The Moon and eclipses

Of all the heavenly bodies visible in the night from the Earth, the Moon is the most spectacular. It far outshines even the most brilliant planet, moves so quickly that you can see its motion against the stars, and provides a wealth of detail in the shadowy features of its disc. Yet its motion is the most difficult to predict and it is for that reason we have left it until last. It is, of course, in orbit about the Earth but the Sun and other members of the Solar System perturb that orbit to such an extent that many corrections are needed to calculate the Moon's position accurately.

In the next few sections we use a simple method to find the position of the Moon. The method takes account only of the principle perturbations to the orbit yet gives results which are accurate enough for most purposes. (We have also provided spreadsheet functions which have much higher accuracy.) We calculate the times of moonrise and moonset, the phases of the Moon, and the circumstances of both solar and lunar eclipses. Finally, we show how to construct an astronomical calendar, bringing together the changing positions of all the Solar-System objects over the course of one year onto a single page. The calculations are lengthy but the satisfaction you feel when you predict, for example, the occurrence of a lunar eclipse, cannot be denied.

64 The Moon's orbit

To an Earth-bound observer, the Moon appears to be in orbit about the Earth, making one complete revolution with respect to the background of stars in 27.321 7 days. This period is called the **sidereal month**. During this time the Earth moves on along its own orbit so that the Sun's position changes with respect to the stars. Hence the Moon has some extra distance to make up to regain its position relative to the Sun. The interval defined by the time taken for the Moon to return to the same position relative to the Sun is called the **synodic month** and is equal to 29.530 6 days. The direction of motion of the Moon in its orbit about the Earth is **prograde**; that is, it is in the same sense as that of all the planets about the Sun.

A celestial observer viewing the Solar System from a great distance would not, however, see the Moon making loops in space about the Earth. Rather, he or she would describe the situation by saying that the Moon is in orbit around the Sun, as is the Earth, and that the effect of the Earth's influence is to make the Moon's orbit wiggle a little as the relative positions of Earth and Moon change (Figure 81). This is because the Sun's gravitational force on the Moon is much greater than that of the Earth, even though the latter is nearer. It is hardly surprising that the orbit of the Moon is so complicated to calculate since it is regulated by two bodies, not one, and the two bodies are themselves tied in orbit about each other.

For the purposes of our calculations, we are going to imagine that both the Sun and the Moon are in orbit about the Earth. We have already calculated the position of the Sun by these means in Section 46. We will need those calculations in the next few sections to find the magnitude of some of the corrections to the Moon's orbit.

There are three main effects of the perturbations caused by the Sun on the Moon's apparent orbit round the Earth. The first of these is called **evection** in which the apparent value of the eccentricity of the Moon's orbit varies slightly. The second is due to the variation of the Earth–Sun distance as the Earth travels in its own ellipse about the Sun. This correction is called the **annual equation**. The third inequality takes account of the motion of the Moon in the Sun's gravitational field. When the Moon is on one side of the Earth it is nearer the Sun so that the Sun's gravitational attraction is slightly more than when the Moon is on the other side of the Earth. This correction is called the **variation**. These corrections alone, together with the usual correction called the **equation of the centre**, can make up to 9° difference in the Moon's mean anomaly, so it is important that they be taken into account. We shall make six corrections in all to

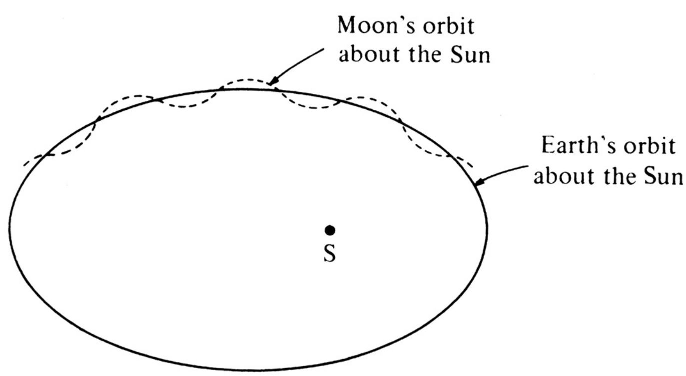


Figure 81. The Moon's orbit, much exaggerated. The Moon is much closer to the Earth than suggested by this diagram; in particular, its orbit is everywhere concave towards the Sun.

find the position of the Moon to within one fifth of a degree. (See spreadsheet MoonPos2 for more precise calculations using spreadsheet functions that employ a better numerical model of the Moon's orbit.)

The apparent motions of the Moon and the Sun about the Earth are drawn in Figure 82. This diagram is similar to that of Figure 63 except that here the Earth is at the centre and both the Sun and the Moon describe ellipses about the Earth. Once again you are to imagine that you are looking at the Solar System from a great distance and, further, that you are moving in such a manner that the Earth appears to be stationary in your view. The large sphere is centred on the Earth, E, and the planes of the orbits of Sun and Moon are projected to cut the sphere along the circles ♈ $N_1'S'N_2'$ and $N_1'P'm'N_2'$ respectively. S' is the projection of the Sun onto this sphere and its longitude, measured from the first point of Aries, ♈, is denoted by $\lambda_\odot$. The Moon's orbit is inclined to the ecliptic at an angle i; N_1' and N_2' are the projections of the ascending and descending nodes, P' is the projection of the Moon's perigee, and m' is the projection of the present position of the Moon. The longitude of the ascending node is ☊, the longitude of the perigee is ☊ $+\,\omega$ and the Moon's true anomaly is ν.

There are two principal effects of the perturbations mentioned above. The first is that the perigee of the Moon's orbit, unlike the (nearly) stationary perihelia of the planets' orbits, advances (prograde) at such a rate that it makes one complete revolution in about 8.85 years. The second is that the line joining the nodes, $N_1'N_2'$, moves backwards (**retrograde**) around the ecliptic so that it makes one complete revolution in about 18.61 years. Yet another month can be defined by the time it takes the Moon to return to its ascending node. This is the **draconic** or **nodal month** and it is equal to 27.212 2 days.

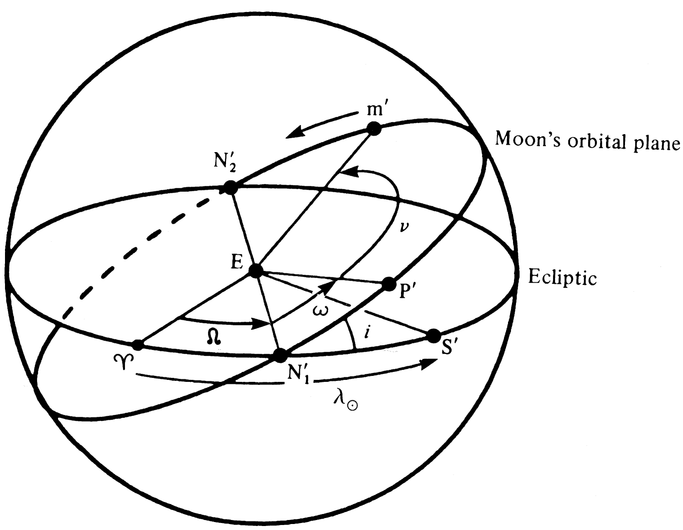


Figure 82. Defining the Moon's orbit.

65 Calculating the Moon's position

The steps involved in the process of finding the position of the Moon are much the same as those involved in calculating the position of a planet, except that (i) correction terms have to be applied at every step and (ii) the longitudes of the ascending node and perigee cannot be regarded as constant. We first determine the Moon's mean anomaly, M_{m}, which refers to the position of a fictitious Moon in uniform circular motion about the Earth. Then we find the longitude, and, by referring it to the plane of the ecliptic, the geocentric ecliptic coordinates λ_{m} and β_{m}. Finally, we convert to right ascension and declination using the method given in Section 27.

Once again we choose the epoch 2010 January 0.0 as our starting point. We calculate the number of days, D, since the epoch to the required date and time, counting the time of day as a fraction of a day. For slightly better accuracy we should use terrestrial time, TT, rather than the universal time, UT (see Section 16). Then we find:

(a) the Sun's ecliptic longitude, $\lambda_{\odot}$, and mean anomaly, $M_{\odot}$, by the method given in Section 46;
(b) the Moon's mean longitude, l, given by

$$l = 13.1763966D + l_0;$$

(c) the Moon's mean anomaly, M_{m}, given by

$$M_{\mathrm{m}} = l - 0.1114041D - P_0;$$

(d) the ascending node's mean longitude, N, given by

$$N = N_0 - 0.0529539D.$$

The symbols l_0, P_0 and N_0 represent the mean longitudes at the epoch.

Next we calculate the corrections for evection, E_{v}, the annual equation, A_{e}, and a third correction, A_3:

$$E_{\mathrm{v}} = 1.2739\sin(2C - M_{\mathrm{m}}),$$

$$A_{\mathrm{e}} = 0.1858\sin(M_{\odot}),$$

$$A_3 = 0.37\sin(M_{\odot}),$$

where $C = l - \lambda_{\odot}$. With these corrections we can find the Moon's corrected anomaly, M'_{m}:

$$M'_{\mathrm{m}} = M_{\mathrm{m}} + E_{\mathrm{v}} - A_{\mathrm{e}} - A_3.$$

We can now find the correction for the equation of the centre:

$$E_{\mathrm{c}} = 6.2886\sin(M'_{\mathrm{m}}).$$

Yet another correction term must be calculated:

$$A_4 = 0.214\sin(2M'_{\mathrm{m}}).$$

Now we can find the value of the Moon's corrected longitude, l', from

$$l' = l + E_{\mathrm{v}} + E_{\mathrm{c}} - A_{\mathrm{e}} + A_4.$$

The final correction to apply to the Moon's longitude is the variation, V, given by

$$V = 0.6583 \sin 2(l' - \lambda_{\odot}).$$

Then the Moon's true orbital longitude, l'', is just

$$l'' = l' + V.$$

Referring the longitude to the ecliptic allows us to calculate the ecliptic latitude, β_{m}, and longitude, λ_{m}. Thus

$$\lambda_{\mathrm{m}} = \tan^{-1}\left\{\frac{\sin(l'' - N')\cos i}{\cos(l'' - N')}\right\} + N',$$

and

$$\beta_{\mathrm{m}} = \sin^{-1}\left\{\sin\left(l'' - N'\right)\sin i\right\},$$

where N' is the corrected longitude of the node, and it is given by

$$N' = N - 0.16 \sin\left(M_{\odot}\right).$$

This is a lengthy calculation! Let us illustrate it with an example: what was the position of the Moon on 1 September 2003 at 0h UT? The values of l_0, P_0, N_0, i, and some other parameters of the Moon's orbit are listed in Table 11.

Moon's mean longitude at the epoch	l_0	=	91.929336 degrees
mean longitude of the perigee at the epoch	P_0	=	130.143076 degrees
mean longitude of the node at the epoch	N_0	=	291.682547 degrees
inclination of Moon's orbit	i	=	5.145396 degrees
eccentricity of the Moon's orbit	e	=	0.0549
semi-major axis of Moon's orbit	a	=	384401 km
Moon's angular diameter at distance a from the Earth	θ_0	=	0.5181 degrees
Moon's parallax at distance a from the Earth	π_0	=	0.9507 degrees

Table 11. Elements of the Moon's orbit, epoch 2010.0.

