The Sun

The nearest star to the Earth is the Sun, being some 91 million miles distant at its closest approach. The sunlight reaching us is already 8 minutes old when we see it, having taken this long to travel the radius of the Earth's orbit. Yet despite this distance the Sun is so huge that it appears as one of the largest celestial objects in the sky, equalled only by the Moon which, by coincidence, has more or less the same angular size. It is certainly the brightest. It dominates the Solar System, controlling the motions of the planets and supplying the energy needed for life on Earth. Although we always know by experience approximately where the Sun is in the sky, we often need to know its position more accurately as, for example, when we wish to calculate an eclipse or the orientation of a sundial. The next few sections deal with methods for calculating the Sun's orbit, distance from the Earth, apparent angular size, the times of sunrise and sunset, the solar elongations of other celestial bodies, the equation of time which you will need if you wish to set your watch by your sundial, and the duration of twilight.

44 Orbits

The motions of the planets around the Sun, and of the satellites about their planets, are all controlled by the action of gravity, that is by the mutual force of attraction between the masses. One of the consequences of the way this force varies with distance is that the planetary orbits trace out the forms of ellipses (Figure 52), geometrical shapes with well-known mathematical properties which enable us to calculate a planet's course precisely. You can imagine an ellipse as a squashed circle; in fact, a circle is a special case of an ellipse where the two **foci** or **focuses**, S and S′, have moved together into the middle. The amount of squashing is measured by the **eccentricity**, e; for a circle, $e = 0$, and the most flattened ellipses have values of e approaching 1. Most planetary orbits have eccentricities less than 0.1, so that their deviations from circular orbits are small. This is fortunate as it enables us to calculate planetary positions quite accurately by relatively simple methods.

All of the planetary orbits have the Sun at one focus, S. In Figure 52 the planet V moves in the direction of the arrow around the ellipse, its distance from the Sun varying from a minimum at A to a maximum at B. These points are called **perihelion** and **aphelion** respectively. The line joining the planet to the Sun, r, is called the **radius vector**, and the angle, l, it makes with a fixed direction in space defines the position of the planet in its orbit at any time. The size of the ellipse is completely defined by the **semi-major axis**, a, and the eccentricity. The length of the **semi-minor axis**, b, is obtained from these two quantities by the equation

$$b^2 = a^2\left(1 - e^2\right).$$

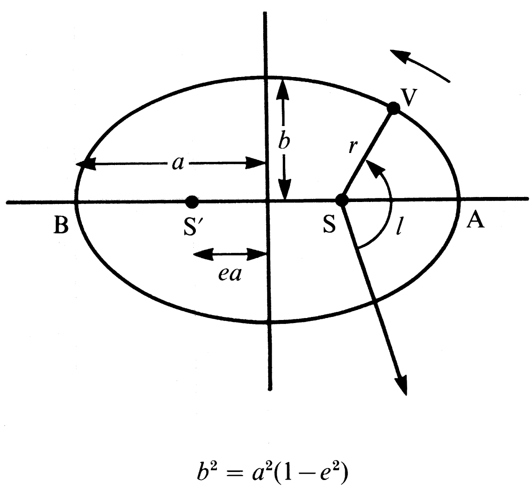


Figure 52. An orbital ellipse.

45 The apparent orbit of the Sun

During the course of a year, the Earth moves in its own elliptical orbit around the Sun, making one complete revolution in about $365\frac{1}{4}$ days. Viewed from the Earth, it seems to us that the Sun is moving in orbit around the Earth and for the purposes of calculating the Sun's position it is convenient to regard this as the case. Hence we now assume that it is we who are at the focus and the Sun describes an ellipse about us. When the Sun is closest to the Earth, we say it is at **perigee** and when it is farthest away it is at **apogee**.

Since the plane which contains the Sun–Earth orbit defines the plane of the ecliptic, it is particularly easy to calculate the Sun's apparent motion as we do not have to worry about deviations from the ecliptic. Once we have calculated the ecliptic longitude, we have defined the Sun's position as the ecliptic latitude is zero.

46 Calculating the position of the Sun

The first thing to do is to define the epoch on which we shall base our calculations; we choose 2010 January 0.0 (JD = 2 455 196.5). The Sun's mean ecliptic longitude at the epoch is $\varepsilon_g = 279.557\,208$ degrees; this is the position it would have had if it had been moving in a circular orbit rather than an ellipse. The value of ε_g represents our starting point. We simply have to add on the correct number of degrees moved by the Sun since then (which may be negative if we are making the calculation for a date before the epoch), and to make due allowance for its elliptical motion, to find where it is at any other time. To do so we need two other constants: $\varpi_g = 283.112\,438$ degrees, the longitude of the Sun at perigee, and $e = 0.016\,705$, the eccentricity of the Sun–Earth orbit. If you wish to find these values for any other epoch, you can do so by using the equations

$$\begin{aligned}
\varepsilon_g &= 279.696\,677\,8 + 36\,000.768\,92T + 0.000\,302\,5T^2 \text{ degrees},\\
\varpi_g &= 281.220\,844\,4 + 1.719\,175T + 0.000\,452\,778T^2 \text{ degrees},\\
e &= 0.016\,751\,04 - 0.000\,041\,8T - 0.000\,000\,126T^2,
\end{aligned}$$

where $T = (\mathrm{JD} - 2\,415\,020.0)/36\,525$ and is the number of Julian centuries since 1900 January 0.5. Note that this is a different definition of T from that used in some other parts of the book.

