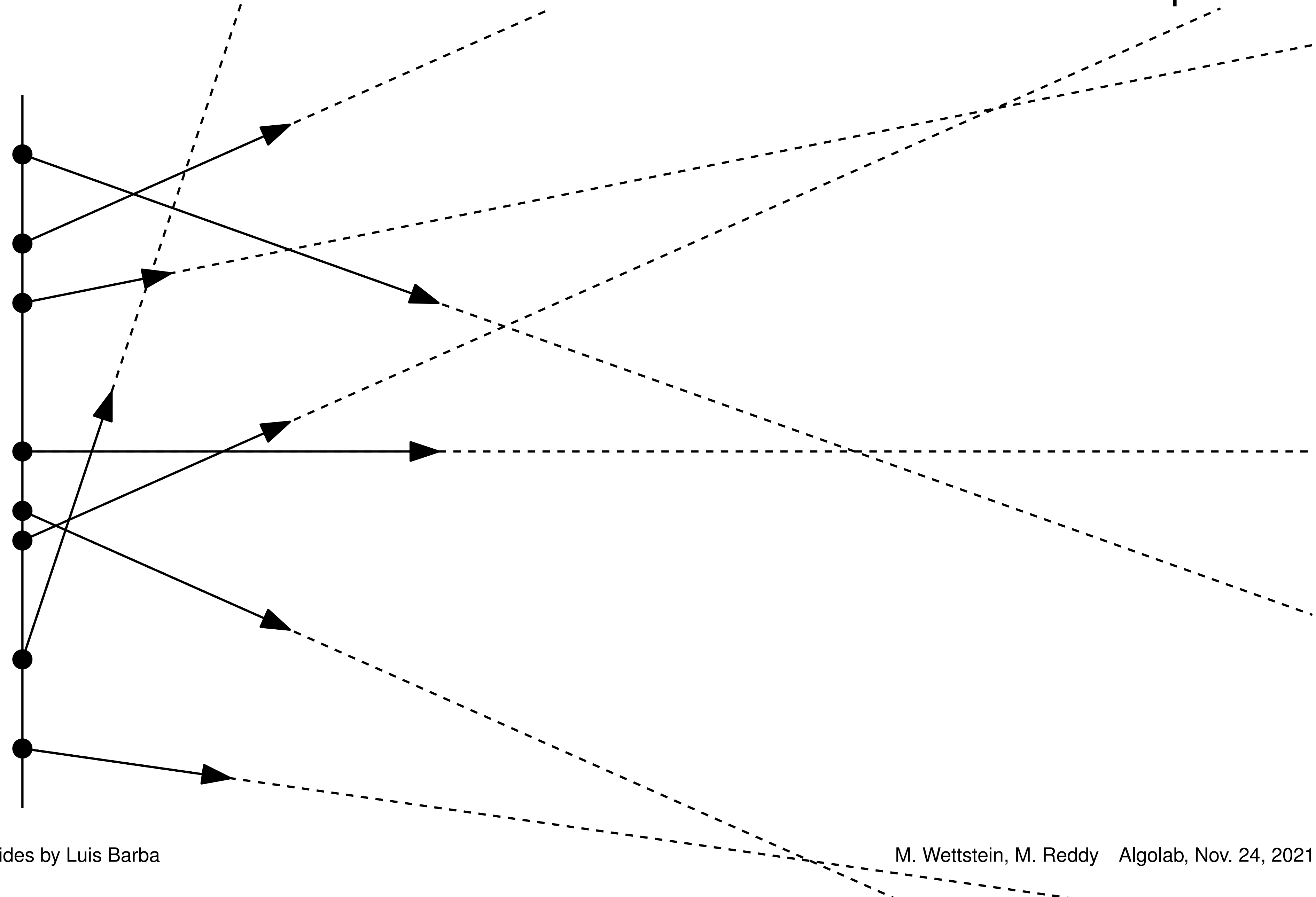

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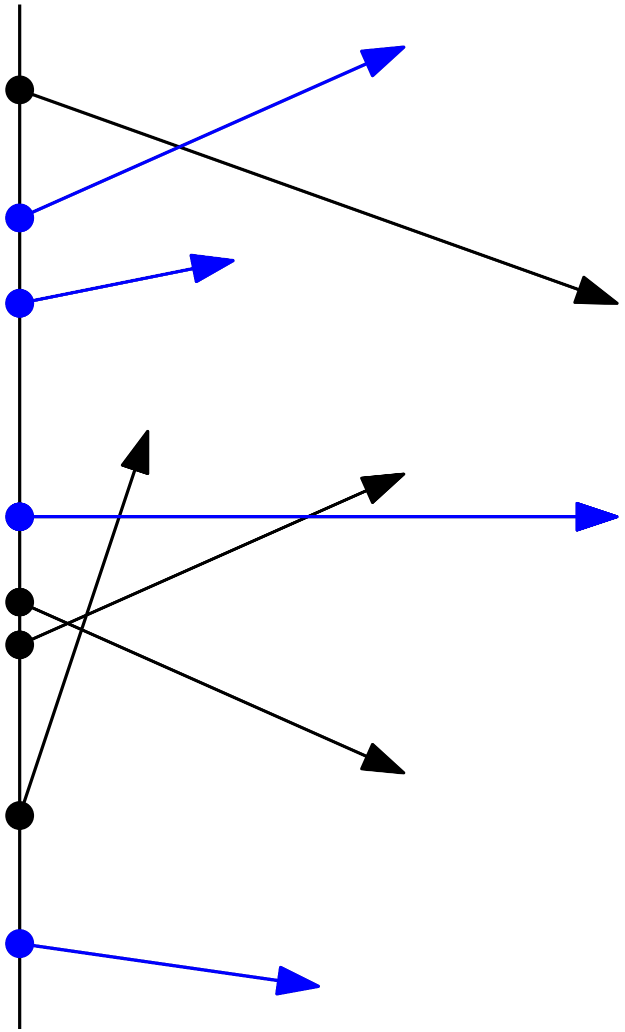
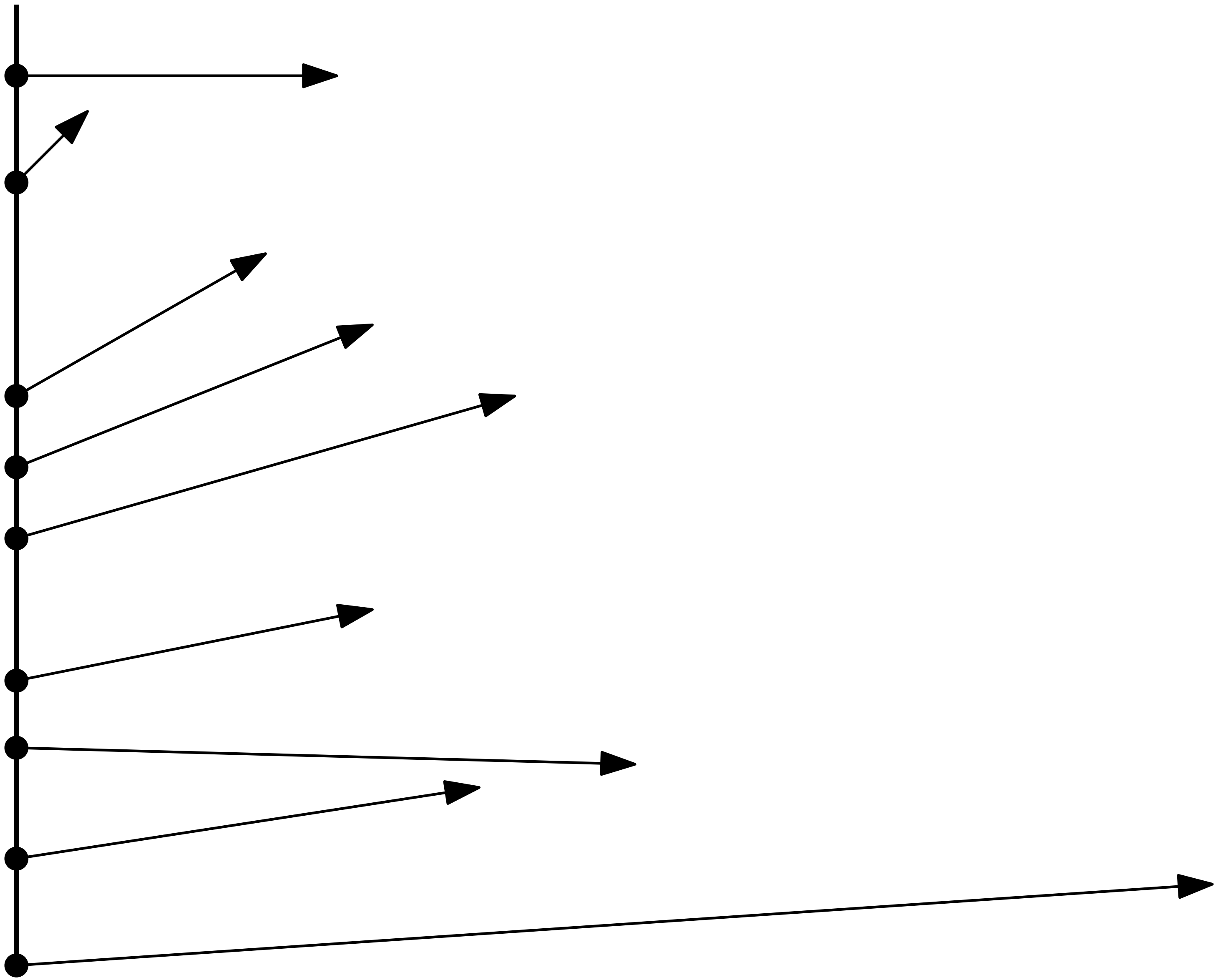


Which bikers get to ride forever?

Admission Control

Admission Control



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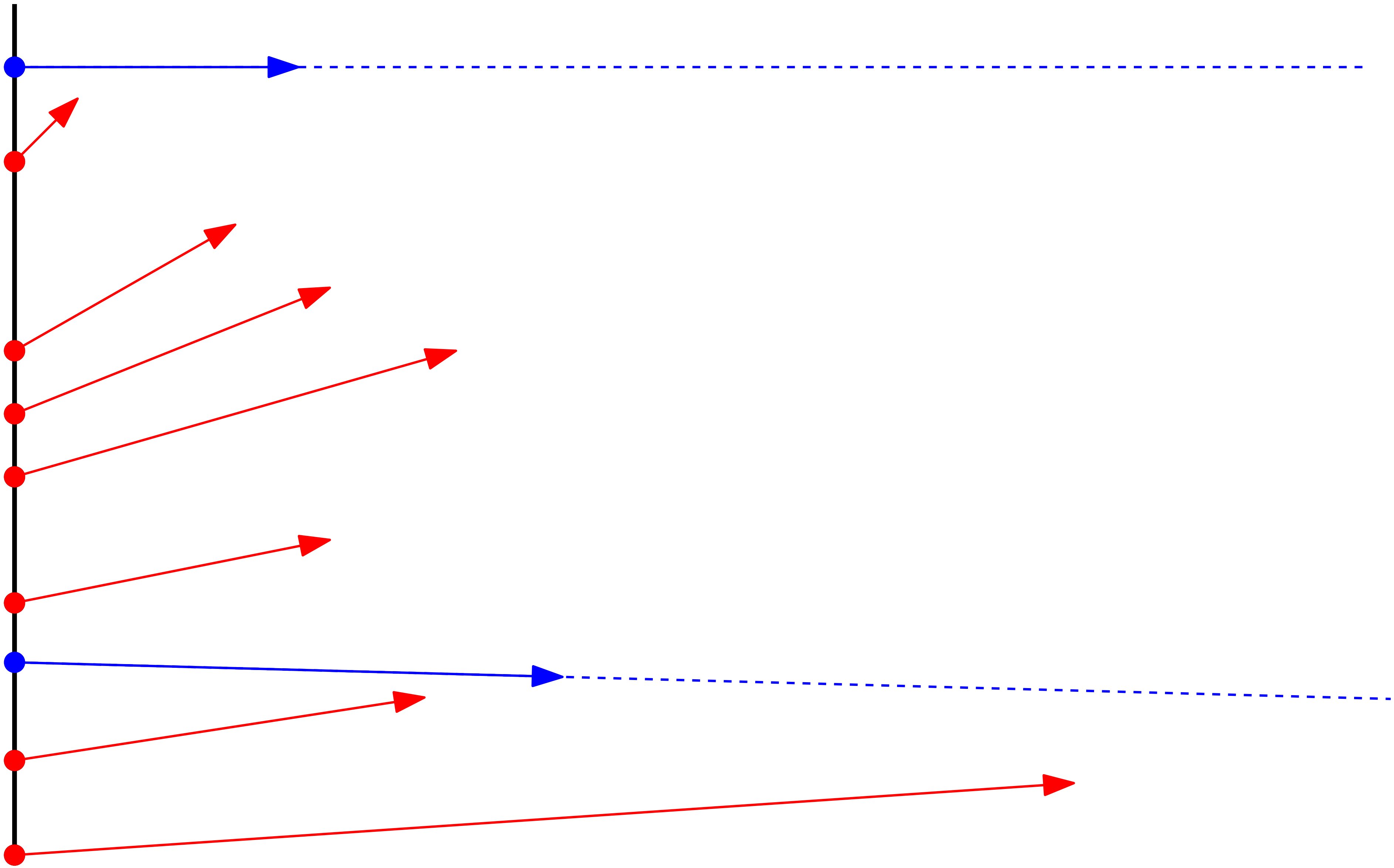

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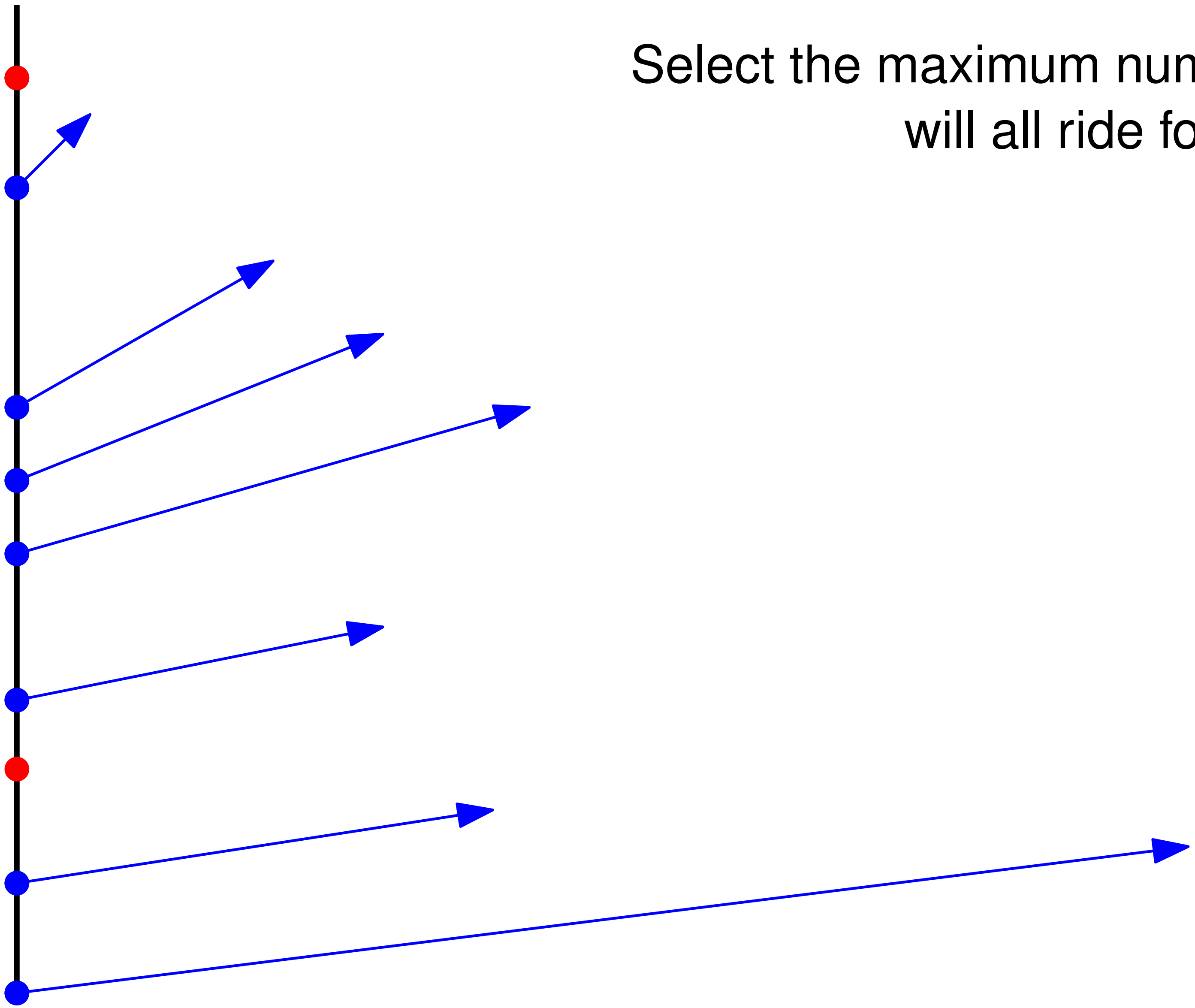
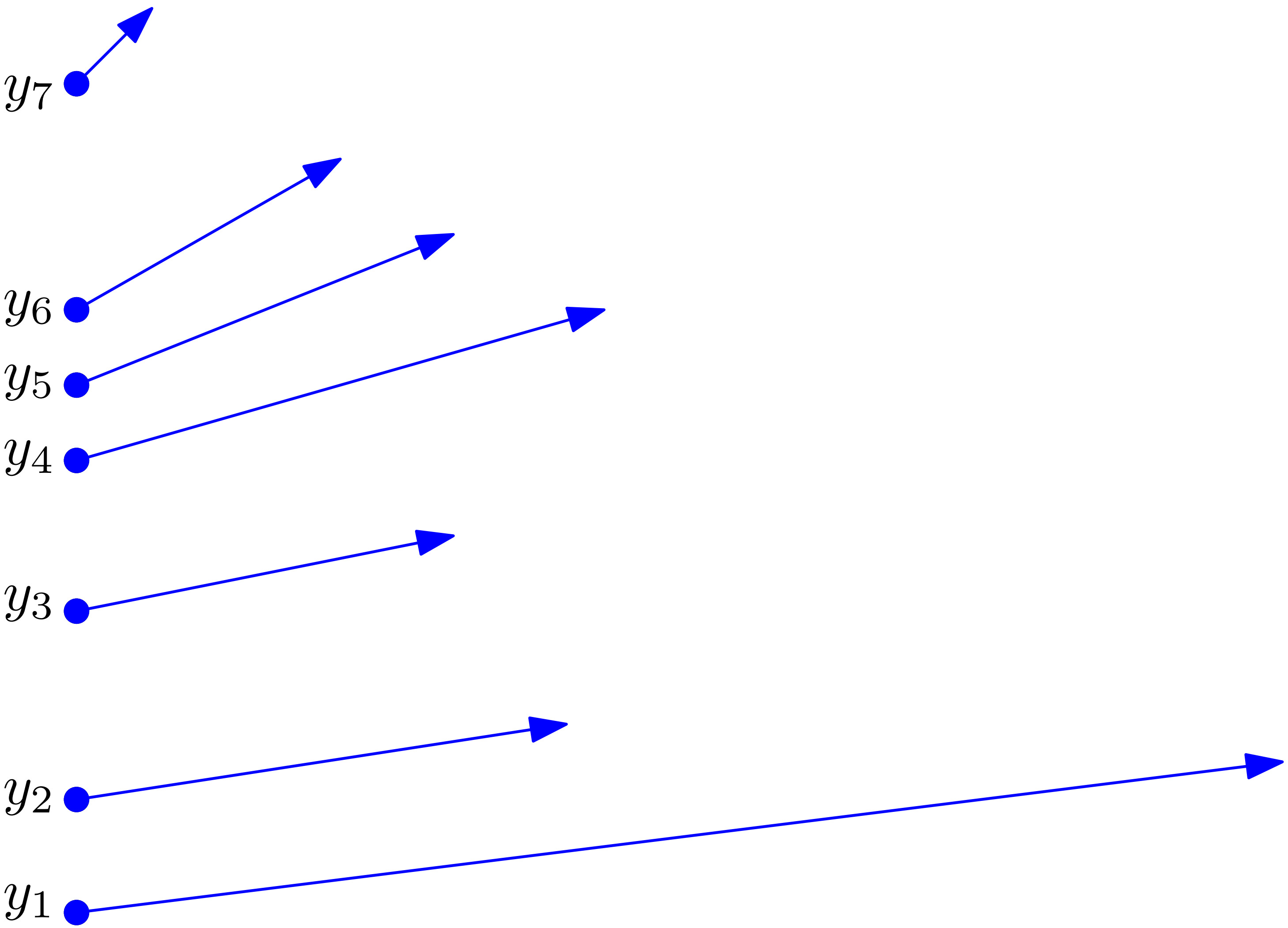


Select the maximum number of bikers that will all ride forever.

Admission Control



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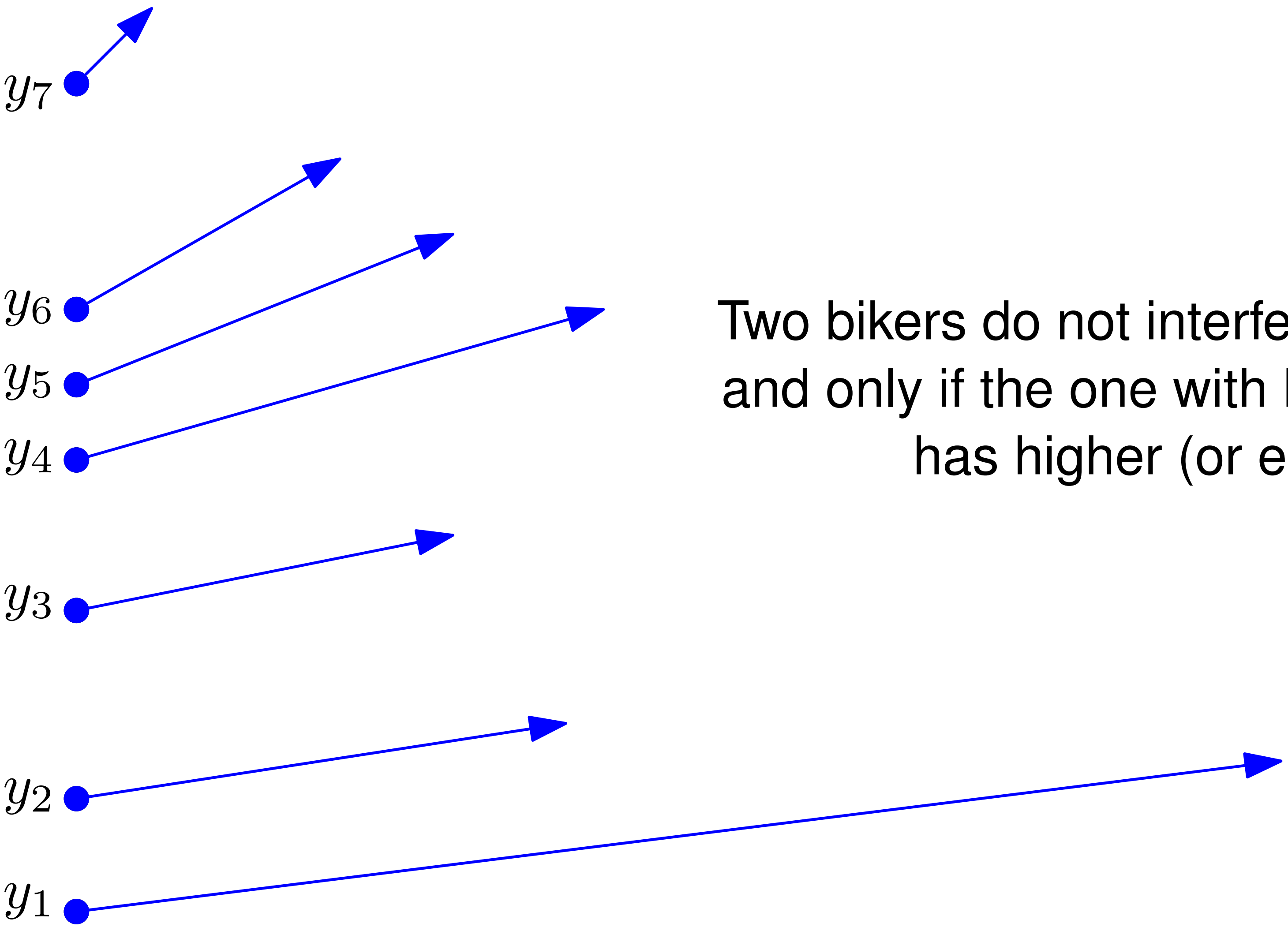
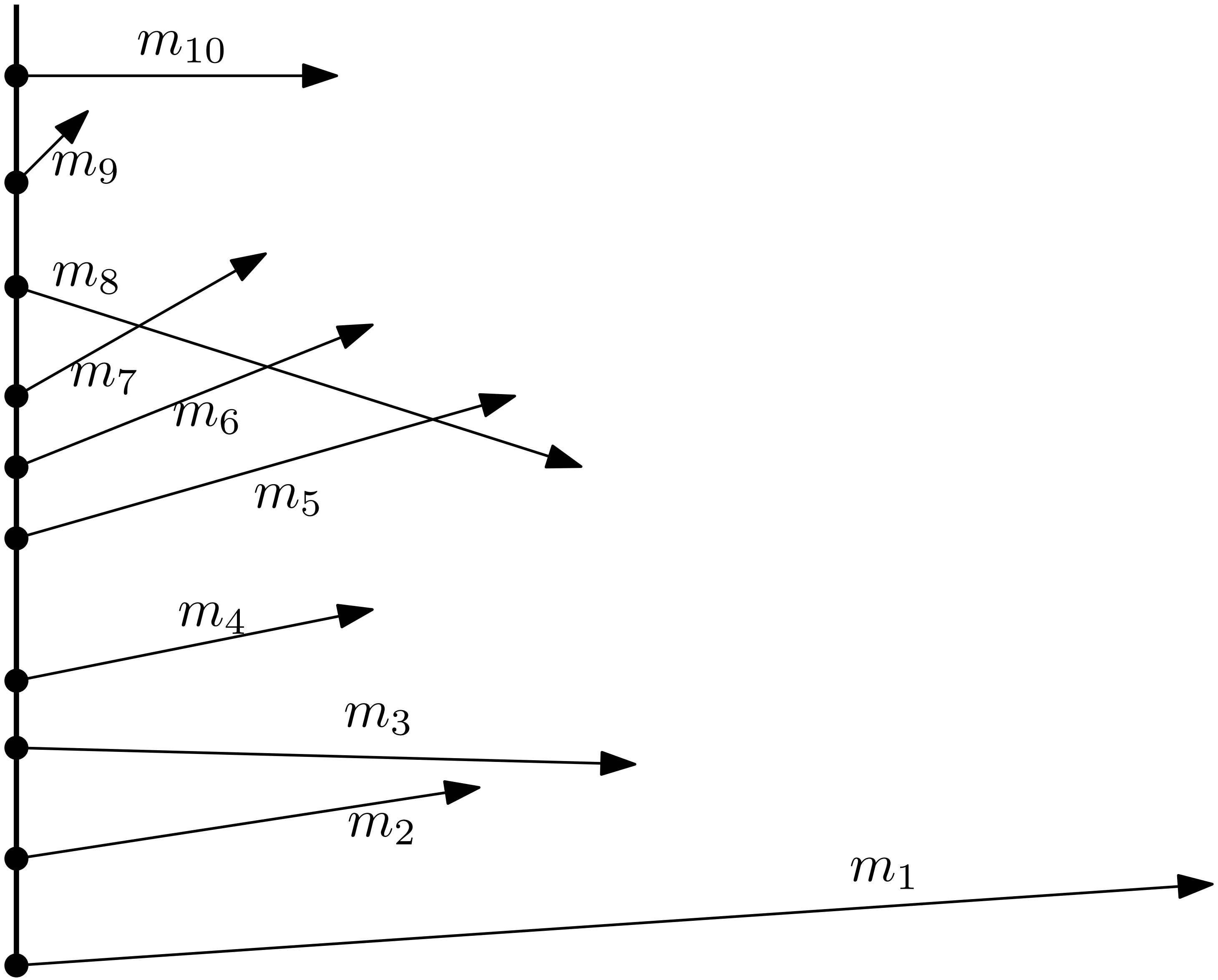


Two bikers do not interfere with each other if and only if the one with higher starting point has higher (or equal) slope.

Admission Control



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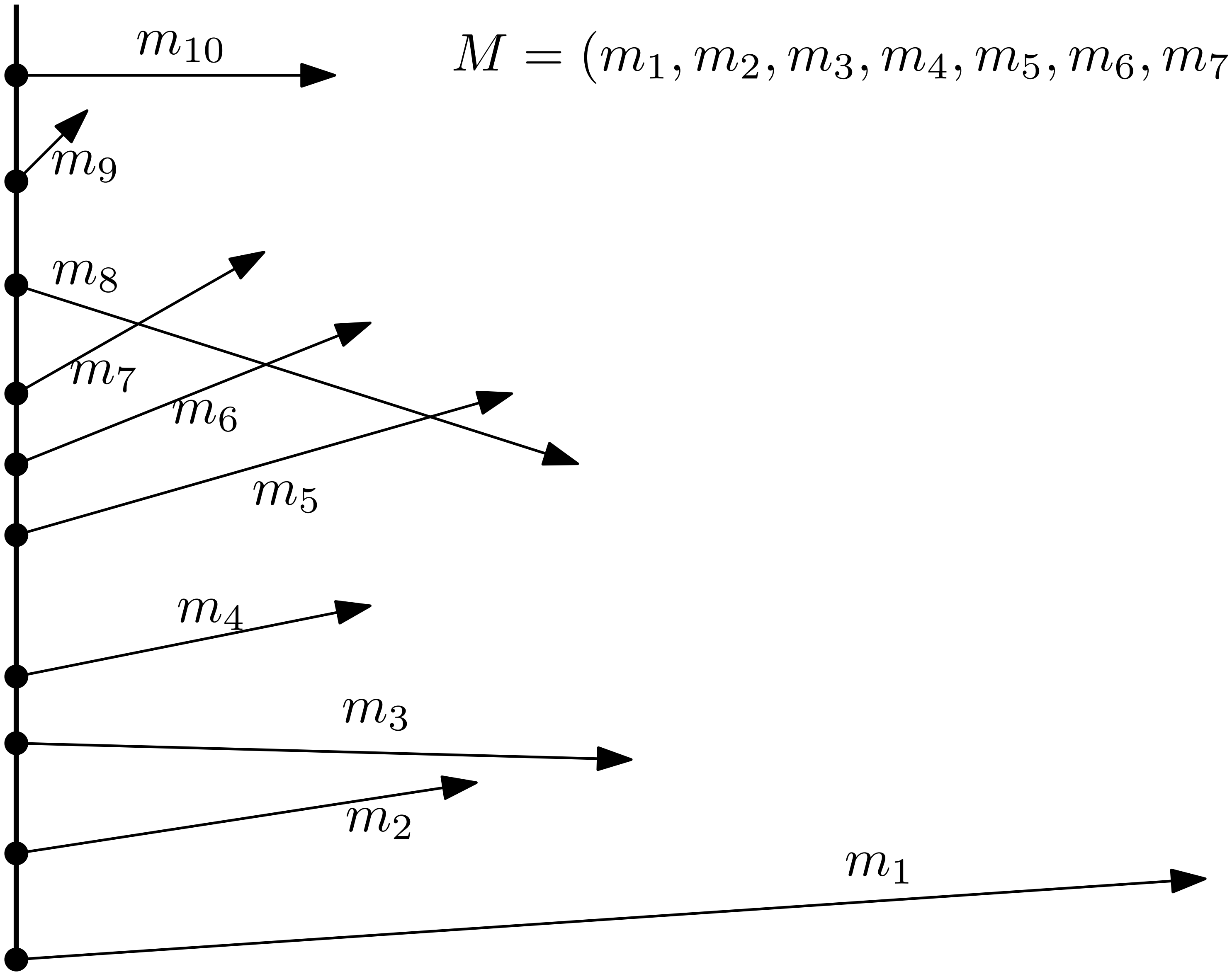
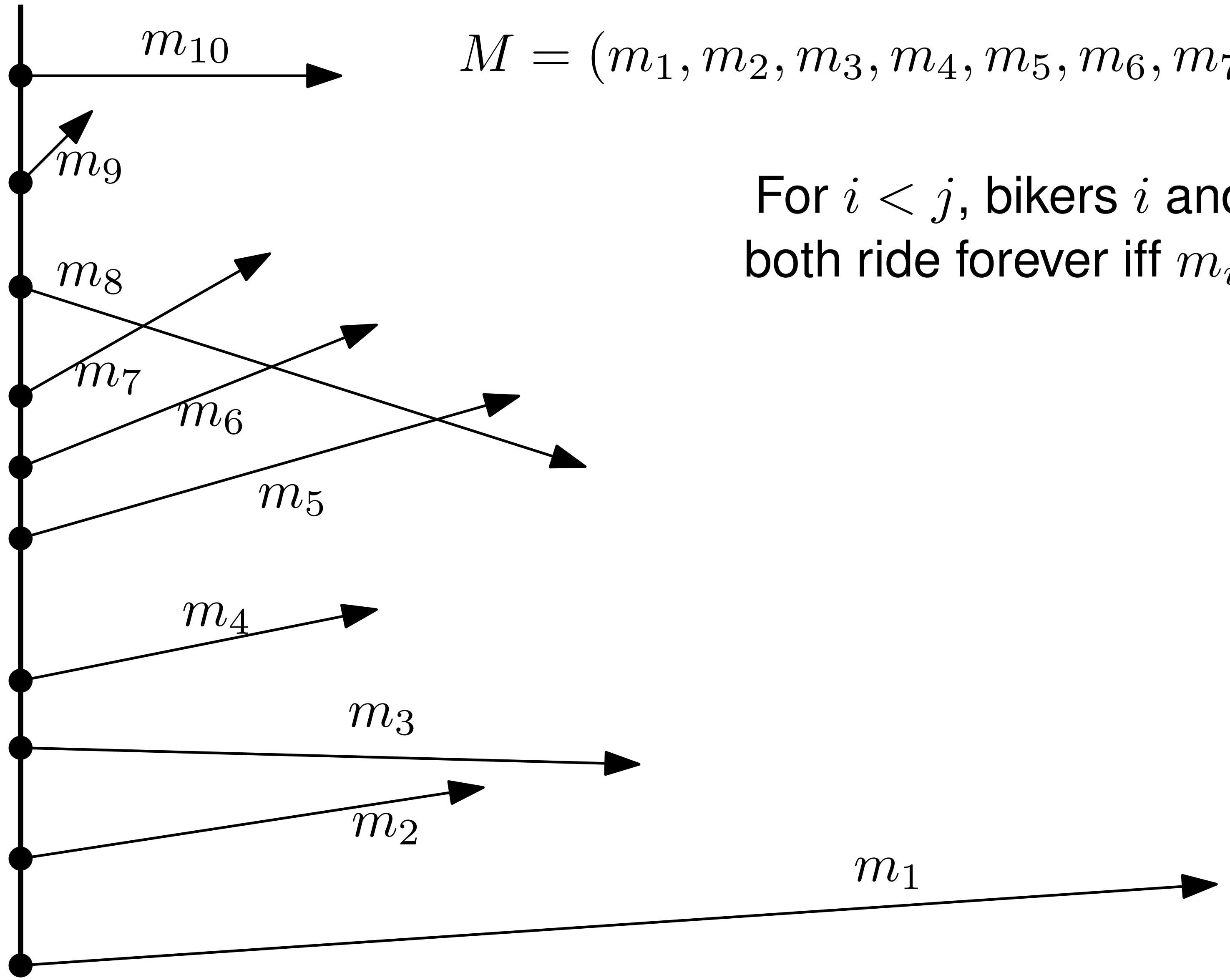