Method		*Example*		
1.	Find the number of days since 2010 January 0.0 (§3). Remember to count the hours, minutes and seconds as a fraction of a day. The total is D.	1 September	=	$243+1$
			=	244
			–	2 557 days
		D	=	$-2\,313$ days
2.	Find $\lambda_\odot$ and $M_\odot$ using the method of §46.	$\lambda_\odot$	=	158.171 829 degrees
		$M_\odot$	=	238.533 547 degrees
3.	Find $l = 13.176\,396\,6D + l_0$. Adjust to the range 0 to 360 by adding or subtracting multiples of 360.	l	=	214.924 000 degrees
4.	Find $M_\mathrm{m} = l - 0.111\,404\,1D - P_0$. Adjust to the range 0 to 360.	M_m	=	342.458 607 degrees
5.	Find $N = N_0 - 0.052\,953\,9D$. Adjust to the range 0 to 360.	N	=	54.164 917 degrees
6.	$E_\mathrm{v} = 1.2739\sin(2C - M_\mathrm{m})$, where $C = l - \lambda_\odot$.	E_v	=	0.960 757 degrees
7.	Find $A_\mathrm{e} = 0.1858\sin(M_\odot)$ and	A_e	=	$-0.158\,477$ degrees
	$A_3 = 0.37\sin(M_\odot)$.	A_3	=	$-0.315\,590$ degrees
8.	Find the corrected anomaly: $M'_\mathrm{m} = M_\mathrm{m} + E_\mathrm{v} - A_\mathrm{e} - A_3$.	M'_m	=	343.893 432 degrees
9.	Calculate $E_\mathrm{c} = 6.2886\sin(M'_\mathrm{m})$.	E_c	=	$-1.744\,614$ degrees
10.	Calculate $A_4 = 0.214\sin(2M'_\mathrm{m})$.	A_4	=	$-0.114\,077$ degrees
11.	Find $l' = l + E_\mathrm{v} + E_\mathrm{c} - A_\mathrm{e} + A_4$.	l'	=	214.184 544 degrees
12.	Find $V = 0.6583\sin\left[2\left(l' - \lambda_\odot\right)\right]$.	V	=	0.610 256 degrees
13.	Hence find the true longitude $l'' = l' + V$.	l''	=	214.794 800 degrees
14.	Find $N' = N - 0.16\sin(M_\odot)$.	N'	=	54.301 389 degrees
15.	Find $y = \sin(l'' - N')\cos i \ldots$	y	=	0.332 570
16.	$\ldots$ and $x = \cos(l'' - N')$.	x	=	$-0.942\,603$
17.	Calculate $\tan^{-1}\left(\frac{y}{x}\right)$. Remove the ambiguity by reference to Figure 29, adding or subtracting 180 or 360 to bring the result into the correct quadrant unless it is already there.	$\tan^{-1}\left(\frac{y}{x}\right)$	=	$-19.433\,905$
			+	180
			=	160.566 095 degrees
18.	Add N' to find λ_m.	λ_m	=	214.867 503 degrees
19.	Find $\beta_\mathrm{m} = \sin^{-1}\{\sin(l'' - N')\sin i\}$.	β_m	=	1.716 074 degrees
20.	Finally, convert λ_m and β_m to right ascension and declination (§27).	α_m	=	**14h 12m 42s**
		δ_m	=	**$-11°\,31'\,38''$**

The *Astronomical Almanac* gives the apparent coordinates of the Moon at 0h TT as α = 14h 12m 10s and $\delta = -11°\ 34'\ 52''$. We may generally expect an error of about a quarter of a degree in ecliptic coordinates (but see MoonPos2 below for a more precise spreadsheet method). This is illustrated in Figure 83 where the error, Δ, between λ_m calculated by this method and that calculated by MoonPos2 is drawn as a function of the date for early 2011.

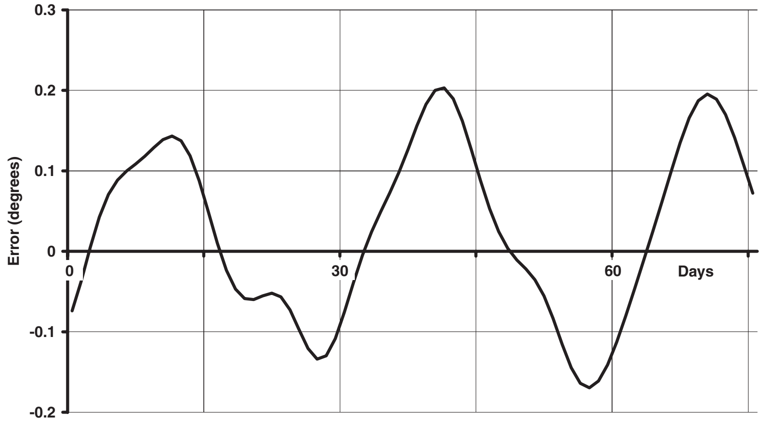


Figure 83. The error, between the ecliptic coordinates of the Moon as calculated by the method given here and those calculated by MoonPos2, for early 2011.

The spreadsheet, labelled MoonPos1 for this calculation, is shown in Figure 84. We have used cells I12 to I15 for the relevant orbital elements of the Moon, rather than having a table on a separate worksheet, since only one body (the Moon) is involved in this case. There is no need, either, to use the VLOOKUP function in this case. We have also cheated a bit by defining new spreadsheet functions SunMeanAnomaly (row 18) and UnwindDeg (e.g. row 19). The former takes eight arguments, which are the local civil time expressed as hours, minutes and seconds, the daylight saving and time zone offsets in hours, and the local civil date as day, month, and year. It returns the Sun's mean anomaly for the given instant in radians. The other function, UnwindDeg, takes a single argument of an angle in degrees and returns the equivalent angle in the range 0 to 360 degrees.

We have provided three additional spreadsheet functions called MoonLong, MoonLat, and MoonHP, returning respectively the Moon's geocentric ecliptic longitude, latitude and horizontal parallax (Section 69) in degrees. All three take the same eight arguments as SunMeanAnomaly above. The algorithms behind the functions use a full numerical model to calculate the respective values, and are accurate to within approximately 10, 5 and 0.5 arcseconds respectively over many hundreds of years. These have been used in the spreadsheet MoonPos2 (Figure 85), which also calculates the Earth–Moon distance in kilometres, and the Moon's horizontal parallax in degrees. Note that these are related (row 21) as

$$\text{distance} = \frac{6378.14}{\sin(\text{horizontal parallax})},$$

so in fact the Moon's orbital parameters a and e are not actually required. The number 6378.14 represents the radius of the Earth in kilometres. Also of note is the correction in row 19 for the effects of nutation in longitude.

	A	B	C	D	E	F	G	H	I	J
1	**The position of the Moon (approximate method)**									
2										
3	*Input*	**local time (hour)**	**0**			*Output*	**Moon's RA (hour)**	**14**	=DHHour(C36)	
4		**local time (min)**	**0**				**Moon's RA (Min)**	**12**	=DHMin(C36)	
5		**local time (sec)**	**0**				**Moon's RA (sec)**	**42.31**	=DHSec(C36)	
6		**daylight saving (hours)**	**0**				**Moon's dec (deg)**	**-11**	=DDDeg(C37)	
7		**zone correction (hours)**	**0**				**Moon's dec (min)**	**31**	=DDMin(C37)	
8		**local date (day)**	**1**				**Moon's dec (sec)**	**38.27**	=DDSec(C37)	
9		**local date (month)**	**9**							
10		**local date (year)**	**2003**							
11										
12	1	Greenwich date (day)	1	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)				***L*0**	**91.92933599**	
13	2	Greenwich date (month)	9	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)				***P*0**	**130.1430763**	
14	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)				***N*0**	**291.6825466**	
15	4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)				***i***	**5.145396**	
16	5	*D* (days)	-2313	=CDJD(C12,C13,C14)-CDJD(0,1,2010)+C15/24						
17	6	Sun long (deg)	158.171829	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)						
18	7	Sun mean anomaly (rad)	4.16319578	=SunMeanAnomaly(C3,C4,C5,C6,C7,C8,C9,C10)						
19	8	*L* m (deg)	214.9240002	=UnwindDeg(13.1763966*C16+I12)						
20	9	*M* m (deg)	342.4586072	=UnwindDeg(C19-0.1114041*C16-I13)						
21	10	*N* (deg)	54.16491734	=UnwindDeg(I14-(0.0529539*C16))						
22	11	*E* v (deg)	0.960757104	=1.2739*SIN(RADIANS(2*(C19-C17)-C20))						
23	12	*A* e (deg)	-0.158477357	=0.1858*SIN(C18)						
24	13	*A* 3 (deg)	-0.315590001	=0.37*SIN(C18)						
25	14	*M* md (deg)	343.8934316	=C20+C22-C23-C24						
26	15	*E* c (deg)	-1.744613565	=6.2886*SIN(RADIANS(C25))						
27	16	*A* 4 (deg)	-0.114077039	=0.214*SIN(2*RADIANS(C25))						
28	17	*L* d (deg)	214.184544	=C19+C22+C26-C23+C27						
29	18	*V* (deg)	0.610255619	=0.6583*SIN(2*RADIANS(C28-C17))						
30	19	*L* dd (deg)	214.7947997	=C28+C29						
31	20	*N* d (deg)	54.30138869	=C21-0.16*SIN(C18)						
32	21	*y*	0.33256969	=SIN(RADIANS(C30-C31))*COS(RADIANS(I15))						
33	22	*x*	-0.942603097	=COS(RADIANS(C30-C31))						
34	23	Moon long (deg)	214.8675028	=UnwindDeg(DEGREES(ATAN2(C33,C32))+C31)						
35	24	Moon lat (deg)	1.716074358	=DEGREES(ASIN(SIN(RADIANS(C30-C31))*SIN(RADIANS(I15))))						
36	25	Moon RA (hours)	14.21175359	=DDDH(ECRA(C34,0,0,C35,0,0,C12,C13,C14))						
37	26	Moon dec (deg)	-11.52729751	=ECDec(C34,0,0,C35,0,0,C12,C13,C14)						

Figure 84. Finding the approximate position of the Moon.

	A	B	C	D	E	F	G	H	I	J
1	**The position of the Moon (higher precision method)**									
2										
3	*Input*	**local time (hour)**	**0**			*Output*	**Moon RA (hour)**	**14**	=DHHour(C22)	
4		**local time (min)**	**0**				**Moon RA (min)**	**12**	=DHMin(C22)	
5		**local time (sec)**	**0**				**Moon RA (sec)**	**10.21**	=DHSec(C22)	
6		**daylight saving (hours)**	**0**				**Moon dec (deg)**	**-11**	=DDDeg(C23)	
7		**zone correction (hours)**	**0**				**Moon dec (min)**	**34**	=DDMin(C23)	
8		**local date (day)**	**1**				**Moon dec (sec)**	**57.83**	=DDSec(C23)	
9		**local date (month)**	**9**				**Earth-Moon dist (km)**	**367964**	=ROUND(C21,0)	
10		**local date (year)**	**2003**				**Moon hor parallax (deg)**	**0.993191**	=ROUND(C20,6)	
11										
12	1	Greenwich date (day)	1	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)						
13	2	Greenwich date (month)	9	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)						
14	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)						
15	4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)						
16	5	Moon ecliptic longitude (deg)	214.7660758	=MoonLong(C3,C4,C5,C6,C7,C8,C9,C10)						
17	6	Moon elciptic latitude (deg)	1.620130541	=MoonLat(C3,C4,C5,C6,C7,C8,C9,C10)						
18	7	nutation in longitude (deg)	-0.003683653	=NutatLong(C12,C13,C14)						
19	8	corrected long (deg)	214.7623922	=C16+C18						
20	9	Moon horizontal parallax (deg)	0.993190567	=MoonHP(C3,C4,C5,C6,C7,C8,C9,C10)						
21	10	Earth-Moon distance (km)	367964.4345	=6378.14/SIN(RADIANS(C20))						
22	11	Moon RA (hours)	14.20283692	=DDDH(ECRA(C19,0,0,C17,0,0,C12,C13,C14))						
23	12	Moon dec (deg)	-11.58273038	=ECDec(C19,0,0,C17,0,0,C12,C13,C14)						

MoonPos2

Figure 85. Finding the position of the Moon using a more precise method.

66 The Moon's hourly motions

The calculation which we have to do to find the position of the Moon is a lengthy affair and needs great care in its execution to avoid making mistakes. It may be that you require the position at several different times during one day and, rather than repeat the calculation several times, it is sufficient to find the position once and then extrapolate to the other times using the values for the hourly motions of the Moon in ecliptic latitude and longitude. These motions are given by the formulas

$$\Delta\beta = 0.05 \cos\left(l'' - N'\right) \text{ degrees/hour,}$$

$$\Delta\lambda = 0.55 + 0.06 \cos\left(M'_{\mathrm{m}}\right) \text{ degrees/hour,}$$

where $\Delta\beta$ is the hourly motion in latitude and $\Delta\lambda$ is the hourly motion in longitude. Given a position λ_0, β_0 at time t_0, the position t hours later is simply

$$\beta = \beta_0 + \Delta\beta t,$$

$$\lambda = \lambda_0 + \Delta\lambda t.$$

Continuing the previous example, what were the Moon's ecliptic coordinates at 3h 30m TT on 1 September 2003?

Method	*Example*		
1. Write down λ_0, β_0, (these are the values of λ_{m}, β_{m} at time t_0), l'', N', M'_{m} at t_0 (§65).	λ_0	=	214.867 503 degrees
	β_0	=	1.716 074 degrees
	l''	=	214.794 800 degrees
	N'	=	54.301 389 degrees
	M'_{m}	=	343.893 432 degrees
	t_0	=	0.0 hours
2. Calculate $\Delta\beta = 0.05\cos(l'' - N')$	$\Delta\beta$	=	−0.047 130 degrees/hour
and $\Delta\lambda = 0.55 + 0.06\cos(M'_{\mathrm{m}})$.	$\Delta\lambda$	=	0.607 645 degrees/hour
3. Find t in hours: t = new time $-t_0$, both times expressed in decimal hours.	t	=	3.5 hours
4. Find the coordinates at the new time:			
$\beta = \beta_0 + \Delta\beta\, t$,	β	=	**1.551 118 degrees**
$\lambda = \lambda_0 + \Delta\lambda\, t$.	λ	=	**216.994 260 degrees**

We have not provided a spreadsheet for this straightforward calculation because, if you are using a spreadsheet to find the Moon's position, you will probably want to calculate the Moon's position directly for every instance rather than use the calculation given above.