We now imagine that the Sun moves in a circle around the Earth at a constant speed, rather than along the ellipse that it actually traces. We can easily calculate the angle, $M_\odot$, called the **mean anomaly**, through which this fictitious **mean Sun** has moved since it passed through perigee by

$$M_\odot = \frac{360}{365.242\,191} d \text{ degrees},$$

where d is the number of days since perigee, because during the course of one tropical year of 365.242 191 days the Sun completes a circle of 360°. But rather than basing our calculations on the moment of perigee, we have decided for convenience to use the epoch 2010.0. Then if D is the number of days since the epoch, the mean anomaly is given by (Figure 53):

$$M_\odot = \frac{360}{365.242\,191} D + \varepsilon_g - \varpi_g \text{ degrees},$$

where ε_g and ϖ_g are the mean longitudes of the Sun at the epoch and perigee respectively (values given in Table 7).

ε_g	(ecliptic longitude at epoch 2010.0)	=	279.557208 degrees
ϖ_g	(ecliptic longitude of perigee at epoch 2010.0)	=	283.112438 degrees
e	(eccentricity of orbit at epoch 2010.0)	=	0.016705
r_0	(semi-major axis)	=	1.495985×10^8 km
θ_0	(angular diameter at $r = r_0$)	=	0.533128 degrees

Table 7. Details of the Sun's apparent orbit at epoch 2010.0.

$M_\odot$ refers to the motion of a mean Sun moving in a circle. We actually need the **true anomaly**, ν, which applies for the true motion of the Sun in an ellipse. This can be found from the **equation of the centre**, which is (to a sufficient accuracy for our purposes; see Figure 68):

$$\nu = M_\odot + \frac{360}{\pi} e \sin M_\odot,$$

where ν and $M_\odot$ are expressed in degrees and $\pi = 3.1415927$. Having found ν, we simply add ϖ_g (see Figure 53) to get the longitude of the Sun, $\lambda_\odot$. Hence

$$\lambda_\odot = \nu + \varpi_g,$$

or

$$\lambda_\odot = \frac{360}{365.242191} D + \frac{360}{\pi} e \sin\left\{\frac{360}{365.242191} D + \varepsilon_g - \varpi_g\right\} + \varepsilon_g.$$

Having calculated $\lambda_\odot$ the method given in Section 27 may be used to find the right ascension and declination (remembering that the ecliptic latitude is zero because the Sun is in the ecliptic).

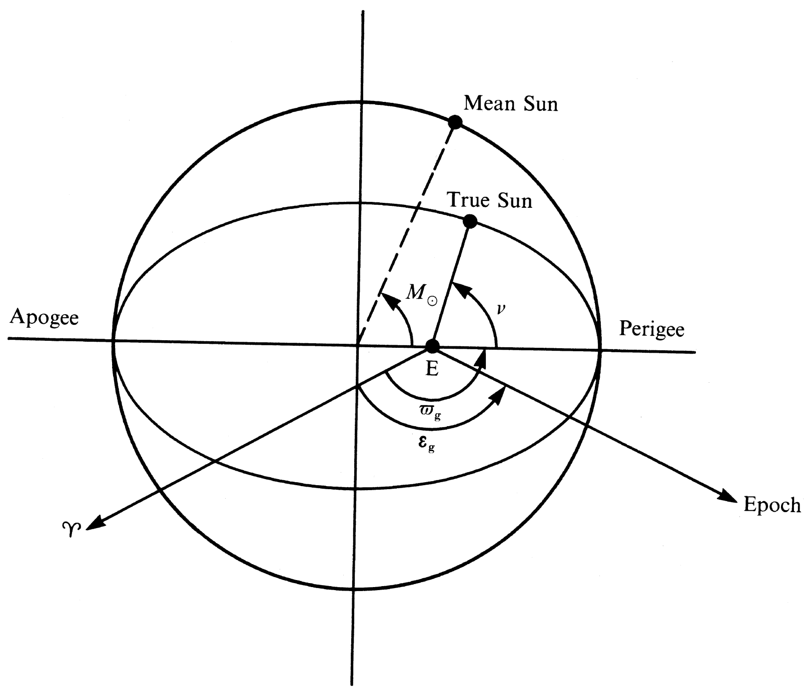


Figure 53. Defining the apparent orbit of the Sun.

Let us clarify all this by working out an example: what were the right ascension and declination of the Sun at 0 h UT on Greenwich date 27 July 2003?

Method		*Example*		
1.	Find the number of days since January 0.0 at the beginning of the year (§3).	27th July	=	181 + 27
			=	208
2.	Add 365 days for every year since 2010 plus 1 extra day for every leap year (see Table 2). The result is D. (Note that we subtract the total in this case as 2003 is before 2010.)		−	2 557
		D	=	−2 349 days
3.	Calculate $N = \frac{360}{365.242\,191}D$; subtract (or add) multiples of 360 until N lies in the range 0 to 360.	N	=	204.714 360 degrees
4.	Find $M_\odot = N + \varepsilon_g - \varpi_g$ (Table 7). If the result is negative, add 360.	$M_\odot$	=	201.159 131 degrees
5.	Find $E_c = \frac{360}{\pi} e \sin M_\odot$ (π = 3.141 592 7 and e from Table 7).	E_c	=	−0.690 967 degrees
6.	Find $\lambda_\odot = N + E_c + \varepsilon_g$. If the result is more than 360, subtract 360. This is the Sun's geocentric ecliptic longitude.	$\lambda_\odot$	=	123.580 601 degrees
7.	Now convert to right ascension and declination (§27). Remember that $\beta_\odot = 0$.	$\alpha_\odot$	=	**8h 23m 34s**
		$\delta_\odot$	=	**19° 21′ 10″**