$$M = (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10})$$

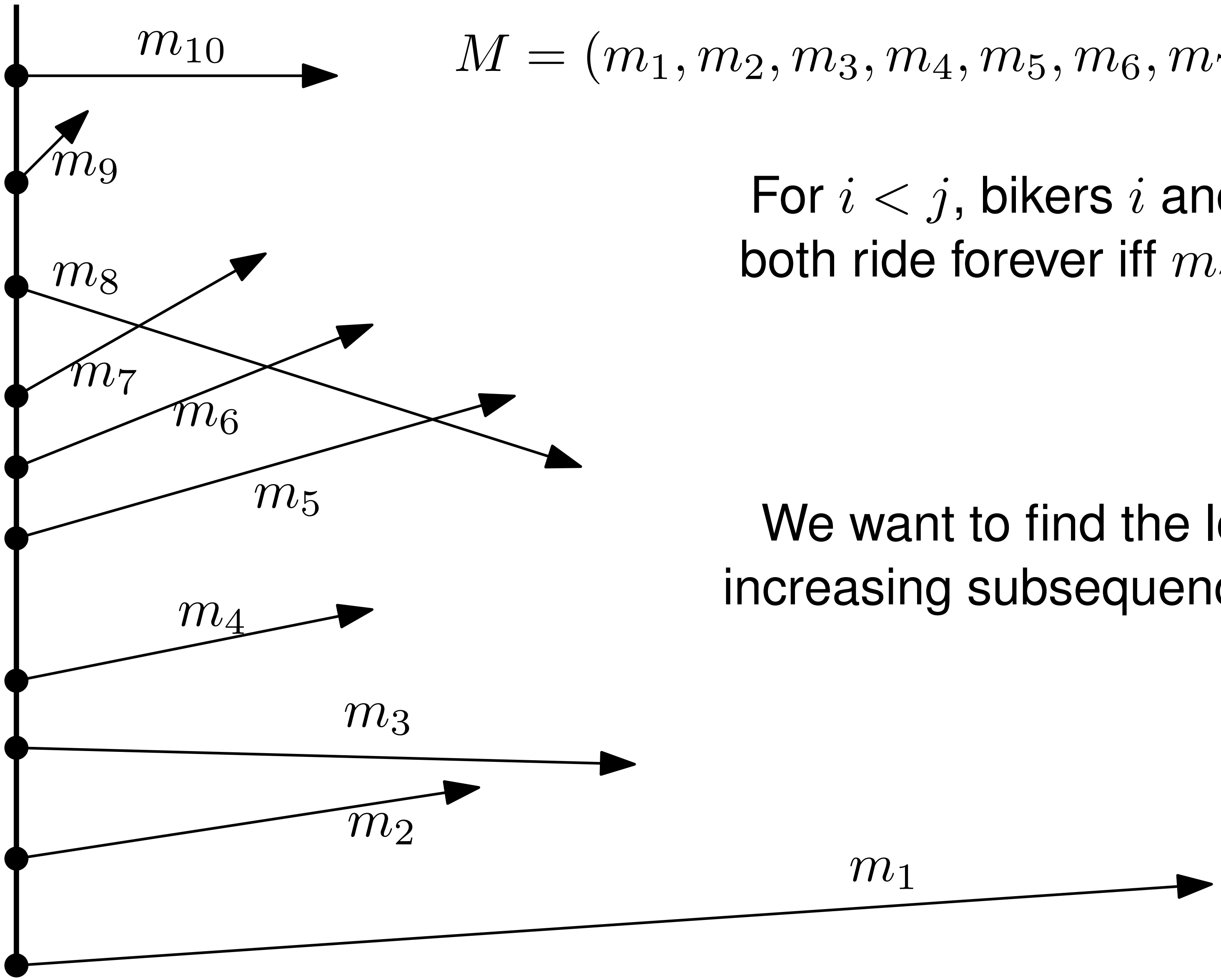
Admission Control



$$M = (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10})$$

For $i < j$, bikers i and j can both ride forever iff $m_i \leq m_j$.

Admission Control



$$M = (m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10})$$

For $i < j$, bikers i and j can both ride forever iff $m_i \leq m_j$.

We want to find the longest increasing subsequence in M.

Longest increasing subsequence

- Classic Dynamic programming leads to $O(n^2)$ time.
- Possible in $O(n \log n)$ time, however.

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$

$L2$

$L3$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (\boxed{0}, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0 $\leftarrow$ increasing subsequence of length 1

$L2$

$L3$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, \boxed{8}, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 8$ $\leftarrow$ increasing subsequence of length 2

$L3$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, \boxed{4}, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ $\quad 0$

$L2$ $\quad 0, 4$ $\quad \leftarrow$ increasing subsequence of length 2 (with smallest possible last entry)

$L3$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, \boxed{12}, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 4$

$L3$ $0, 4, 12$ $\leftarrow$ increasing subsequence of length 3

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, \boxed{2}, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 4$

$L3$ $0, 4, 12$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, \boxed{2}, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0 $\leftarrow$ cannot improve since $L1$ ends in 0 (< 2)

$L2$ $0, 4$

$L3$ $0, 4, 12$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, \boxed{2}, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0 $\leftarrow$ cannot improve since $L1$ ends in 0 (< 2)

$L2$ $0, 4$

$L3$ $0, 4, 12$ $\leftarrow$ cannot improve since $L2$ ends in 4 (> 2)

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, \boxed{2}, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0 $\leftarrow$ cannot improve since $L1$ ends in 0 (< 2)

$L2$ $0, 4$ $\leftarrow$ improves by adding 2 at the end of $L1$

$L3$ $0, 4, 12$ $\leftarrow$ cannot improve since $L2$ ends in 4 (> 2)

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, \boxed{2}, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0 ← cannot improve since $L1$ ends in 0 (< 2)

$L2$ $0, 2$ ← improves by adding 2 at the end of $L1$

$L3$ $0, 4, 12$ ← cannot improve since $L2$ ends in 4 (> 2)

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, \boxed{10}, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$

$L3$ $0, 4, 12$ $\leftarrow$ improves by adding 10 at the end of $L2$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, \boxed{10}, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$

$L3$ $0, 2, 10$ $\leftarrow$ improves by adding 10 at the end of $L2$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, \boxed{6}, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$

$L3$ $0, 2, 10$ $\leftarrow$ improves by adding 6 at the end of $L2$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, \boxed{6}, 14, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$

$L3$ $0, 2, 6$ $\leftarrow$ improves by adding 6 at the end of $L2$

$L4$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, \boxed{14}, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$

$L3$ $0, 2, 6$

$L4$ $\leftarrow$ "improves" by adding 14 at the end of $L3$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, \boxed{14}, 1, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$

$L3$ $0, 2, 6$

$L4$ $0, 2, 6, 14$ $\leftarrow$ "improves" by adding 14 at the end of $L3$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, \boxed{1}, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 2$ $\leftarrow$ improves by adding 1 at the end of $L1$

$L3$ $0, 2, 6$

$L4$ $0, 2, 6, 14$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, \boxed{1}, 9, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$ $\leftarrow$ improves by adding 1 at the end of $L1$

$L3$ $0, 2, 6$

$L4$ $0, 2, 6, 14$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, \boxed{9}, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 2, 6$

$L4$ $0, 2, 6, 14$ $\leftarrow$ improves by adding 9 at the end of $L3$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, \boxed{9}, 5, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 2, 6$

$L4$ $0, 2, 6, 9$ $\leftarrow$ improves by adding 9 at the end of $L3$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, \boxed{5}, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 2, 6$ $\leftarrow$ improves by adding 5 at the end of $L2$

$L4$ $0, 2, 6, 9$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, \boxed{5}, 13, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 5$ $\leftarrow$ improves by adding 5 at the end of $L2$

$L4$ $0, 2, 6, 9$

$L5$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, \boxed{13}, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 5$

$L4$ $0, 2, 6, 9$

$L5$ $\leftarrow$ "improves" by adding 13 at the end of $L4$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, \boxed{13}, 3, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 5$

$L4$ $0, 2, 6, 9$

$L5$ $0, 2, 6, 9, 13$ $\leftarrow$ "improves" by adding 13 at the end of $L4$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, \boxed{3}, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 5$ $\leftarrow$ improves by adding 3 at the end of $L2$

$L4$ $0, 2, 6, 9$

$L5$ $0, 2, 6, 9, 13$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, \boxed{3}, 11, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$ $\leftarrow$ improves by adding 3 at the end of $L2$

$L4$ $0, 2, 6, 9$

$L5$ $0, 2, 6, 9, 13$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, \boxed{11}, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 2, 6, 9$

$L5$ $0, 2, 6, 9, 13$ $\leftarrow$ improves by adding 11 at the end of $L4$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, \boxed{11}, 7, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 2, 6, 9$

$L5$ $0, 2, 6, 9, 11$ $\leftarrow$ improves by adding 11 at the end of $L4$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, \boxed{7}, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 2, 6, 9$ $\leftarrow$ improves by adding 7 at the end of $L3$

$L5$ $0, 2, 6, 9, 11$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, \boxed{7}, 15)$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 1, 3, 7$ $\leftarrow$ improves by adding 7 at the end of $L3$

$L5$ $0, 2, 6, 9, 11$

$L6$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, \boxed{15})$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 1, 3, 7$

$L5$ $0, 2, 6, 9, 11$

$L6$ $\leftarrow$ “improves” by adding 15 at the end of $L5$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, \boxed{15})$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 1, 3, 7$

$L5$ $0, 2, 6, 9, 11$

$L6$ $0, 2, 6, 9, 11, 15$ $\leftarrow$ "improves" by adding 15 at the end of $L5$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, \boxed{15})$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 1, 3, 7$

$L5$ $0, 2, 6, 9, 11$

$L6$ $0, 2, 6, 9, 11, 15$

In every iteration, exactly one row changes $\Rightarrow$ time $O(n^2)$

Longest increasing subsequence

$$M = (0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, \boxed{15})$$

$L1$ 0

$L2$ $0, 1$

$L3$ $0, 1, 3$

$L4$ $0, 1, 3, 7$

$L5$ $0, 2, 6, 9, 11$

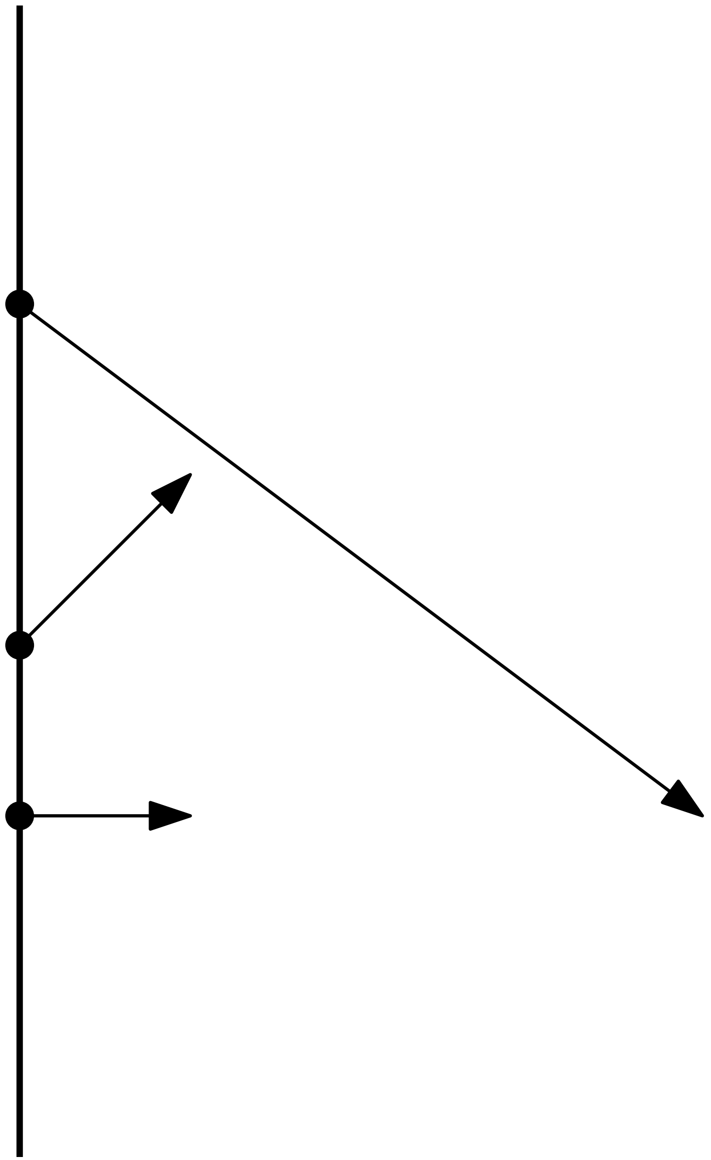
$L6$ $0, 2, 6, 9, 11, 15$

In every iteration, exactly one row changes $\Rightarrow$ time $O(n^2)$

We can also compute only the main diagonal $\Rightarrow$ time $O(n \log n)$

Starting Schedules

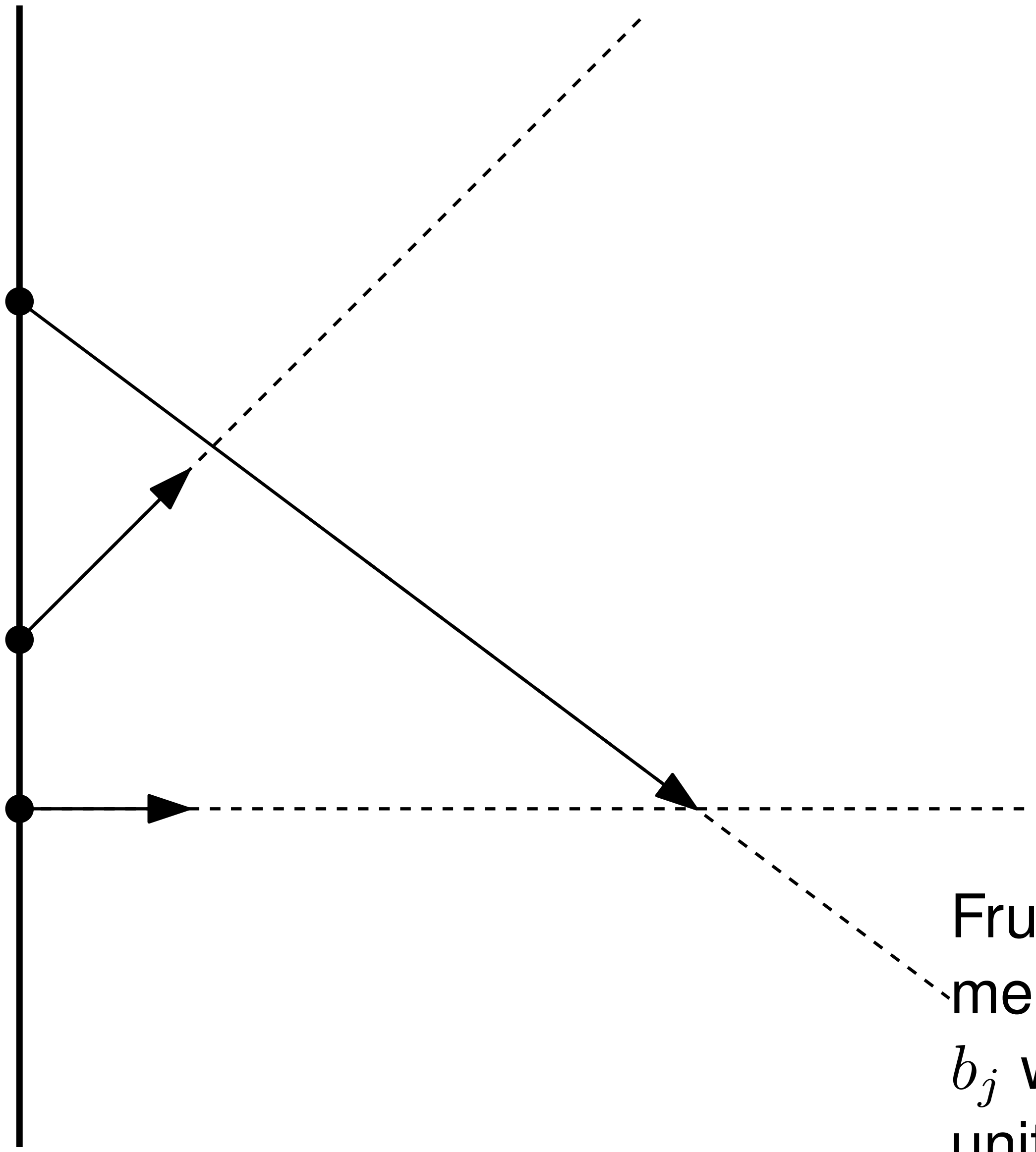
We are allowed to modify the starting time s_i of each bike b_i.



Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Starting Schedules

We are allowed to modify the starting time s_i of each bike b_i.



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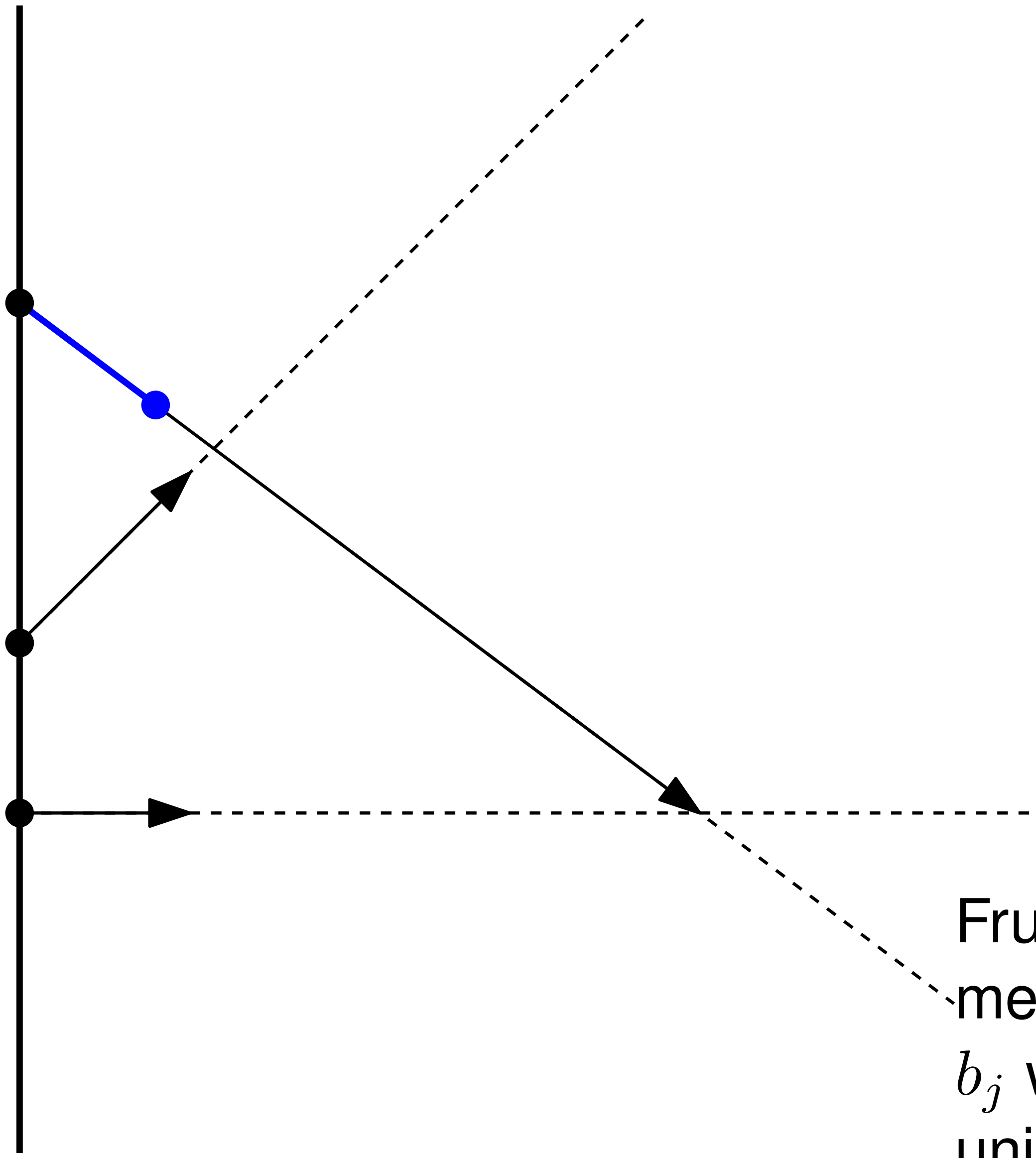

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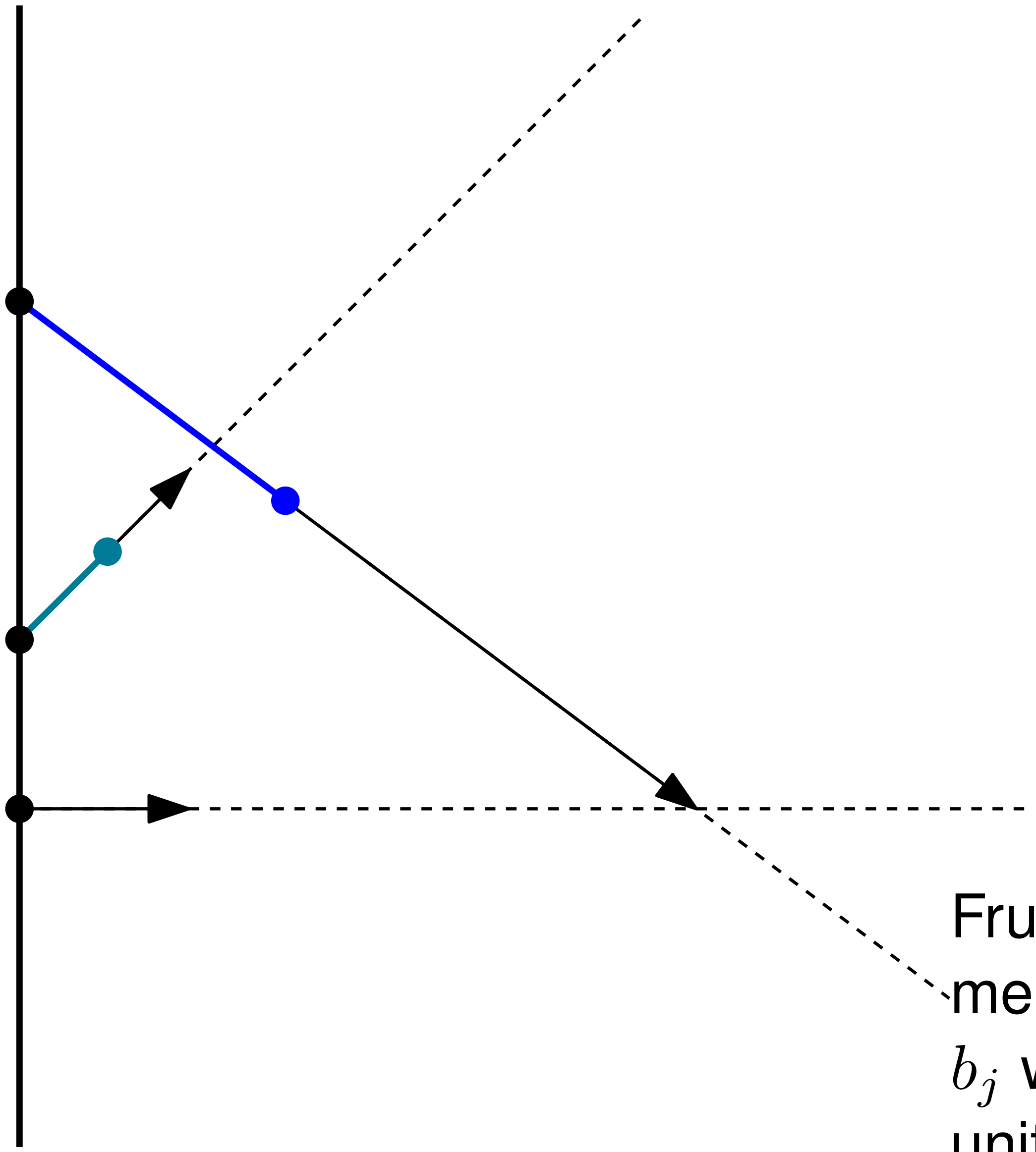

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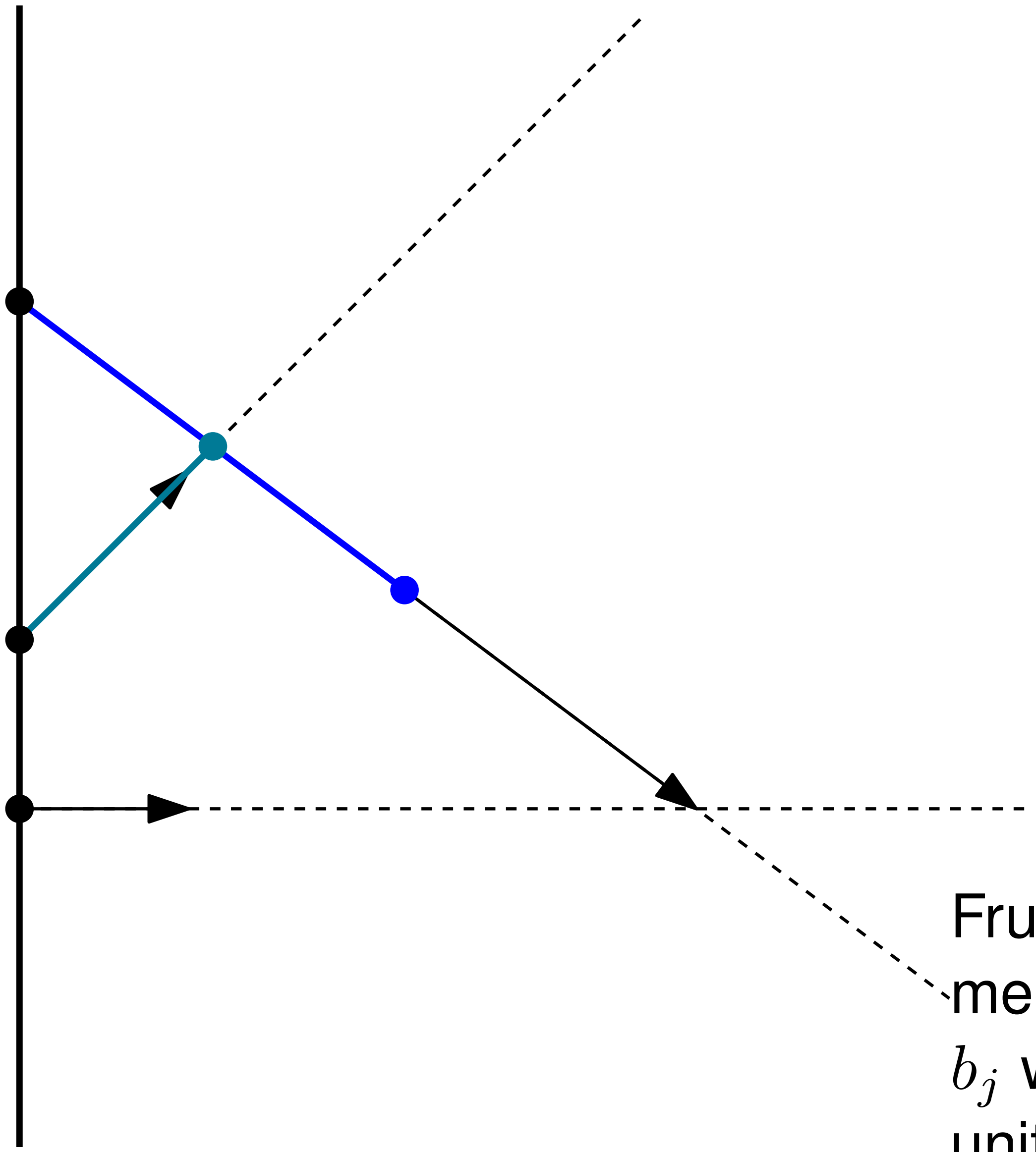

Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

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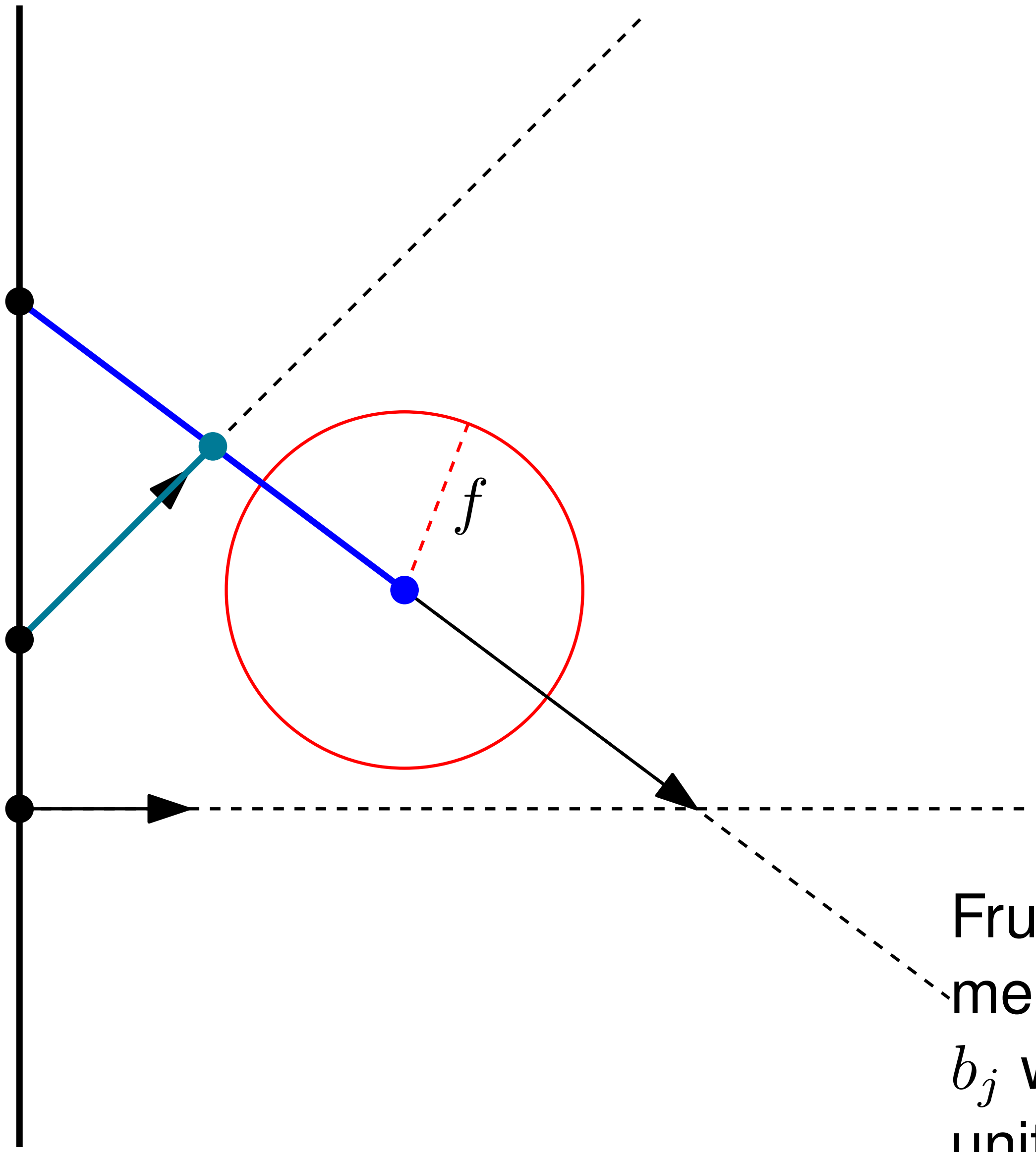
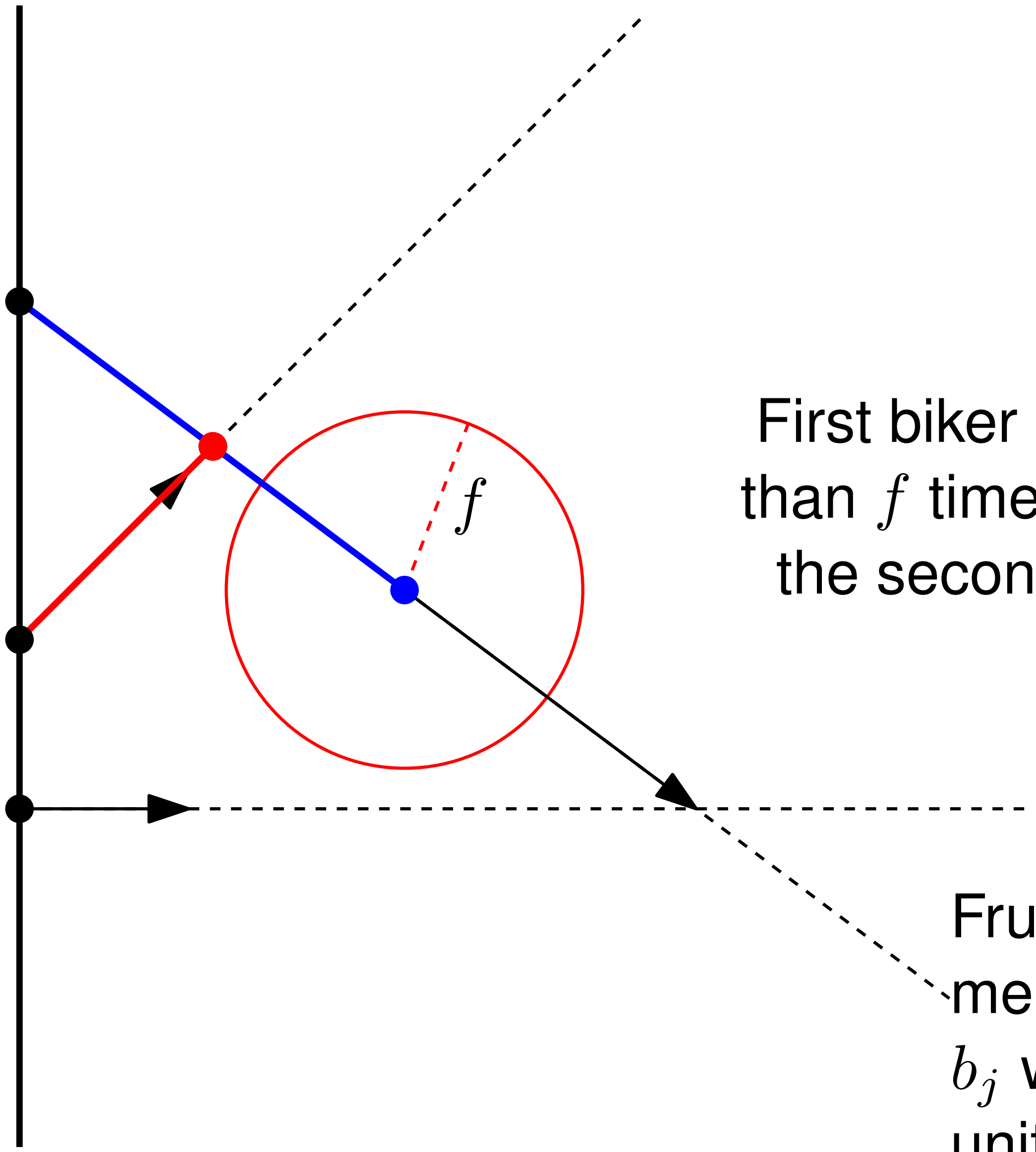


Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Starting Schedules

We are allowed to modify the starting time s_i of each bike b_i.