67 The phases of the Moon

The relative positions of the Sun and the Moon as viewed from the Earth change during the course of one month. It is always the hemisphere of the Moon facing towards the Sun which is brightly illuminated but we on the Earth see only that half which faces us. Unless the Moon is in opposition to the Sun, the time of full Moon, our half is not uniformly illuminated but overlaps both the bright and dark sides; hence we see only a segment of the disc. The area of the segment expressed as a fraction of the whole disc is called the **phase**.

The variation of phase with the Moon's position is illustrated in Figure 86, showing a plan view of the Moon's orbit about the Earth, E. The Moon is drawn in five positions marked 1 to 5. At 1, the whole of the dark side is turned towards us so that unless the Moon is illuminated by sufficient **earthshine** it is invisible. This is the new Moon. One week later the Moon has reached position 3 and is said to be in **quadrature**. This is the first quarter. Position 4 is the full Moon, the point of opposition with the Sun. At position 5, the Moon is again in quadrature; this is the third quarter. Between positions 3 and 5 more than half of the Moon's face is illuminated and the Moon is said to be **gibbous**.

In Figure 86, the angle D is called the **age** of the Moon, varying from 0° to 360° as the Moon completes one cycle of its orbit. Sometimes this angle is expressed in days, 1 day being equivalent to about 13°. The phase, F, is given by

$$F = \frac{1}{2}(1 - \cos D).$$

We have already made most of the calculations to find D in Section 65. Referring again to Figure 86, we find

$$D = l'' - \lambda_{\odot} \text{ degrees.}$$

Continuing the example of Section 65, we will find the phase of the Moon at 0h TT on 1 September 2003.

Method		*Example*		
1.	Find the values of $\lambda_{\odot}$ and l'', using the methods of §§46 and 65.	$\lambda_{\odot}$	=	158.171 829 degrees
		l''	=	214.794 800 degrees
2.	Find $D = l'' - \lambda_{\odot}$.	D	=	56.622 971 degrees
3.	Calculate $F = \frac{1}{2}(1 - \cos D)$.	F	=	**0.225**

The *Astronomical Almanac* reports the fraction of the disc which is illuminated (the phase) as 0.226 for this time.

The spreadsheet MoonPhase, Figure 87, provides a spreadsheet version of this calculation. The relevant rows are 12 to 18, the rest (19 to 26) being to do with calculating the position-angle of the bright limb of the Moon (next section). We have also provided a (slightly more accurate) spreadsheet function to calculate the phase explicitly. It is called MoonPhase and it takes eight arguments, namely the local civil time as hours, minutes and seconds, the daylight saving and time zone offsets in hours, and the calendar date as day, month, year. Thus, having saved a copy of the spreadsheet, you could delete rows 12 to 18, and place the following spreadsheet formula in cell H3:

```
=ROUND(MoonPhase(C3,C4,C5,C6,C7,C8,C9,C10),2).
```

You can play a similar trick with rows 19 to 26 (see the next section).

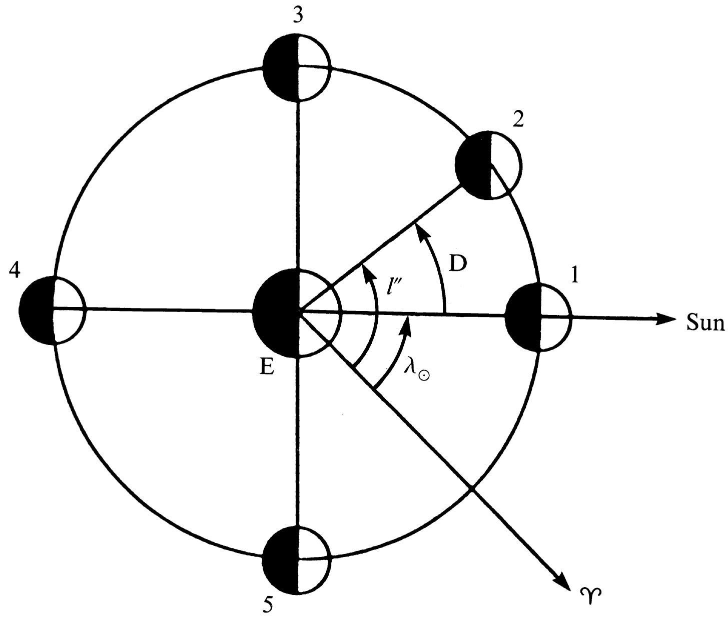


Figure 86. The phases of the Moon.

One of the important steps in calculating an eclipse of the Sun or Moon (Sections 71–74) is to find the times of new Moon and full Moon, that is the moments when the Moon is in **conjunction** or **opposition** to the Sun respectively. To help you to do this, we have provided a spreadsheet called MoonNewFull (Figure 88), and spreadsheet functions called NewMoon and FullMoon. These functions are used in the spreadsheet at rows 16 and 17, and each takes five arguments, namely the daylight saving and time zone offsets in hours, and the calendar date as day, month, year. The functions return the Julian dates of the instances of new Moon and full Moon associated with the lunar cycle (the **lunation**) in progress on the given date. The rest of the spreadsheet, rows 18 to 29, turns the Julian dates into local civil times and calendar dates.

	A	B	C	D	E	F	G	H	I
1	**The phase and the position angle of the bright limb of the Moon**								
2									
3	*Input*	**local time (hour)**	**0**			*Output*	**Moon phase (1 is full)**	**0.22**	=ROUND(C18,2)
4		**local time (min)**	**0**				**PA bright limb (deg)**	**-71.58**	=ROUND(C26,2)
5		**local time (sec)**	**0**						
6		**daylight saving (hours)**	**0**						
7		**zone correction (hours)**	**0**						
8		**local date (day)**	**1**						
9		**local date (month)**	**9**						
10		**local date (year)**	**2003**						
11									
12	1	Greenwich date (day)	1	=LCTGDay(C3,C4,C5,C6,C7,C8,C9,C10)					
13	2	Greenwich date (month)	9	=LCTGMonth(C3,C4,C5,C6,C7,C8,C9,C10)					
14	3	Greenwich date (year)	2003	=LCTGYear(C3,C4,C5,C6,C7,C8,C9,C10)					
15	4	Sun longitude (deg)	158.171829	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)					
16	5	MoonLong (deg)	214.7660758	=MoonLong(C3,C4,C5,C6,C7,C8,C9,C10)					
17	6	*D* (rad)	0.987755945	=RADIANS(C16-C15)					
18	7	Moon phase (0-1)	0.224717717	=(1-COS(C17))/2					
19	8	Moon latitude (deg)	1.620130541	=MoonLat(C3,C4,C5,C6,C7,C8,C9,C10)					
20	9	Sun RA (rad)	2.789425109	=RADIANS(ECRA(C15,0,0,0,0,0,C12,C13,C14))					
21	10	Moon RA (rad)	3.718355856	=RADIANS(ECRA(C16,0,0,C19,0,0,C12,C13,C14))					
22	11	Sun dec (rad)	0.148454791	=RADIANS(ECDec(C15,0,0,0,0,0,C12,C13,C14))					
23	12	Moon dec (rad)	-0.20217822	=RADIANS(ECDec(C16,0,0,C19,0,0,C12,C13,C14))					
24	13	*y*	-0.792170114	=COS(C22)*SIN(C20-C21)					
25	14	*x*	0.263794374	=COS(C23)*SIN(C22)-SIN(C23)*COS(C22)*COS(C20-C21)					
26	15	chi (deg)	-71.58212927	=DEGREES(ATAN2(C25,C24))					

MoonPhase

Figure 87. Finding the phase and position-angle of the bright limb of the Moon.

	A	B	C	D	E	F	G	H	I
1	The times of new Moon and full Moon								
2									
3	*Input*	daylight saving (hours)	0			*Output*	new Moon instant:		
4		zone correction (hours)	0				local time (hour)	17	=DHHour(C28)
5		local date (day)	1				local time (min)	27	=DHMin(C28)
6		local date (month)	9				local date (day)	27	=UTLCDay(C26,0,0,C6,C7,C19,C20,C21)
7		local date (year)	2003				local date (month	8	=UTLcMonth(C26,0,0,C6,C7,C19,C20,C21)
8							local date (year)	2003	=UTLcYear(C26,0,0,C6,C7,C19,C20,C21)
9							full Moon instant:		
10							local time (hour)	16	=DHHour(C29)
11							local time (min)	36	=DHMin(C29)
12							local date (day)	10	=UTLCDay(C27,0,0,C6,C7,C23,C24,C25)
13							local date (month	9	=UTLcMonth(C27,0,0,C6,C7,C23,C24,C25)
14							local date (year)	2003	=UTLcYear(C27,0,0,C6,C7,C23,C24,C25)
15									
16	1	JD of new Moon (days)	2452879.227	=NewMoon(C3,C4,C5,C6,C7)					
17	2	JD of full Moon (days)	2452893.192	=FullMoon(3,C4,C5,C6,C7)					
18	3	G date of new Moon (day)	27.72680637	=JDCDay(C16)					
19	4	… integer day	27	=INT(C18)					
20	5	G date of new Moon (month)	8	=JDCMonth(C16)					
21	6	G date of new Moon (year)	2003	=JDCYear(C16)					
22	7	G date of full Moon (day)	10.69174652	=JDCDay(C17)					
23	8	… integer day	10	=INT(C22)					
24	9	G date of full Moon (month)	9	=JDCMonth(C17)					
25	10	G date of full Moon (year)	2003	=JDCYear(C17)					
26	11	UT of new Moon (hours)	17.44335286	=24*(C18-C19)					
27	12	UT of full Moon (hours)	16.60191639	=24*(C22-C23)					
28	13	LCT of new Moon (hours)	17.45168586	=UTLCT(C26+0.008333,0,0,C6,C7,C19,C20,C21)					
29	14	LCT of full Moon (hours)	16.61024939	=UTLCT(C27+0.008333,0,0,C6,C7,C23,C24,C25)					

MoonNewFull

Figure 88. Finding the instances of new Moon and full Moon.

68 The position-angle of the Moon's bright limb

In Section 59 we saw how to calculate the position-angle, χ, of the bright limb of a planet. χ is defined to be the angle of the midpoint of the illuminated limb measured eastwards from the north point of the disc (see Figure 71). We can do the same for the Moon. χ is given by

$$\chi = \tan^{-1}\left\{\frac{\cos\delta_\odot \sin\left(\alpha_\odot - \alpha_m\right)}{\cos\delta_m \sin\delta_\odot - \sin\delta_m \cos\delta_\odot \cos\left(\alpha_\odot - \alpha_m\right)}\right\},$$

where $\alpha_\odot$, $\delta_\odot$ and α_m, δ_m are the equatorial coordinates of the Sun and Moon respectively.

For example, what was the position-angle of the Moon's bright limb on 1 September 2003 at 0h TT? The coordinates of the Sun and Moon that day were $\alpha_\odot$ = 10h 39m 17s, $\delta_\odot$ = 8° 30′ 21″, α_m = 14h 12m 10s, and δ_m = −11° 34′ 58″. (These can be calculated by the methods given in Sections 46 and 65.)

	Method	*Example*		
1.	Convert $\alpha_\odot$ and α_m first to decimal hours (§7) and then to degrees (§22).	$\alpha_\odot$	=	10.654 722 hours
			=	159.820 833 degrees
		α_m	=	14.202 778 hours
			=	213.041 667 degrees
2.	Convert $\delta_\odot$ and δ_m to decimal degrees (§21).	$\delta_\odot$	=	8.505 833 degrees
		δ_m	=	−11.582 778 degrees
3.	Find $y = \cos\delta_\odot \sin\left(\alpha_\odot - \alpha_m\right)\ldots$	y	=	−0.792 170
4.	$\ldots$ and $x = \cos\delta_m \sin\delta_\odot - \sin\delta_m \cos\delta_\odot \cos\left(\alpha_\odot - \alpha_m\right)$.	x	=	0.263 794
5.	Find $\chi = \tan^{-1}\left(\frac{y}{x}\right)$. Remove the ambiguity by referring to Figure 29, adding or subtracting 180 to bring the result into the correct quadrant if not already there, according to the signs of x and y.	χ	=	−71.582 151 degrees
		(already in the right quadrant)		
		χ	=	**−71.58 degrees**

As mentioned in the previous section, the spreadsheet MoonPhase (Figure 87) also performs this calculation. We have provided a spreadsheet function called MoonPABL and it takes eight arguments, namely the local civil time as hours, minutes and seconds, the daylight saving and time zone offsets in hours, and the calendar date as day, month, year. Thus, having saved a copy of the spreadsheet, you could delete rows 19 to 26, and place the following spreadsheet formula in cell H4:

```
=ROUND(MoonPABL(C3,C4,C5,C6,C7,C8,C9,C10),2).
```

Note that the angles -71.58 and $360 + (-71.58) = 288.42$ are equivalent to each other (all expressed in degrees).