The *Astronomical Almanac* gives α = 8h 23m 33s and δ = 19° 21′ 16″ so our result is really quite accurate. In general, we should find that we can calculate α to within about 10 s and δ to within a few arcminutes by this method. The inaccuracies arise because we have only used the first term in the **equation of the centre**, and we have not taken account of all sorts of tiny perturbations due to the influences of the other planets in the Solar System.

Figure 54 shows the spreadsheet for this calculation, called SunPos1. It follows the method outlined above, but we have cheated slightly in that we have defined the spreadsheet functions SunElong, SunPeri, and SunEcc to calculate the values of ε_g, ϖ_g, and e respectively using the formulas given above for the Greenwich calendar date supplied in the three arguments as day, month, year. In particular, we have used the values of ϖ_g and e for 27 July 2003 (see rows 20 and 22) and not the values given in Table 7, so the result is a bit more accurate. In the next section, we will see how to get even better accuracy.

	A	B	C	D	E	F	G	H	I	J
1	**Finding the position of the Sun (approximate method)**									
2										
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Sun RA (hour)**	**8**	=DHHour(C26)	
4		**local civil time (min)**	**0**				**Sun RA (min)**	**23**	=DHMin(C26)	
5		**local civil time (sec)**	**0**				**Sun RA (sec)**	**33.73**	=DHSec(C26)	
6		**local date (day)**	**27**				**Sun dec (deg)**	**19**	=DDDeg(C27)	
7		**local date (month)**	**7**				**Sun dec (min)**	**21**	=DDMin(C27)	
8		**local date (year)**	**2003**				**Sun dec (sec)**	**14.33**	=DDSec(C27)	
9		**daylight saving**	**0**							
10		**zone correction**	**0**							
11										
12	1	Greenwich date (day)	27	=LctGDay(C3,C4,C5,C9,C10,C6,C7,C8)						
13	2	Greenwich date (month)	7	=LctGMonth(C3,C4,C5,C9,C10,C6,C7,C8)						
14	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C9,C10,C6,C7,C8)						
15	4	UT (hours)	0	=LctUT(C3,C4,C5,C9,C10,C6,C7,C8)						
16	5	UT (days)	0	=C15/24						
17	6	JD (days)	2452847.5	=CDJD(C12,C13,C14)+C16						
18	7	*D* (days)	-2349	=C17-CDJD(0,1,2010)						
19	8	*N* (deg)	-2315.28564	=360*C18/365.242191						
20	10	*M* (deg)	-2318.840869	=C19+SunElong(0,1,2010)-SunPeri(C12,C13,C14)						
21	11	*M* (deg)	201.1591307	=C20-360*INT(C20/360)						
22	12	*E* c (deg)	-0.690963565	=360*SunEcc(0,1,2010)*SIN(RADIANS(C21))/PI()						
23	13	*L* s (deg)	-2036.419395	=C19+C22+SunElong(0,1,2010)						
24	14	*L* s (deg)	123.5806048	=C23-360*INT(C23/360)						
25	15	RA (deg)	125.8905256	=ECRA(C24,0,0,0,0,0,C12,C13,C14)						
26	16	RA (hours)	8.392701704	=DDDH(C25)						
27	17	dec (deg)	19.35398081	=ECDec(C24,0,0,0,0,0,C12,C13,C14)						

Figure 54. Finding the position of the Sun by an approximate method.

47 Calculating orbits more precisely

In this section we discover how to find the true anomaly, ν, by a slightly more accurate method.† For most purposes the accuracy of the simpler method given in Section 46 will suffice, but if you have a good programmable calculator, or access to a computer, you may find this section useful.

As before, we find the mean anomaly, but this time we use the equations for ε_g and ϖ_g given in the previous section. This requires us to find the number of Julian centuries since 1900 January 0.5, T, and to use that value in the formulas. We calculate the mean anomaly from

$$M_\odot = \varepsilon_g - \varpi_g,$$

where $M_\odot$, ε_g and ϖ_g are all expressed in degrees. Then we find the **eccentric anomaly**, E, which is defined in Figure 55. E is given by **Kepler's equation**, named after the famous German astronomer, Johannes Kepler (1571–1630), who made a detailed study of the planets:

$$E - e\sin E = M_\odot \text{ radians},$$

where this time E and $M_\odot$ are expressed in circular measure, or **radians**. Unfortunately, this equation is not easily solved, but with the aid of an iterative routine a numerical solution may be reached. Such a routine is given in Routine R2 and is appropriate for values of e less than about 0.1 (but see Section 61 for comets, which have larger values of e). You can recast the equation to use degrees rather than radians by noting that 1 radian is $180/\pi$ degrees, so if E and $M_\odot$ are expressed in degrees, we have

$$E - \frac{180}{\pi} e\sin E = M_\odot \text{ degrees}.$$

The value of e for the Sun's orbit is given in Table 7.