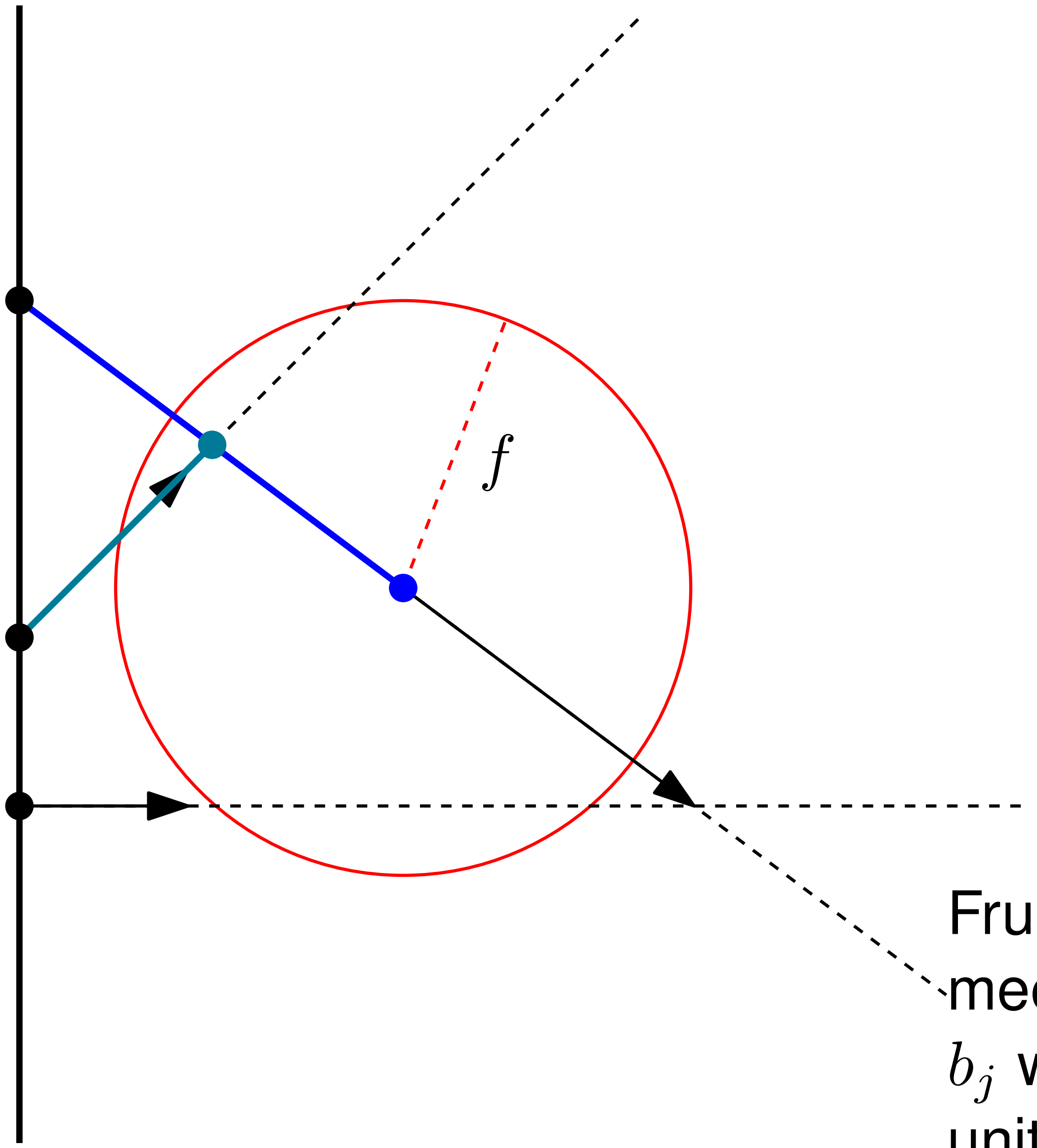


First biker passed more than f time units ago, so the second must stop.

Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Starting Schedules

We are allowed to modify the starting time s_i of each bike b_i.



Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Starting Schedules

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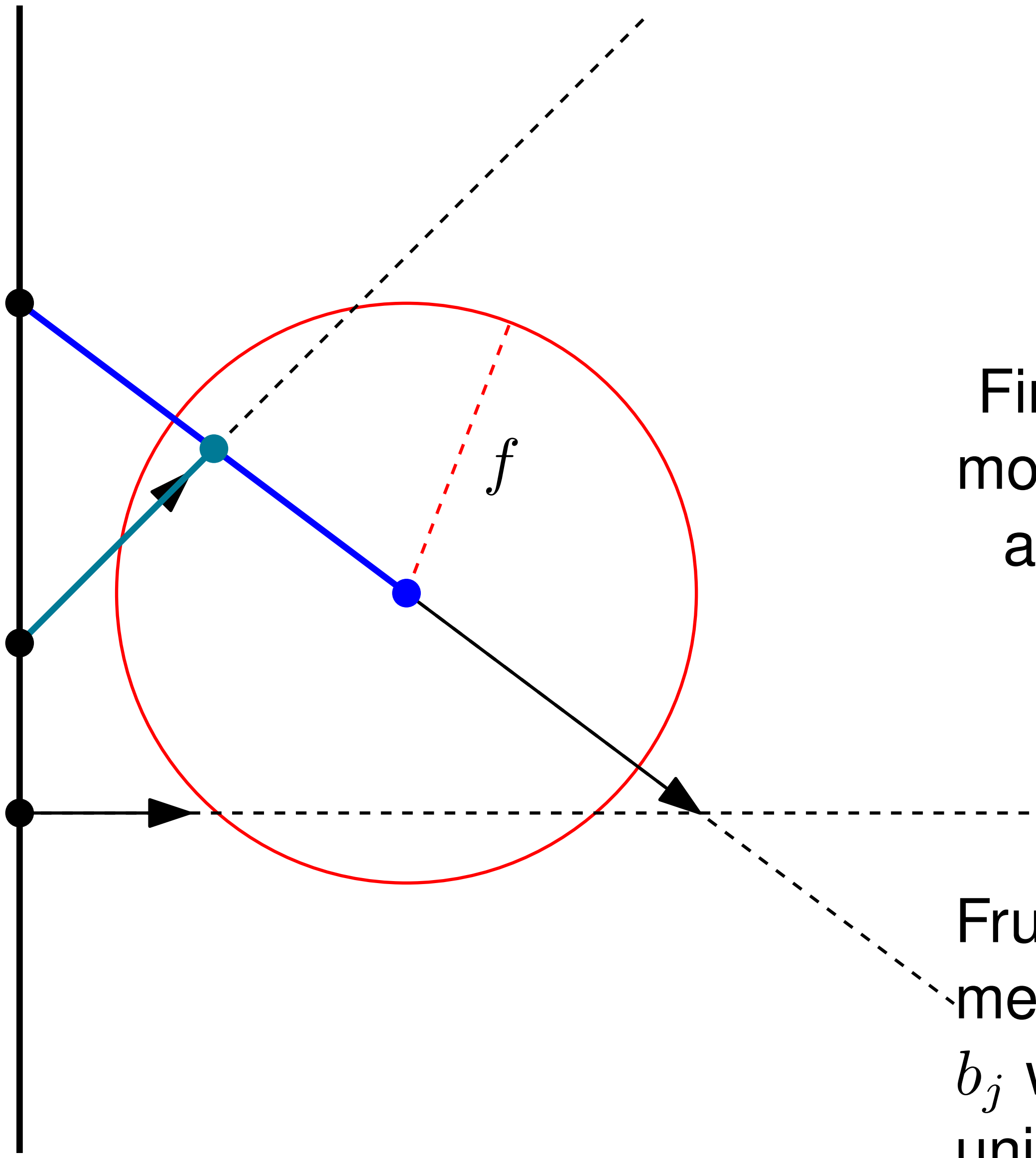
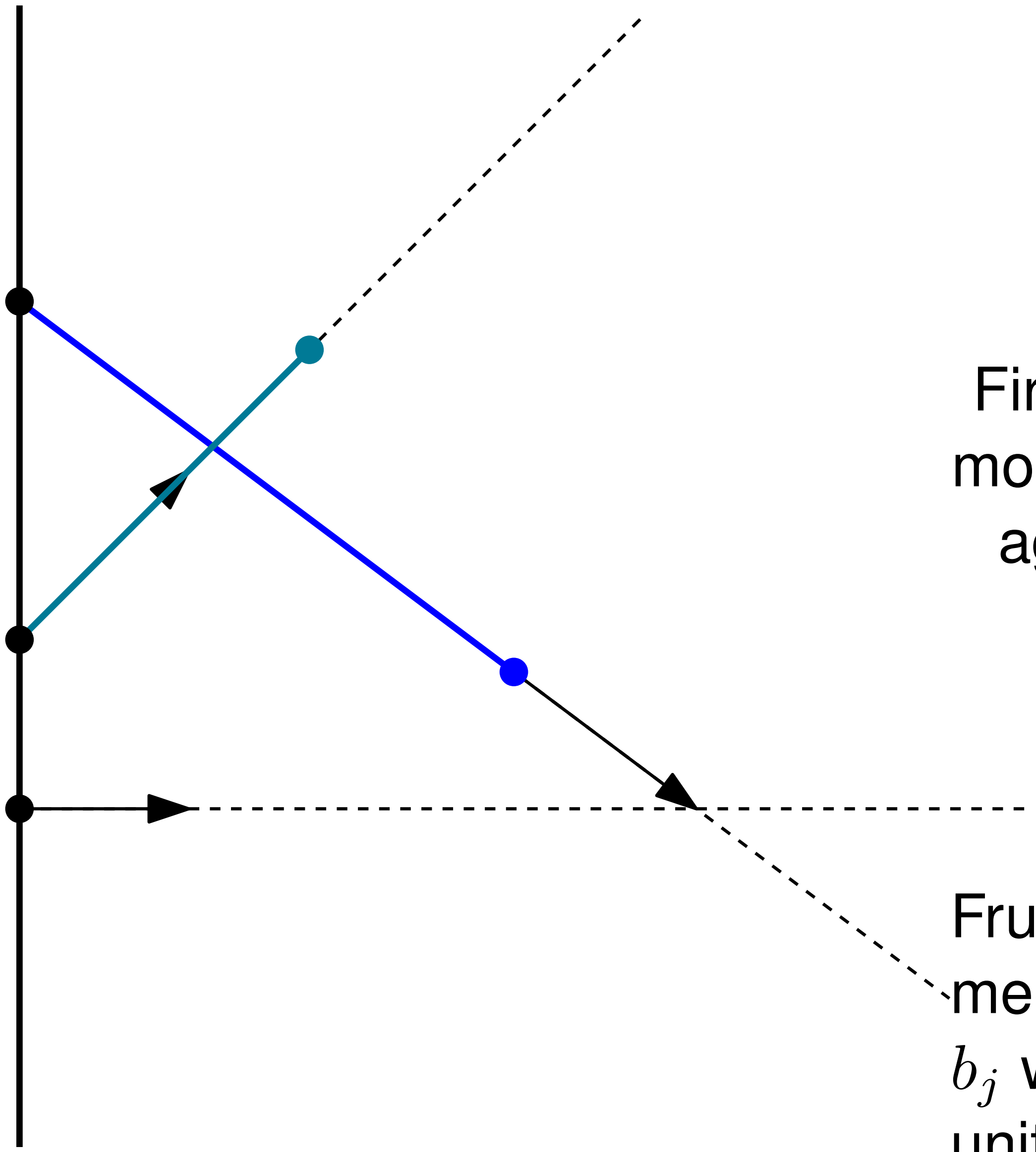


First biker passed no more than f time units ago, so the second keeps riding.

Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Starting Schedules

We are allowed to modify the starting time s_i of each bike b_i.



First biker passed no more than f time units ago, so the second keeps riding.

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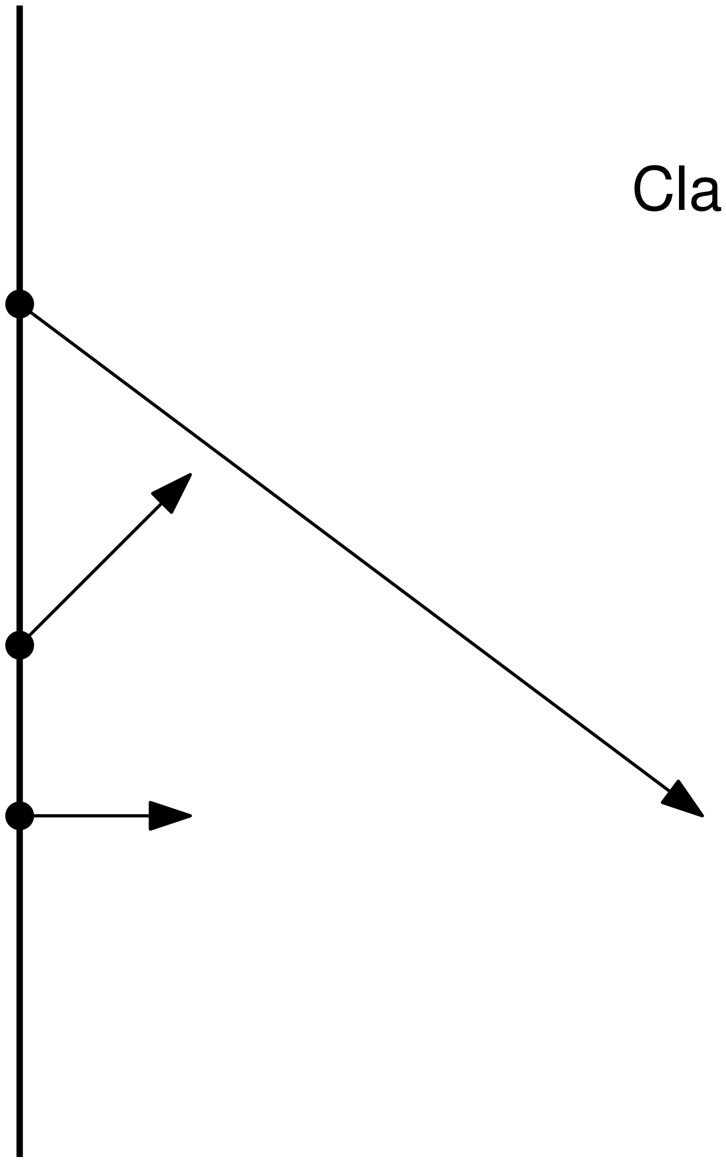
Starting Schedules

We are allowed to modify the starting time s_i of each bike b_i.

Claim: We can solve this instance with $f = 0$.



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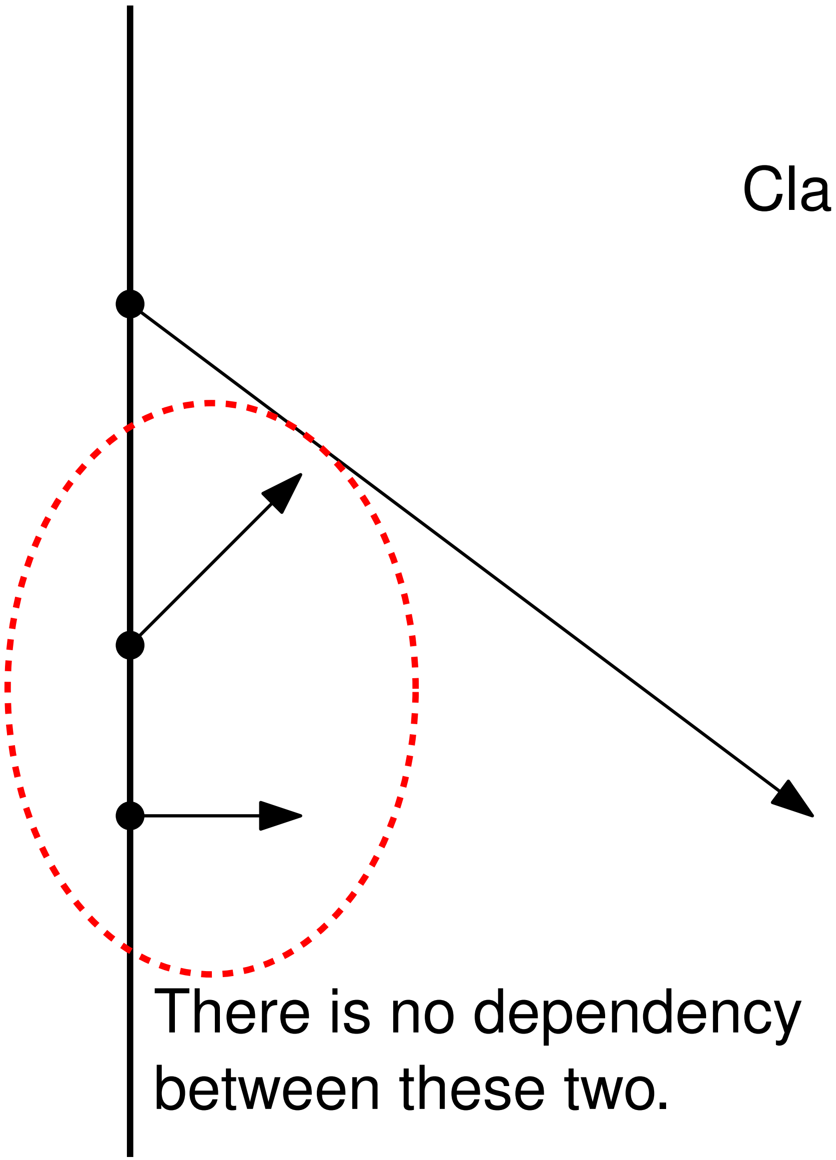

There is no dependency between these two.

Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Starting Schedules

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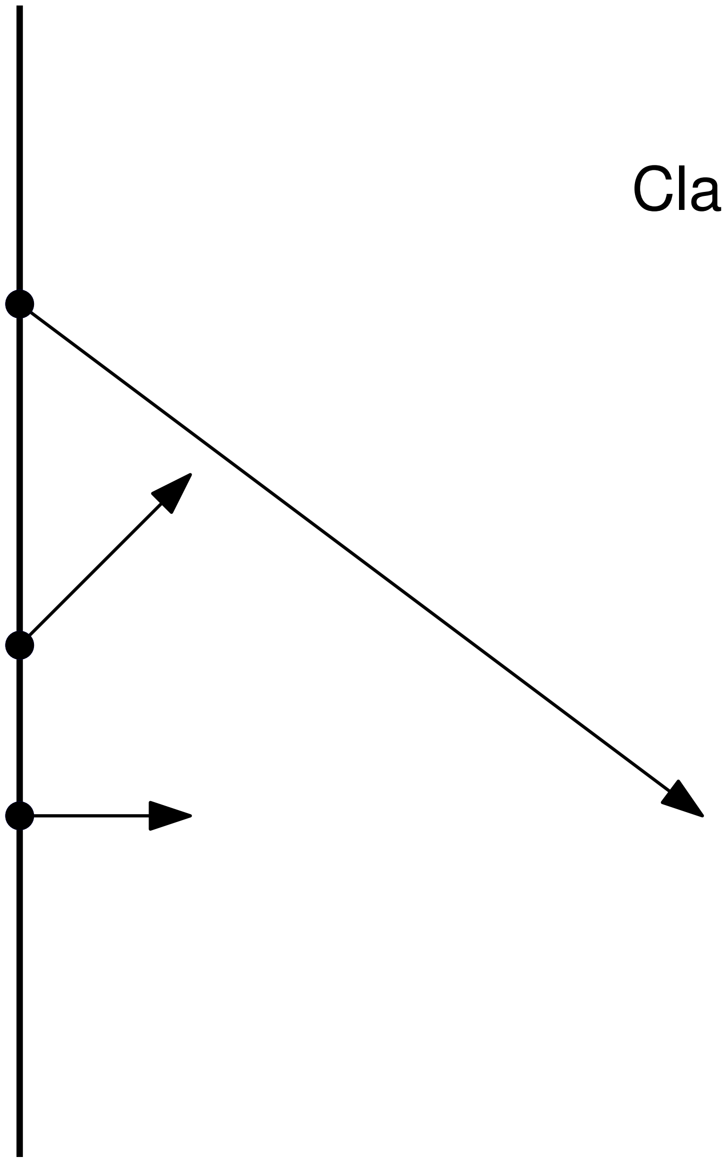

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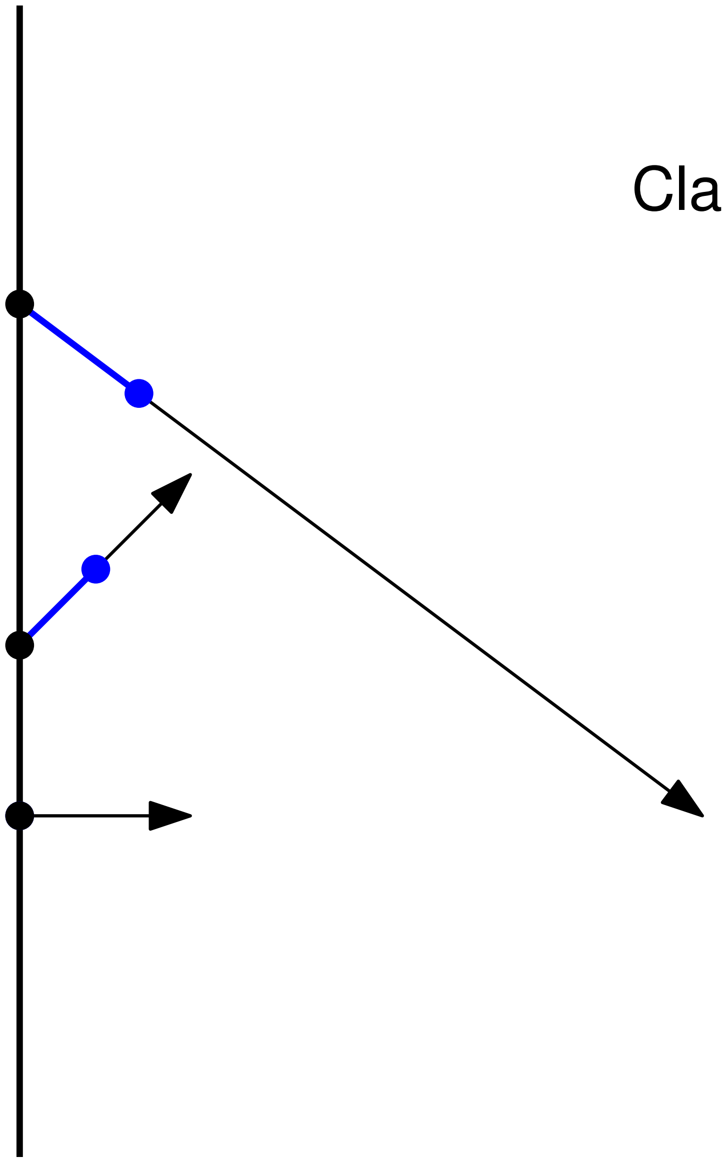

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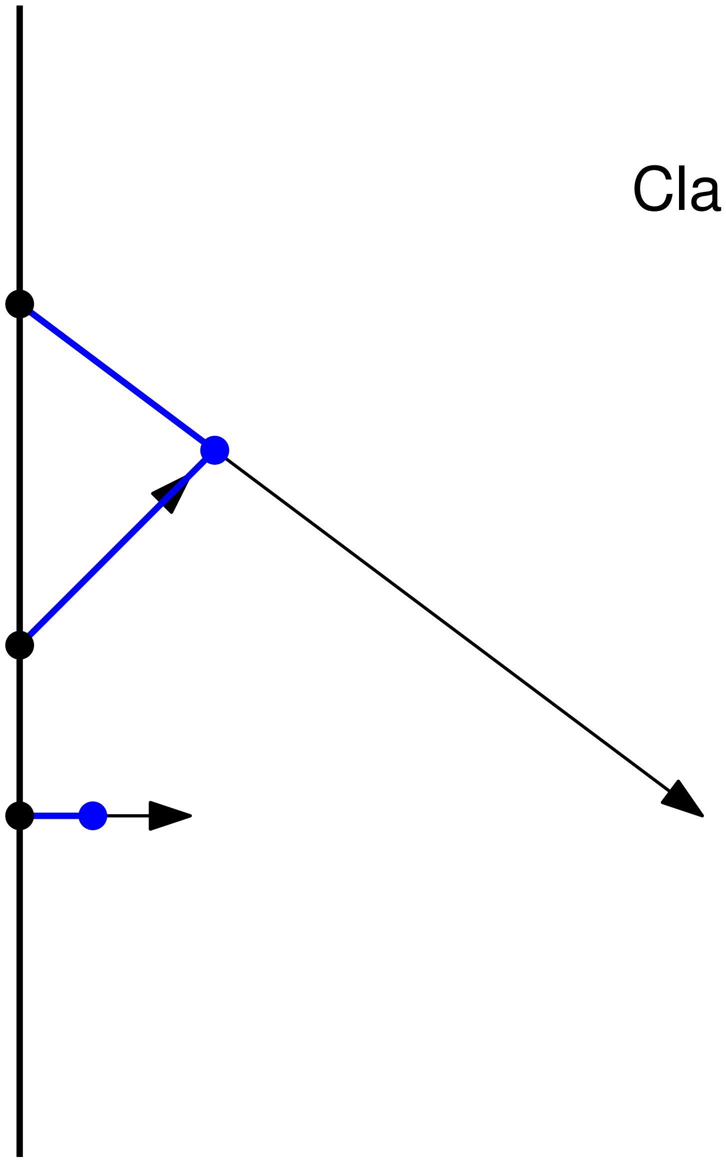

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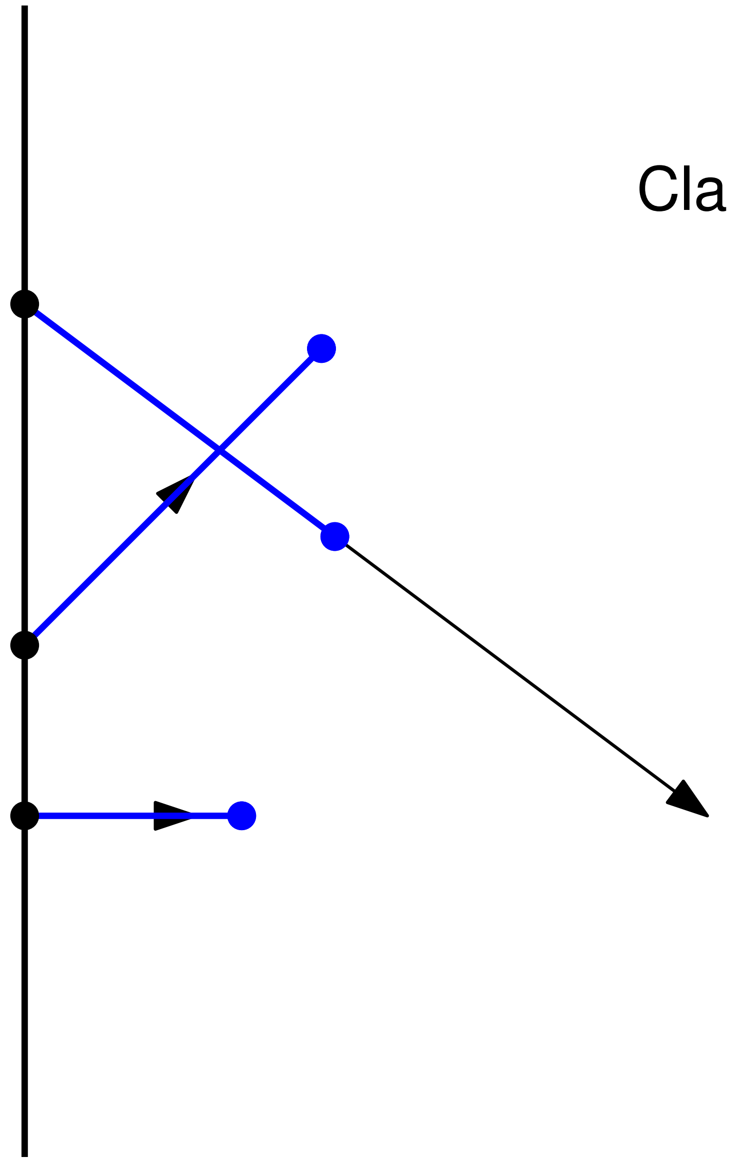

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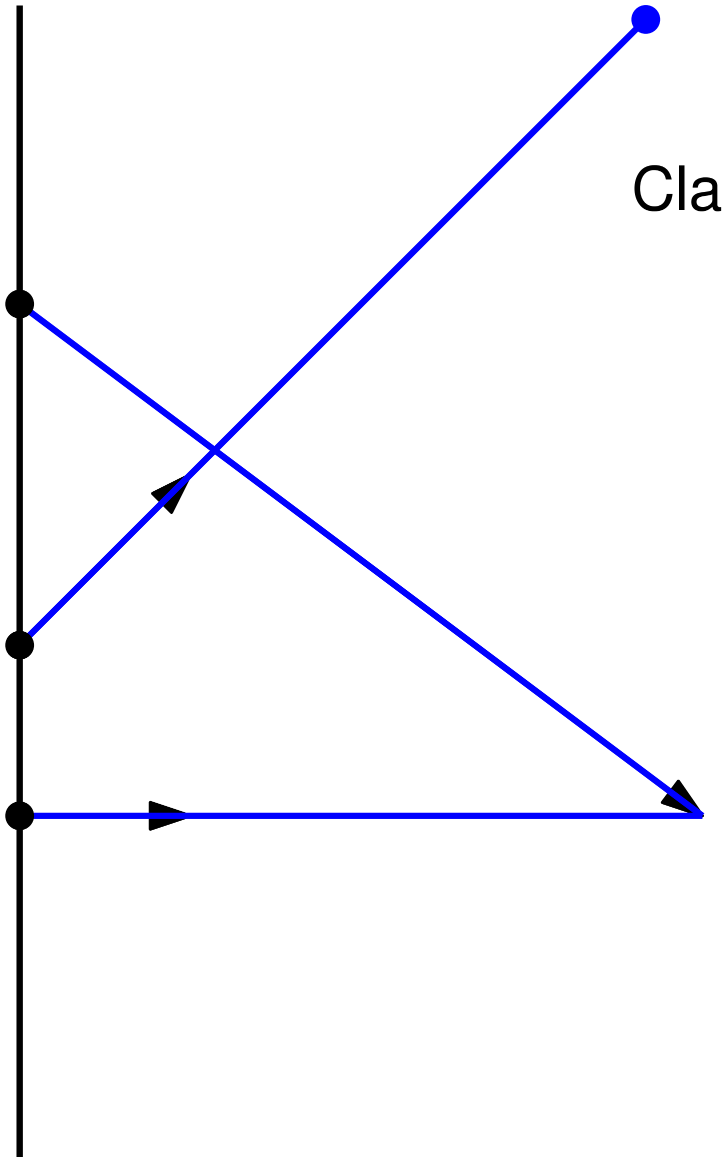

Claim: We can solve this instance with $f = 0$.