69 The Moon's distance, angular size and horizontal parallax

During the course of one complete circuit of its orbit, the Moon's distance, ρ, from the Earth varies quite considerably. Its point of closest approach, the perigee, is about 356 000 km from the Earth while the furthest point, the apogee, is at a distance of 407 000 km. We can calculate its distance at any other point quite easily, as it is given by the formula

$$\rho = \frac{a\left(1-e^2\right)}{1+e\cos\left(M'_{\rm m}+E_{\rm c}\right)},$$

where $M'_{\rm m}$ is the corrected anomaly, $E_{\rm c}$ is the correction for the equation of the centre (defined in Section 65), e is the eccentricity (about 0.054 900) and a is the semi-major axis of the Moon's orbit (see Table 11). We usually wish to express the distance as a fraction of a so we write

$$\rho' = \frac{\rho}{a} = \frac{\left(1-e^2\right)}{1+e\cos\left(M'_{\rm m}+E_{\rm c}\right)}.$$

The units of ρ are the same as those of a; if a is expressed in kilometres, so is ρ.

The Moon's apparent angular diameter, θ, follows directly from the value of ρ'. It is given by

$$\theta = \frac{\theta_0}{\rho'},$$

where θ_0 is the Moon's apparent angular diameter when it is at a distance a from Earth. The value of θ_0 is given in Table 11 (approximately 0.518 1 degrees).

The Moon's horizontal parallax is defined to be the angle subtended at the Moon by the Earth's radius. In Figure 89 it is given by the symbol π (not to be confused with the constant 3.141 592 7). The formula is

$$\pi = \frac{\pi_0}{\rho'},$$

where π_0 is the horizontal parallax at distance a from the Earth (Table 11; about 0.950 7 degrees). Note that we have provided a spreadsheet function, MoonHP (see Section 65) that calculates the value of π directly. From this you can calculate ρ, given the radius of the Earth (6378.14 km):

$$\rho = \frac{6378.14}{\sin\left(\pi\right)}.$$

For example, what were the values of ρ', θ and π on 1 September 1979 at 0h UT?

Method	*Example*		
1. Find $M'_{\rm m}$ and $E_{\rm c}$ by the methods in §65.	$M'_{\rm m}$	=	343.893 432 degrees
	$E_{\rm c}$	=	−1.744 614 degrees
2. Calculate $\rho' = \frac{\rho}{a} = \frac{1-e^2}{1+e\cos(M'_{\rm m}+E_{\rm c})}$.	ρ'	=	**0.947 474**
3. Find $\theta = \frac{\theta_0}{\rho'}$.	θ	=	0.546 822 degrees
		=	**32′ 49″**
4. Find $\pi = \frac{\pi_0}{\rho'}$.	π	=	1.003 405 degrees
		=	**1° 00′ 12″**

We have provided spreadsheet functions to make these calculations rather more accurately. They are called MoonDist, which returns the distance between the Earth's centre and the Moon's centre in kilometres, MoonSize, which returns the angular diameter of the Moon as seen from the centre of the Earth, in degrees, and MoonHP, which gives us the Moon's horizontal parallax in degrees. Each of them takes the same set of eight arguments, namely the local civil time as hours, minutes and seconds, the daylight saving and time zone offsets in hours (W negative), and the local calendar date as day, month and year. We have used these in a spreadsheet called MoonDist (Figure 90). The *Astronomical Almanac* gives the values of the distance, angular diameter and horizontal parallax respectively as 367 948 km, $32' \, 29''$ and $0° \, 59' \, 36''$. Note that the apparent angular size of the Moon will be slightly different depending on where you are on the Earth, being largest when you are nearest to it, i.e. when the Moon is directly overhead.

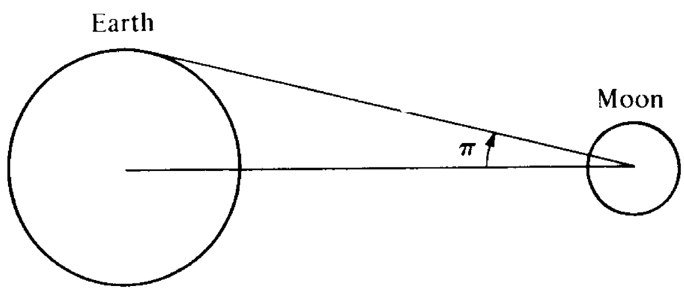


Figure 89. Lunar parallax.

	A	B	C	D	E	F	G	H	I	J	K
1	**The Moon's distance, angular diameter, and horizontal parallax**										
2											
3	*Input*	**local time (hour)**	**0**			*Output*	**Earth-Moon dist (km)**	**367960**	=ROUND(C12,-1)		
4		**local time (min)**	**0**				**ang diameter (deg)**	**0**	=DDDeg(C13+0.008333)		
5		**local time (sec)**	**0**				**(min)**	**32**	=DDMin(C13+0.008333)		
6		**daylight saving (hours)**	**0**				**hor parallax (deg)**	**0**	=DDDeg(C14)		
7		**zone correction (hours)**	**0**				**(min)**	**59**	=DDMin(C14)		
8		**local date (day)**	**1**				**(sec)**	**35.49**	=DDSec(C14)		
9		**local date (month)**	**9**								
10		**local date (year)**	**2003**								
11											
12	1	Moon's distance	367964.4345	=MoonDist(C3,C4,C5,C6,C7,C8,C9,C10)							
13	2	Moon's angular diameter	0.541242956	=MoonSize(C3,C4,C5,C6,C7,C8,C9,C10)							
14	3	Moon horizontal parallax	0.993190567	=MoonHP(C3,C4,C5,C6,C7,C8,C9,C10)							

MoonDist

Figure 90. Finding the Moon's distance, diameter and horizontal parallax.

70 Moonrise and moonset

In Section 33 we found how to calculate the times of rising or setting of a star given its equatorial coordinates. We can apply the same method to the Moon to find the times of moonrise and moonset but the problem is complicated by three factors. One is that the Moon is in rapid motion so that its right ascension and declination are continually changing. To find the time of moonset, for instance, we require the coordinates of the Moon at that time; but to find these coordinates we need the time of moonset, and so we go round in a circle.

The second problem is also associated with the Moon's rapidly changing position. In order to find the circumstances of moonrise and moonset at a given location on the Earth for a given calendar date, we need to know the corresponding Greenwich dates. We adopt an iterative scheme here, in which we find the position of the Moon at, say, midday, use that position to calculate the times of moonrise and moonset, then recalculate the Moon's positions at those times, then recalculate the times of moonrise and moonset, and so on until the changes are small enough for us to ignore. We need to take account of the fact that the calendar date might change, and that the conversion from Greenwich sidereal time to UT (an essential step) might be ambiguous (see Section 13). And then there is the fact that the Moon might not rise or set on a particular date!

The third major complication with the Moon is that it is, astronomically speaking, very close to the Earth. The coordinates which we work out are correct for the centre of the Earth, but when we observe from the Earth's surface the apparent coordinates change slightly; this effect is called **parallax** (see Section 38). In the case of the Moon the parallax can be as much as a whole degree. Taking this into account, together with the corrections for atmospheric refraction and the finite size of the Moon's disc (times quoted are for the upper limb), we can find the times of moonrise and moonset correct to within a minute or two of time.

Let us clarify all this with an example: what were the times of moonrise and moonset on 6 March 1986 as observed from Boston, Massachusetts, longitude $71^\circ\ 03'$ W and latitude $42^\circ\ 22'$ N, time zone -5 hours? The *Old Farmer's Almanac* (see Bibliography on page 208) lists these times as EST rising = 4h 20m am and EST setting = 1h 08m pm.

The calculation is lengthy, even for one iteration, so we use spreadsheets straight away. The first of these, Figure 91, is called MoonRiseSet1 and it provides a sort of do-it-yourself iterative method. The local calendar date and a time (called the starting time – cell C11) are first converted to the Greenwich calendar date (rows 14 to 16), and then the Moon's position and horizontal parallax are calculated (rows 17 to 21). The right ascension and declination, and the vertical displacement (row 22), are used to find the local sidereal times (rows 23 to 24) and then the Greenwich sidereal times (rows 26 and 27) of both rising and setting. The times are converted to universal times in rows 28 to 31. Note that by this time we have already acquired two status flags, one to do with rising and setting (row 25) and the other to do with conversion from Greenwich sidereal time to universal time (rows 29 and 31). Finally, the universal times are converted to local calendar dates and times (rows 32 to 39) and the azimuths calculated (rows 40 and 41).

Figure 91 shows the results of carrying out this series of calculations using a starting time of midday, 12 o'clock (cell C11). Comparing the time of moonrise, say, with that given in the *Old Farmer's Almanac*, we see that our result (4h 36m) is some 16 minutes later than the quoted time. We can now realise the power of the spreadsheet by inserting this time back into cell C11 as the starting time. Figure 92 shows what happens. The time of moonrise is now found to be 4h 21m, within a minute of the *Old Farmer's* calculation. We would need to repeat this exercise for the time of moonset, substituting 13.066 7 (13h 4m) into the cell C11.

	A	B	C	D	F	G	H
1	Moonrise and moonset (manual method)						
2							
3	*Input*	**local date (day)**	**6**		*Output*	**moonrise: (OK: OK)**	
4		**local date (month)**	**3**			**local time (hour)**	**4**
5		**local date (year)**	**1986**			**local time (min)**	**36**
6		**daylight saving (hours)**	**0**			**local date**	**6/3/1986**
7		**zone correction (hours)**	**-5**			**azimuth (deg)**	**126.14**
8		**geog long (deg)**	**-71.05**			**moonset: (OK: OK)**	
9		**geog lat (deg)**	**42.3667**			**local time (hour)**	**13**
10						**local time (min)**	**4**
11		**starting time (hours)**	**12**			**local date**	**6/3/1986**
12						**azimuth (deg)**	**233.86**
13							
14	1	G date (day)	6	=LCTGDay(C11,0,0,C6,C7,C3,C4,C5)			=CONCATENATE("moonrise: (",C25,": ",C29,")")
15	2	G date (month)	3	=LCTGMonth(C11,0,0,C6,C7,C3,C4,C5)			=DHHour(C32+0.008333)
16	3	G date (year)	1986	=LCTGYear(C11,0,0,C6,C7,C3,C4,C5)			=DHMin(C32+0.008333)
17	4	Moon long (deg)	297.2909149	=MoonLong(C11,0,0,C6,C7,C3,C4,C5)			=CONCATENATE(TEXT(C33,"##"),"/",TEXT(C34,"##"),"/",TEXT(C35,"####"))
18	5	Moon lat (deg)	-5.127456775	=MoonLat(C11,0,0,C6,C7,C3,C4,C5)			=ROUND(C40,2)
19	6	Moon HP (rad)	0.016952465	=RADIANS(MoonHP(C11,0,0,C6,C7,C3,C4,C5))			=CONCATENATE("moonset: (",C25,": ",C31,")")
20	7	Moon RA (hours)	20.030675	=DDDH(ECRA(C17,0,0,C18,0,0,C14,C15,C16))			=DHHour(C36+0.008333)
21	8	Moon dec (deg)	-25.7304396	=ECDec(C17,0,0,C18,0,0,C14,C15,C16)			=DHMin(C36+0.008333)
22	9	vertical displacement (deg)	-0.139979829	=DEGREES(0.27249*SIN(C19)+0.0098902-C19)			=CONCATENATE(TEXT(C37,"##"),"/",TEXT(C38,"##"),"/",TEXT(C39,"####"))
23	10	local sid time rising (hours)	15.7846104	=RSLSTR(C20,0,0,C21,0,0,C22,C9)			=ROUND(C41,2)
24	11	ocal sid time setting (hours)	0.276739591	=RSLSTS(C20,0,0,C21,0,0,C22,C9)			
25	12	rising & setting status	OK	=eRS(C20,0,0,C21,0,0,C22,C9)			
26	13	GST rising (hours)	20.52127706	=LSTGST(C23,0,0,C8)			
27	14	GST setting (hours)	5.013406257	=LSTGST(C24,0,0,C8)			
28	15	UT rising (hours)	9.599378064	=GSTUT(C26,0,0,C14,C15,C16)			
29	16	UT rising status	OK	=eGSTUT(C26,0,0,C14,C15,C16)			
30	17	UT setting (hours)	18.06832006	=GSTUT(C27,0,0,C14,C15,C16)			
31	18	UT setting status	OK	=eGSTUT(C27,0,0,C14,C15,C16)			
32	19	LCT rising (hours)	4.599378068	=UTLCT(C28,0,0,C6,C7,C14,C15,C16)			
33	20	LCT rising date (day)	6	=UTLCDay(C28,0,0,C6,C7,C14,C15,C16)			
34	21	LCT rising date (month)	3	=UTLCMonth(C28,0,0,C6,C7,C14,C15,C16)			
35	22	LCT rising (year)	1986	=UTLCYear(C28,0,0,C6,C7,C14,C15,C16)			
36	23	LCT setting (hours)	13.06832006	=UTLCT(C30,0,0,C6,C7,C14,C15,C16)			
37	24	LCT setting date (day)	6	=UTLCDay(C30,0,0,C6,C7,C14,C15,C16)			
38	25	LCT setting date (month)	3	=UTLCMonth(C30,0,0,C6,C7,C14,C15,C16)			
39	26	LCT setting date (year)	1986	=UTLCYear(C30,0,0,C6,C7,C14,C15,C16)			
40	27	azimuth rising (deg)	126.1440993	=RSAzR(C20,0,0,C21,0,0,C22,C9)			
41	28	azimuth setting (deg)	233.8559007	=RSAzS(C20,0,0,C21,0,0,C22,C9)			

Figure 91. A manual method of calculating moonrise and moonset.