Having found a solution to Kepler's equation, we can find the true anomaly, ν, from

$$\tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2},$$

and then, as in the previous section, the ecliptic longitude is given by

$$\lambda_\odot = \nu + \varpi_g \text{ degrees}.$$

†Figure 68 (Section 56) shows the error introduced by using only the first term in the equation of the centre.

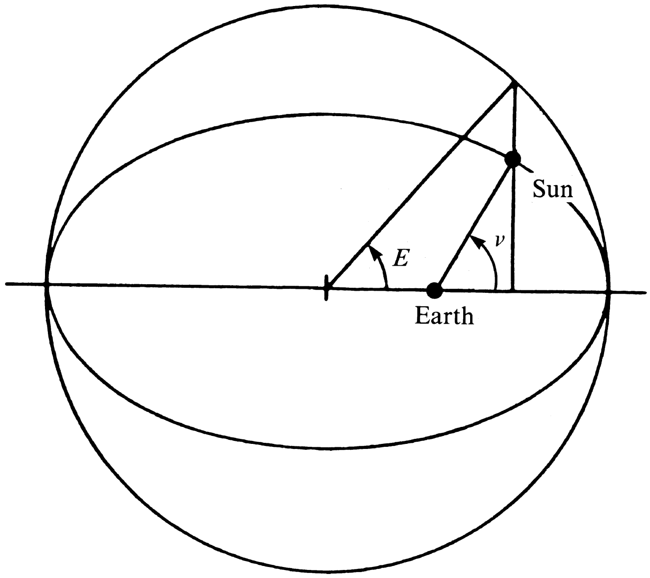


Figure 55. True and eccentric anomalies.

Routine R2: To find a solution to Kepler's equation $E - e\sin E = M$ for small values of e. All angles are expressed in radians.

1. First guess, put $E = E_0 = M$.
2. Find the value of $\delta = E - e\sin E - M$.
3. If† $|\delta| \leq \varepsilon$ go to step 6.
   If $|\delta| > \varepsilon$ proceed with step 4.
   ε is the required accuracy (= 10^{-6} radians).
4. Find $\Delta E = \delta/(1 - e\cos E)$.
5. Take new value $E_1 = E - \Delta E$. Go to step 2.
6. The present value of E is the solution, correct to within ε of the true value.

† $|\delta|$ is the absolute value of δ.

Let us use this method to solve another example: what were the right ascension and declination of the Sun on Greenwich date 27 July 1988 at 0 h UT?

Method		*Example*		
1.	Find the Julian date (§4).	JD	=	2 447 369.5
2.	Subtract 2 415 020.0 and divide by 36 525. The result is T.	T	=	0.885 681 centuries
3.	Find ε_g, ϖ_g and e using the formulas given in the previous section. Reduce to the range 0 to 360 by adding or subtracting multiples of 360.	ε_g	=	124.895 390 degrees
		ϖ_g	=	282.743 840 degrees
		e	=	0.016 714
4.	Find $M_\odot = \varepsilon_g - \varpi_g$. Add or subtract 360 to bring into	$M_\odot$	=	202.151 550 degrees
	the range 0 to 360. Multiply by $\frac{\pi}{180}$ to convert to radians.	$M_\odot$	=	3.528 210 radians
5.	Use Routine R2 to find the eccentric anomaly.	E	=	3.522 004 radians
	Divide by $\frac{\pi}{180}$ to convert to degrees.	E	=	201.795 968 degrees
6.	Find $\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$.	$\tan\frac{\nu}{2}$	=	−5.281 459
7.	Take the inverse tan and multiply by 2 to find ν. Add 360 if negative.	ν	=	201.443 110 degrees
8.	Find $\lambda_\odot = \nu + \varpi_g$. If the result is more than 360, subtract 360. This is the Sun's geocentric ecliptic longitude.	$\lambda_\odot$	=	124.186 950 degrees
9.	Now convert to right ascension and declination (§27).	$\alpha_\odot$	=	**8h 26m 4s**
	Remember that $\beta_\odot = 0$.	$\delta_\odot$	=	**19° 12′ 43″**

The *Astronomical Almanac* gives α = 8h 26m 3s and δ = 19° 12′ 52″.

The spreadsheet for a more precise calculation of the position of the Sun is shown in Figure 56. It incorporates the method given above, together with many correction terms for slight perturbations to the Earth's orbit, all of which are calculated by a single spreadsheet function SunLong (row 15). This returns the Sun's ecliptic longitude as seen at a given local calendar date and local civil time. It takes eight arguments, namely the local civil time in hours, minutes and seconds, the daylight saving correction and time zone offset, both in hours, and the local calendar date as day, month and year.