Frustration tolerance f: Whenever b_i meets the tracks of b_j, it stops unless b_j was there no more than f time units ago.

Solving it with Linear Programming

What are the variables?

Solving it with Linear Programming

What are the variables?

- Starting time s_i of each biker
- Frustration tolerance f

Solving it with Linear Programming

What are the variables?

- Starting time s_i of each biker
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From the specification, we know there are at most 101 variables, which is still OK for linear programming.

- It starts with a line that contains a single integer `n` so that $1 \leq n \leq 10^2$. Here n denotes the number of bikers.

Solving it with Linear Programming

What are the variables?

- Starting time s_i of each biker
- Frustration tolerance f

What are the constraints?

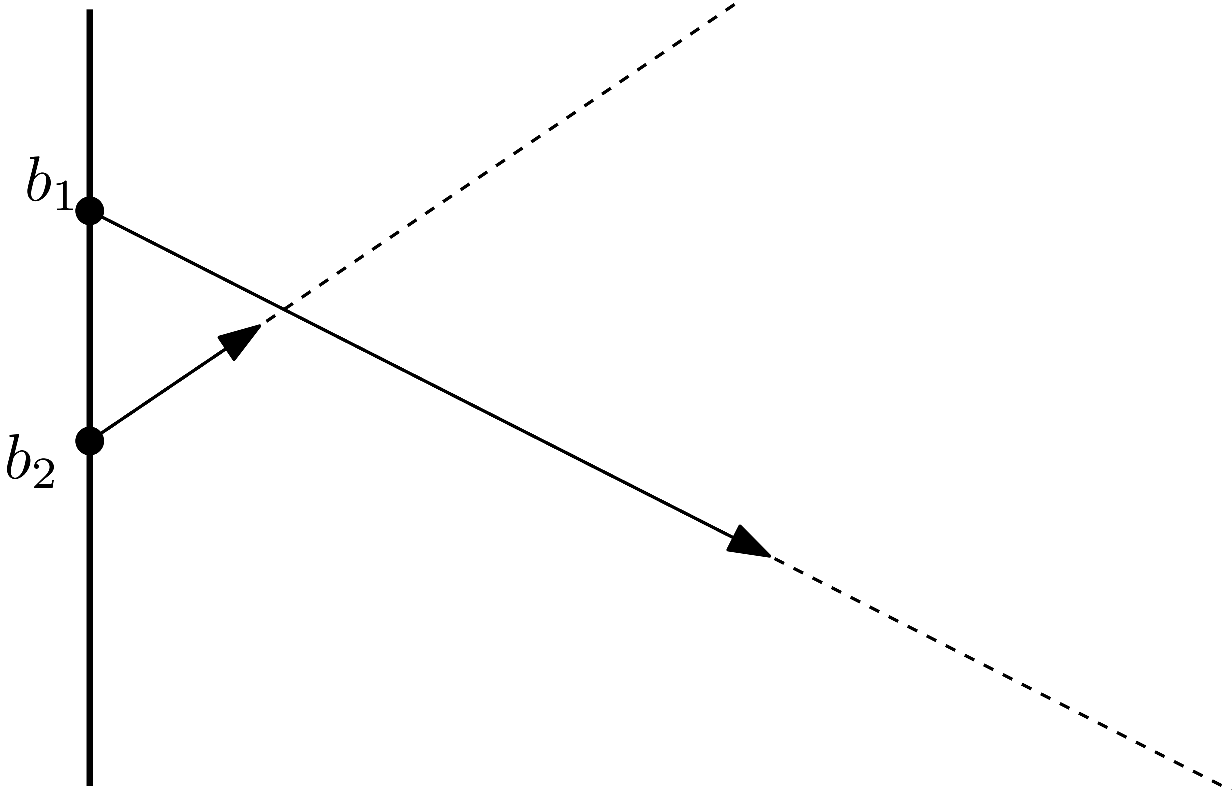
Solving it with Linear Programming

What are the variables?

- Starting time s_i of each biker
- Frustration tolerance f

One constraint for each pair b_i, b_j with crossing paths.

What are the constraints?



Solving it with Linear Programming

What are the variables?

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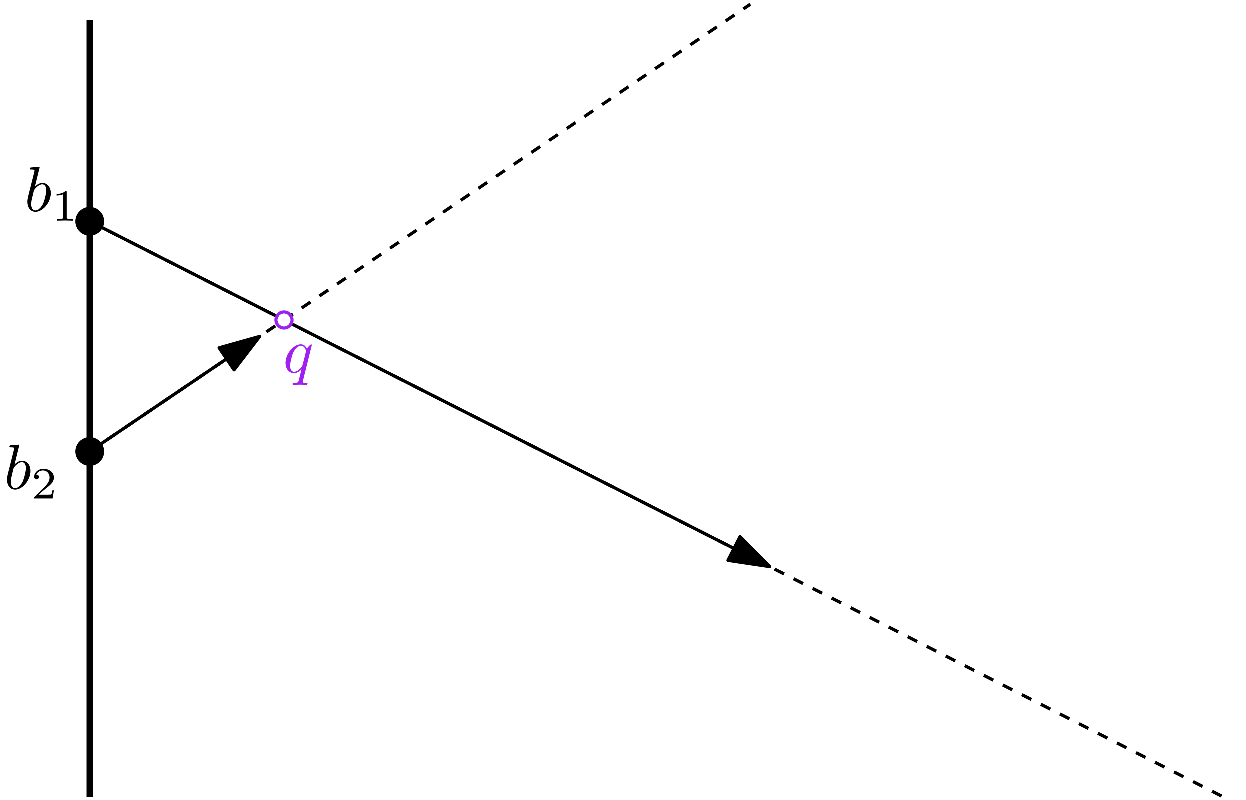


One constraint for each pair b_i, b_j with crossing paths.

Solving it with Linear Programming

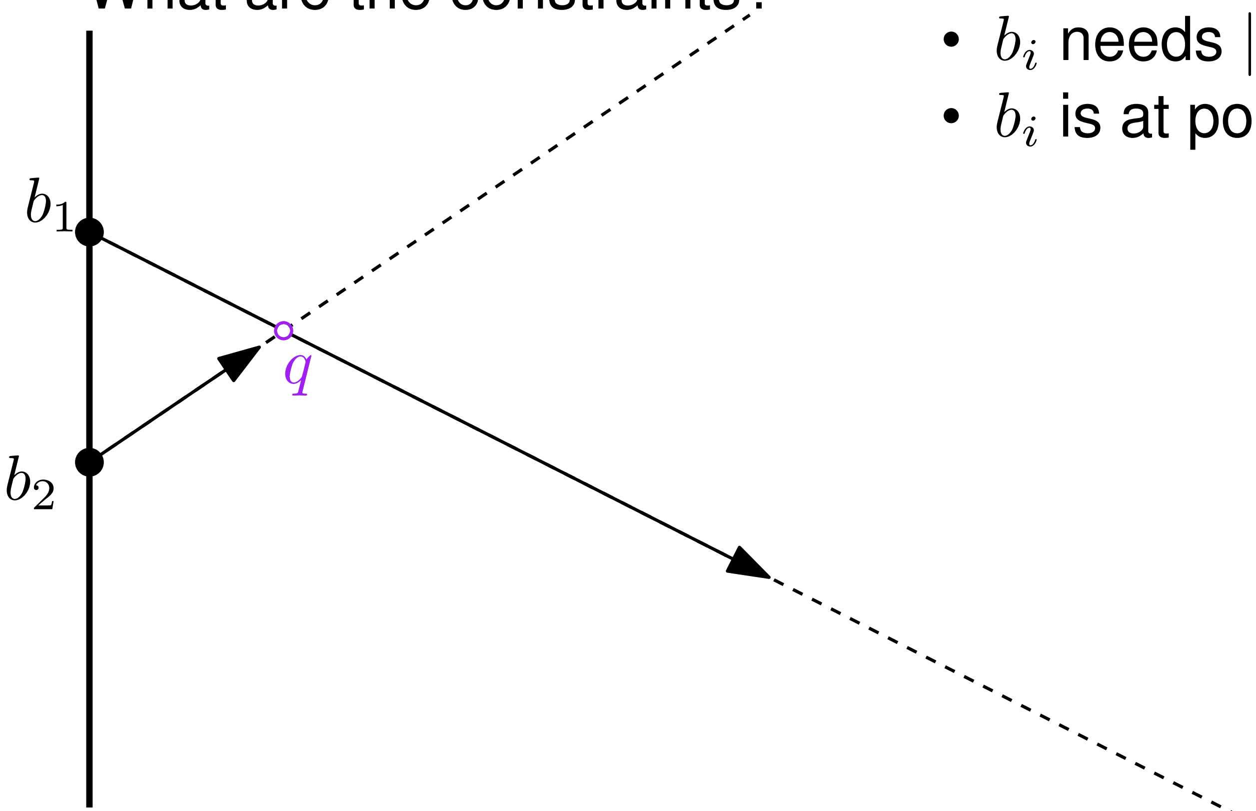
What are the variables?

- Starting time s_i of each biker
- Frustration tolerance f

What are the constraints?

One constraint for each pair b_i, b_j with crossing paths.

- b_i needs $||b_i - q||$ time to reach q.
- b_i is at position q at time $s_i + ||b_i - q||$.



Solving it with Linear Programming

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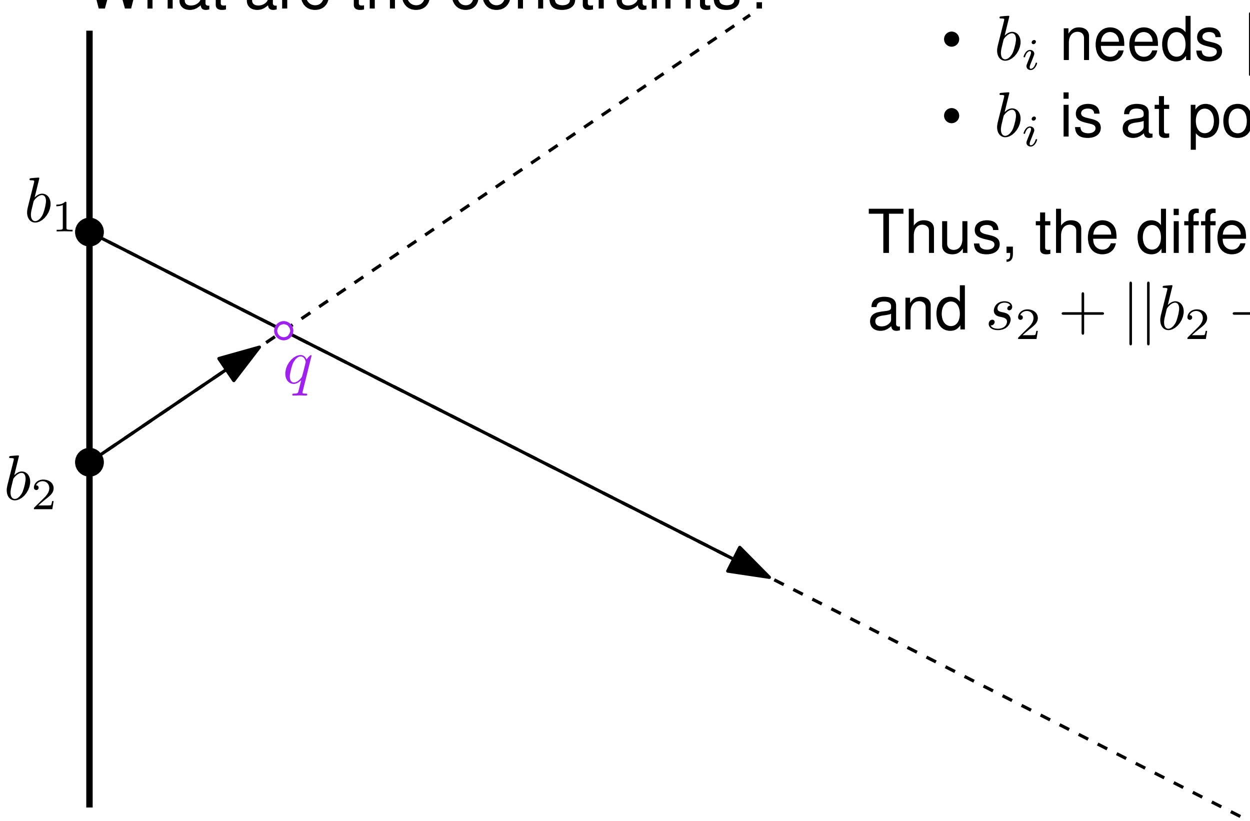


One constraint for each pair b_i, b_j with crossing paths.

- b_i needs $||b_i - q||$ time to reach q.
- b_i is at position q at time $s_i + ||b_i - q||$.

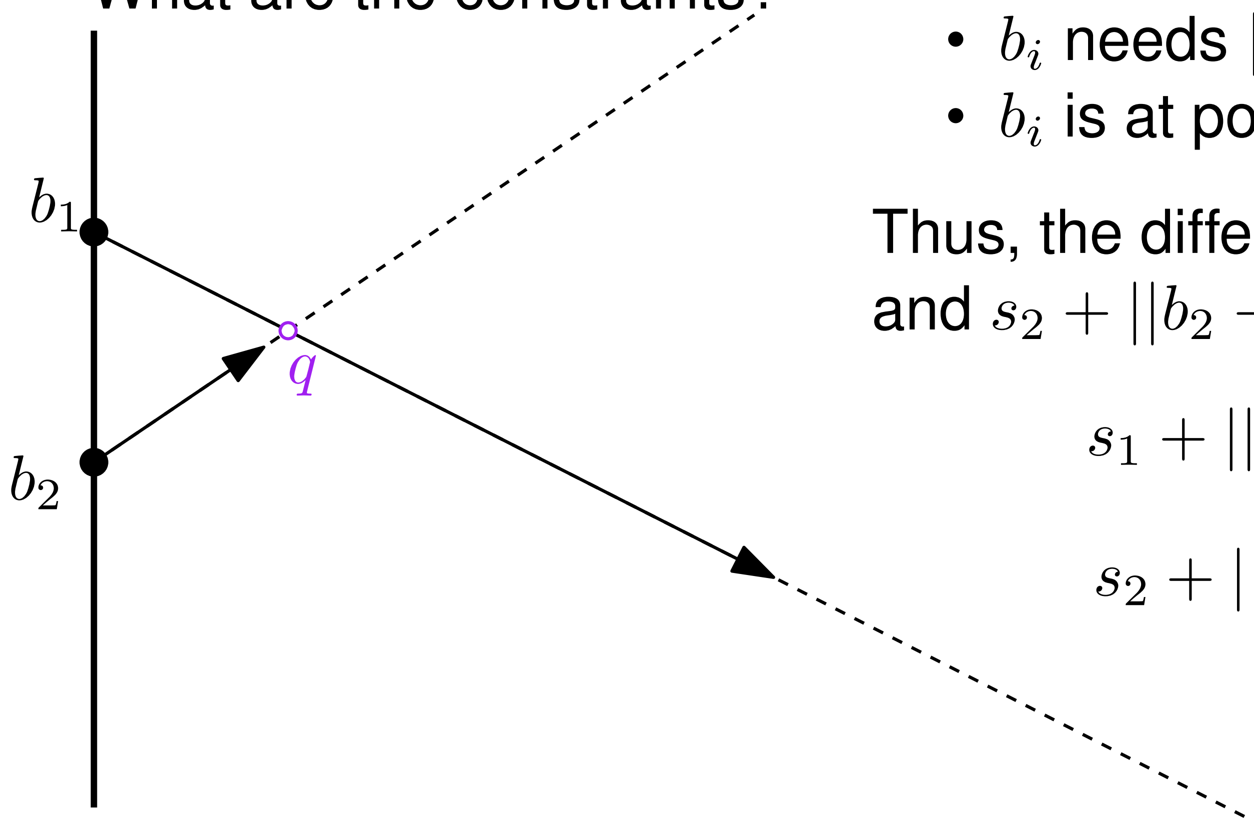
Thus, the difference between $s_1 + ||b_1 - q||$ and $s_2 + ||b_2 - q||$ can be at most f.

Solving it with Linear Programming

What are the variables?

- Starting time s_i of each biker
- Frustration tolerance f

What are the constraints?



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$$s_1 + ||b_1 - q|| \leq s_2 + ||b_2 - q|| + f,$$

$$s_2 + ||b_2 - q|| \leq s_1 + ||b_1 - q|| + f$$

Solving it with Linear Programming

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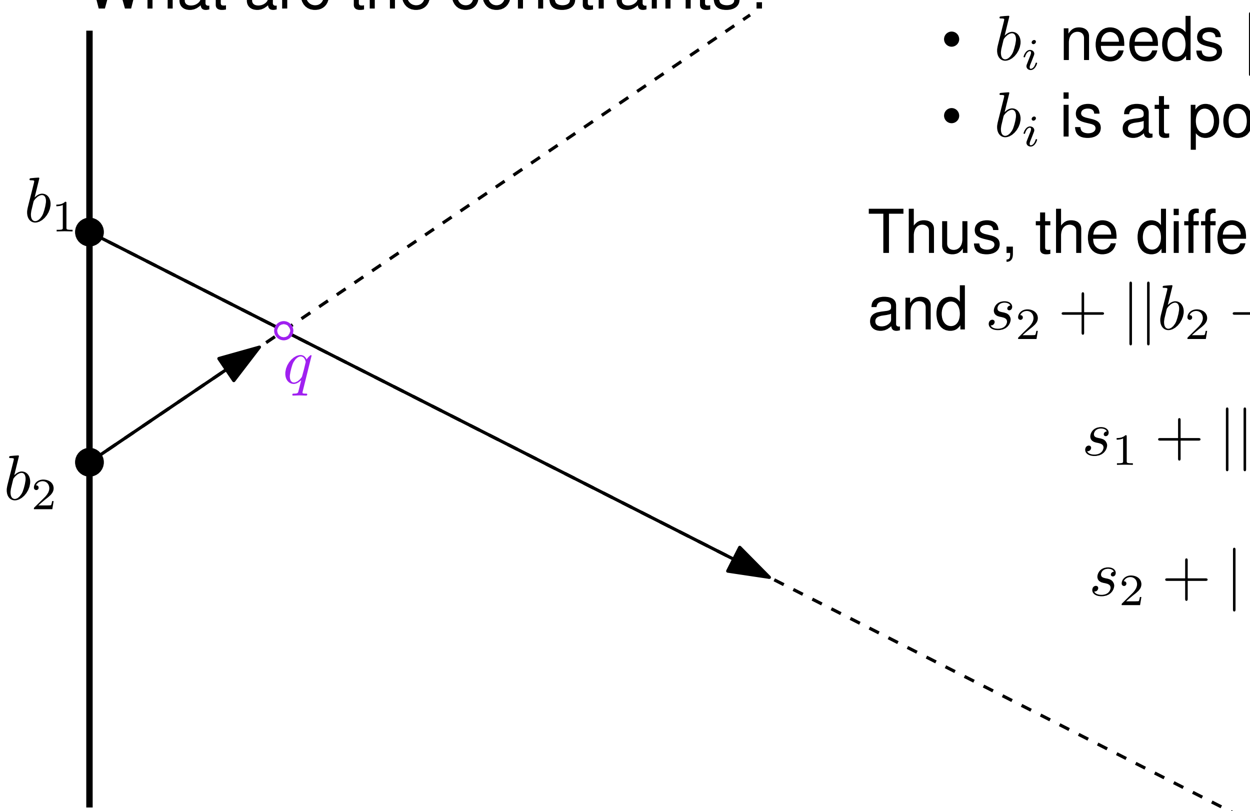

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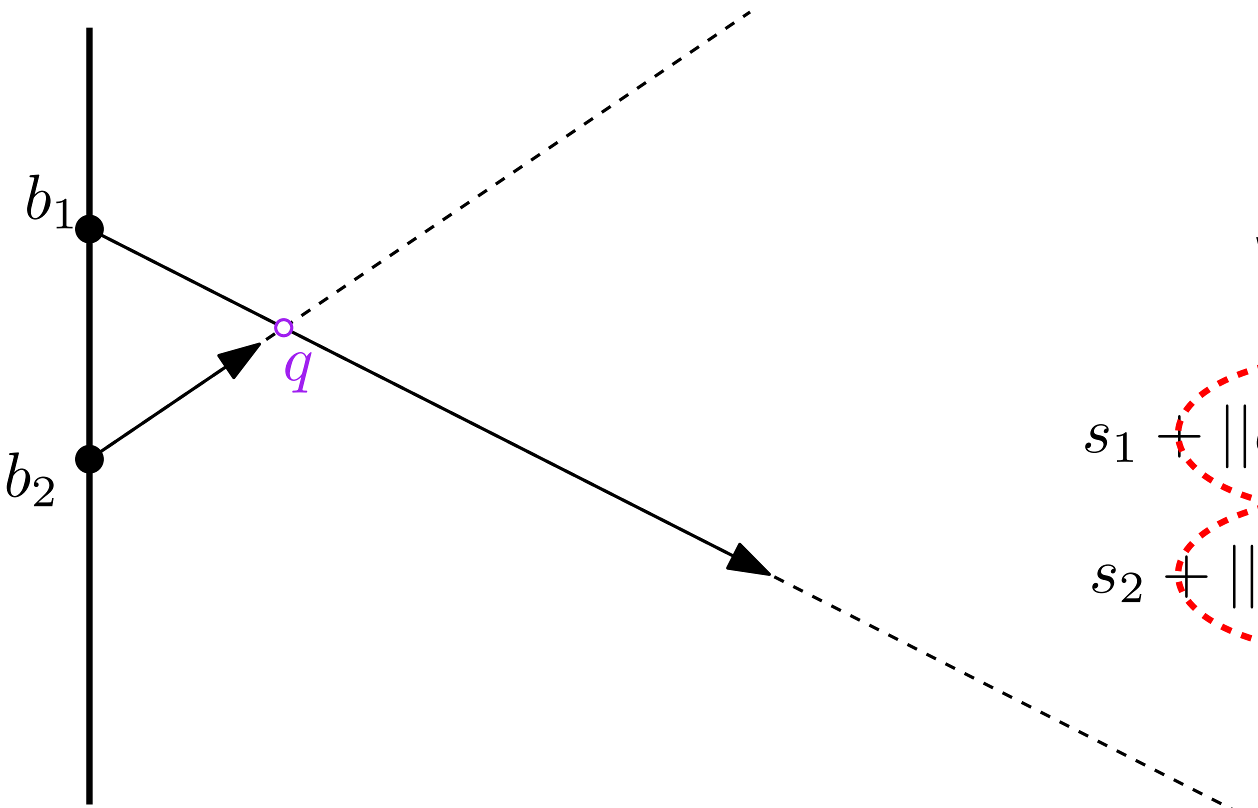
$$s_2 + ||b_2 - q|| \leq s_1 + ||b_1 - q|| + f$$

$O(n^2)$ constraints, which is at most 10,000.

Solving it with Linear Programming

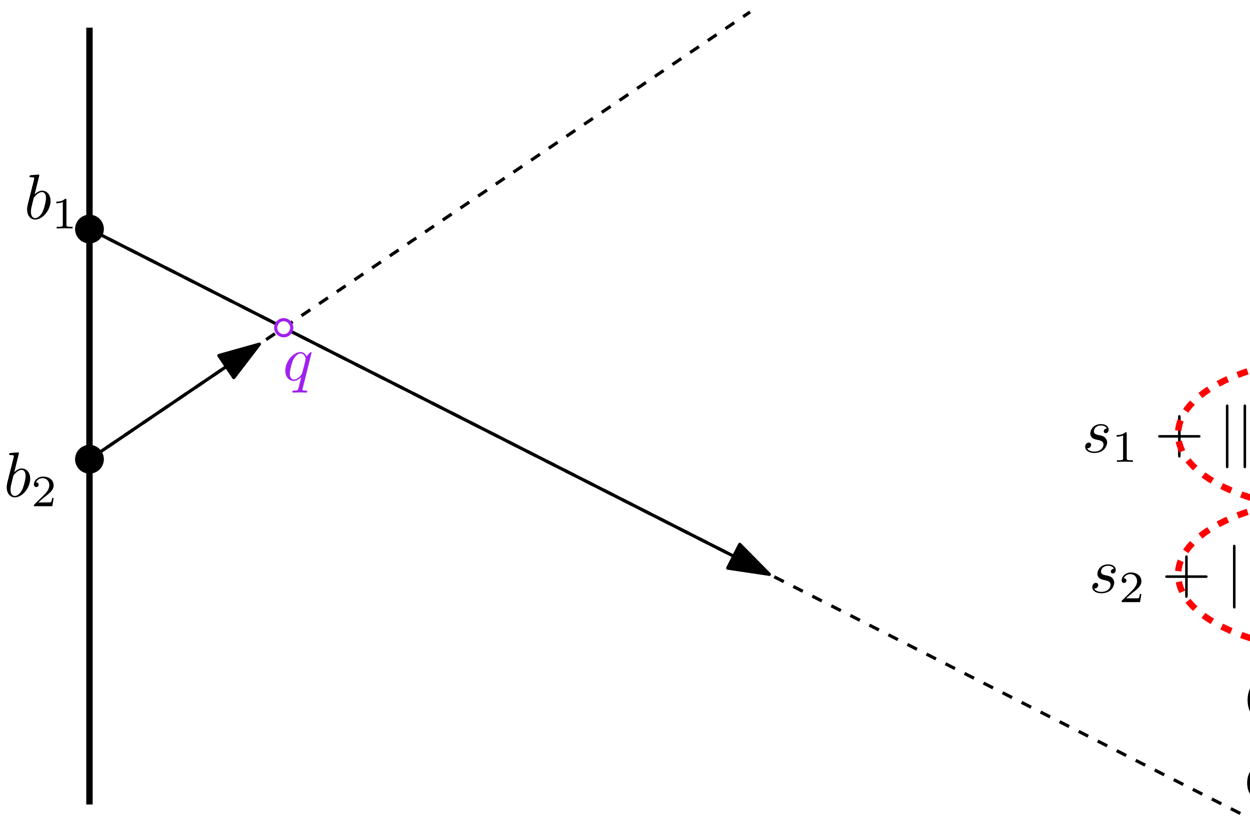


One constraint for each pair b_i, b_j with crossing paths.

What are these terms?

$$s_1 + ||b_1 - q|| \leq s_2 + ||b_2 - q|| + f,$$
$$s_2 + ||b_2 - q|| \leq s_1 + ||b_1 - q|| + f$$

Solving it with Linear Programming



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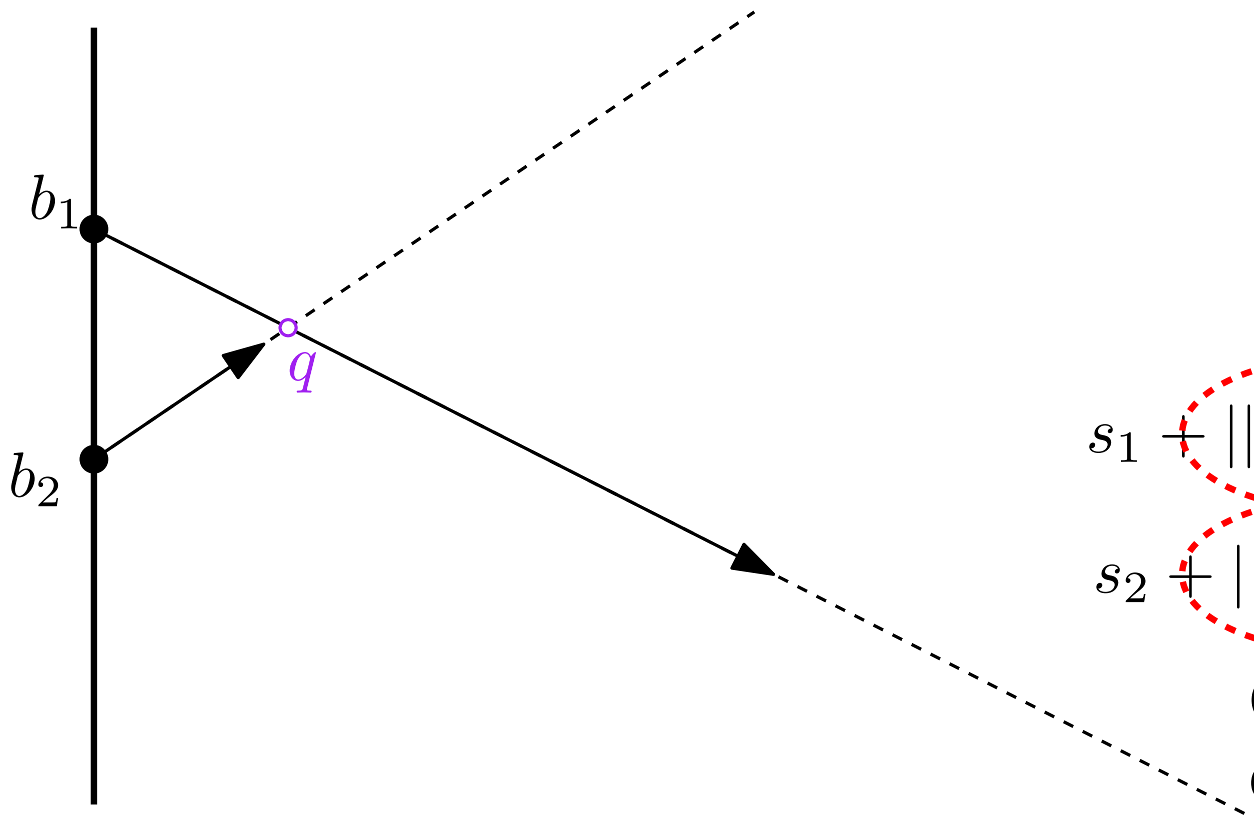
One needs square roots to compute these constants.

Solving it with Linear Programming

In addition, all variables are non-negative.

One constraint for each pair b_i, b_j with crossing paths.