	A	B	C	F	G	H
1	Moonrise and moonset (manual method)					
2						
3	*Input*	**local date (day)**	**6**	*Output*	**moonrise: (OK: OK)**	
4		**local date (month)**	**3**		**local time (hour)**	**4**
5		**local date (year)**	**1986**		**local time (min)**	**21**
6		**daylight saving (hours)**	**0**		**local date**	**6/3/1986**
7		**zone correction (hours)**	**-5**		**azimuth (deg)**	**127.3**
8		**geog long (deg)**	**-71.05**		**moonset: (OK: OK)**	
9		**geog lat (deg)**	**42.3667**		**local time (hour)**	**12**
10					**local time (min)**	**42**
11		**starting time (hours)**	**4.6**		**local date**	**6/3/1986**
12					**azimuth (deg)**	**232.7**

Figure 92. Another iteration in the calculation of moonrise.

We have also provided spreadsheet formulas to carry all this out behind the scenes. Each of them makes many iterations, and should generally work, though they may break down in particularly pathological circumstances. The formulas are MoonRiseLCT, MoonRiseLCDay, MoonRiseLCMonth, MoonRiseLCYear, MoonRiseAz, eMoonRise for moonrise, returning, respectively, the local civil time of moonrise in hours, the local calendar date, month and year, the azimuth, and a status string telling us about any error conditions. The string is set to OK if everything is fine, but if not it will tell us about Greenwich sidereal time conversion uncertainties as well as conditions under which the Moon never rises or never sets. Each function takes the same seven arguments, namely the local calendar date as day, month, year (but note that the functions may give you the result for the day before or after), the daylight saving and time zone offsets in hours (W negative), and the geographical longitude (W negative) and latitude (S negative) in degrees. There is an equivalent set of functions for moonset, identical in every respect except that the word Set replaces the word Rise in the name, e.g. MoonSetAz etc. A spreadsheet which uses these functions, called MoonRiseSet2, is shown in Figure 93. You can see that it delivers results which are very close to those quoted in the almanac.

Note that in Figures 91 to 93, the contents of cells H4 to H12 are shown below in cells H15 to H23 in order to save space. The contents of cell G3 is shown in cell H14. Some of these cells use the CONCATENATE function which joins together strings from various parts of the spreadsheet. Thus, in cell H6, we have

```
=CONCATENATE(TEXT(C22,##),''/'',TEXT(C23,##),''/'',TEXT(C24,####))
```

to produce the date in the format dd/mm/yyyy. The function TEXT converts a number (first argument) into a text string containing the number of digits identified in the second argument by the identifiers ## (for two digits) and #### for four digits. Then CONCATENATE produces just one string with all its arguments joined together with no spaces.

	A	B	C	F	G	H
1	Moonrise and moonset (automatic method)					
2						
3	*Input*	local date (day)	6	*Output*	moonrise: (OK)	
4		local date (month)	3		local time (hour)	4
5		local date (year)	1986		local time (min)	21
6		daylight saving (hours)	0		local date	6/3/1986
7		zone correction (hours)	-5		azimuth (deg)	127.34
8		geog long (deg)	-71.05		moonset: (OK)	
9		geog lat (deg)	42.3667		local time (hour)	13
10					local time (min)	8
11					local date	6/3/1986
12					azimuth (deg)	234.05
13						
14	1	local time of moonrise (hours)	4.345838	=MoonRiseLCT(C3,C4,C5,C6,C7,C8,C9)		=CONCATENATE("moonrise: (",C15,")")
15	2	local moonrise status	OK	=eMoonRise(C3,C4,C5,C6,C7,C8,C9)		=DHHour(C14+0.008333)
16	3	local date of moonrise (day)	6	=MoonRiseLcDay(C3,C4,C5,C6,C7,C8,C9)		=DHMin(C14+0.008333)
17	4	local date of moonrise (month)	3	=MoonRiseLcMonth(C3,C4,C5,C6,C7,C8,C9)		=CONCATENATE(TEXT(C16,"##"),"/",TEXT(C17,"##"),"/",TEXT(C18,"####"))
18	5	local date of moonrise (year)	1986	=MoonRiseLcYear(C3,C4,C5,C6,C7,C8,C9)		=ROUND(C19,2)
19	6	local azimuth (deg)	127.3362	=MoonRiseAz(C3,C4,C5,C6,C7,C8,C9)		=CONCATENATE("moonset: (",C21,")")
20	7	local time of moonset (hours)	13.1254	=MoonSetLCT(C3,C4,C5,C6,C7,C8,C9)		=DHHour(C20+0.008333)
21	8	local moonset status	OK	=eMoonset(C3,C4,C5,C6,C7,C8,C9)		=DHMin(C20+0.008333)
22	9	local date of moonset (day)	6	=MoonSetLcDay(C3,C4,C5,C6,C7,C8,C9)		=CONCATENATE(TEXT(C22,"##"),"/",TEXT(C23,"##"),"/",TEXT(C24,"####"))
23	10	local date of moonset (month)	3	=MoonSetLcMonth(C3,C4,C5,C6,C7,C8,C9)		=ROUND(C25,2)
24	11	local date of moonset (year)	1986	=MoonSetLcYear(C3,C4,C5,C6,C7,C8,C9)		
25	12	local azimuth (deg)	234.0493	=MoonSetAz(C3,C4,C5,C6,C7,C8,C9)		

MoonRiseSet2

Figure 93. An automatic method of calculating moonrise and moonset using spreadsheet functions.

71 Eclipses

Both the Earth and the Moon cast long shadows into space. The Earth's shadow lies exactly in the plane of the ecliptic opposite the Sun whereas that of the Moon may be above or below the ecliptic depending on the position of the Moon (Figure 94). The shadows are always present but we are usually unaware of them since we cannot see them from the Earth. Occasionally, however, one of the bodies passes through the shadow of the other and then we observe an eclipse: if the Moon passes through the Earth's shadow it is an eclipse of the Moon, or a **lunar eclipse**; when the Moon casts its shadow upon the Earth, we see the Sun partially or totally obscured and it is then a **solar eclipse**.

An eclipse of the Moon can only happen at full Moon and an eclipse of the Sun at new Moon. We do not see an eclipse on every such occasion, however, since the Moon's orbit does not lie in the plane of the ecliptic. Only when the Moon is also near one of its nodes can an eclipse occur.

A lunar eclipse begins with the **penumbral phase** when the Moon enters the penumbra of the Earth's shadow, and the Moon's disc becomes a little fainter. You probably wouldn't notice this unless you were looking for it. As the Moon enters the umbra the **partial phase** begins; when it has all moved inside the umbra the Sun's light is entirely cut off and the **total phase** begins. The only light reaching the Moon is then that refracted round the edges of the Earth, giving the Moon a coppery hue. The Earth's shadow extends well beyond the Moon's orbit so that it is always possible for a total lunar eclipse to occur, if other circumstances are favourable (Figure 95(*a*)).

A solar eclipse begins with the partial phase when the Earth enters the penumbra of the Moon's shadow. We see a 'bite' missing out of the Sun's disc and as the eclipse progresses the size of the bite increases. If you are favourably situated, you will see the Moon eventually cover the whole Sun. The eclipse is then total. Since the Moon is so much smaller than the Earth, its umbra extends a much shorter distance into space, in fact only just far enough to reach the Earth when the conditions are right (Figure 95(*b*)). The tip of the umbra casts a small shadow on the face of the Earth which moves across it as the Moon and Sun change their relative positions. Never is the umbra sufficiently large to engulf the whole Earth. Consequently any total eclipse can only be seen along a narrow strip of the Earth's surface.

Sometimes, however, the umbra does not reach the Earth at all (Figure 95(*c*)). In this case an **annular eclipse** can occur with the Moon not quite obscuring the whole of the Sun's disc at maximum eclipse but leaving a ring of light round its edge.

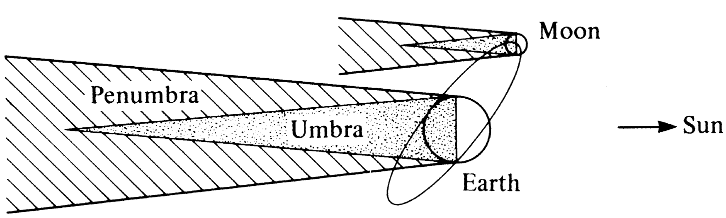


Figure 94. Shadows cast by the Moon and the Earth.

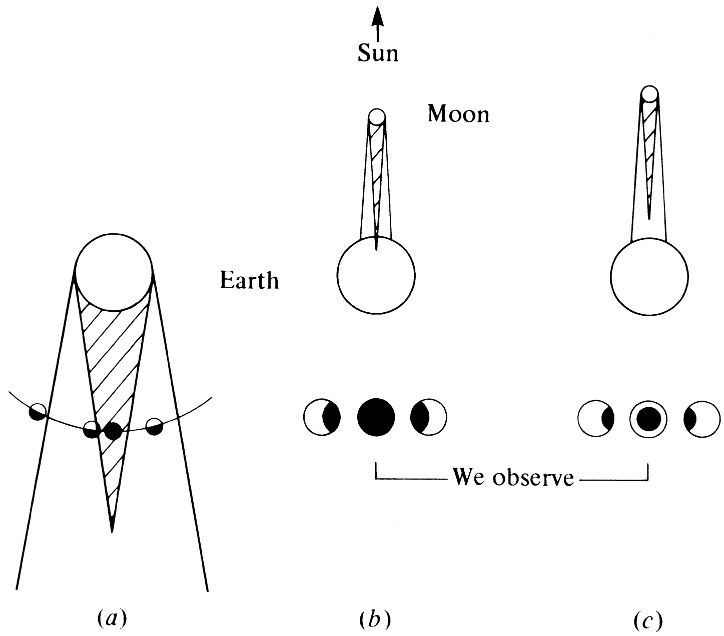


Figure 95. (*a*) Lunar eclipse, (*b*) total solar eclipse and (*c*) annular solar eclipse.

72 The 'rules' of eclipses

Here is a summary of the most important 'rules' which appear to govern the occurrence of eclipses.

(a) A lunar eclipse can only occur at full Moon and a solar eclipse at new Moon. There is not an eclipse every month.
(b) At least two solar eclipses occur every year, and never more than five. There is a maximum of three lunar eclipses in a year. The highest total number of eclipses in a year, lunar and solar, is seven.
(c) Eclipses tend to go in pairs or threes: solar–lunar–solar. A lunar eclipse is always preceded or followed by a solar eclipse (with two weeks in between them).
(d) The pattern of eclipses tends to recur in cycles of 18 years 11 days and 8 hours, the so-called 'Saros' cycle. The pattern is not repeated exactly.
(e) At the moment of greatest eclipse the Sun and Moon are nearly in opposition or conjunction. If the angle between the line of nodes and the Sun or Moon is greater than $12^\circ\ 15'$ a total lunar eclipse is not possible, while if it is less than $9^\circ\ 30'$ a lunar eclipse *must* occur. If the angle is more than $18^\circ\ 31'$ a solar eclipse cannot happen, while if it is less than $15^\circ\ 31'$ a solar eclipse *must* occur.
(f) In a lunar eclipse, the total phase can last for a maximum time of 1 hour 40 minutes, and the umbral phase, partial–total–partial, for a maximum time of 3 hours 40 minutes. The maximum time of total solar eclipse (at the equator) is 7 minutes 40 seconds and an annular eclipse can last at most for 12 minutes and 24 seconds.

A total solar eclipse is a special event in the history of the Earth, for the Moon is gradually moving away from the Earth as tidal forces are continuously transferring angular momentum from the Earth to the Moon's orbit. Right now we live at a particular time when the Sun and the Moon just happen to have more-or-less the same angular size in the sky. A very long time in the future will see the Moon and Earth locked together so that the period of the Moon's orbit around the Earth is exactly the same as the time for the Earth to spin once on its axis. The length of the day will then be nearly two months of present time and the Moon will be much smaller than the Sun in the sky so that total solar eclipses will never occur.

73 Calculating a lunar eclipse

There are two questions to be asked before proceeding with the calculation of an eclipse. The first is, 'Is an eclipse likely to occur?' If the answer is yes, the second is, 'Will I be able to see it?' You may predict an eclipse of the Moon at a certain time, but if the Moon hasn't risen or has already set you won't be able to see it!