	A	B	C	D	E	F	G	H	I	J
1	**Finding the Position of the Sun (more precise method)**									
2										
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Sun RA (hour)**	**8**	=DHHour(C17)	
4		**local civil time (min)**	**0**				**Sun RA (min)**	**26**	=DHMin(C17)	
5		**local civil time (sec)**	**0**				**Sun RA (sec)**	**3.83**	=DHSec(C17)	
6		**local date (day)**	**27**				**Sun dec (deg)**	**19**	=DDDeg(C18)	
7		**local date (month)**	**7**				**Sun dec (min)**	**12**	=DDMin(C18)	
8		**local date (year)**	**1988**				**Sun dec (sec)**	**49.72**	=DDSec(C18)	
9		**daylight saving**	**0**							
10		**zone correction**	**0**							
11										
12	1	Gyear	27	=LctGDay(C3,C4,C5,C9,C10,C6,C7,C8)						
13	2	Gmonth	7	=LctGMonth(C3,C4,C5,C9,C10,C6,C7,C8)						
14	3	Gyear	1988	=LctGYear(C3,C4,C5,C9,C10,C6,C7,C8)						
15	4	Sun's ecliptic longitude (deg)	124.1873516	=SunLong(C3,C4,C5,C9,C10,C6,C7,C8)						
16	5	RA (deg)	126.515956	=ECRA(C15,0,0,0,0,0,C12,C13,C14)						
17	6	RA (hours)	8.434397066	=DDDH(C16)						
18	7	dec (deg)	19.21381243	=ECDec(C15,0,0,0,0,0,C12,C13,C14)						

Figure 56. Finding the position of the Sun by a more precise method.

48 Calculating the Sun's distance and angular size

Having found the true anomaly, ν, by the method of Sections 46 or 47, we can easily calculate the Sun–Earth distance, r, and the Sun's angular size (i.e. its angular diameter), θ. The formulas are:

$$r = r_0 \left(\frac{1-e^2}{1+e\cos\nu} \right),$$

$$\theta = \theta_0 \left(\frac{1+e\cos\nu}{1-e^2} \right),$$

where r_0 is the semi-major axis, θ_0 is the angular diameter when $r = r_0$, and e is the eccentricity of the orbit. These constants are given in Table 7. Continuing the example of Section 47 we can find r and θ for the Sun on Greenwich date 27 July 1988 at 0 h UT.

Method		*Example*		
1.	Find the true anomaly, ν (§§46 or 47).	ν	=	201.443 110 degrees
2.	Find $f = \frac{1+e\cos\nu}{(1-e^2)}$. (See Table 7 for e.)	f	=	0.984 726
3.	Then $r = \frac{r_0}{f}$. (See Table 7 for r_0.)	r	=	**$1.519\,189 \times 10^8$ km**
4.	And $\theta = f\theta_0$. (See Table 7 for θ_0.)	θ	=	**$0°\ 31'\ 30''$**

The *Astronomical Almanac* gives $\theta = 0°\ 31'\ 30''$ and, in general, we should be within a few arcseconds of the correct value. It is interesting to note that the Sun's light took r/c seconds to reach us, where $c = 3 \times 10^5$ km s^{-1}. In this case the light travel time was 506 seconds, during which interval the Sun moved about 21 arcseconds. Strictly speaking, we should subtract this from the calculated position to find the Sun's apparent position.

Spreadsheet Sundist (Figure 57) also shows how to make this calculation. We have cheated a bit by using a function, called SunTrueAnomaly (row 15), to find the Sun's true anomaly without going through the fuss of solving Kepler's equation etc. separately. This function takes the following arguments: the local civil time in hours, minutes and seconds, the daylight saving correction and time zone offset in hours, and the local calendar date as day, month and year. We have also used the function SunEcc (row 17; defined in Section 46) to get the eccentricity of the Sun's apparent orbit about the Earth.

We have provided spreadsheet functions SunDist and SunDia to make these calculations more simply. The functions return the values of the Sun's distance from the Earth in astronomical units (multiply by r_0 to get kilometres), and the Sun's angular diameter in decimal degrees respectively. Both functions take the same arguments as SunTrueAnomaly (see above). Thus, having saved a copy, you could modify the spreadsheet Sundist as follows: delete rows 12 to 20 inclusive, and insert the following spreadsheet formulas into cells H3 to H6:

```
=1.495985E8*SunDist(C3,C4,C5,C9,C10,C6,C7,C8)†
=DDDeg(SunDia(C3,C4,C5,C9,C10,C6,C7,C8))
=DDMin(SunDia(C3,C4,C5,C9,C10,C6,C7,C8))
=DDSec(SunDia(C3,C4,C5,C9,C10,C6,C7,C8)).
```

	A	B	C	D	E	F	G	H	I	J
1	**Sun's distance and angular size**									
2										
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Sun's dist (km)**	**1.519201E+08**	=ROUND(C19,-2)	
4		**local civil time (min)**	**0**				**Sun's ang size (deg)**	**0**	=DDDeg(C20)	
5		**local civil time (sec)**	**0**				**Sun's ang size (min)**	**31**	=DDMin(C20)	
6		**local date (day)**	**27**				**Sun's ang size (sec)**	**29.93**	=DDSec(C20)	
7		**local date (month)**	**7**							
8		**local date (year)**	**1988**							
9		**daylight saving**	**0**							
10		**zone correction**	**0**							
11										
12	1	Gyear	27	=LctGDay(C3,C4,C5,C9,C10,C6,C7,C8)						
13	2	Gmonth	7	=LctGMonth(C3,C4,C5,C9,C10,C6,C7,C8)						
14	3	Gyear	1988	=LctGYear(C3,C4,C5,C9,C10,C6,C7,C8)						
15	4	true anomaly (deg)	-158.556892	=SunTrueAnomaly(C3,C4,C5,C9,C10,C6,C7,C8)						
16	5	true anomaly (rad)	-2.767339817	=RADIANS(C15)						
17	6	eccentricity	0.01671392	=SunEcc(C12,C13,C14)						
18	7	*f*	0.984718087	=(1+C17*COS(C16))/(1-C17*C17)						
19	8	*r* (km)	151920130.3	=149598500/C18						
20	9	theta (deg)	0.524980784	=C18*0.533128						

Figure 57. Finding the Sun's distance and angular size.