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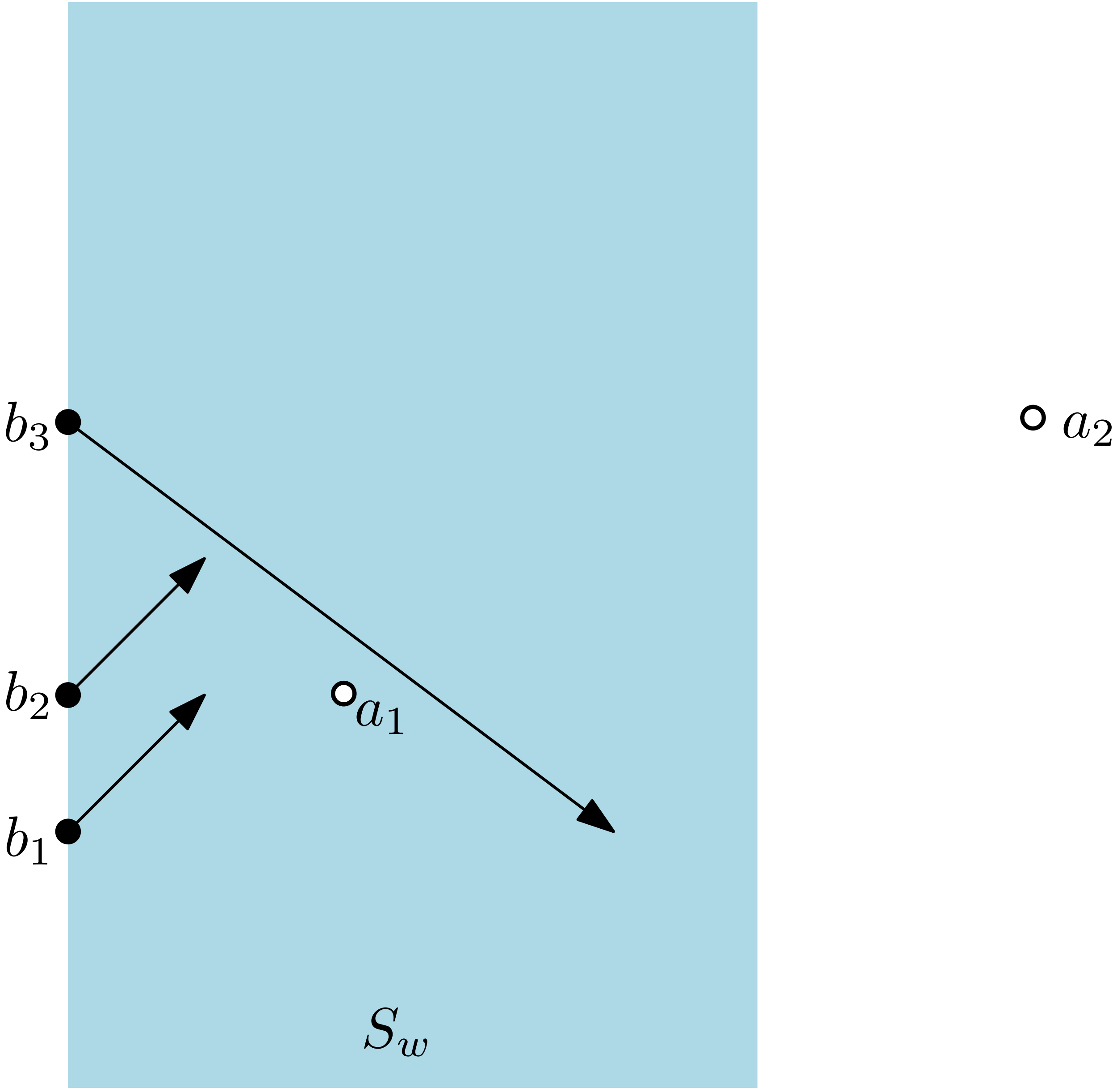
Beethoven



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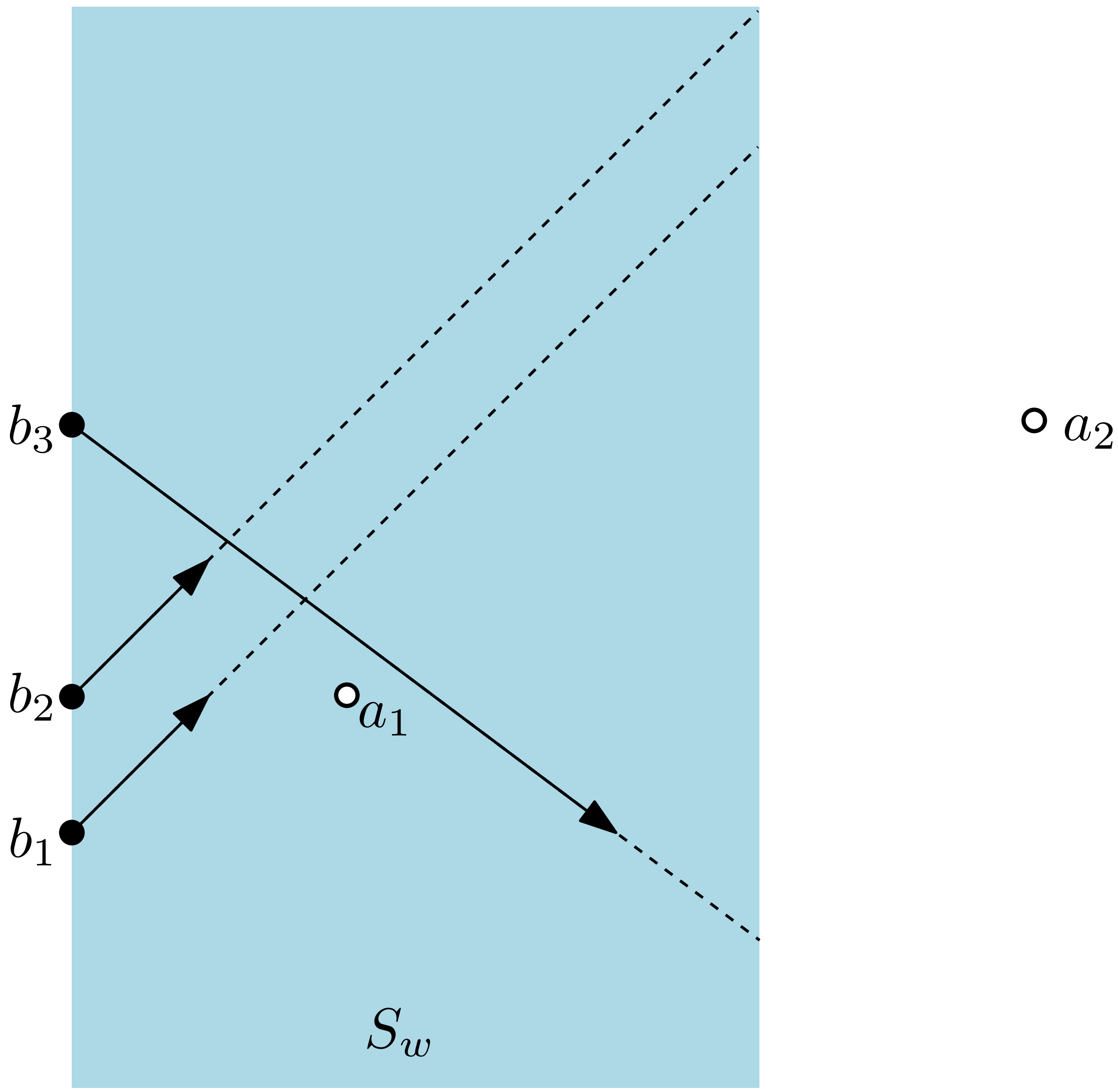

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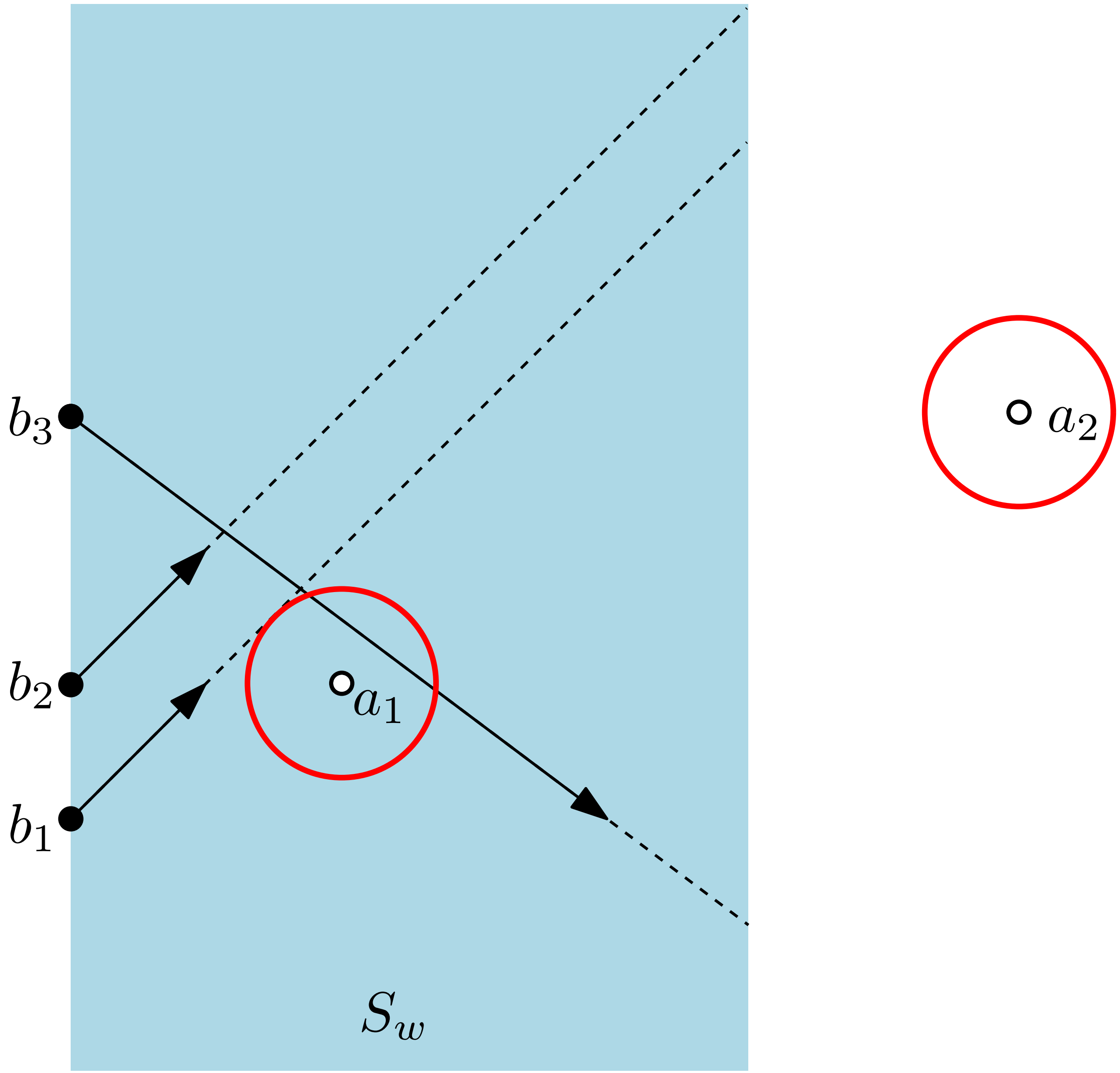

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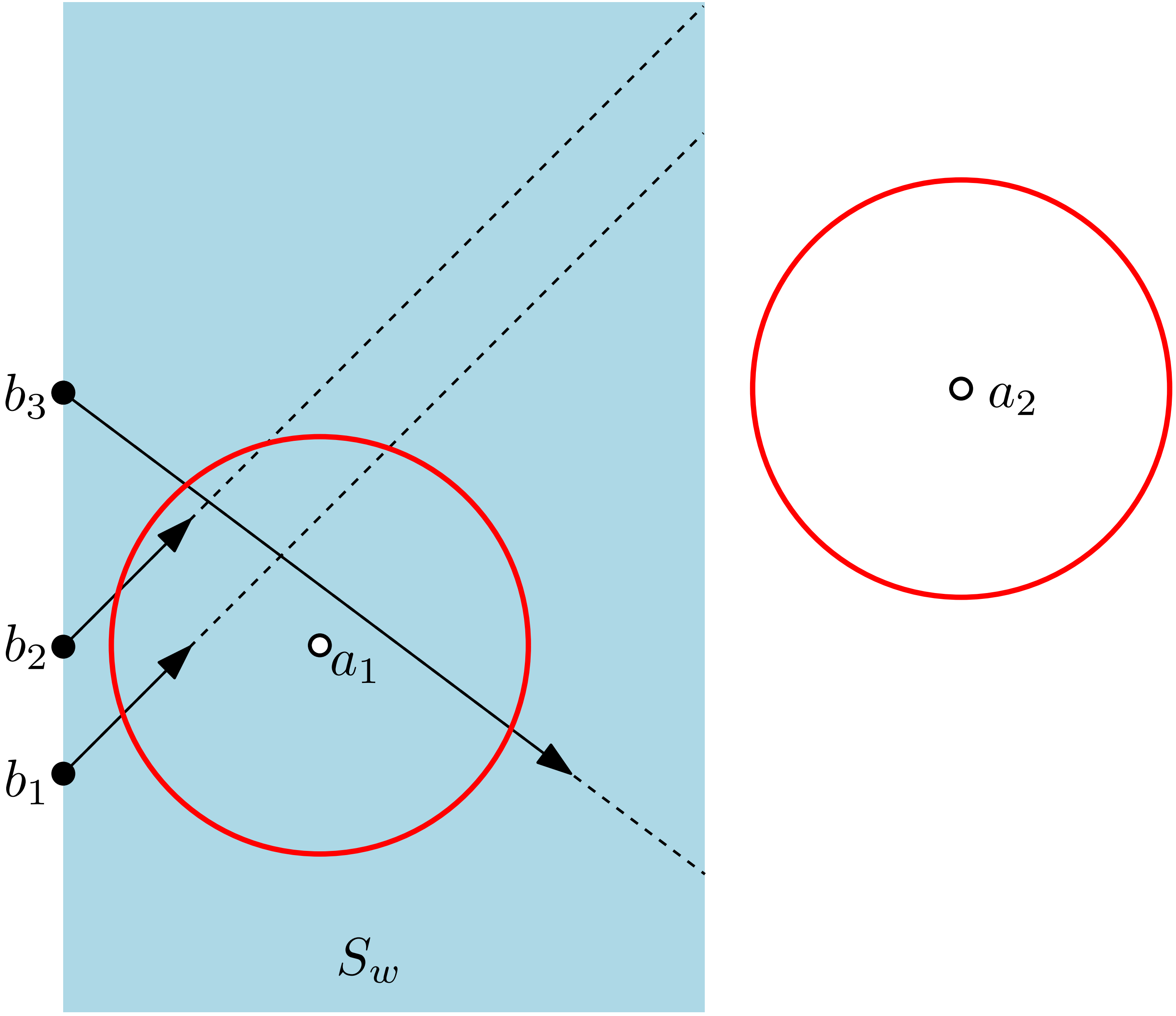

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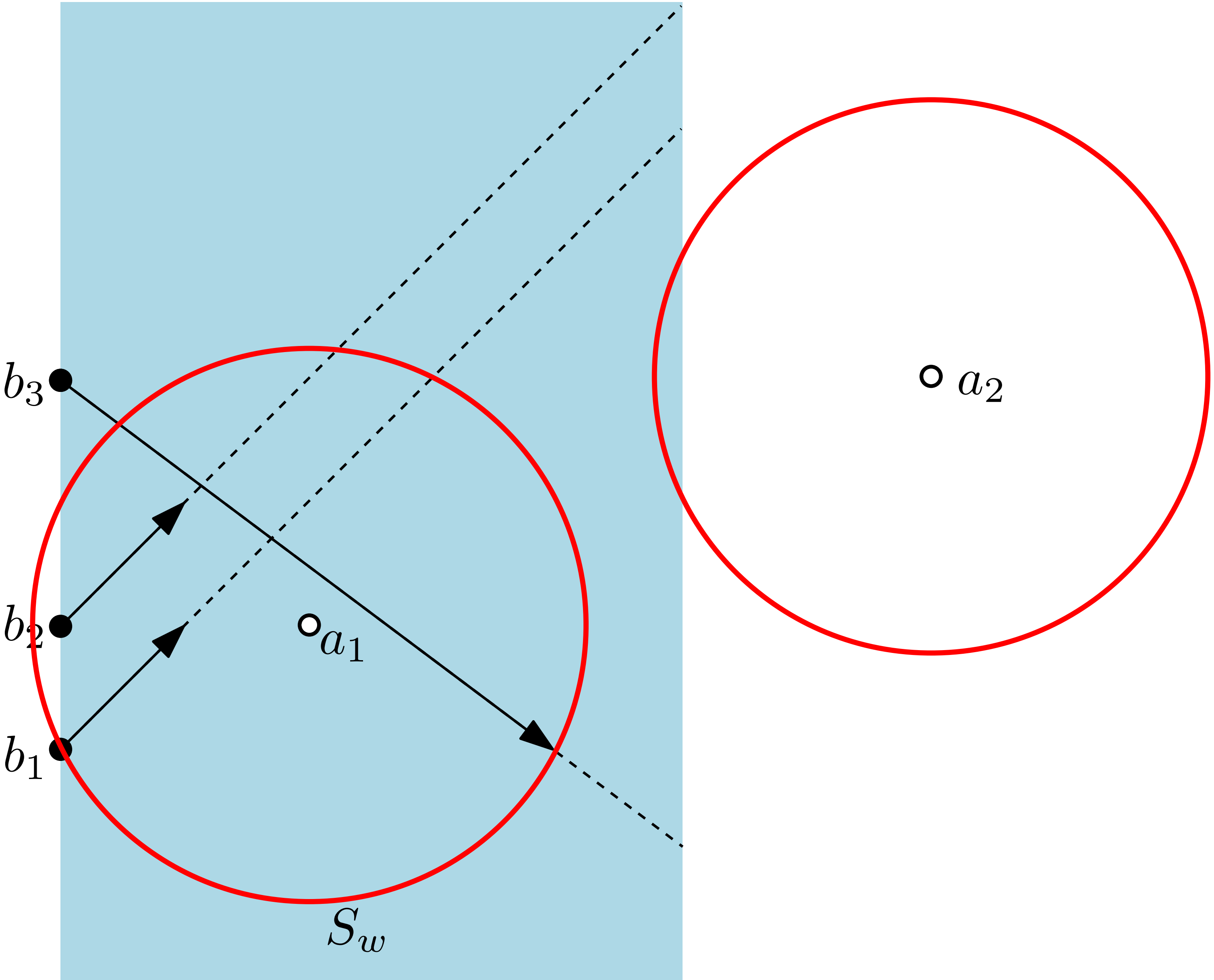

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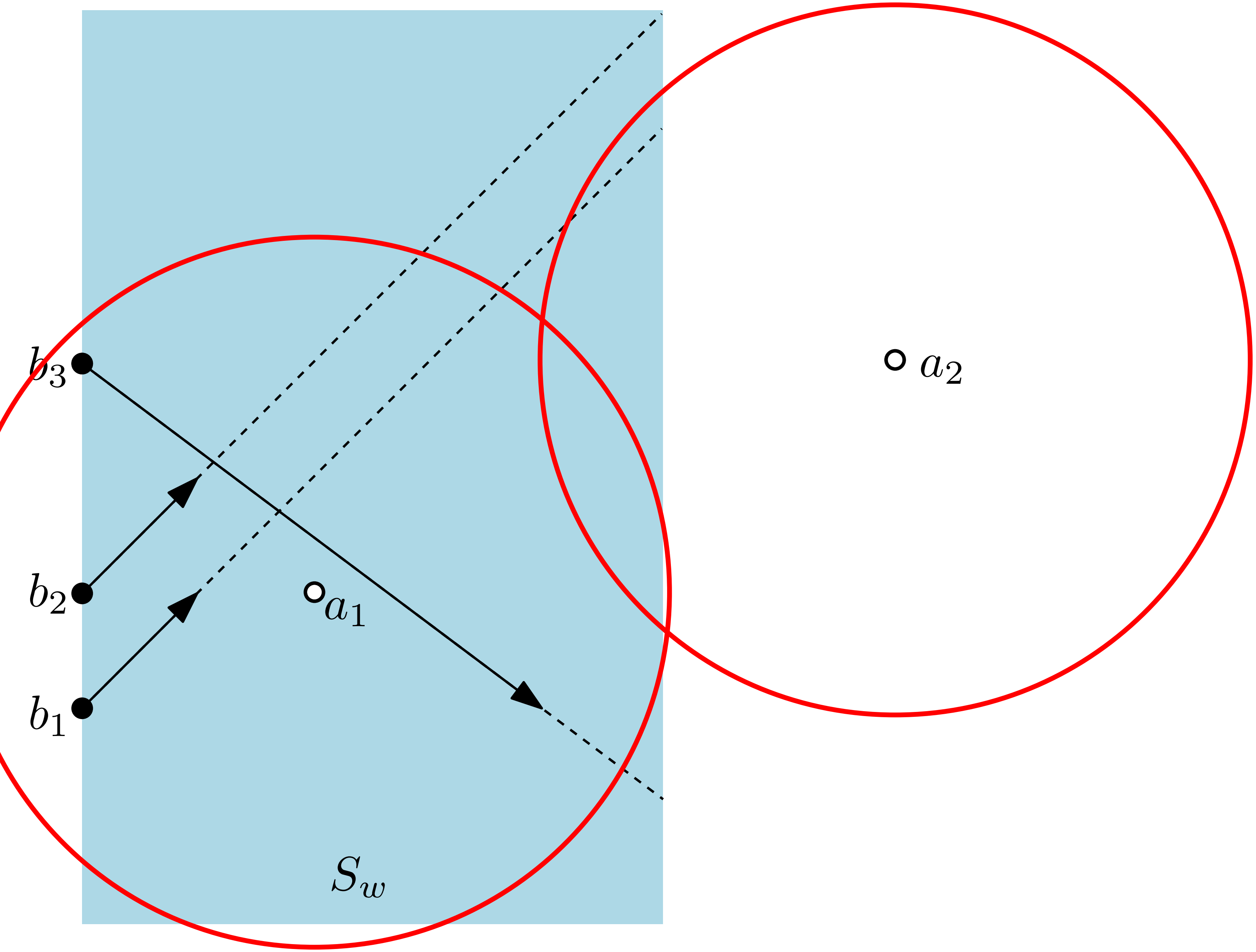

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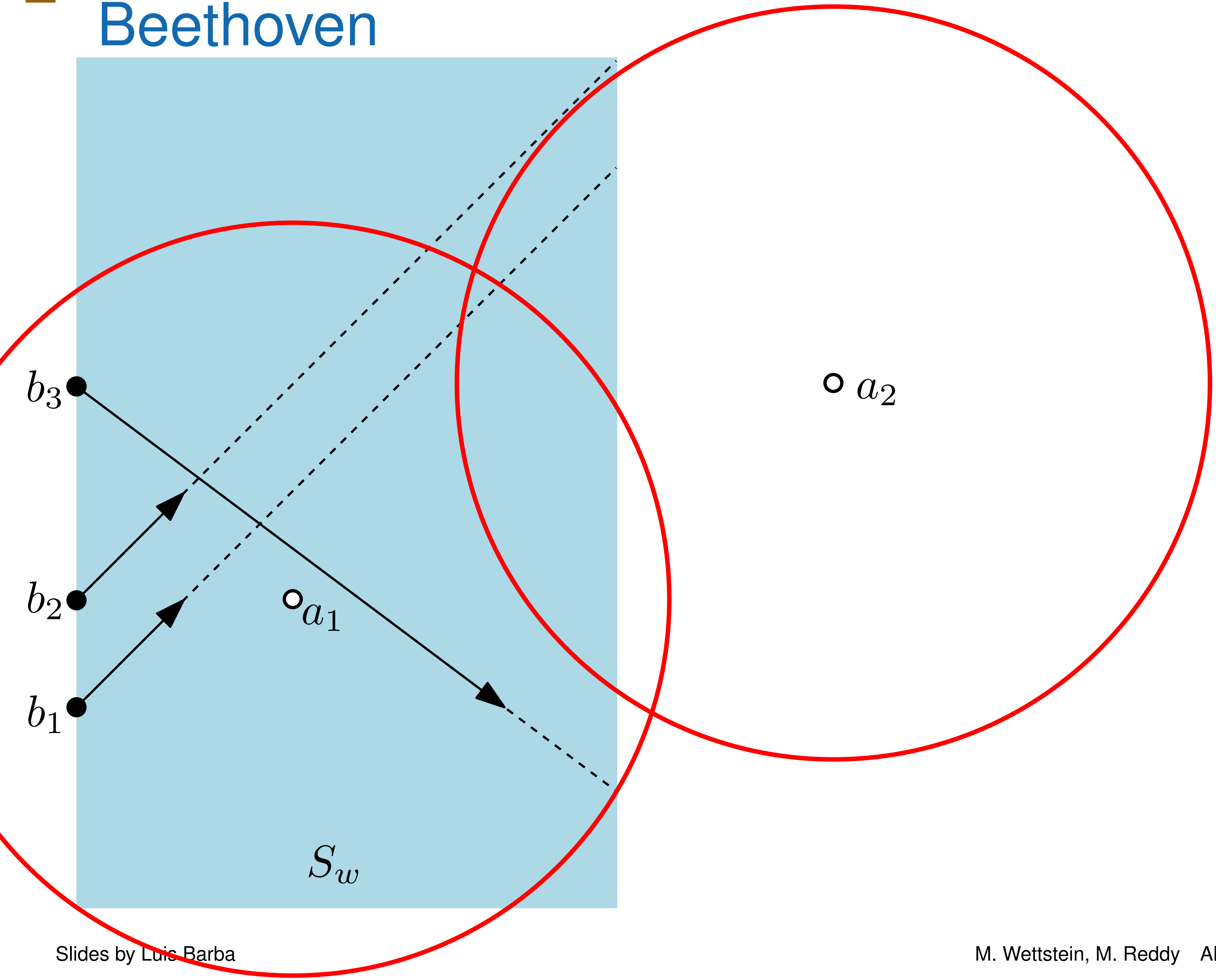

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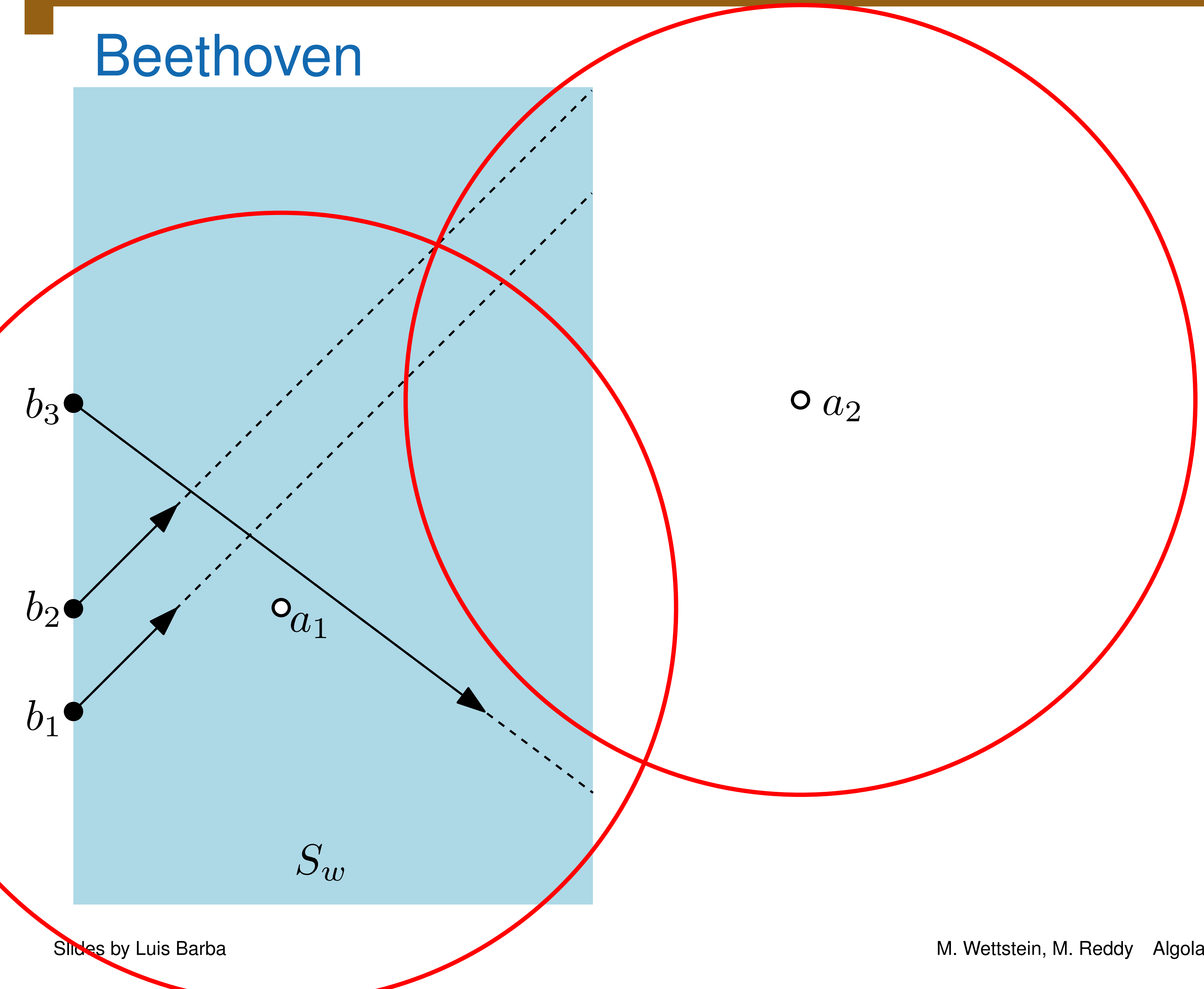
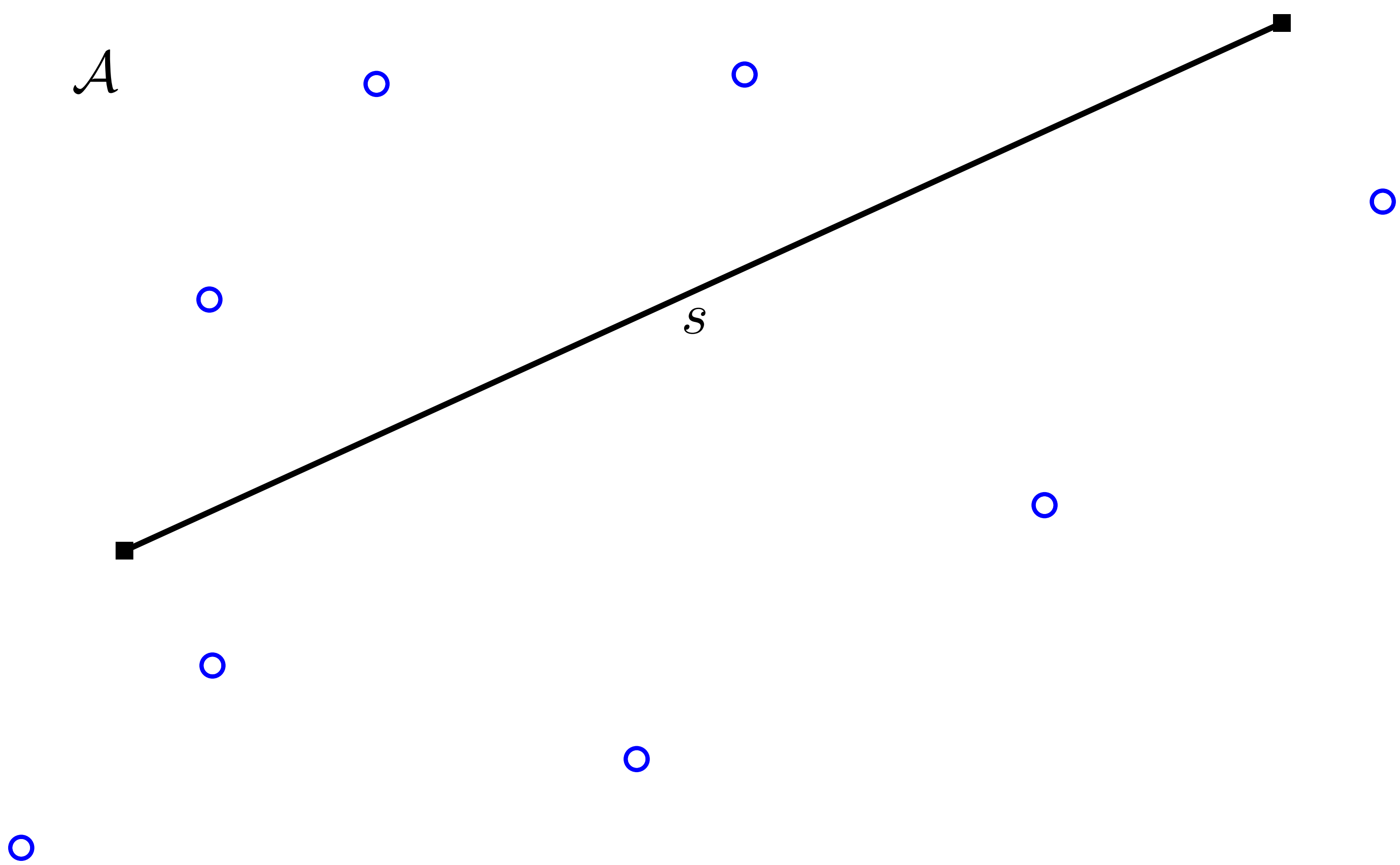