First, then, to spot the likely time of occurrence of a lunar eclipse. From rule (a) above there must be a full Moon, that is the angle $\lambda_m - \lambda_\odot = 180°$. From rule (e) the angle between the line of nodes and the Moon must be within 12° 15′ of 0° or 180° at that time. This is the angle $l'' - N'$ in the calculations of Section 65.

For example, is there a lunar eclipse associated with the full Moon on 4 April 2015? In order to find out, we need to calculate the time of opposition, that is when the Moon and Sun are opposite each other in ecliptic longitude, and then we need to find the Moon's distance from the line of nodes. Using spreadsheet MoonNewFull (Section 67), we see that the UT of the full Moon on this date is 12.118 660 h (12h 07m). Hence we find, using the spreadsheet MoonPos1 of Section 65, that

$$\lambda_\odot = 14.407 \text{ degrees},$$
$$\lambda_m = 194.400 \text{ degrees},$$

(hence $\lambda_m - \lambda_\odot = 179.993$ degrees), and

$$l'' = 194.419 \text{ degrees},$$
$$N' = 189.824 \text{ degrees},$$

(hence $l'' - N' = 4.595$ degrees). We see that $\lambda_m - \lambda_\odot$ is very nearly 180° so that we are almost at the point of full Moon, and that $l'' - N' = 4.595$ degrees, well within the limit of 12° 15′. There is indeed a total lunar eclipse that day.

We have also provided a spreadsheet function called LEOccurrence which tests the angle between the Moon and the nearest node and returns with one of the string messages Lunar eclipse certain, Lunar eclipse possible, or No lunar eclipse depending on its magnitude. The function takes five arguments which are the daylight saving and time zone offsets in hours, and the local calendar date as day, month, year. It tests the full Moon of the lunation in progress on the date you specify. Use the function FullMoon to find the time and date of the full Moon. Spreadsheet LunarEclipseOccurrence, Figure 96, shows how it goes.

Having predicted the occurrence of an eclipse, the second question is, can you see it? The answer obviously depends on your position on the Earth. We can calculate, using Section 70, the times of moonrise and moonset on the day of the eclipse for an observer on the Greenwich meridian at latitude 52° N. We find that the Moon does not rise until 18h 49m UT while the eclipse is in progress at 10h 41m UT. Our observer cannot therefore see the eclipse. However, repeating the calculation for someone in Sydney, Australia, shows that moonrise is at 17h 39m local time, in time zone +10 hours, while the eclipse is in progress at midday$+10 =$ 22h local time, giving Sydney dwellers an excellent view if the night is clear.

To calculate the circumstances of an eclipse we need to know the Moon's position at some particular time near to full Moon, its hourly motions in longitude and latitude, its angular distance from the Sun (the angle $\lambda_m - \lambda_\odot$, its angular diameter, and the angular radius of the Earth's shadow at the distance of the Moon's orbit. This last is given by the following simple formulas with sufficient accuracy for our purposes:

$$S_p = \text{radius of penumbra} = \pi + 0.27 \text{ degrees},$$
$$S_u = \text{radius of umbra} = \pi - 0.27 \text{ degrees},$$

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Lunar eclipse occurrence**											
2												
3	*Input*	**local date (day)**	**1**			*Output*	**Lunar eclipse certain**		=C19			
4		**local date (month)**	**4**				**… on**	**4/4/2015**	=CONCATENATE(C16,"/",C17,"/",C18)			
5		**local date (year)**	**2015**									
6		**daylight saving (hours)**	**0**									
7		**zone correction (hours)**	**10**									
8												
9	1	Julian date of full Moon	2457117.005	=FullMoon(C6,C7,C3,C4,C5)								
10	2	G date of full Moon (day)	4.50494417	=JDCDay(C9)								
11	3	… integer day	4	=INT(C10)								
12	4	G date of full Moon (month)	4	=JDCMonth(C9)								
13	5	G date of full Moon (year)	2015	=JDCYear(C9)								
14	6	UT of full Moon (hours)	12.11866009	=(C10-C11)*24								
15	7	local civil time (hours)	22.11866008	=UTLCT(C14,0,0,C6,C7,C11,C12,C13)								
16	8	local civil date (day)	4	=UTLCDay(C14,0,0,C6,C7,C11,C12,C13)								
17	9	local civil date (month)	4	=UTLCMonth(C14,0,0,C6,C7,C11,C12,C13)								
18	10	local civil date (year)	2015	=UTLCYear(C14,0,0,C6,C7,C11,C12,C13)								
19	11	eclipse occurrence?	Lunar eclipse certain	=LEOccurrence(C6,C7,C3,C4,C5)								

Figure 96. Predicting a lunar eclipse.

where π is the Moon's horizontal parallax (Section 69). Let us now calculate the circumstances of the lunar eclipse on 4 April 2015. First, we write down all the details:

2015 April 4 at 12h 07m TT:

$$
\begin{aligned}
\lambda_{\mathrm{m}} &= 194.400 \text{ degrees;}\\
\beta_{\mathrm{m}} &= 0.412 \text{ degrees;}\\
\lambda_{\odot} &= 14.407 \text{ degrees;}\\
\Delta\lambda &= 0.500 \text{ degrees/hour;}\\
\Delta\beta &= 0.050 \text{ degrees/hour;}\\
\pi &= 0.907 \text{ degrees;}\\
S_{\mathrm{p}} = \pi + 0.27 &= 1.177 \text{ degrees;}\\
S_{\mathrm{u}} = \pi - 0.27 &= 0.637 \text{ degrees;}\\
\theta_{\mathrm{m}} &= 0.494 \text{ degrees.}
\end{aligned}
$$

Add to this the fact that the Sun is also moving in longitude at a rate of $360/(365.2422 \times 24)$ degrees/hour, that is at 0.041 degrees/hour.

Next, we calculate the precise point of opposition. This is the moment when the angle $\lambda_{\mathrm{m}} - \lambda_{\odot} = 180°$. At 12h 07m TT the angle is $194.400 - 14.407 = 179.993$ degrees, just 0.007 degrees short of 180°. The Earth's shadow, always directly opposite the Sun, moves at the same rate and in the same direction as the Sun. The Moon is moving at a rate of 0.500 degrees/hour, also in the same direction, and the Earth's shadow at 0.041 degrees/hour. Thus the Moon is catching up on the shadow at a rate of $0.500 - 0.041 = 0.459$ degrees/hour, so that it takes $0.007/0.459$ hours to catch up by 0.007 degrees, that is almost 1 minute. Opposition (in ecliptic coordinates) is therefore at 12h 08m TT.

Now we are in a position to construct the **eclipse diagram**, Figure 97. Draw a horizontal line to represent the plane of the ecliptic. Below this, draw a parallel line and, choosing a suitable scale (say 1 hour = 2 cm), mark it in hours. Find the position on the scale corresponding to the moment of opposition and mark it (point P) on the ecliptic line. (This is near the moment of maximum eclipse.) Next draw circles to scale centred on P to represent the umbra and the penumbra. Their radii should correspond with the angular radii of the shadows they represent. For example, we found above that $S_p = 1.177$ degrees. The difference between the hourly motions in longitude of the Earth's shadow and the Moon was 0.459 degrees/hour. Thus, on our scale of 1 hour: 2 cm, 0.459 degrees also equals 2 cm. Hence the scale factor between degrees and centimetres is 1 degree = 2/0.459 = 4.357 cm, and we should draw a circle of radius $1.177 \times 4.357 = 5.129$ cm to represent the penumbra. The radius of the circle marking the umbra should be $0.637 \times 4.357 = 2.775$ cm.

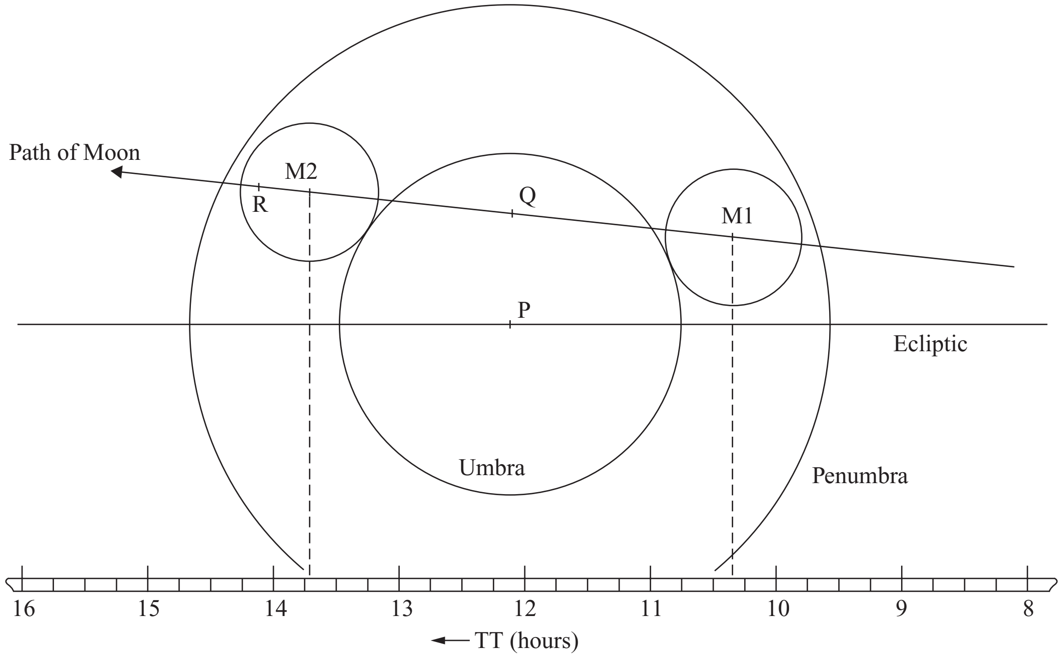


Figure 97. The lunar eclipse of 4 April 2015.

Now we are ready to plot the path of the Moon through the Earth's shadow. First, we mark the position, Q, of the Moon at 12h 07m TT for which we have calculated its ecliptic coordinates. This position lies at $\beta_\mathrm{m} = 0.412$ degrees above the ecliptic, corresponding to $0.412 \times 4.357 = 1.795$ cm on the scale of our diagram. Then we calculate the Moon's position at some other time, say 2 hours later at 14h 07m TT. Mark this point R. You can use the spreadsheet MoonPos1 to find the Moon's ecliptic latitude (giving $\beta_\mathrm{m} = 0.502$ degrees). Alternatively, you can use the Moon's hourly motions to find its approximate position as

$$\beta_\mathrm{m} = \beta_0 + \Delta\beta t = 0.412 + (0.050 \times 2) = 0.512 \text{ degrees.}$$

The difference of 0.01 degrees between the two calculations corresponds to a difference of about 4 mm on the diagram, so for best accuracy you should always use the spreadsheet results if you have them. Using 1 degree = 4.357 cm, we need to mark point R at $0.502 \times 4.357 = 2.187$ cm above the ecliptic line at the 14h 07m on our horizontal timescale. Joining Q and R with a straight line gives the path of the Moon.

Finally, we simply have to draw circles centred on the line RQ to represent the Moon at any point. The radii of these circles must correspond with the calculated angular radius of the Moon, $\theta_\mathrm{m} = 0.247$ degrees or 1.08 cm on the scale of the diagram. We mark two such positions: M1 represents the point where the Moon enters the umbra, and M2 where the Moon leaves the umbra. The corresponding times may be found from the scale below. They are:

Calculated
M1 at 10h 22m
M2 at 13h 44m

We have provided spreadsheet functions to calculate these times automatically with higher precision than you can obtain from using pencil and paper. The functions are

UTFirstContactLunarEclipse,
UTStartUmbralLunarEclipse,
UTStartTotalLunarEclipse,
UTMaxLunarEclipse,
UTEndTotalLunarEclipse,
UTEndUmbralLunarEclipse,
UTLastContactLunarEclipse, and
MagLunarEclipse.

Each of these takes the same set of five arguments, namely the local calendar date expressed as day, month, year, and the daylight saving and time zone offsets in hours. All but the last of them return the universal times, respectively, of first contact (when the Moon first touches the penumbra), the start of the umbral phase (M1 on our diagram), the start of the total phase (M2), mid-eclipse (when the eclipse is at its maximum), the end of the total phase (M3), the end of the umbral phase (M4) and the end of the eclipse (when the Moon leaves the penumbra). The last function, MagLunarEclipse, returns the **magnitude** of the eclipse, that is the fraction of the lunar diameter obscured by the Earth's shadow at the moment of greatest eclipse, measured along the common diameter. Its value is 1 or greater if the eclipse is total. These functions will return a value of -99 if the particular phenomenon does not occur. For example, UTEndUmbralLunarEclipse returns -99 if there is no umbral phase.

We have drawn all these functions together in the spreadsheet LunarEclipseCircumstances (Figure 98). This calculates the local civil date and universal times of an eclipse, if one occurs, for the lunation in progress on the specified local calendar date. We have used time zone 10 for Sydney, Australia. It would be a small task to modify the spreadsheet so that local civil times were displayed instead of universal times. Try it for yourself. You will need to use the function UTLCT. Note that the contents of cells G3, and H3 to H11, are shown respectively in cells H13 to H22 in order to save space.