† 1.495985E8 is the number $1.495\,985 \times 10^8$

49 Sunrise and sunset

In Section 33 we found how to calculate the rising and setting times of any celestial object whose equatorial coordinates were known. We have calculated the right ascension and declination of the Sun (Sections 46 and 47) so that we can apply the same method to find the times of sunrise and sunset. The problem is complicated, however, by the fact that the Sun is in continual motion along the ecliptic, and its equatorial coordinates are therefore continuously changing. The values of α and δ we have calculated are correct only for the time we have chosen. (In the example of Section 46, this time was the midnight between 26 July and 27 July. By the time the Sun had risen the next morning it had moved by about a quarter of a degree from its midnight position, and by sunset about three quarters of a degree.) Provided that we do not require high accuracy in our calculations, we can ignore the Sun's motion and simply take the position at midday as correct for both sunrise and sunset. The results are then within a few minutes of their correct values.

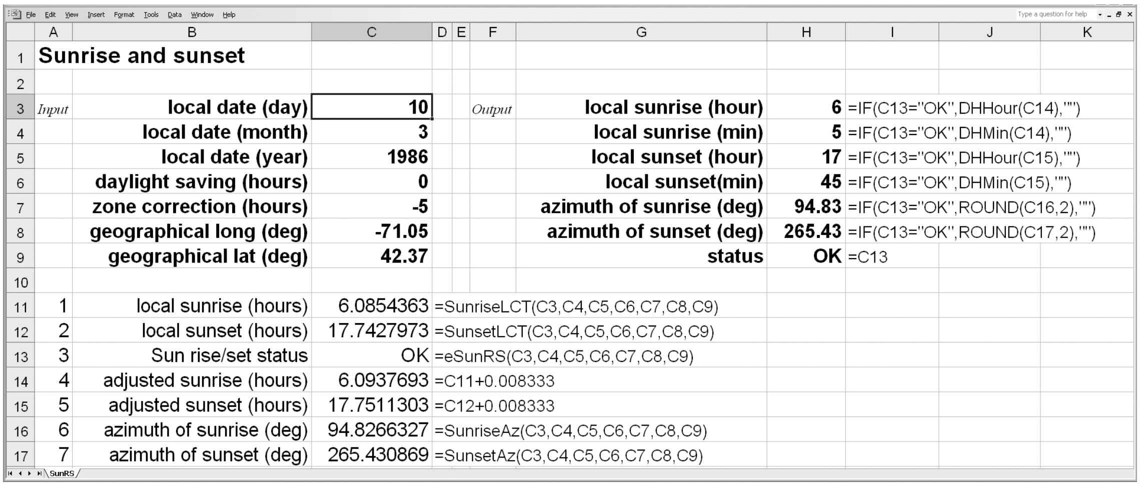
Further refinements include taking account of refraction by the Earth's atmosphere (Section 37) and geocentric parallax (Section 38). We must also consider the finite diameter of the Sun's disc; times of sunrise and sunset are usually quoted as those appropriate to the upper limb.

For our example, let us calculate the times of sunrise and sunset (upper limb) over a level horizon at sea-level on 10 March 1986, as observed from Boston, Massachusetts, at longitude 71.05° W and latitude 42.37° N. We shall take the Sun's angular diameter to be 0.533 degrees, its horizontal parallax to be 8.79 arcseconds, and the refraction due to the atmosphere as 34 arcminutes and, having added on half of the Sun's angular diameter and a small correction for parallax, we arrive at a total vertical shift at the horizon of the upper limb of 0.833 333 degrees. The time zone correction (Section 9) is −5 hours.

Method		*Example*		
1.	Calculate the Sun's position at midday	$\alpha_\odot$	=	23.375 999 hours
	(§§46 or 47; we've used SunPos1).	$\delta_\odot$	=	−4.034 153 degrees
2.	Calculate the corresponding rising and setting times	$\mathrm{UT_r}$	=	11h 6m
	(§33; we've used RiseSet).	$\mathrm{UT_s}$	=	22h 43m
3.	Convert these times to local civil times (§10).	$\mathrm{LCT_r}$	=	**6h 6m**
		$\mathrm{LCT_s}$	=	**17h 43m**

The *Old Farmer's Almanac* (see the Bibliography on page 208) lists these times as $\mathrm{EST_r}$ = 6h 5m am, and $\mathrm{EST_s}$ = 5h 45m pm. In general, we should be within a few minutes of the actual times using this approximate method.

The spreadsheet shown in Figure 58, called SunRS, performs this calculation rather more precisely. It uses the five spreadsheet functions called SunriseLCT, SunsetLCT, SunriseAz, SunsetAz and eSunRS, which calculate the local civil times of sunrise and sunset, their azimuths, and a status word respectively. Each has the same seven arguments, namely the local calendar date as day, month, year, the daylight saving correction and time zone offset (both in hours), and the geographical longitude and latitude (both in degrees). These functions start out by performing the calculation based on the Sun's position at midday, but then use the calculated times of sunrise and sunset to find new positions for the Sun at those times, before recalculating the times of sunrise and sunset using the new Sun positions. The result should be accurate to within a minute of time. If an error is detected, such that the Sun never rises or never sets, as can happen near the poles (try a latitude of + or − 89°), the function eSunRS returns the status word ** never rises, or ** circumpolar, respectively, otherwise it returns with OK. If not OK, the other functions return with −99, a disallowed value of time (0 to 24 hours) or angle (0 to 360 degrees). The result of eSunRS is used to gate the writing of the output values in cells H3 to H8 so that numerical results only appear if they are valid. You can see that the times of rising and setting calculated by spreadsheet SunRS agree with the values given in the *Old Farmer's Almanac* in this example.