Working with a single segment



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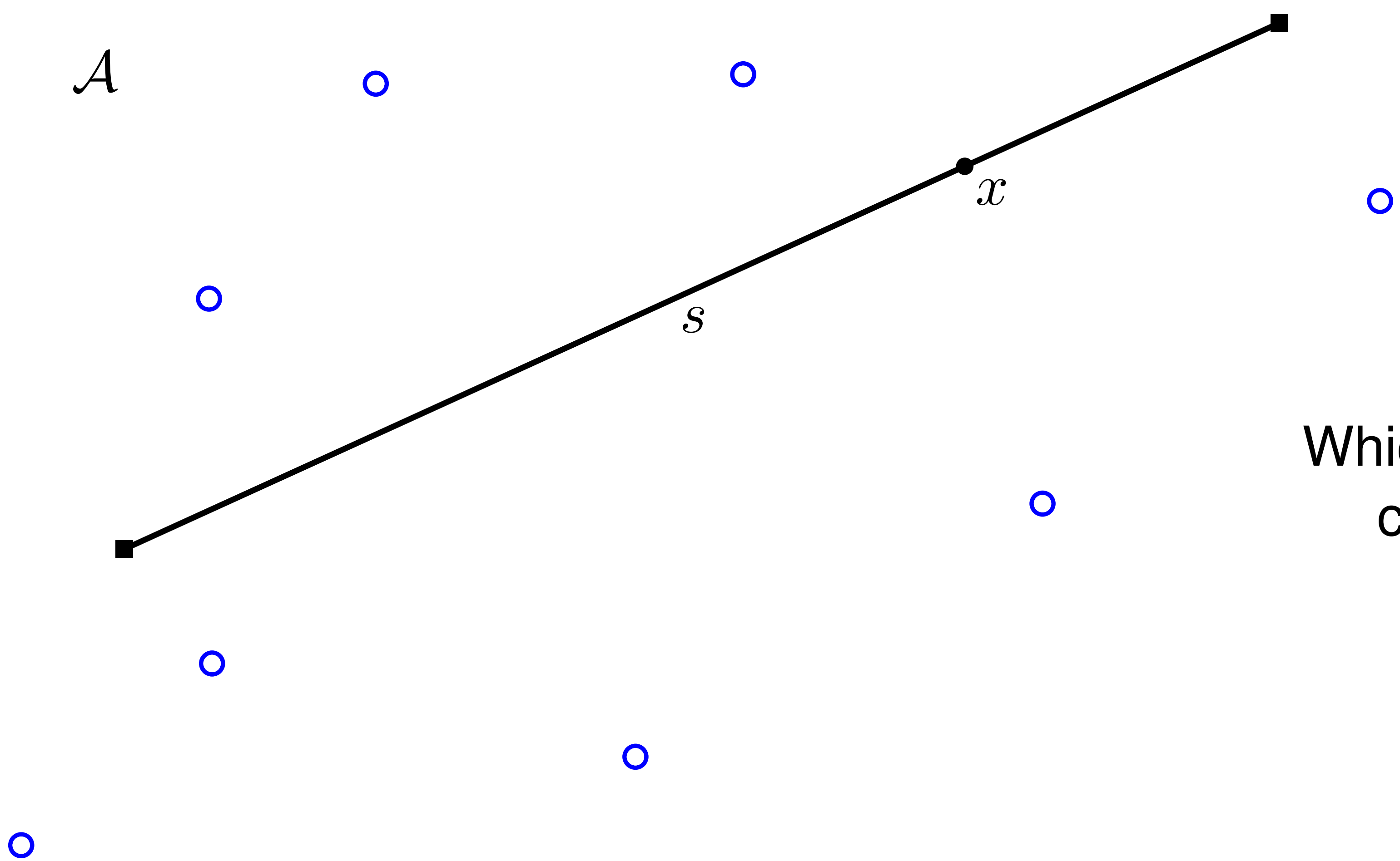
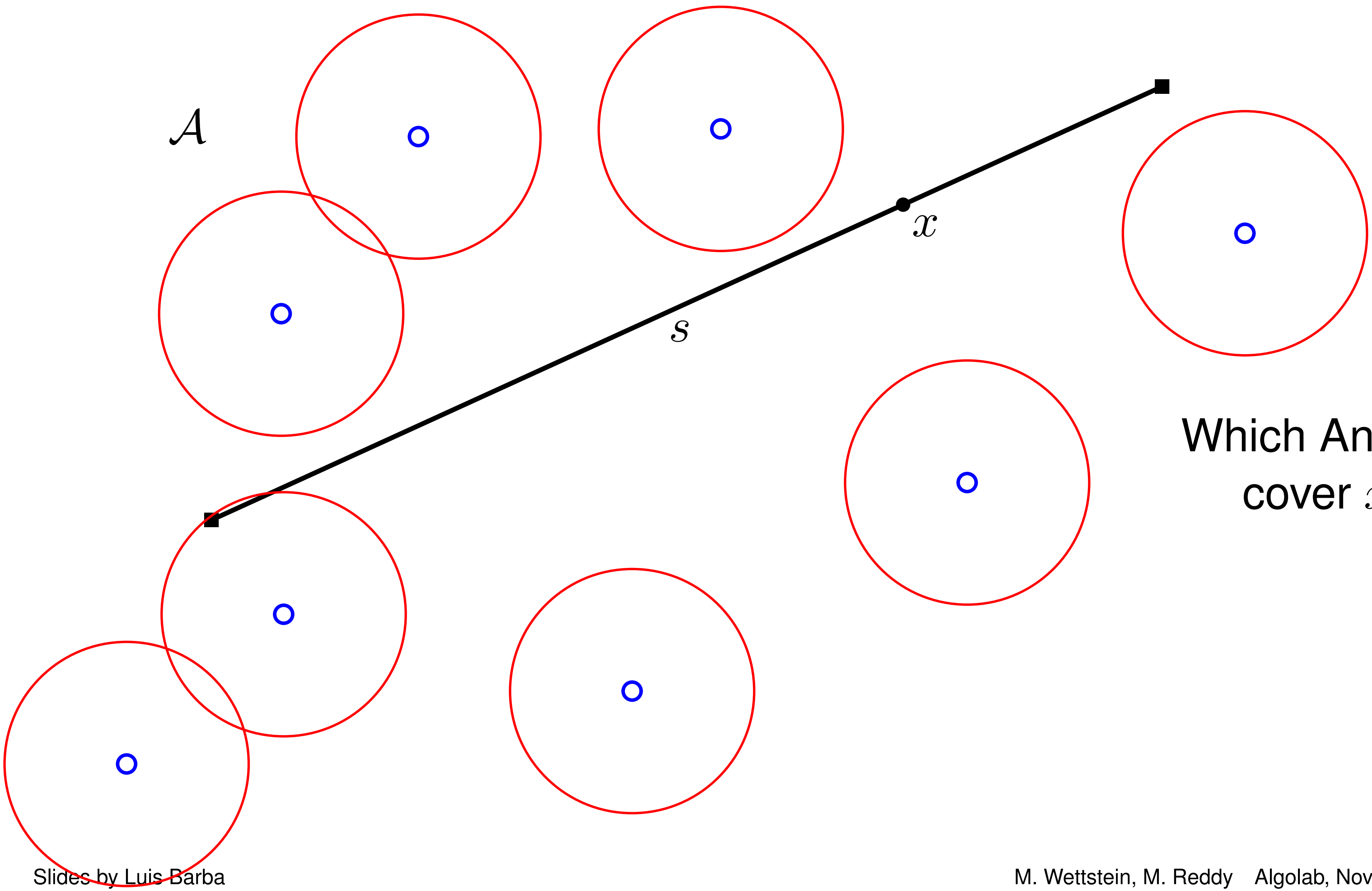


Which Antenna will cover x first?

Working with a single segment



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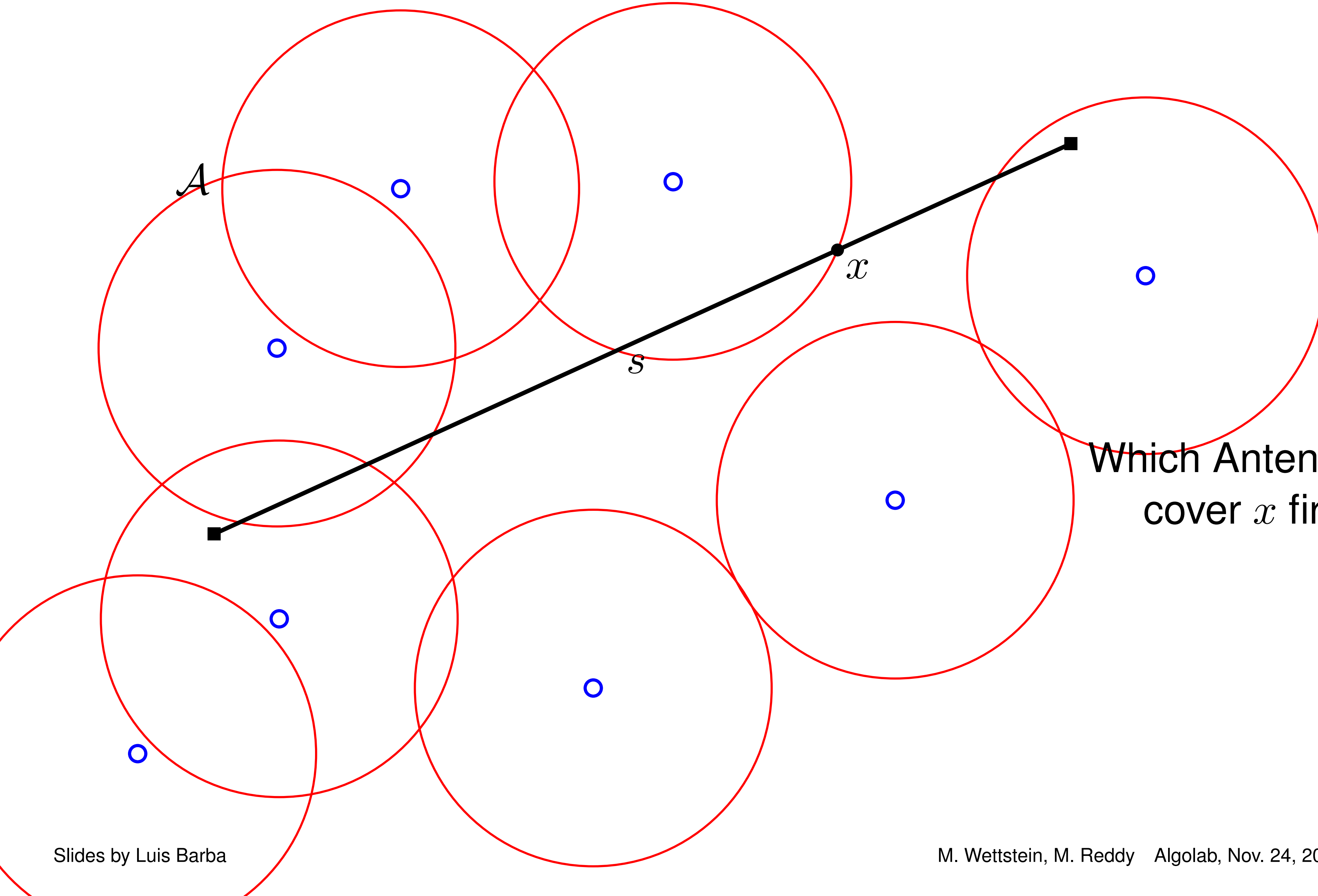

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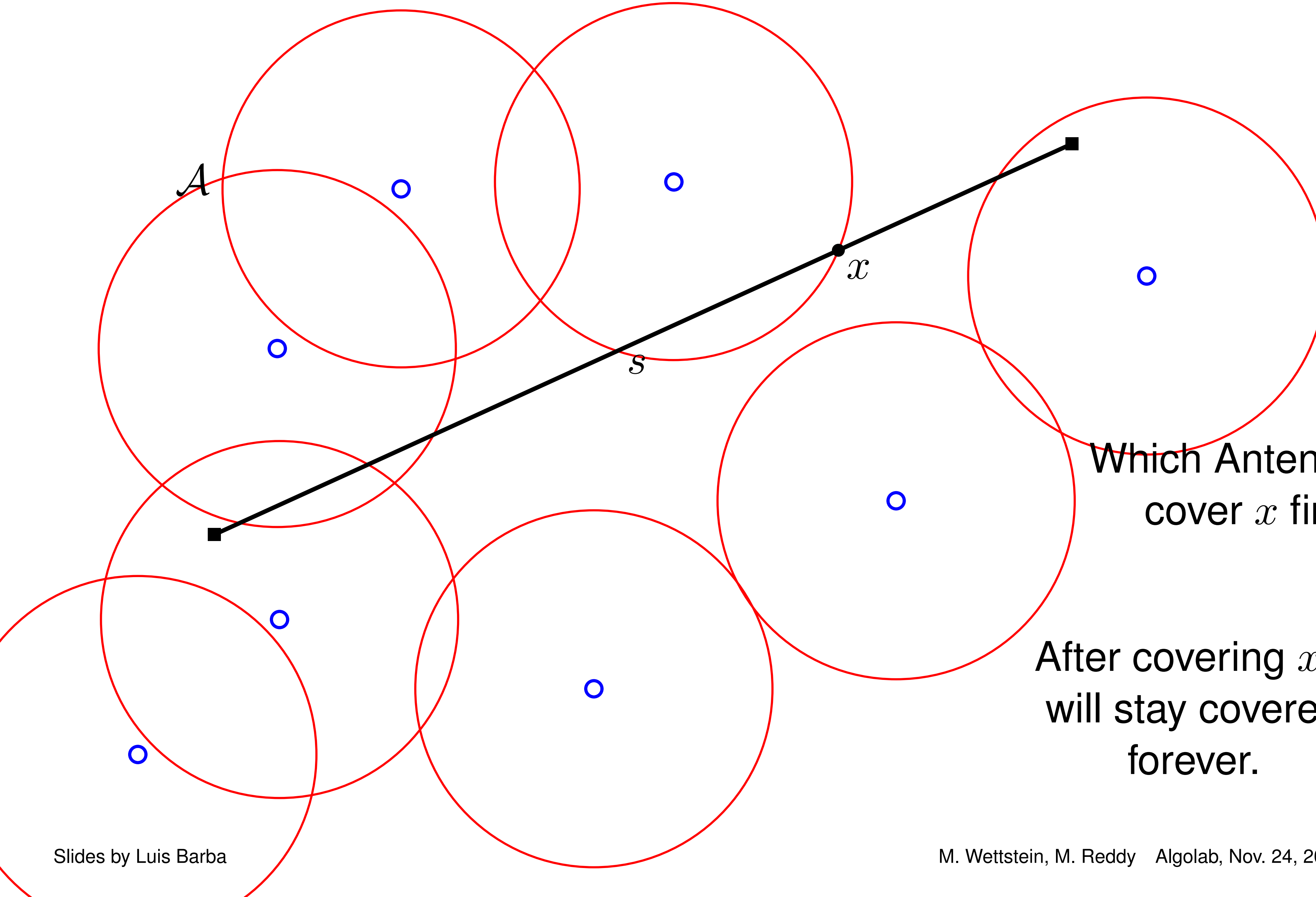
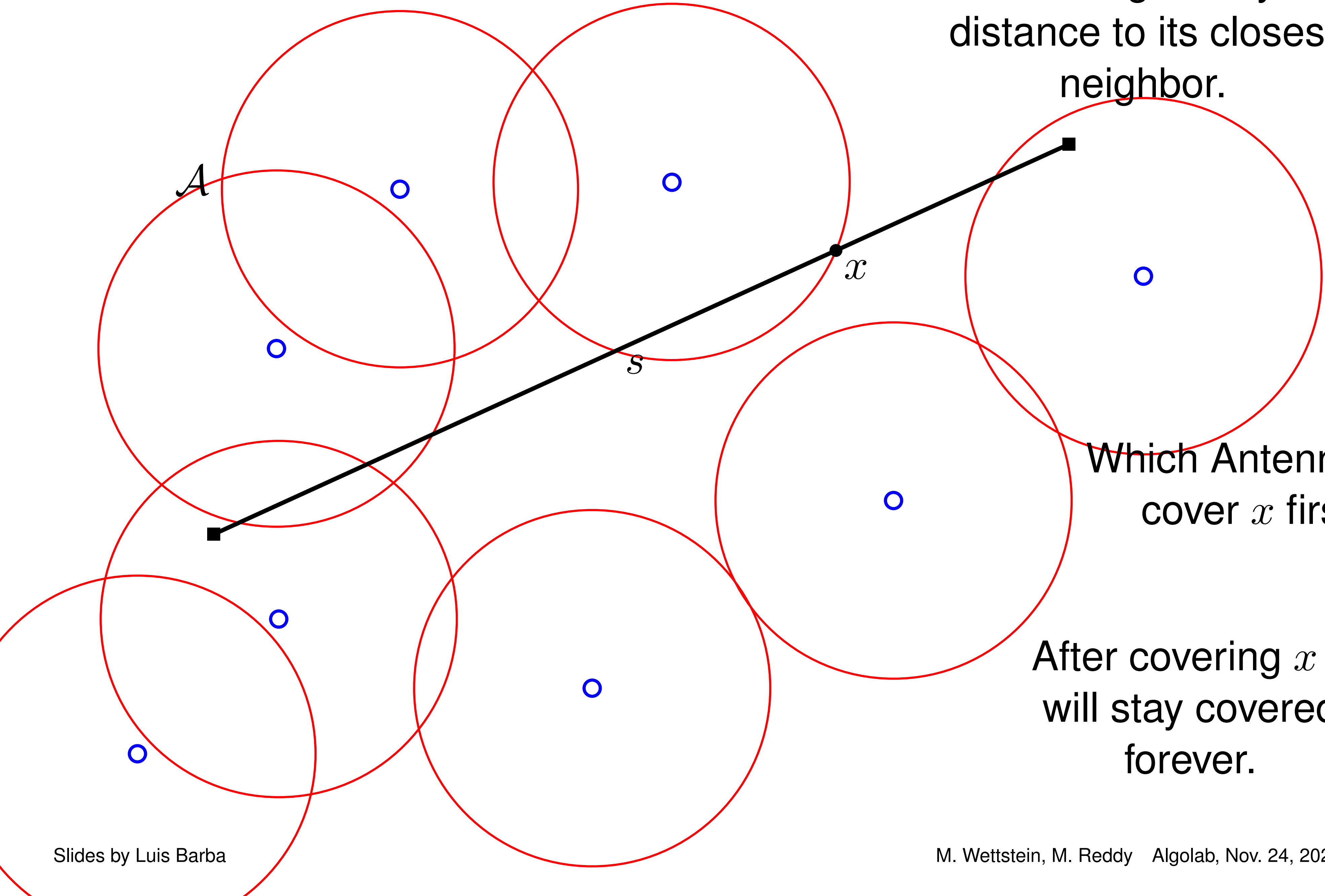


Which Antenna will cover x first?

After covering x it will stay covered forever.

Working with a single segment

The minimum radius to cover x is given by the distance to its closest neighbor.



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ETH zürich

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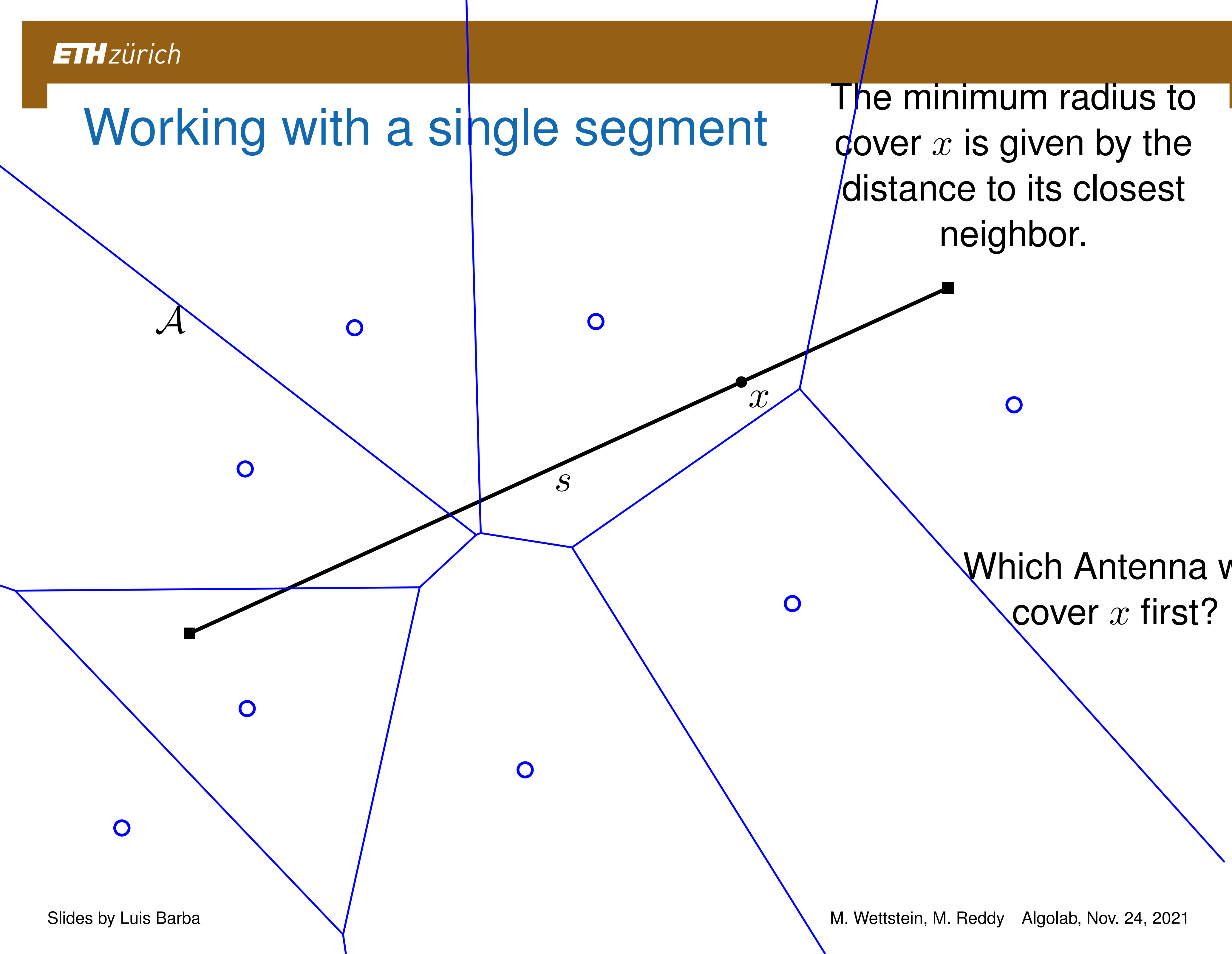

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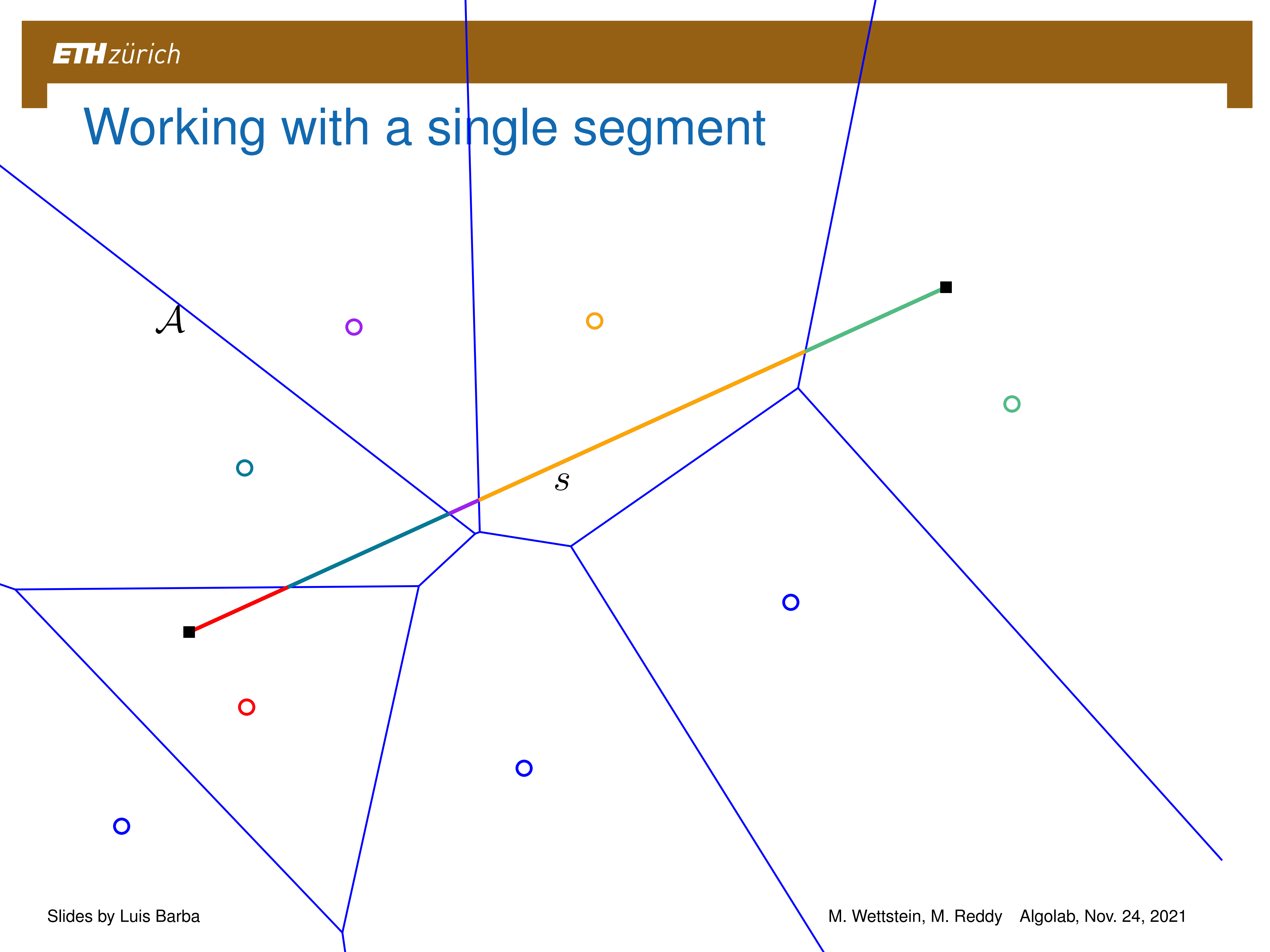
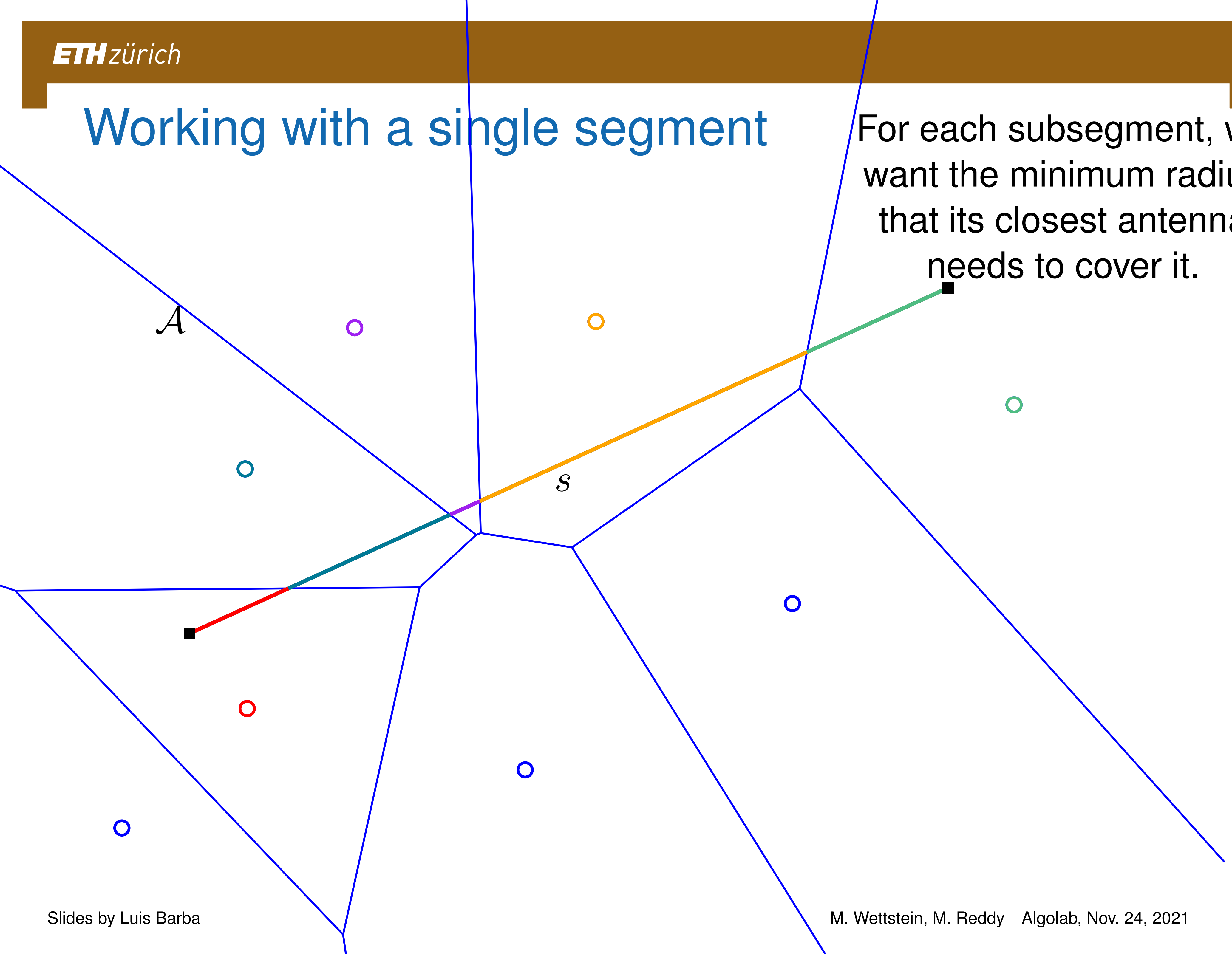


Working with a single segment

For each subsegment, we want the minimum radius that its closest antenna needs to cover it.



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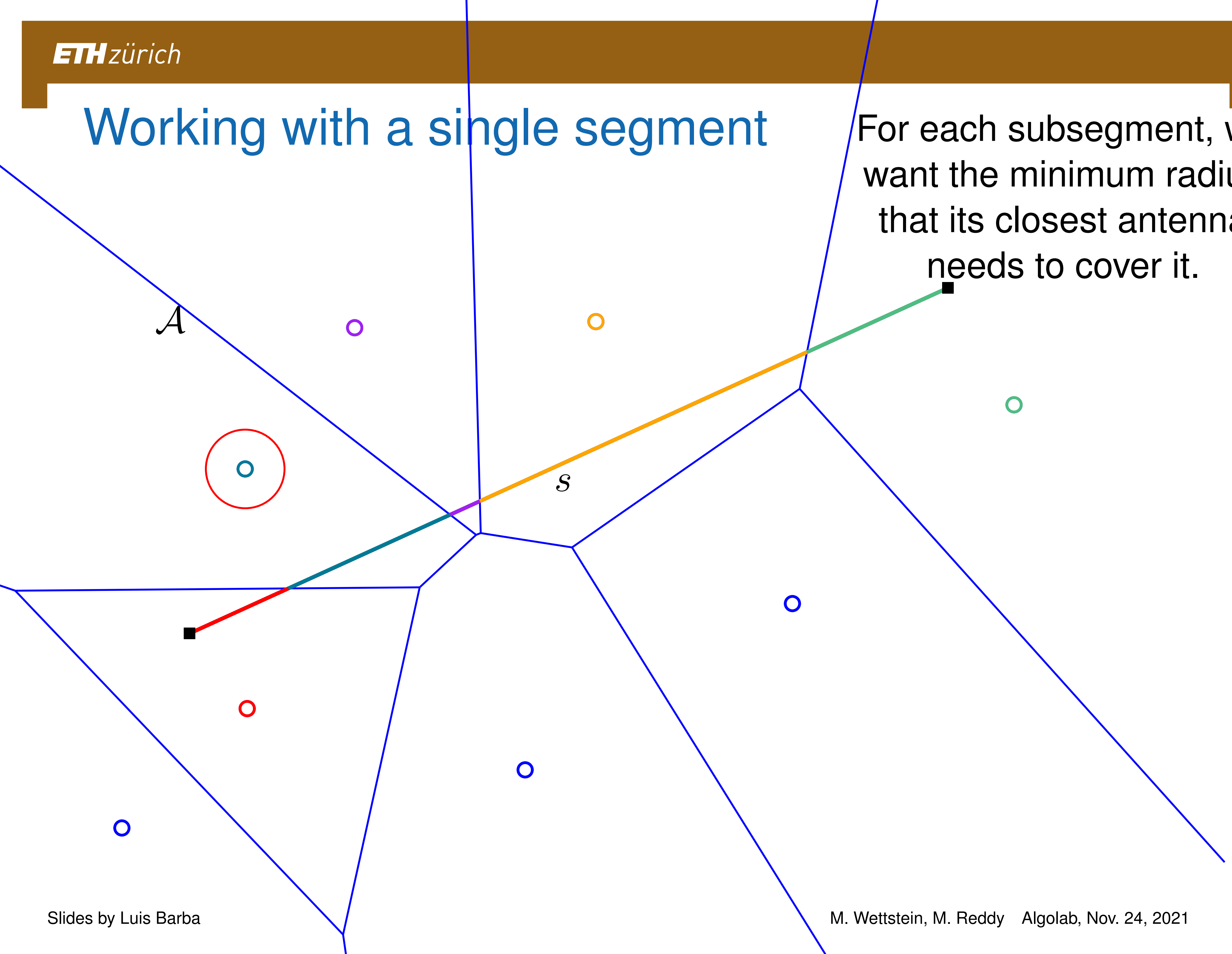