	A	B	C	D	F	G	H
1	The circumstances of a lunar eclipse						
2							
3	*Input*	local date (day)	1		*Output*	Lunar eclipse certain on	4/4/2015
4		local date (month)	4			UT start of pen. phase	9:0
5		local date (year)	2015			UT start of umbral phase	10:16
6		daylight saving (hours)	0			UT start of total phase	11:55
7		zone correction (hours)	10			UT of mid eclipse	12:1
8						UT end of total phase	12:7
9						UT end of umbral phase	13:46
10						UT end of pen. phase	15:1
11						eclipse magnitude	1.005
12							
13	1	Julian date of full Moon	2457117.005	=FullMoon(C6,C7,C3,C4,C5)			=CONCATENATE(C23," on ")
14	2	G date full Moon (day)	4.50494417	=JDCDay(C13)			=CONCATENATE(C20,"/",C21,"/",C22)
15	3	… integer day	4	=INT(C14)			=IF(C25=-99,"",CONCATENATE(DHHour(C25+0.008333),":",DHMin(C25+0.008333)))
16	4	G date full Moon (month)	4	=JDCMonth(C13)			=IF(C27=-99,"",CONCATENATE(DHHour(C27+0.008333),":",DHMin(C27+0.008333)))
17	5	G date full Moon (year)	2015	=JDCYear(C13)			=IF(C29=-99,"",CONCATENATE(DHHour(C29+0.008333),":",DHMin(C29+0.008333)))
18	6	UT of full Moon (hours)	12.11866009	=(C14-C15)*24			=IF(C24=-99,"",CONCATENATE(DHHour(C24+0.008333),":",DHMin(C24+0.008333)))
19	7	local civil time (hours)	22.11866008	=UTLCT(C18,0,0,C6,C7,C15,C16,C17)			=IF(C30=-99,"",CONCATENATE(DHHour(C30+0.008333),":",DHMin(C30+0.008333)))
20	8	local civil date (day)	4	=UTLCDay(C18,0,0,C6,C7,C15,C16,C17)			=IF(C28=-99,"",CONCATENATE(DHHour(C28+0.008333),":",DHMin(C28+0.008333)))
21	9	local civil date (month)	4	=UTLCMonth(C18,0,0,C6,C7,C15,C16,C17)			=IF(C26=-99,"",CONCATENATE(DHHour(C26+0.008333),":",DHMin(C26+0.008333)))
22	10	local civil date (year)	2015	=UTLCYear(C18,0,0,C6,C7,C15,C16,C17)			=IF(C31=-99,"",ROUND(C31,3))
23	11	eclipse occurrence?	eclipse certain	=LEOccurrence(C6,C7,C3,C4,C5)			
24	12	UT max eclipse	12.01218263	=UTMaxLunarEclipse(C3,C4,C5,C6,C7)			
25	13	UT first contact	9.003272842	=UTFirstContactLunarEclipse(C3,C4,C5,C6,C7)			
26	14	UT last contact	15.02109241	=UTLastContactLunarEclipse(C3,C4,C5,C6,C7)			
27	15	UT start umbral phase	10.26520797	=UTStartUmbralLunarEclipse(C3,C4,C5,C6,C7)			
28	16	UT end umbral phase	13.75915728	=UTEndUmbralLunarEclipse(C3,C4,C5,C6,C7)			
29	17	UT start total phase	11.91406936	=UTStartTotalLunarEclipse(C3,C4,C5,C6,C7)			
30	18	UT end total phase	12.11029589	=UTEndTotalLunarEclipse(C3,C4,C5,C6,C7)			
31	19	eclipse magnitude	1.005100347	=MagLunarEclipse(C3,C4,C5,C6,C7)			

LunarEclipseCircumstances

Figure 98. Calculating the circumstances of a lunar eclipse.

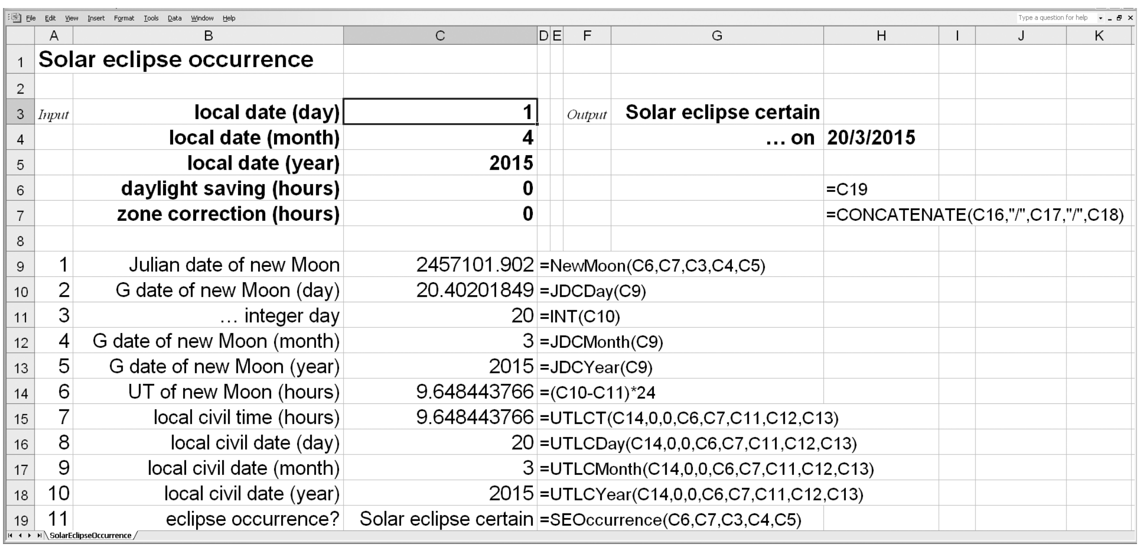
74 Calculating a solar eclipse

A solar eclipse is rather more difficult to calculate than a lunar eclipse. If you look up a solar eclipse in the *Astronomical Almanac* you will find a map of the world showing the path and duration of the eclipse at each point; we shall not attempt such detail here. Our simple calculations will be made for just one location but will give a good guide of what to expect.

Once again we need to answer the question 'Is an eclipse likely?' Rule (a) in Section 72 tells us that we have to be at new Moon, that is the angle $\lambda_m - \lambda_\odot$ equals 0° (or equivalently 360°). Rule (e) tells us that the angle between the Sun or the Moon and a node must be within 18° 31′ of 0° or 180° at that time; this is the angle $\lambda_\odot - N'$ or $l'' - N'$. We can find the time of new Moon using spreadsheet NewMoon (Section 67), but we have also provided a spreadsheet function called SEOccurrence which tests the angle between the Moon and the nearest node and returns with one of the string messages "Solar eclipse certain", "Solar eclipse possible", or "No solar eclipse" depending on its magnitude. The function takes five arguments which are the daylight saving and time zone offsets in hours, and the local calendar date as day, month, year. It tests the new Moon of the lunation in progress on the date you specify. Spreadsheet SolarEclipseOccurrence, Figure 99, shows how to use both of these functions to search for a solar eclipse. Note that the contents of cells G3 and H4 are shown in cells H6 and H7 respectively, moved here in order to save space.

We see from Figure 99 that there is a solar eclipse on 20 March 2015. We shall illustrate the method by working out the circumstances of that eclipse, as observed by someone on longitude 0° and at latitude 68° 39′ N.

First, we must work through the calculations of Sections 65, 66 and 69. The results for the time of new Moon (Figure 99), 9.648 h = 9h 39 m TT (taken to be equal to UT) on 20 March 2015 are as follows (we have used spreadsheet functions to find these values):



	A	B	C	D	E	F	G	H	I	J	K
1	Solar eclipse occurrence										
2											
3	Input	local date (day)	1			Output	Solar eclipse certain				
4		local date (month)	4				… on	20/3/2015			
5		local date (year)	2015								
6		daylight saving (hours)	0					=C19			
7		zone correction (hours)	0					=CONCATENATE(C16,"/",C17,"/",C18)			
8											
9	1	Julian date of new Moon	2457101.902	=NewMoon(C6,C7,C3,C4,C5)							
10	2	G date of new Moon (day)	20.40201849	=JDCDay(C9)							
11	3	… integer day	20	=INT(C10)							
12	4	G date of new Moon (month)	3	=JDCMonth(C9)							
13	5	G date of new Moon (year)	2015	=JDCYear(C9)							
14	6	UT of new Moon (hours)	9.648443766	=(C10-C11)*24							
15	7	local civil time (hours)	9.648443766	=UTLCT(C14,0,0,C6,C7,C11,C12,C13)							
16	8	local civil date (day)	20	=UTLCDay(C14,0,0,C6,C7,C11,C12,C13)							
17	9	local civil date (month)	3	=UTLCMonth(C14,0,0,C6,C7,C11,C12,C13)							
18	10	local civil date (year)	2015	=UTLCYear(C14,0,0,C6,C7,C11,C12,C13)							
19	11	eclipse occurrence?	Solar eclipse certain	=SEOccurrence(C6,C7,C3,C4,C5)							

Figure 99. Predicting a solar eclipse.

$\lambda_\odot$ = 359.461 degrees;

λ_m = 359.473 degrees;

($\lambda_m - \lambda_\odot = +0.012$ degrees);

β_m = 0.964 degrees;

$\Delta\lambda$ = 0.608 degrees/hour;

$\Delta\beta$ = -0.049 degrees/hour;

θ_m = 0.556 degrees;

π = 1.021 degrees; and

hourly motion of the Sun = 0.041 degrees/hour.

We now have to take account of geocentric parallax. The coordinates λ_m and β_m that we have just calculated are those that would be observed at the centre of the Earth. We are observing from the surface of the Earth at longitude 0° and at latitude 68° 39′ N, and we see slightly different ecliptic coordinates which can be calculated as follows (values from spreadsheet functions):

Method	*Example*
1. Transform λ_m and β_m to equatorial coordinates (§27).	α_m = 23.942 hours δ_m = 0.675 degrees
2. Find the apparent right ascension and declination after allowing for the effects of geocentric parallax (§39). Use the value of π for *P*.	α'_m = 23.957 hours δ'_m = -0.274 degrees
3. Convert α'_m and δ'_m back to ecliptic coordinates (§28).	λ'_m = **359.301 degrees** β'_m = **0.005 degrees**

Next we calculate the precise moment of conjunction in ecliptic coordinates when $\lambda'_m - \lambda_\odot = 0°$. Strictly, we ought to apply the correction for parallax to the Sun's coordinates as well, but we shall ignore this small correction here. At 09h 39m TT, $\lambda'_m - \lambda_\odot = -0.160$ degrees so that the Moon has still a little distance to catch up with the Sun. Its speed in longitude is $\Delta\lambda = 0.608$ degrees/hour so it is gaining on the Sun at $0.608 - 0.041 = 0.567$ degrees/hour. The difference of 0.160 degrees is made up in 0.160/0.567 hours = 0.282 hours = 17 minutes. Conjunction therefore occurs at 09h 56m TT. At this moment the Sun's longitude is $359.461 + (0.282 \times 0.041) = 359.473$ degrees.

Now we are ready to construct the eclipse diagram (Figure 100). We proceed exactly as we did for the lunar eclipse, drawing two horizontal lines, one to represent the ecliptic and the lower one to represent time. Choosing a suitable scale (say 2 cm = 1 hour) we mark off the lower line in hours such that the time of conjunction is roughly in the middle of the diagram. Next we find point P on the ecliptic corresponding to conjunction and we draw a circle centred on P of the correct radius to represent the Sun. In this calculation, we assume that the angular radius of the Sun is 0.268 degrees. (We can find this value using spreadsheet SunDist, Section 48.) The relative motion between Sun and Moon in this case is 0.567 degrees/hour. Thus

2 cm on our scale, which represents 1 hour, also represents 0.567 degrees. Hence 1 degree = $2/0.567$ = 3.527 cm. The Sun's radius converts to $0.268 \times 3.527 = 0.945$ cm on the scale of the diagram.