	A	B	C	D	E	F	G	H	I
1	**Sunrise and sunset**								
2									
3	*Input*	**local date (day)**	**10**			*Output*	**local sunrise (hour)**	**6**	=IF(C13="OK",DHHour(C14),"")
4		**local date (month)**	**3**				**local sunrise (min)**	**5**	=IF(C13="OK",DHMin(C14),"")
5		**local date (year)**	**1986**				**local sunset (hour)**	**17**	=IF(C13="OK",DHHour(C15),"")
6		**daylight saving (hours)**	**0**				**local sunset(min)**	**45**	=IF(C13="OK",DHMin(C15),"")
7		**zone correction (hours)**	**-5**				**azimuth of sunrise (deg)**	**94.83**	=IF(C13="OK",ROUND(C16,2),"")
8		**geographical long (deg)**	**-71.05**				**azimuth of sunset (deg)**	**265.43**	=IF(C13="OK",ROUND(C17,2),"")
9		**geographical lat (deg)**	**42.37**				**status**	**OK**	=C13
10									
11	1	local sunrise (hours)	6.0854363	=SunriseLCT(C3,C4,C5,C6,C7,C8,C9)					
12	2	local sunset (hours)	17.7427973	=SunsetLCT(C3,C4,C5,C6,C7,C8,C9)					
13	3	Sun rise/set status	OK	=eSunRS(C3,C4,C5,C6,C7,C8,C9)					
14	4	adjusted sunrise (hours)	6.0937693	=C11+0.008333					
15	5	adjusted sunset (hours)	17.7511303	=C12+0.008333					
16	6	azimuth of sunrise (deg)	94.8266327	=SunriseAz(C3,C4,C5,C6,C7,C8,C9)					
17	7	azimuth of sunset (deg)	265.430869	=SunsetAz(C3,C4,C5,C6,C7,C8,C9)					

Figure 58. Calculating the circumstances of sunrise and sunset.

50 Twilight

Whenever the Sun is less than a certain amount below the horizon, after sunset or before sunrise, the light scattered by the upper atmosphere illuminates the Earth. The intensity of the scattered sunlight falls sharply as the Sun dips lower below the horizon, and is almost negligible by the time the Sun's zenith angle reaches 108°, i.e. 18° below the horizon. The period after sunset or before sunrise during which the Sun's zenith angle is less than an agreed amount is called **twilight**: 96° for **civil twilight**, 102° for **nautical twilight**, and 108° for **astronomical twilight**.

We can calculate the duration of morning or evening twilight quite simply. We first find the hour angles, H, of the Sun at rising or setting by

$$\cos H = -\tan\phi\tan\delta,$$

where ϕ is the geographic latitude and δ is the Sun's declination. Then we calculate its hour angle, H', at the point when its zenith angle is $\theta_\mathrm{t} = 96$, 102, or 108 degrees using

$$\cos H' = \frac{\cos\theta_\mathrm{t} - \sin\phi\sin\delta}{\cos\phi\cos\delta}.$$

Then the duration of twilight in sidereal hours is simply

$$t = \frac{H' - H}{15}\text{ hours.}$$

We should multiply this by 0.9973 to obtain the equivalent time interval in terms of normal (UT) hours.

During the course of one year the Sun's declination ranges from about −23.5° to +23.5°. Latitudes north of +48.5° or south of −48.5° will therefore experience a twilight which lasts all night during the summer.

For example, on the latitude 60° N the twilight lasts all night from about 23 April until 22 August. When this is so, the value of $\cos H'$ lies outside its allowed range of −1 to +1, and your calculator should respond with 'error' if you attempt inverse cos. For example, let us calculate the beginning of morning astronomical twilight and the end of evening astronomical twilight on 7 September 1979 for an observer at latitude 52° N and longitude 0°.

Method		*Example*		
1.	Calculate the declination of the Sun; its value at midday will do as this is not a very precise calculation. We have used SunPos1.	$\delta_\odot$	=	6.189 592 degrees
2.	Find the Sun's hour angle at setting from $H = \cos^{-1}\{-\tan\phi\tan\delta\}$.	H	=	97.979 045 degrees
3.	Find the Sun's hour angle at $z = 108°$ from $H' = \cos^{-1}\left\{\frac{\cos 108 - \sin\phi\sin\delta}{\cos\phi\cos\delta}\right\}$.	H'	=	130.066 840 degrees
4.	Calculate $t = \frac{H'-H}{15}$ hours and multiply by 0.9973 to convert to interval of UT.	t	=	2.133 411 UT hours
5.	Add or subtract this from the time of sunset or sunrise respectively to find the end of evening twilight or the start of morning twilight. We used SunRS.	$\mathrm{UT_s}$ $\mathrm{UT_r}$	= =	18.582 658 hours 5.338 170 hours evening twilight ends at **20h 43m** UT morning twilight begins at **3h 12m** UT

The *Astronomical Ephemeris* gives these times as 3h 17m and 20h 37m, having taken due account of the Sun's changing coordinates throughout the day, as well as refraction and parallax.