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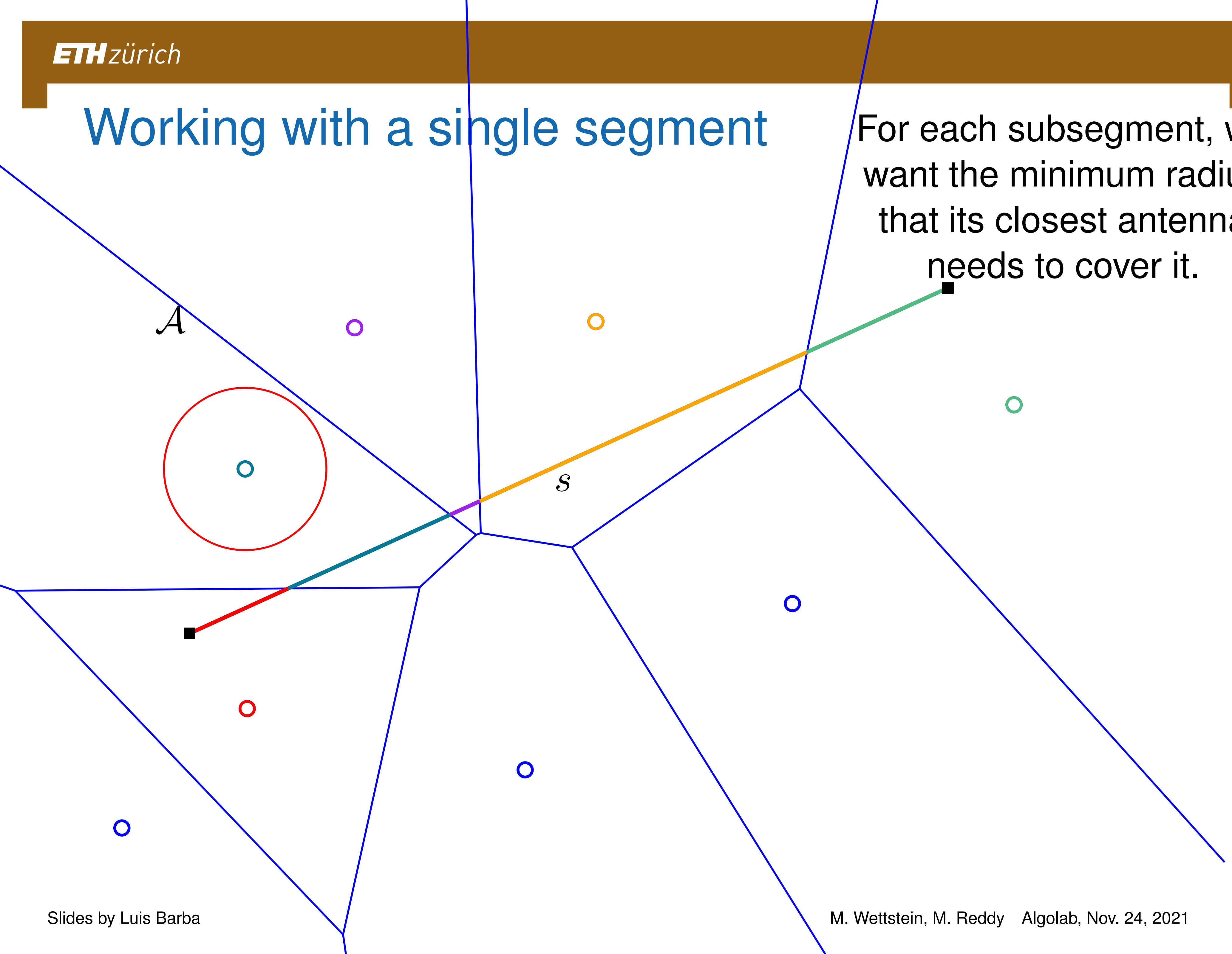

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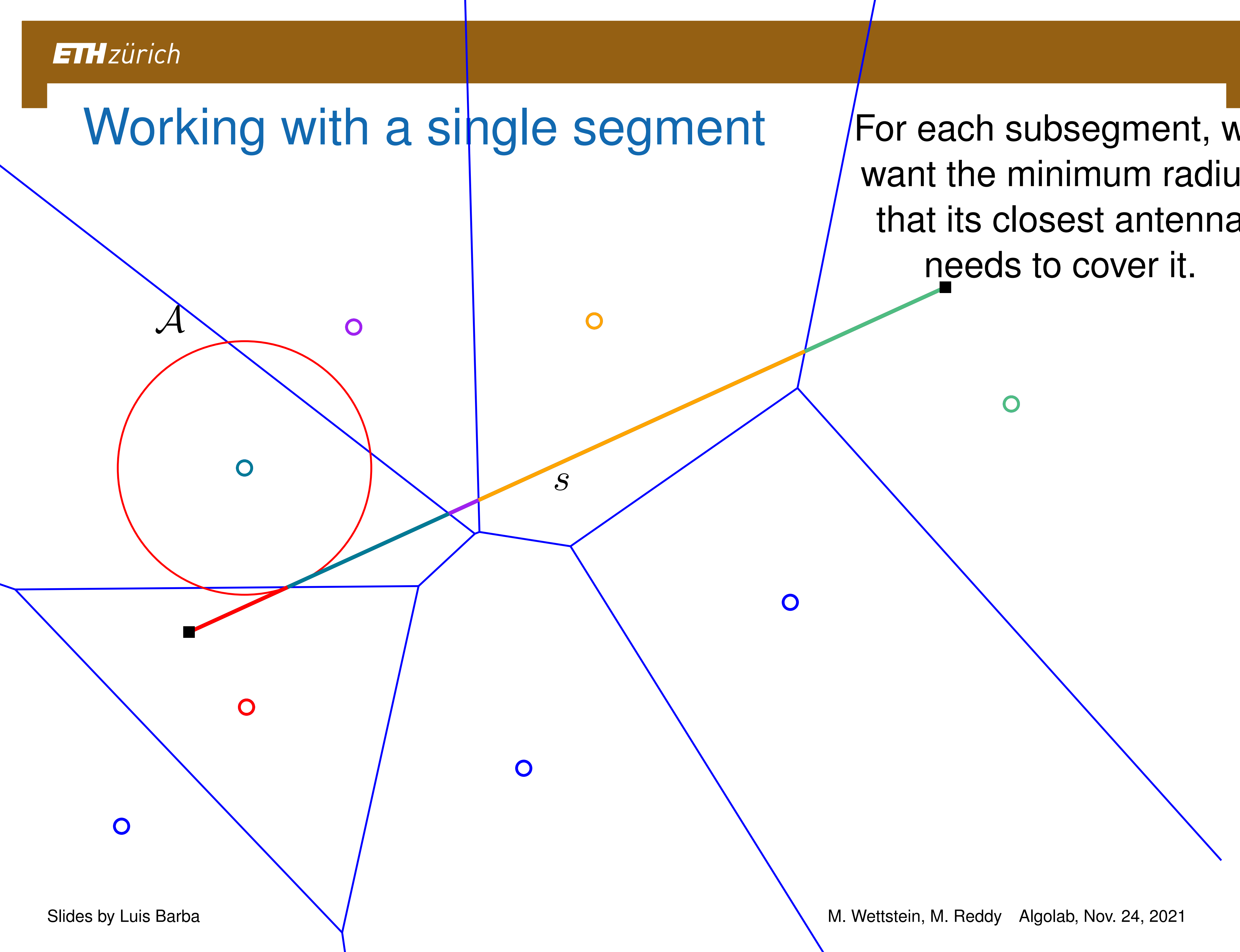

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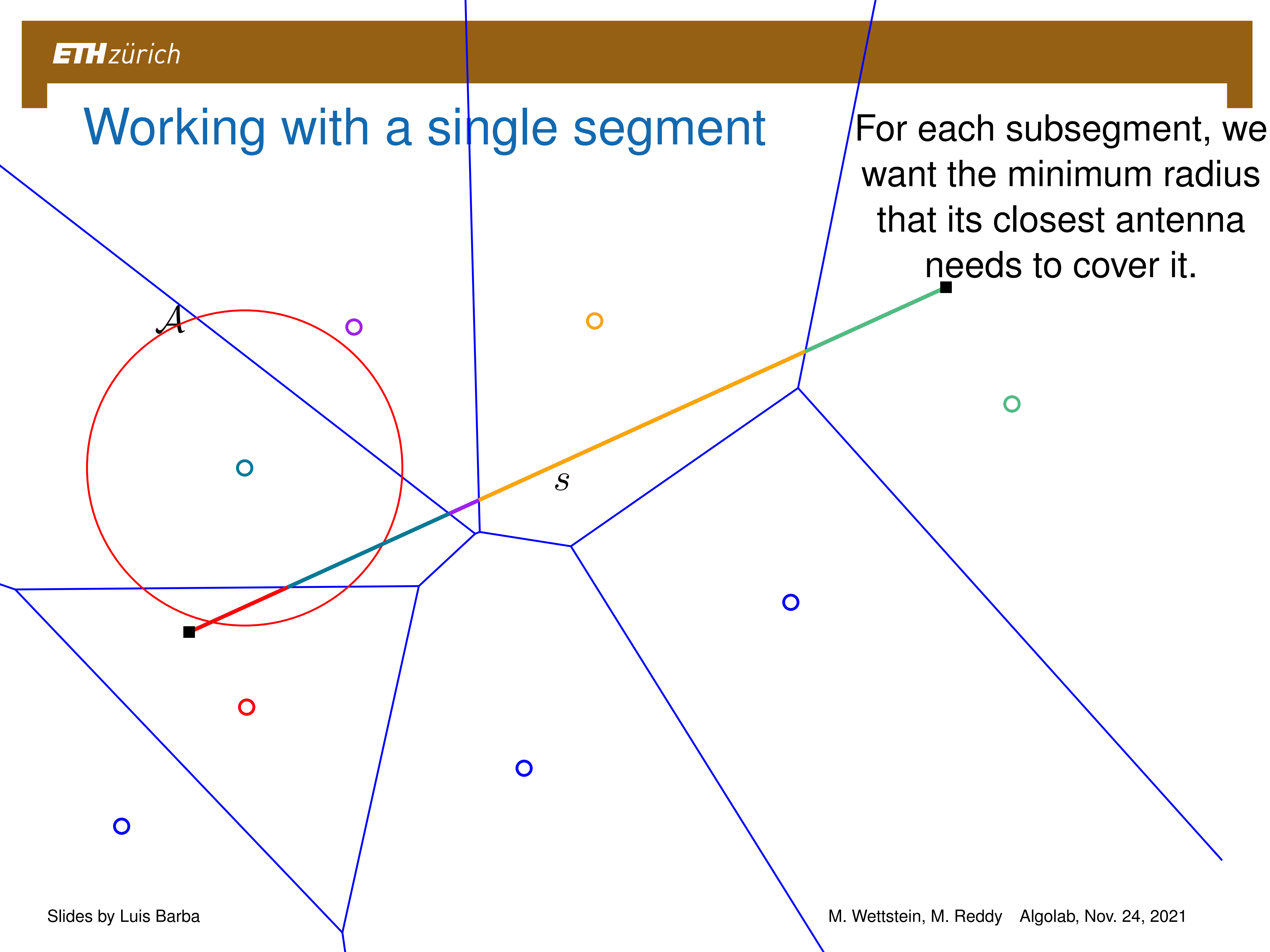

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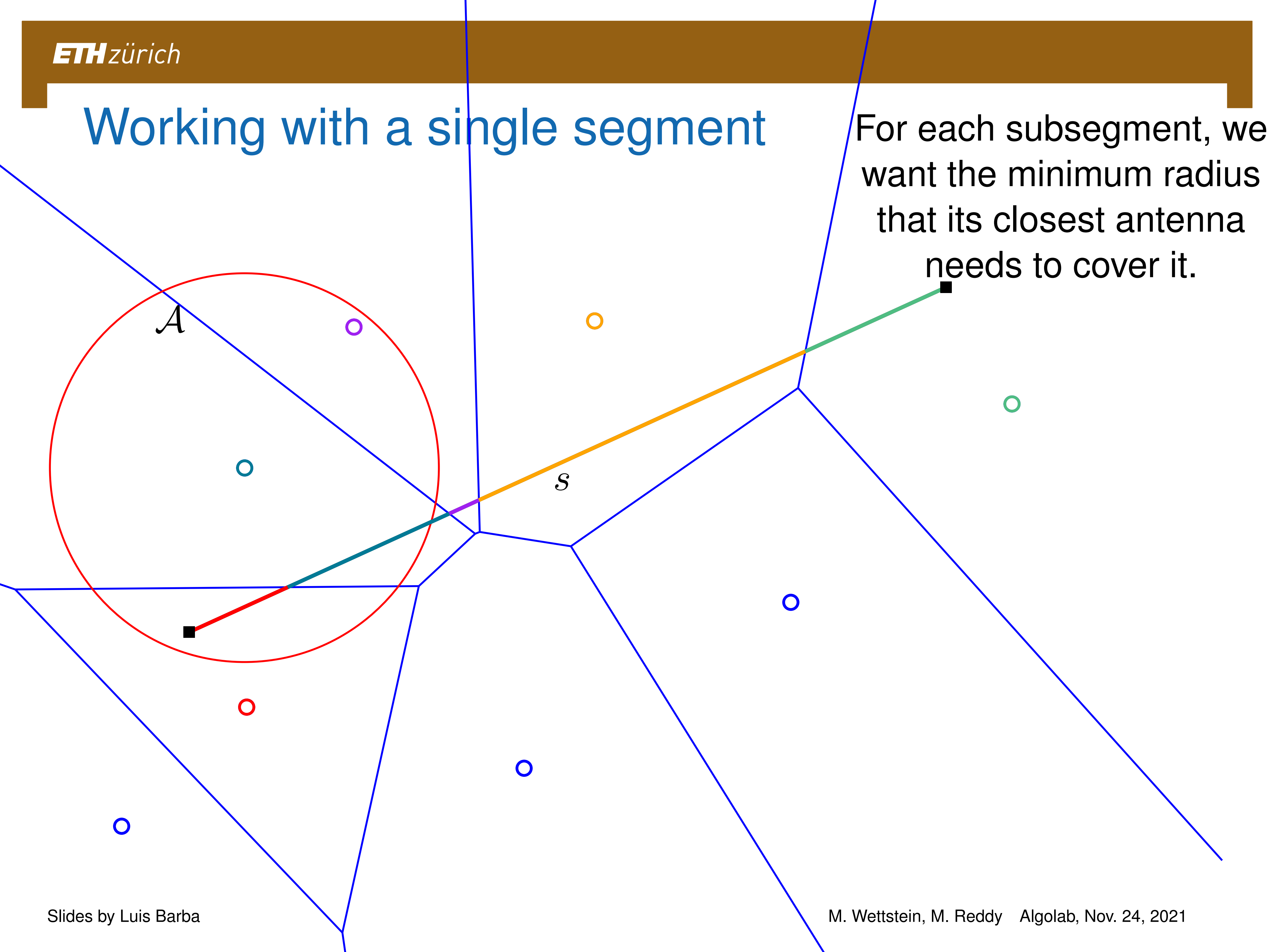

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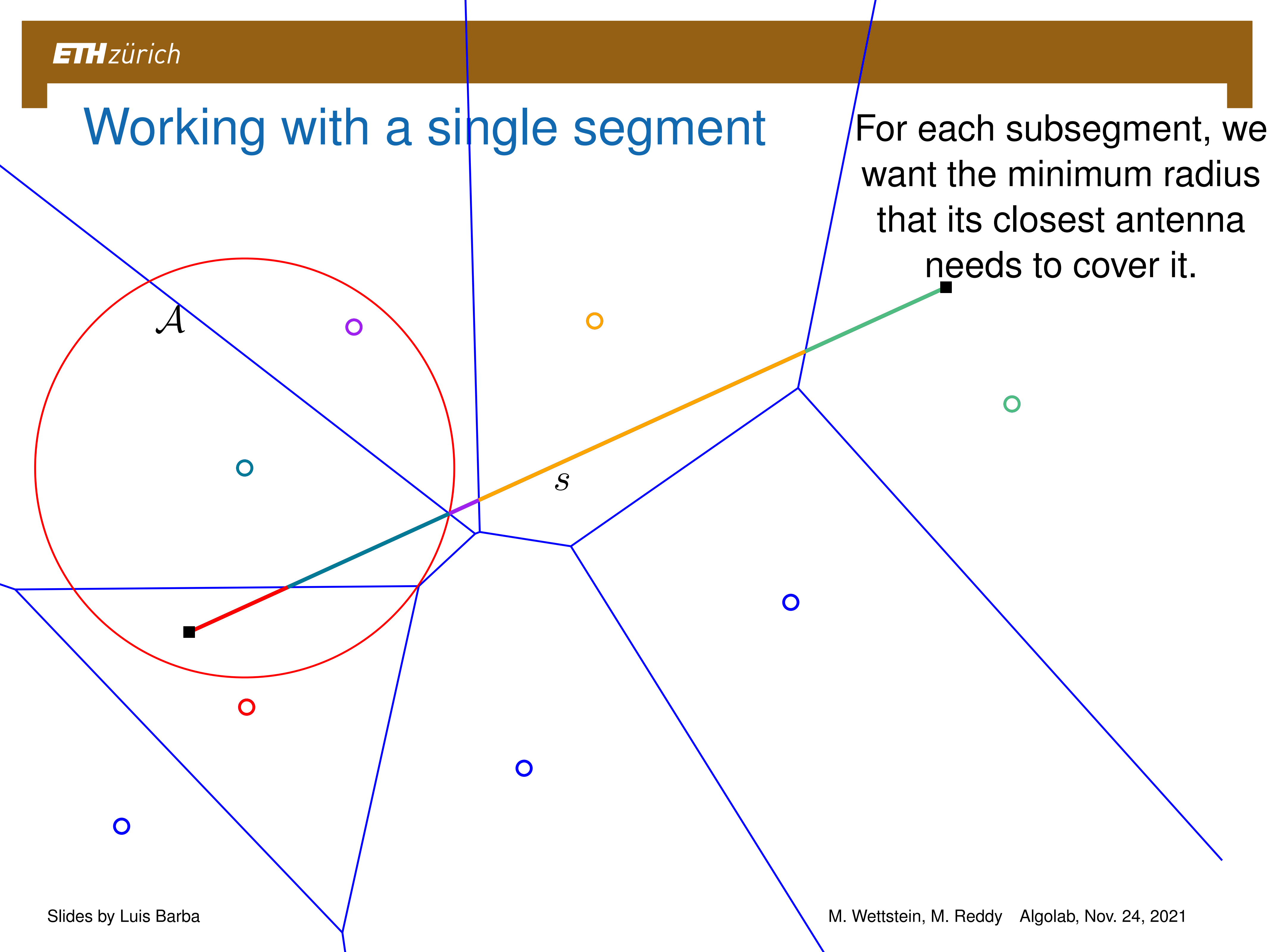

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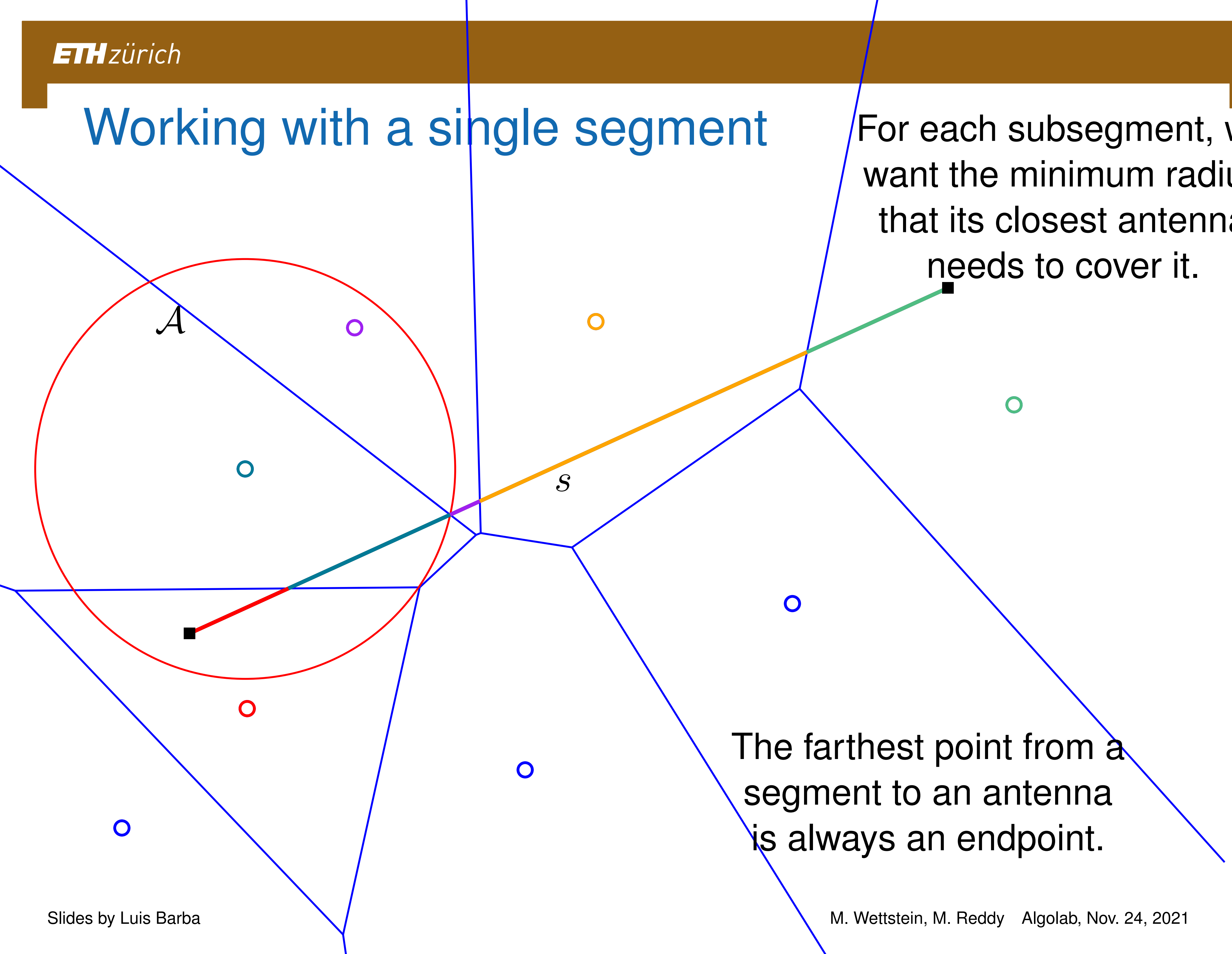

The farthest point from a segment to an antenna is always an endpoint.

Working with a single segment

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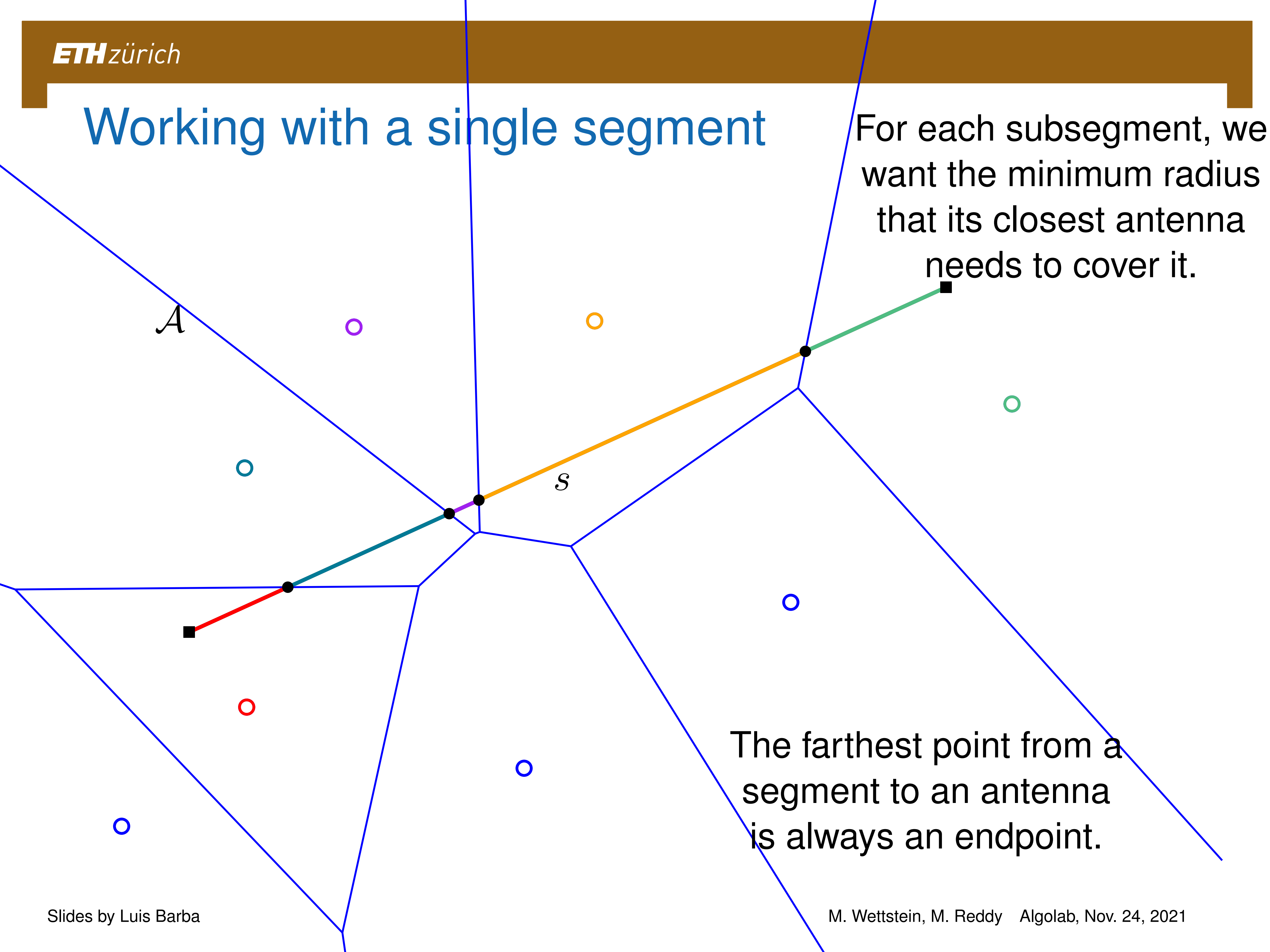

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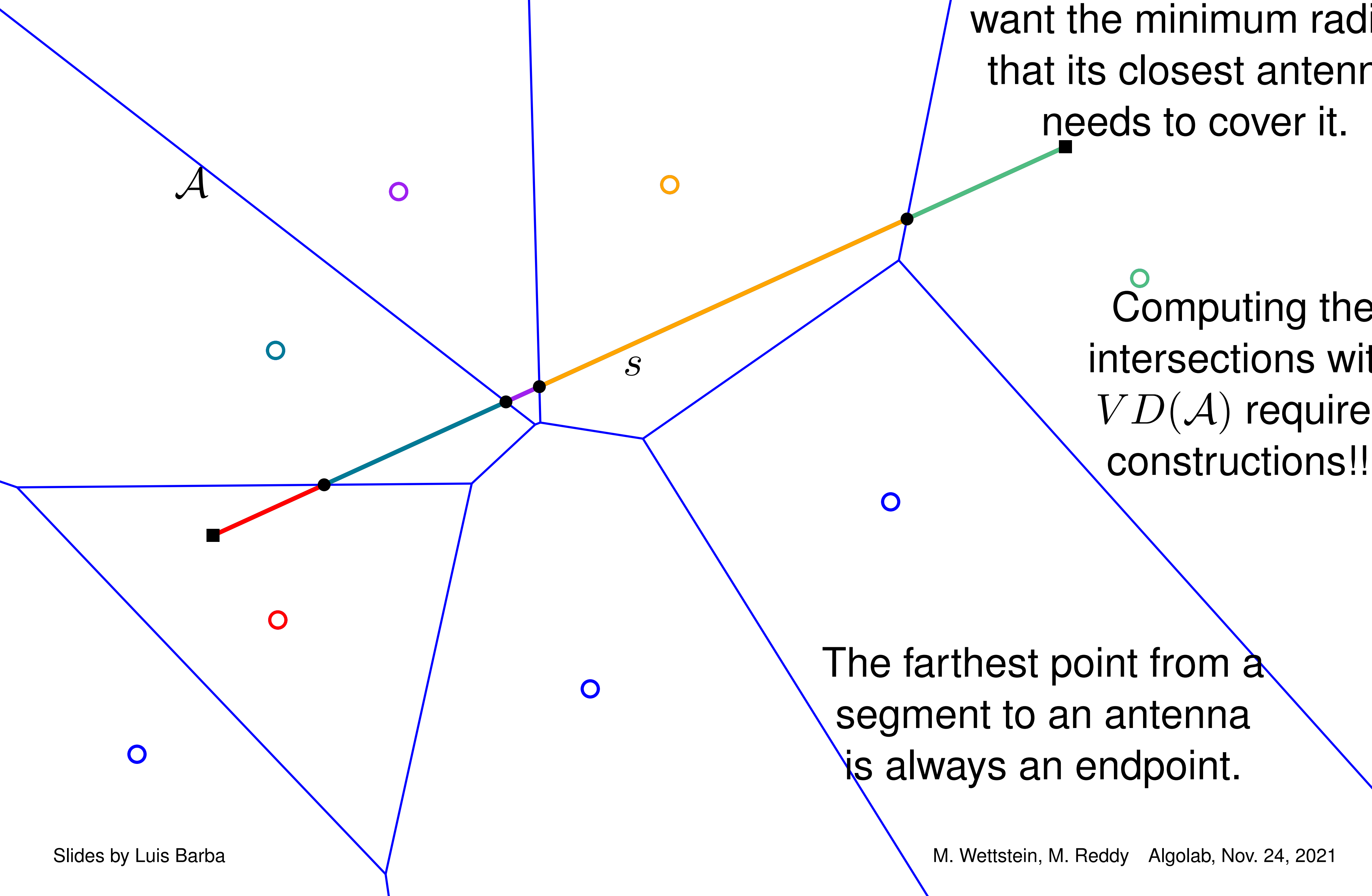
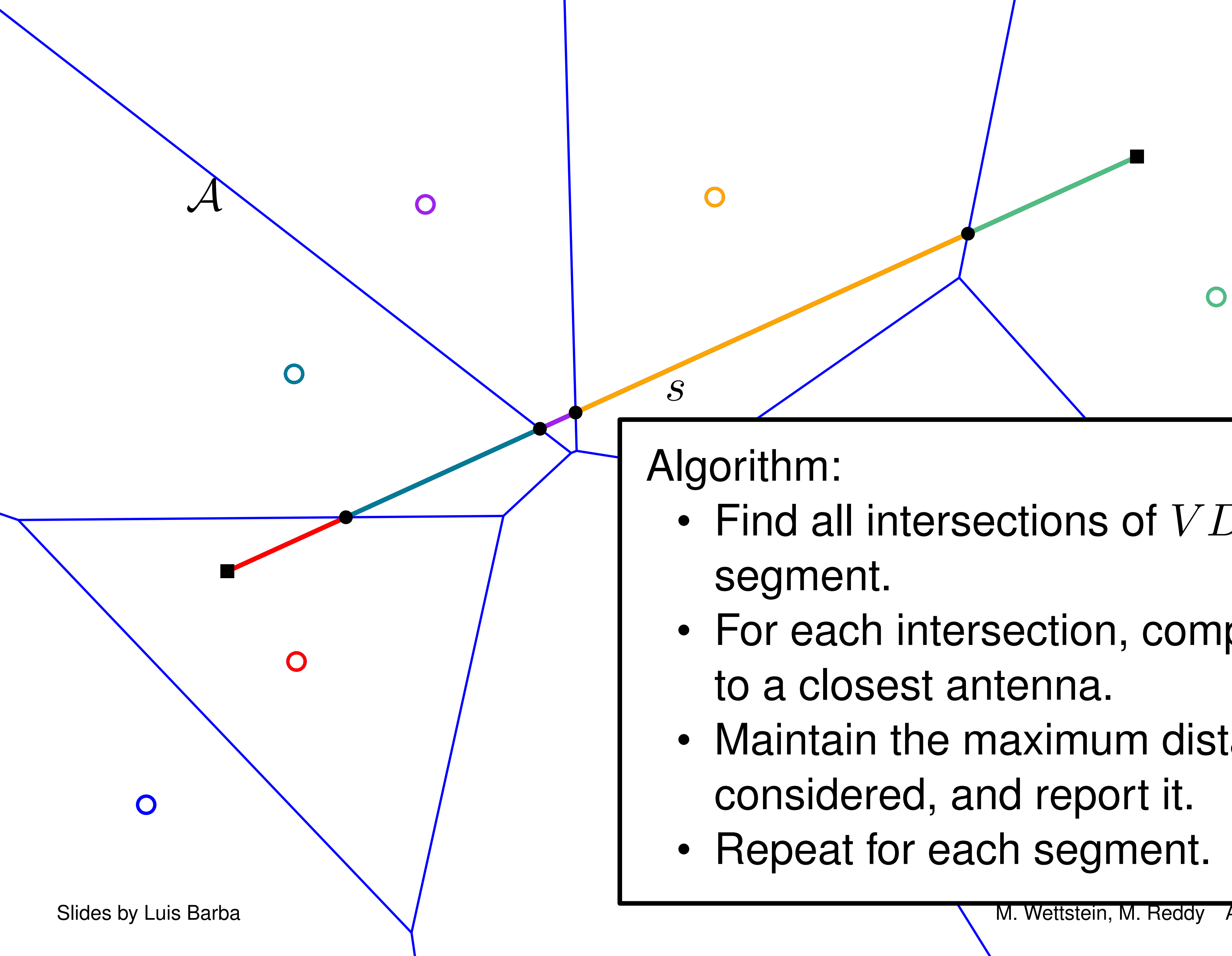


Computing the intersections with $VD(\mathcal{A})$ requires constructions!!!

The farthest point from a segment to an antenna is always an endpoint.