Next we plot the position of the Moon which we have calculated, using the corrected value β'_m. We find the point corresponding to 09h 39m TT and $\beta'_m = 0.005$ degrees (= 0.526 cm), and mark it Q. Then we find the Moon's position, say, 2 hours later, preferably using spreadsheets to repeat the calculation of finding the apparent position corrected for parallax, or approximately (but more easily) using the hourly motion in latitude:

$$\beta'_m = 0.005 - (0.049 \times 2) = -0.093 \text{ degrees} = -0.33 \text{ cm}.$$

Mark this point R. Joining Q and R by a straight line gives the path of the Moon relative to the Sun. We need only mark off circles centred on the line of the correct radius ($\theta_m/2$) to represent the Moon to determine the aspect of the eclipse at any time. In Figure 100 we have marked two positions, M1 and M2, corresponding to the start and end of the eclipse. The circles are of radius $(0.556/2) \times 3.527 = 0.98$ cm. Here are the results:

Calculated
M1 at 08h 59m
M2 at 10h 54m

You will notice from Figure 100 that the eclipse was total. Comparison of our calculated times with those

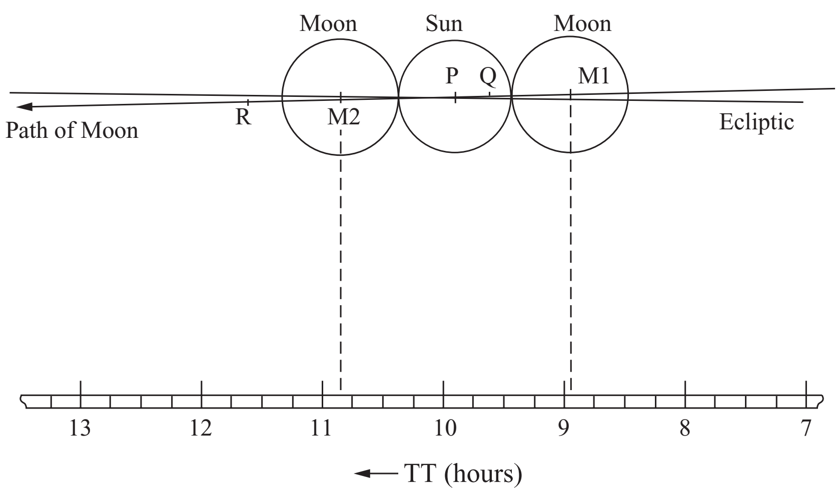


Figure 100. Solar eclipse of 20th March 2015 as observed from longitude 0° and latitude 68° 39′ N.

deduced from a more-accurate program shows that we are within a few minutes of the correct results. Even our comparatively simple method allows us to make quite accurate predictions of what is surely the heavens' most awe-inspiring phenomenon.

As in the previous section, we have provided spreadsheet functions to make these calculations more accurately and with a lot less effort. The functions are:

UTFirstContactSolarEclipse,

UTMaxSolarEclipse,

UTLastContactSolarEclipse, and

MagLunarEclipse.

The first three functions return the universal times of first contact (i.e. the moment when the limbs of the Sun and Moon first intersect with each other), mid-eclipse (which may or may not be total), and last contact (when the limbs no longer intersect). The last function provides the magnitude of the solar eclipse, defined to be the fraction of the solar diameter obscured by the Moon at the moment of greatest eclipse, measured along the common diameter. Total eclipses have magnitudes greater than 1, as in this case. All four functions take the same seven arguments, namely the local calendar date as day, month, year, the daylight saving and time zone offsets in hours, and the geographical longitude and latitude in degrees (W and S negative). The functions return the value −99 if an eclipse does not occur.

A spreadsheet called SolarEclipseCircumstances (Figure 101) shows how to make use of these functions. For our observer on the Greenwich meridian, the universal times are also the local times. For other observers, it would be useful to display local civil times, and you can do this quite easily using the spreadsheet function UTLCT. Note that the contents of cells G3, and H3 to H7, are shown respectively in cells H9 to H14 in order to save space.

	A	B	C	D	F	G	H
1	The circumstances of a solar eclipse						
2							
3	*Input*	local date (day)	20		*Output*	Solar eclipse certain on	20/3/2015
4		local date (month)	3			UT of first contact	8:55
5		local date (year)	2015			UT of mid eclipse	9:57
6		daylight saving (hours)	0			UT of last contact	10:58
7		zone correction (hours)	0			magnitude of eclipse	1.016
8		geog longitude (deg)	0				
9		geog latitude (deg)	68.65				=CONCATENATE(C21," on ")
10							=CONCATENATE(C18,"/",C19,"/",C20)
11	1	Julian date of new Moon	2457101.902	=NewMoon(C6,C7,C3,C4,C5)			=IF(C23=-99,"",CONCATENATE(DHHour(C23+0.008333),":",DHMin(C23+0.008333)))
12	2	G date of new Moon (day)	20.40201849	=JDCDay(C11)			=IF(C22=-99,"",CONCATENATE(DHHour(C22+0.008333),":",DHMin(C22+0.008333)))
13	3	… integer day	20	=INT(C12)			=IF(C24=-99,"",CONCATENATE(DHHour(C24+0.008333),":",DHMin(C24+0.008333)))
14	4	G date new Moon (month)	3	=JDCMonth(C11)			=IF(C25=-99,"",ROUND(C25,3))
15	5	G date of new Moon (year)	2015	=JDCYear(C11)			
16	6	UT of new Moon (hours)	9.648443766	=(C12-C13)*24			
17	7	local civil time (hours)	9.648443766	=UTLCT(C16,0,0,C6,C7,C13,C14,C15)			
18	8	local civil date (day)	20	=UTLCDay(C16,0,0,C6,C7,C13,C14,C15)			
19	9	local civil date (month)	3	=UTLCMonth(C16,0,0,C6,C7,C13,C14,C15)			
20	10	local civil date (year)	2015	=UTLCYear(C16,0,0,C6,C7,C13,C14,C15)			
21	11	eclipse occurrence?	Solar eclipse certain	=SEOccurrence(C6,C7,C3,C4,C5)			
22	12	UT max eclipse	9.947296821	=UTMaxSolarEclipse(C3,C4,C5,C6,C7,C8,C9)			
23	13	UT first contact	8.920277911	=UTFirstContactSolarEclipse(C3,C4,C5,C6,C7,C8,C9)			
24	14	UT last contact	10.97431573	=UTLastContactSolarEclipse(C3,C4,C5,C6,C7,C8,C9)			
25	15	magnitude	1.016286817	=MagSolarEclipse(C3,C4,C5,C6,C7,C8,C9)			

SolarEclipseCircumstances

Figure 101. Calculating the circumstances of a solar eclipse.

75 The Astronomical Calendar

It is often useful to have a chart that shows, at a glance, the relative configurations of the Sun, Moon and planets and the likely times of occurrence of eclipses. The astronomical calendar is just such a chart, displaying the right ascension of each heavenly body for every day in the year; the chart for 2015 is drawn in Figure 102.

It is convenient (though not essential) to construct the chart on graph paper. Mark the vertical axis in days (1 to 365 or 366) on a scale to make best use of the paper, and the horizontal axis in hours (0 to 24) such that time increases towards the left; this is a convention often adopted by astronomers as the chart then more nearly represents the relative positions of the bodies in the sky as seen from the Earth. Lines representing the times of the Sun and midnight may now be drawn. To do so it is necessary only to calculate the right ascension (RA) of the Sun on two days of the year several months apart by the method of Section 46, and to join the points by a straight line. Where the line goes off the edge of the chart (as at A and B in Figure 102) it should be continued from the points exactly opposite (A′ and B′). The resulting lines should slope down towards the right; mark them with the symbol ⊙ to represent the Sun. The tracks of midnight, marked by the symbol ●, are parallel to those of the Sun but displaced by 12 hours on the RA scale.

Next, the track of the Moon should be marked in. Again, this can be done by calculating the Moon's right ascension on two days (a week or so apart) every month using the method of Section 65 and joining the points by straight lines. However, the calculations are lengthy and somewhat tedious unless you have a programmable calculator, or you use spreadsheets, so you may like to cheat a bit. The position of the Moon is given for every hour of the year in the *Astronomical Almanac* which you can consult in your local library, but it can also be deduced from the information given in most diaries, the dates of new Moon and full Moon. We know that when the Moon is full it is in opposition to the Sun and, conversely, it is in conjunction with the Sun at new Moon. We have already marked the tracks of conjunction (⊙) and opposition (●) so that we can easily plot the Moon's position from the dates of new Moon and full Moon. For example, our diary (in 2009) indicated that new and full Moons occurred on 22 July and 6 August, so if we were making an astronomical calendar for 2009 we would be able to deduce that the right ascension of the Moon was the same as that of the Sun on 22 July and the same as midnight on 6 August. Join these two points with a straight line to mark the track of the Moon.

Next we must mark in the tracks of the Moon's ascending node (☊) and descending node (☋). The mean longitude of the former is given by the value of N in Section 65, and of the latter by $N+180$. Find these values on two days separated by six months or so and convert to right ascension by the method of Section 27 (setting $\beta = 0$). Join each pair of points by a straight line.

We are now in a position to make predictions about eclipses. As explained in Section 72, an eclipse can only occur when the Moon is near one of its nodes at full Moon (lunar eclipse) or new Moon (solar eclipse). We must therefore find points on the chart where the tracks of the Moon, Sun or midnight, and either node pass close to one another. In Figure 102 these points are marked '+' together with the dates on which the eclipses occur as follows:

20 March 2015:	eclipse of the Sun;
4 April 2015:	eclipse of the Moon;
13 September 2015:	eclipse of the Sun;
28 September 2015:	eclipse of the Moon.

To complete the chart, we can mark on the tracks of the major planets: Mercury (☿), Venus (♀), Mars (♂), Jupiter (♃), Saturn (♄), Uranus (⛢) and Neptune (♅). Their positions can be calculated by the method given in Section 54.

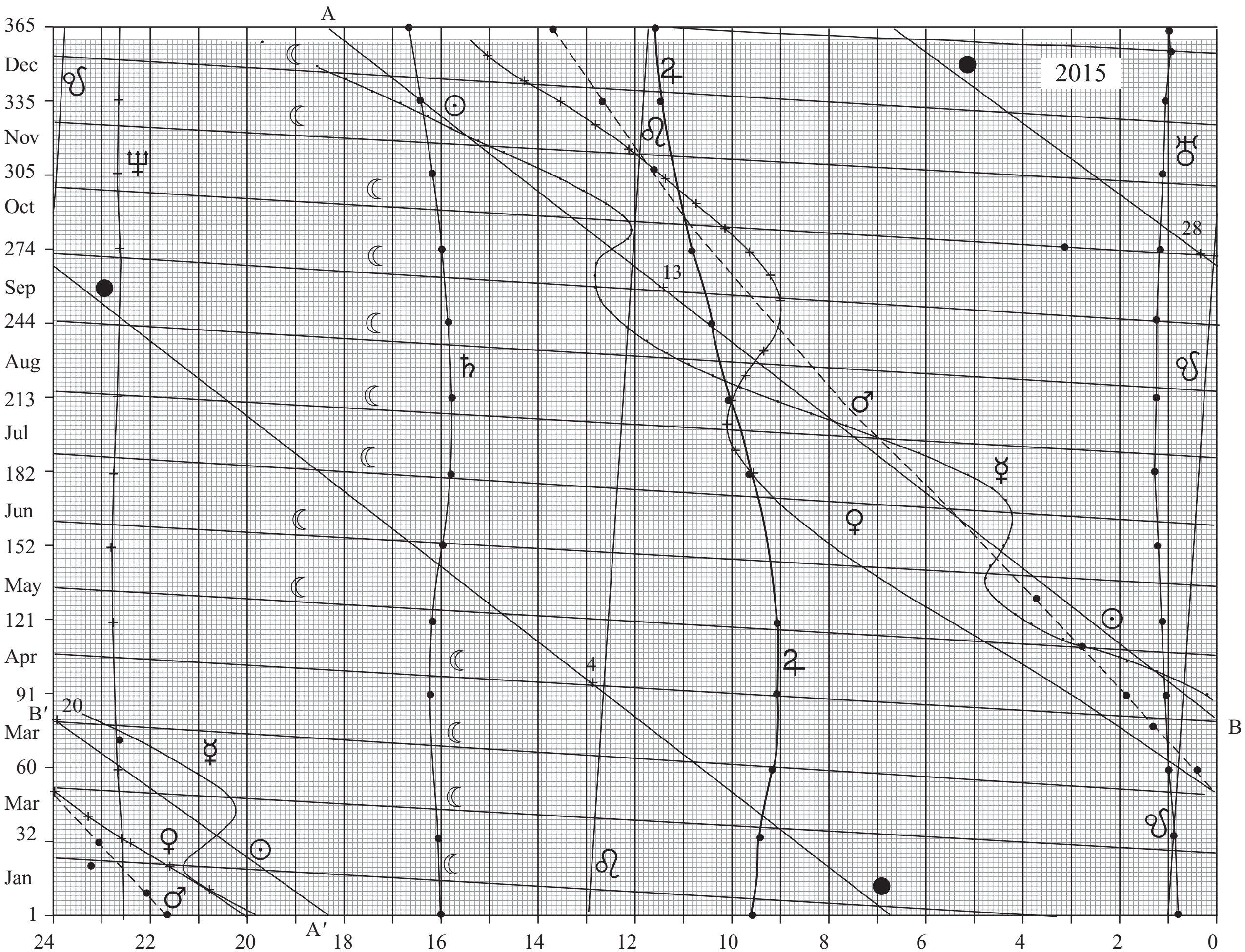


Figure 102. The astronomical calendar for 2015.