Working with a single segment

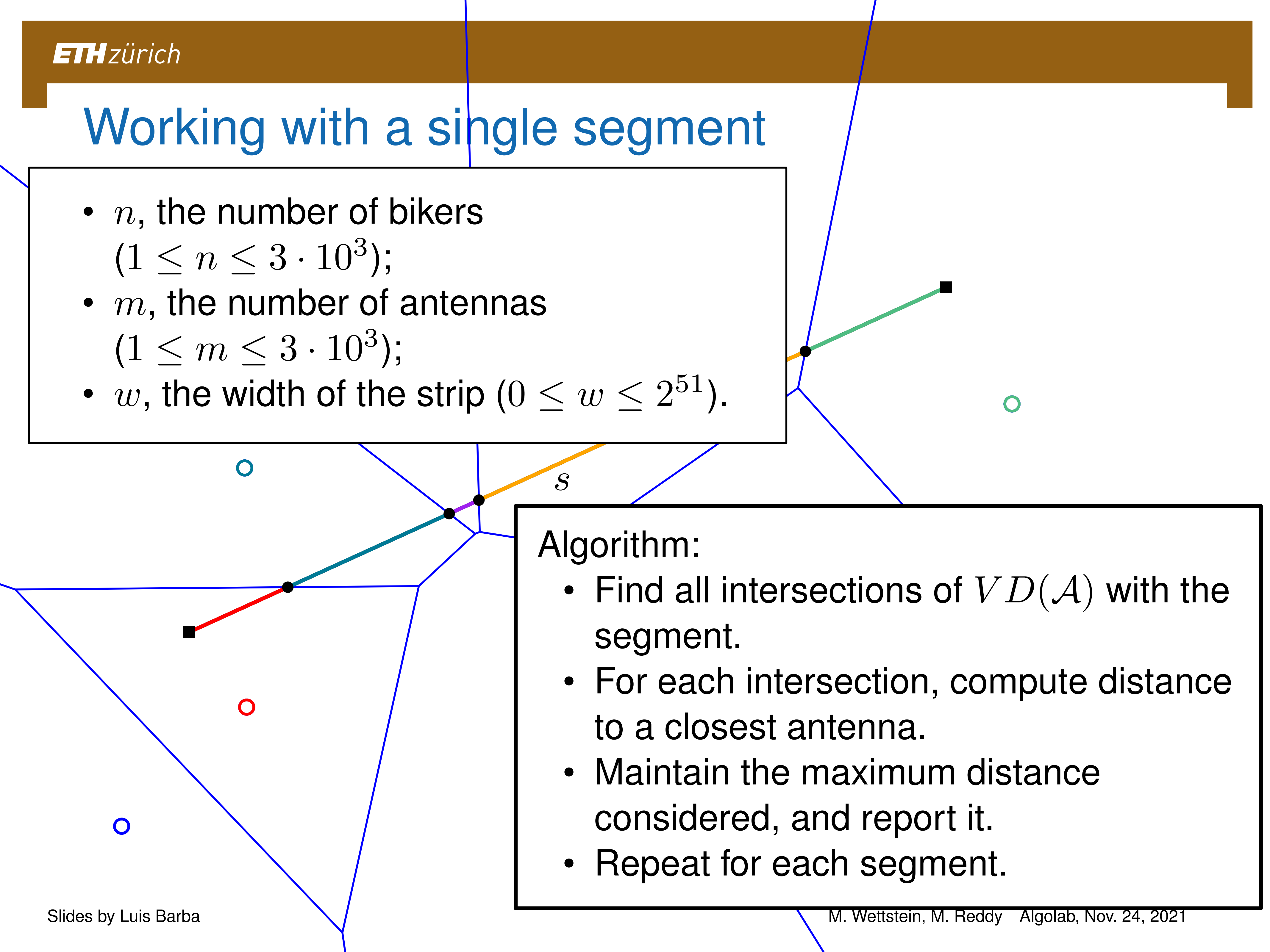


Algorithm:

- Find all intersections of $VD(\mathcal{A})$ with the segment.
- For each intersection, compute distance to a closest antenna.
- Maintain the maximum distance considered, and report it.
- Repeat for each segment.

Working with a single segment

- n, the number of bikers $(1 \leq n \leq 3 \cdot 10^3)$;
- m, the number of antennas $(1 \leq m \leq 3 \cdot 10^3)$;
- w, the width of the strip $(0 \leq w \leq 2^{51})$.



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- m, the number of antennas $(1 \leq m \leq 3 \cdot 10^3)$;
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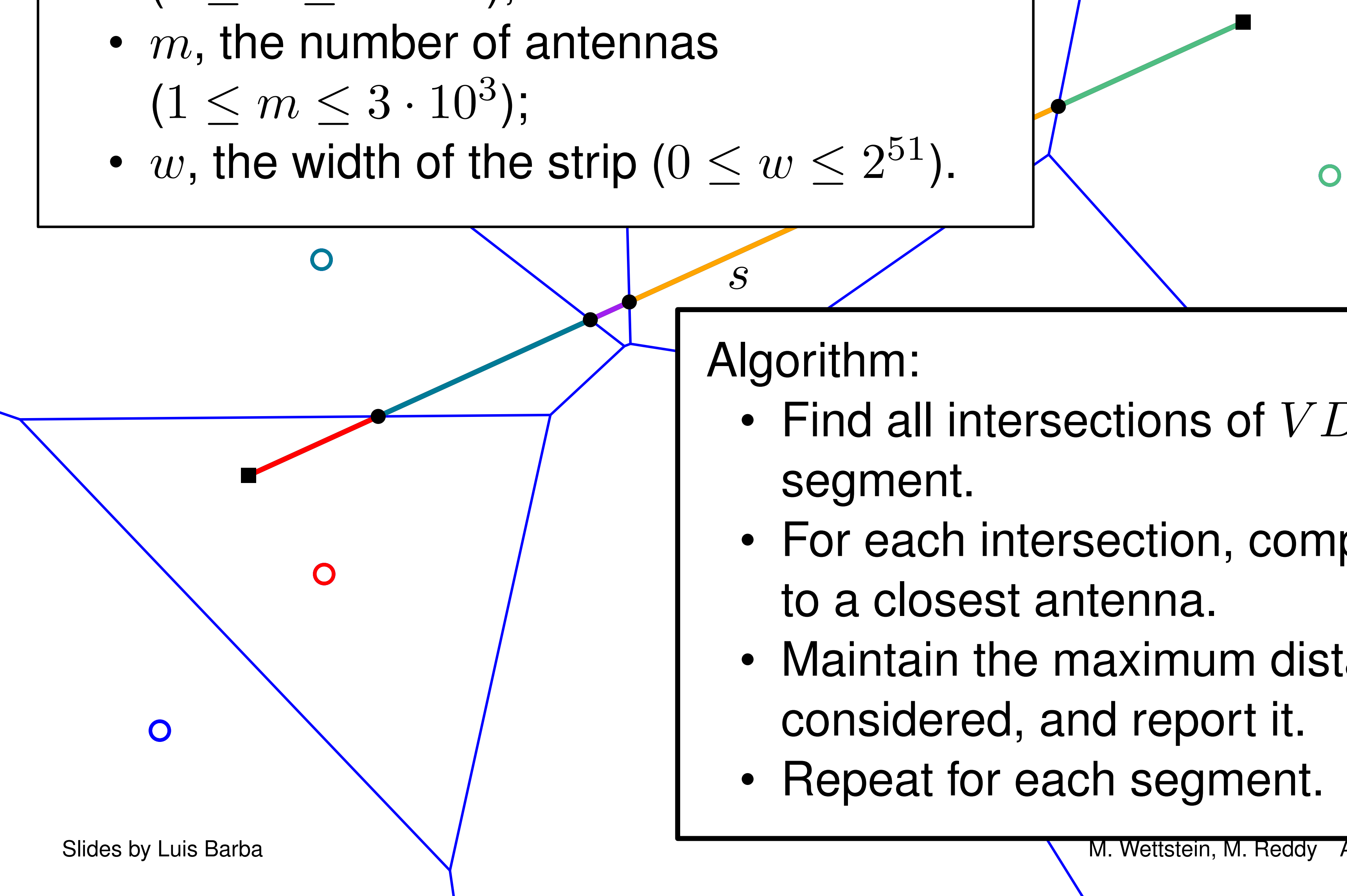
- $O(m \log m)$ time to Compute $VD(\mathcal{A})$.
- $O(m)$ time per segment.
- $O(nm)$ time in total.



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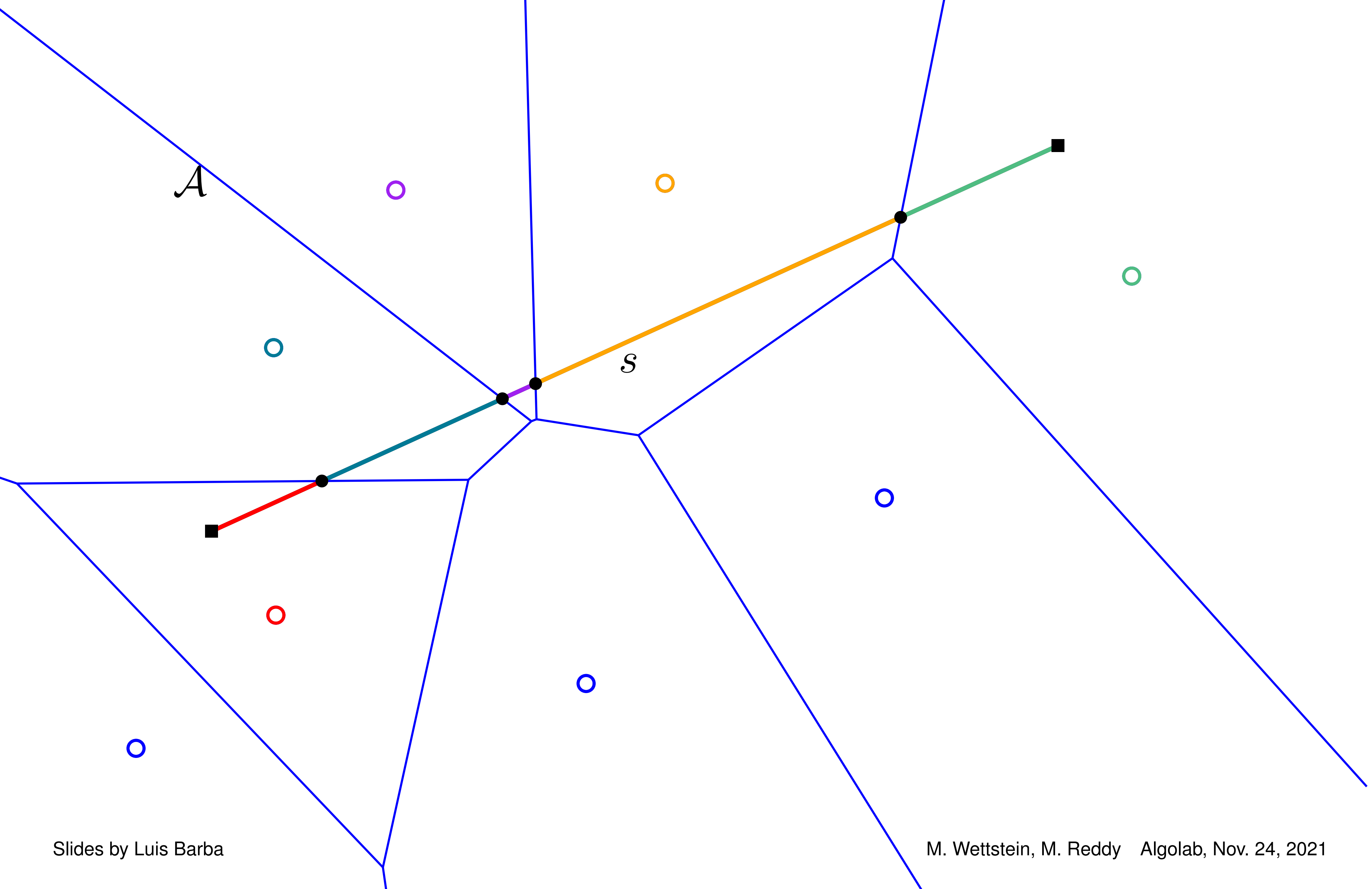
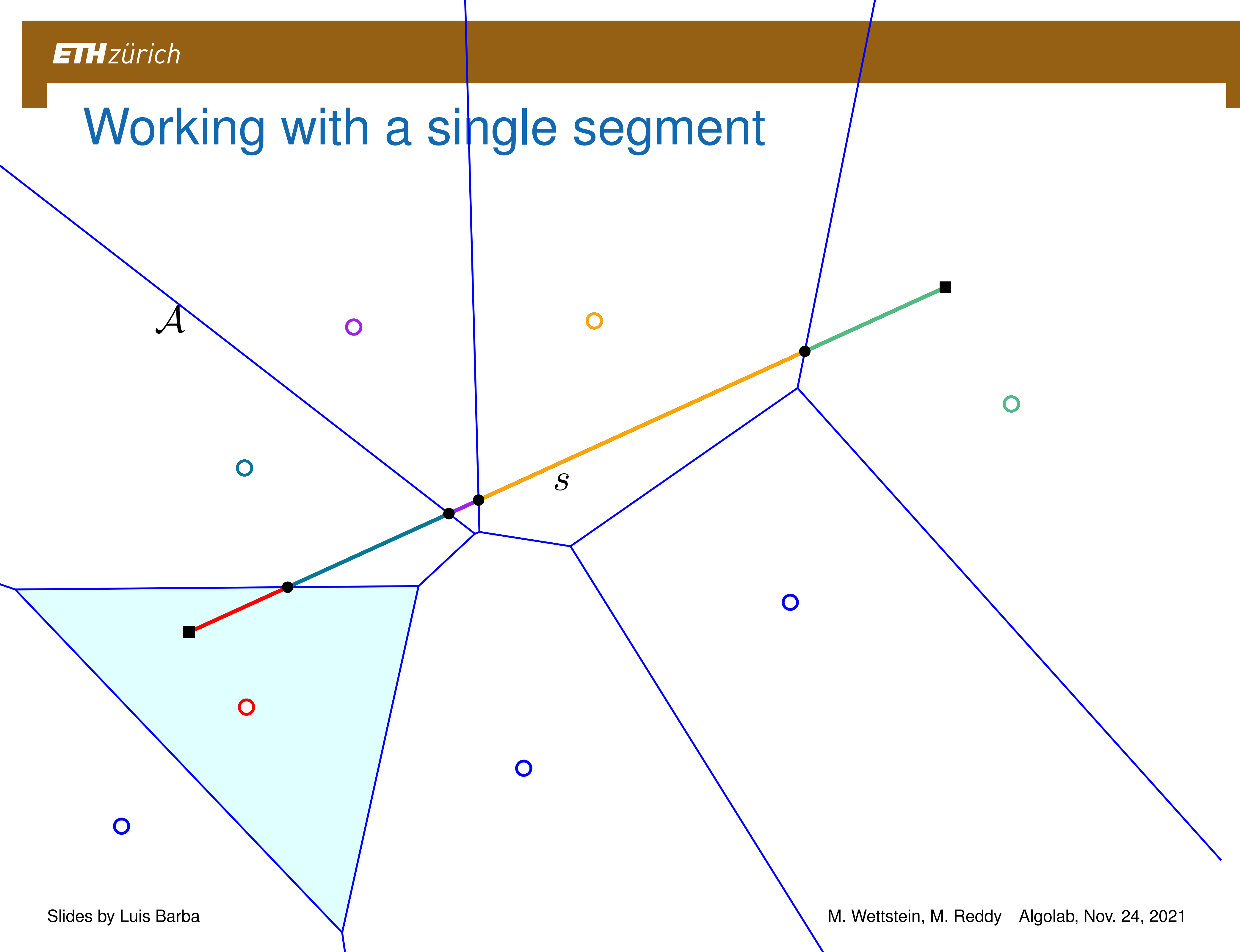
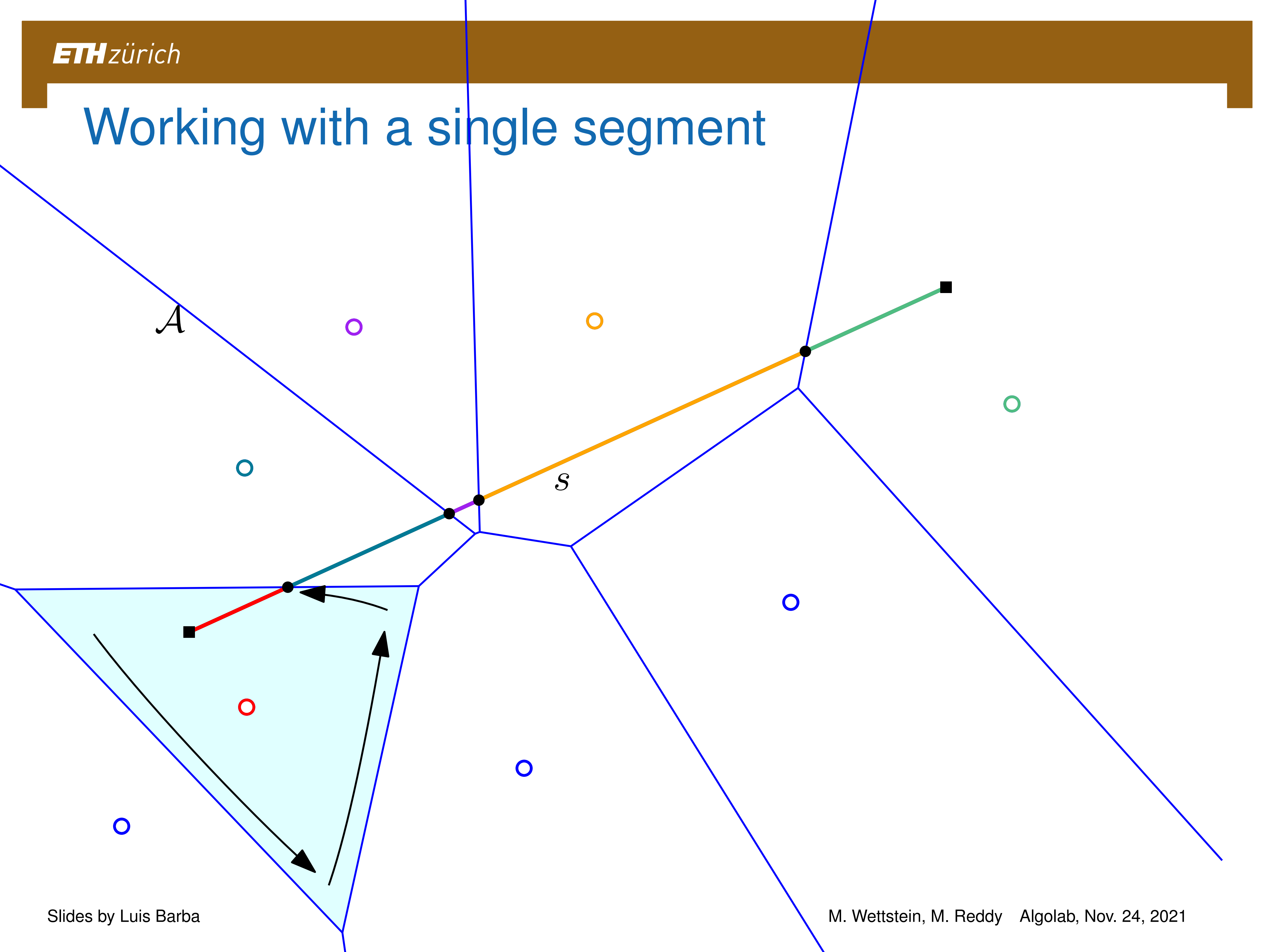
Working with a single segment



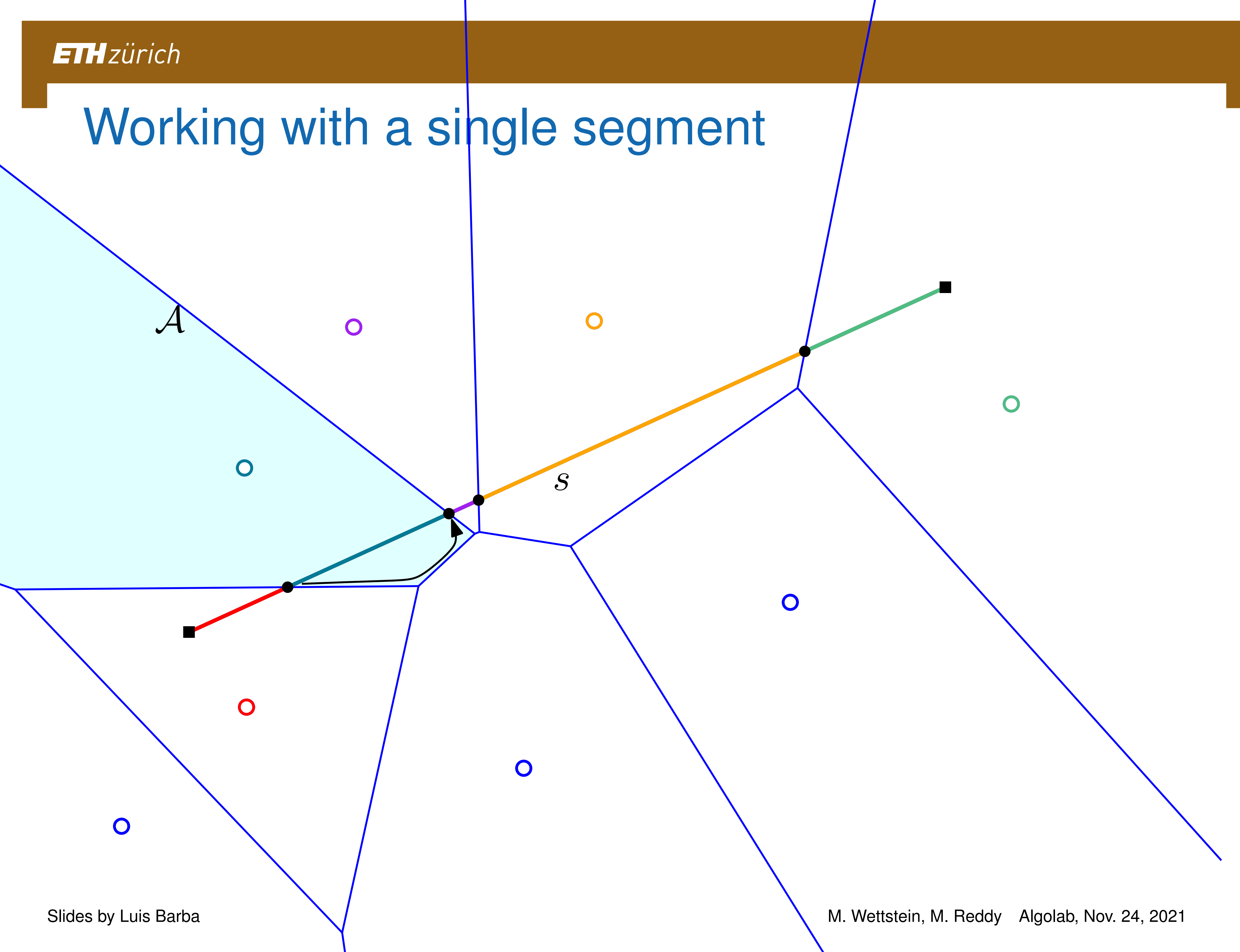
Working with a single segment



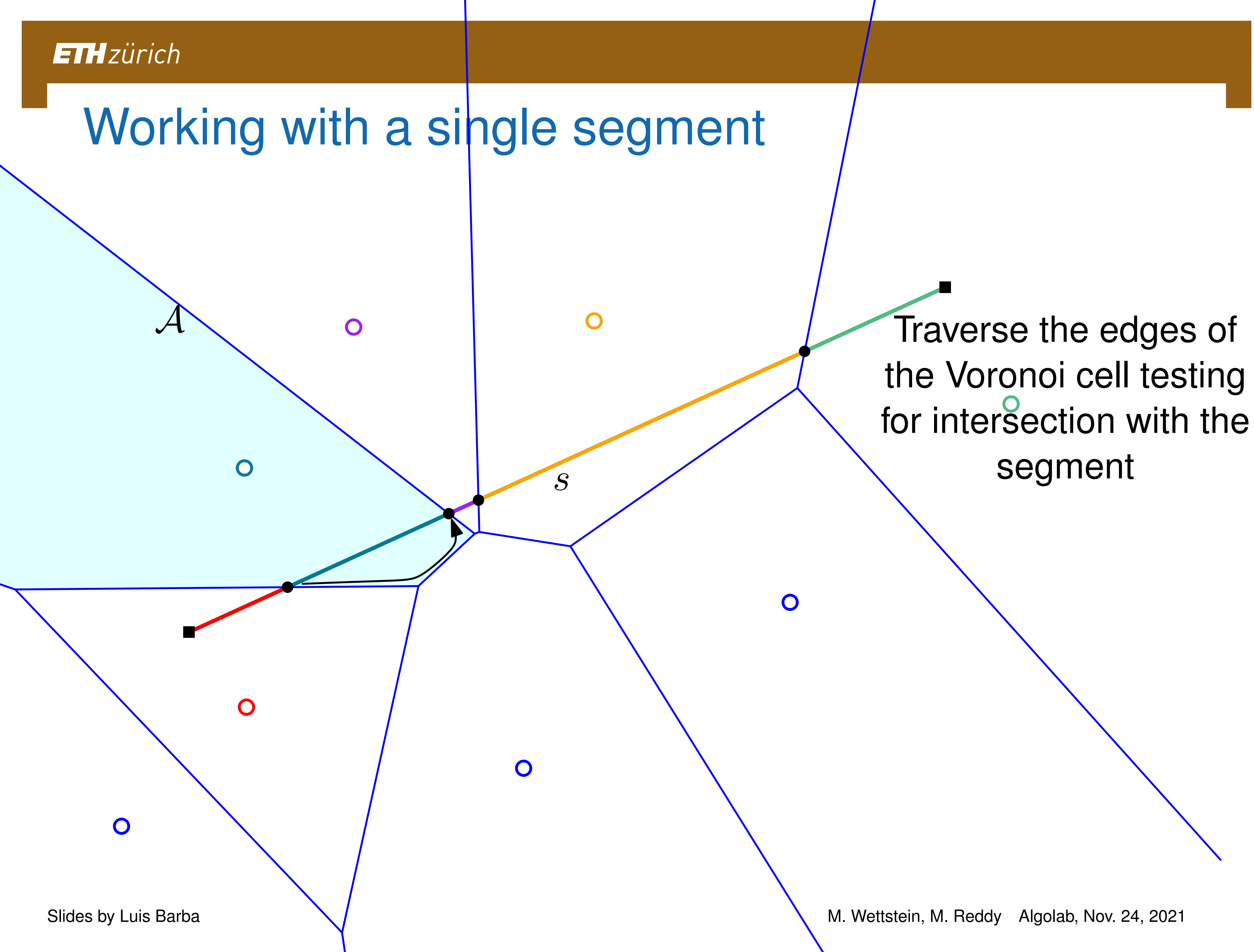
Working with a single segment



Working with a single segment

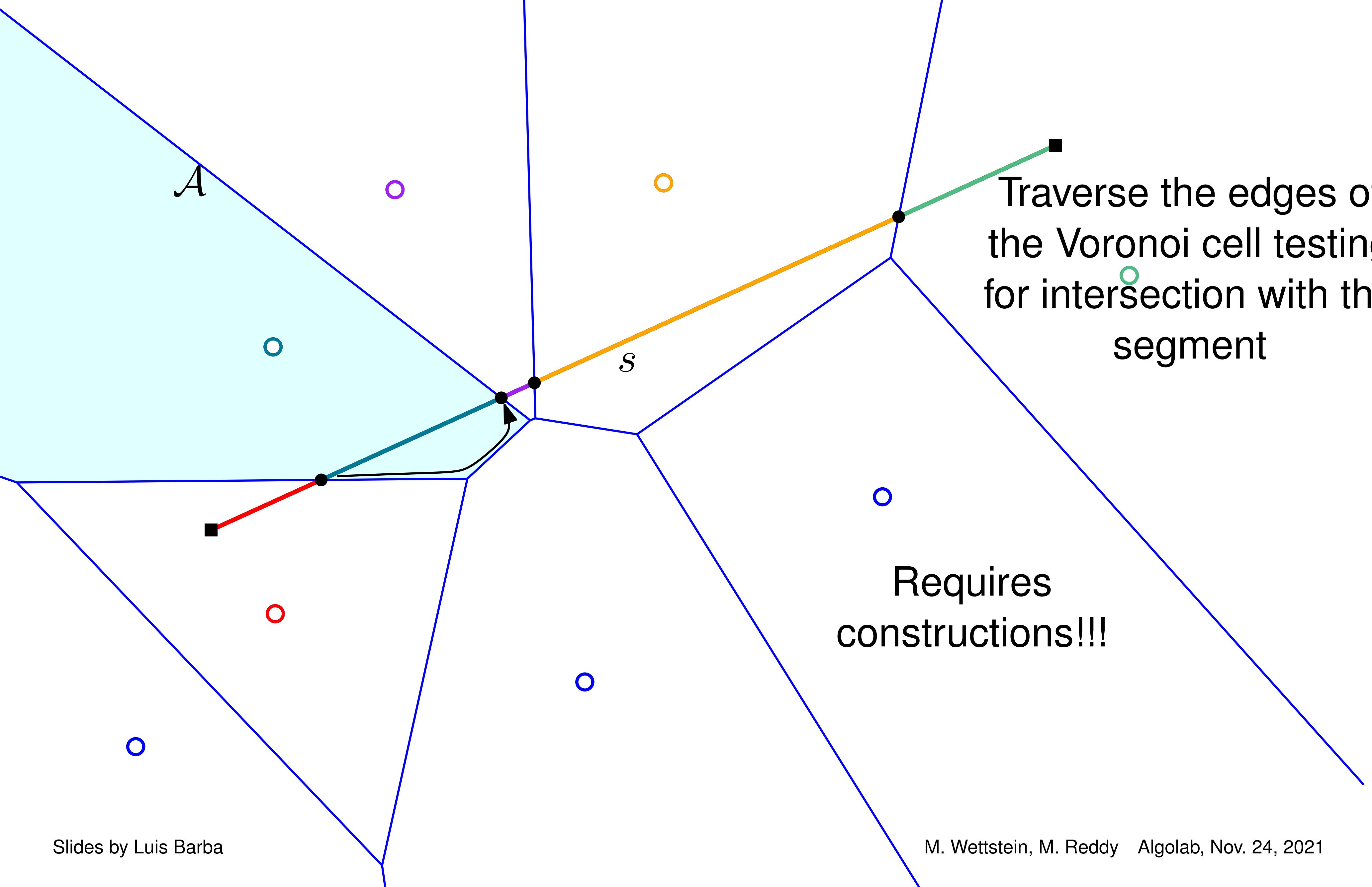


Working with a single segment



Traverse the edges of the Voronoi cell testing for intersection with the segment

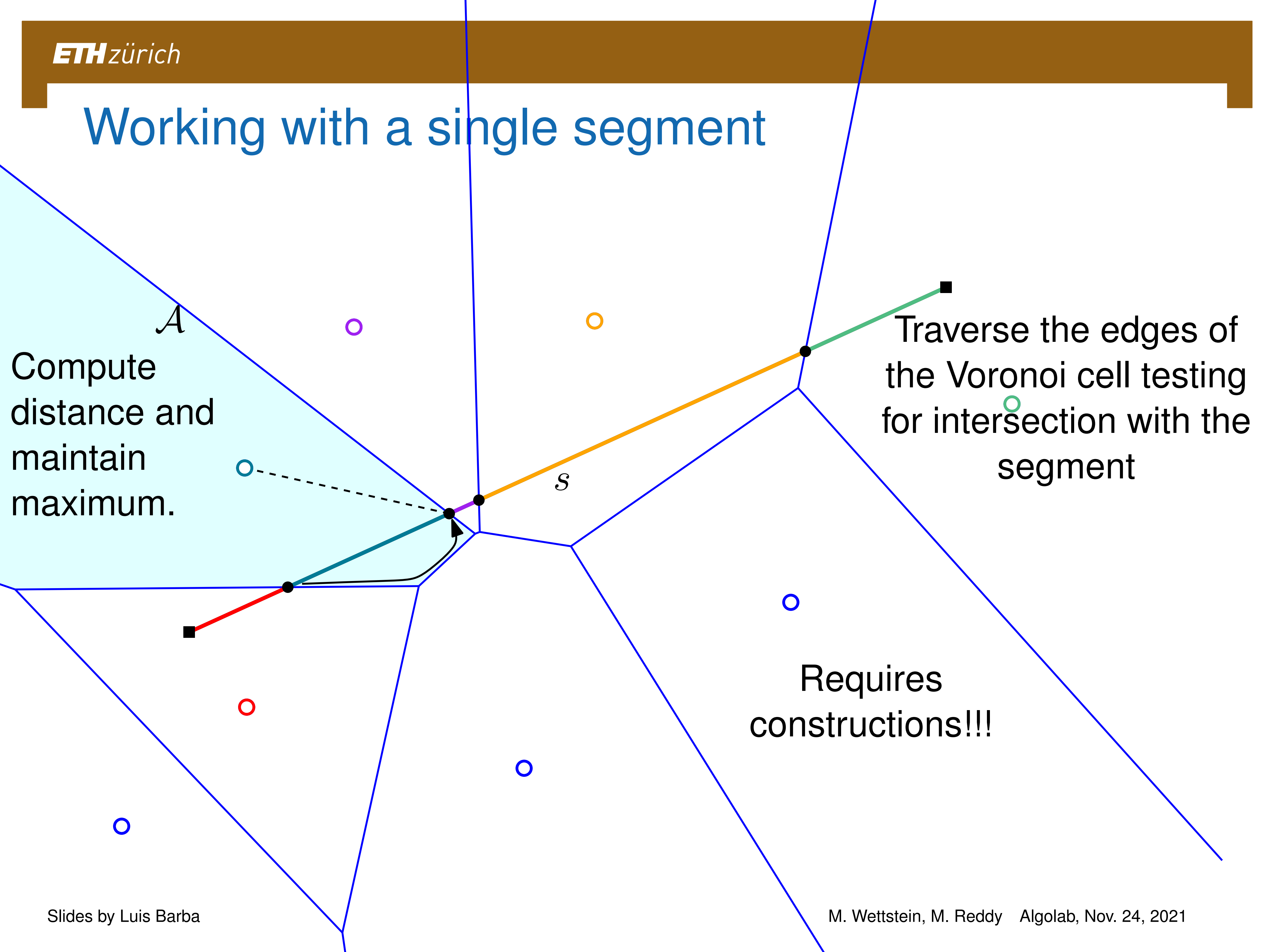
Working with a single segment



Traverse the edges of the Voronoi cell testing for intersection with the segment

Requires constructions!!!

Working with a single segment



Compute distance and maintain maximum.

Traverse the edges of the Voronoi cell testing for intersection with the segment

Requires constructions!!!

Working with a single segment