You can make a more-accurate estimate of the time of the start or end of twilight using the spreadsheet Twilight shown in Figure 59. To do this, we need to take account of the change in the Sun's position between the start of morning twilight and end of evening twilight, in effect treating the problem in the same way as for sunrise and sunset. In the spreadsheet we have used the spreadsheet functions TwilightAMLCT, TwilightPMLCT and eTwilight to carry out the calculation behind the scenes, returning respectively the local civil times of the start of morning twilight, the end of evening twilight, both in hours, and a status word (really a string) of OK, ** lasts all night, or ** Sun too far below horizon. All three functions take the same eight arguments which are the local date as day, month, year, the daylight saving correction and time zone offset in hours, the geographical longitude and latitude in degrees (W and S are negative), and a switch in the form of the single character C, N, or A to specify whether civil, nautical, or astronomical twilight is to be calculated respectively. You can see that the result produced by the spreadsheet agrees exactly with the *Astronomical Ephemeris* in this case.

	A	B	C	D	E	F	G	H	I	J	K
1	**Morning and evening twilight**										
2											
3	*Input*	**local date (day)**	**7**			*Output*	**am twilight begins (hour)**	**3**	=IF(C14="OK",DHHour(C15),"")		
4		**local date (month)**	**9**				**am twilight begins (min)**	**17**	=IF(C14="OK",DHMin(C15),"")		
5		**local date (year)**	**1979**				**pm twilight ends (hour)**	**20**	=IF(C14="OK",DHHour(C16),"")		
6		**daylight saving (hours)**	**0**				**pm twilight ends(min)**	**37**	=IF(C14="OK",DHMin(C16),"")		
7		**zone correction (hours)**	**0**				**status**	**OK**	=C14		
8		**geographical long (deg)**	**0**								
9		**geographical lat (deg)**	**52**								
10		**twilight type (C, N, A)**	**A**								
11											
12	1	start of am twilight (hours)	3.2844351	=TwilightAMLCT(C3,C4,C5,C6,C7,C8,C9,C10)							
13	2	end of pm twilight (hours)	20.6228	=TwilightPMLCT(C3,C4,C5,C6,C7,C8,C9,C10)							
14	3	twilight status	OK	=eTwilight(C3,C4,C5,C6,C7,C8,C9,C10)							
15	4	adjusted am start time	3.2927681	=C12+0.008333							
16	5	adjusted pm end time	20.631133	=C13+0.008333							

Figure 59. Calculating twilight.

51 The equation of time

The apparent motion of the Sun along the plane of the ecliptic is not regular. This is rather surprising at first because we are used to thinking of the Sun as a time-keeper by which we can set our watches. In fact, it is really quite a bad time-keeper by quartz-crystal watch standards; it can at any moment in the year be as much as 16 minutes out compared with a regular clock whose time increases at a uniform rate. The Sun's non-uniform motion is caused by two effects:

(a) The Earth's orbit is not circular but elliptical. Its speed therefore varies throughout the year, being maximum at perihelion and minimum at aphelion. Viewed from the Earth, the Sun's speed in its apparent orbit varies from a maximum at perigee to a minimum at apogee.
(b) The Earth's axis is tilted at an angle to the perpendicular of the plane of the ecliptic. The angle is the same as the obliquity of the ecliptic, $\varepsilon = 23° \; 26'$ (Section 27). The Earth acts as a huge gyroscope keeping its rotation axis in a fixed direction in space, making the Sun's altitude at noon vary throughout the year from a maximum at midsummer to a minimum at midwinter. This variation in altitude has a small effect on the time of transit of the Sun.

To take account of the Sun's apparent aberrations from perfect time-keeping we imagine a fictitious Sun, called the **mean Sun**, which moves at a uniform rate along the equator. Noon is defined to be the instant when the mean Sun crosses the meridian, and two successive passages of the mean Sun across it define the length of the day. Time measured by the mean Sun corresponds to UT.

The difference between the real Sun time and the mean Sun time is called the **equation of time**. Hence

$$\Delta t = \mathrm{RST} - \mathrm{MST},$$

where Δt is the value of the equation of time, MST is the mean Sun time and RST is the real Sun time. It is plotted in Figure 60.

We can calculate the equation of time on any day quite easily by first finding the Sun's right ascension at noon and then, remembering that the right ascension is the sidereal time at transit, converting the right ascension to UT. The result is the UT at which the real Sun transits; by subtracting 12h 00m from this, the UT at which the mean Sun transits, we have the value of the equation of time.

For example, what was the value of the equation of time on 27 July 2010? (Remember that noon is July 27.5.)

Method		*Example*		
1.	Calculate the right ascension of the Sun in decimal hours. We have used SunPos2 (§47) to do this.	$\alpha_\odot$	=	8.446 350 hours
2.	Taking this as the GST, convert it to UT (§13).	UT	=	12.108 755 hours
3.	Subtract 12 and convert to hours, minutes and seconds (§8). This is the value of the equation of time.	Δt	=	**6m 32s**

If you have a sundial, you will need the equation of time to convert the sundial's reading into mean time (UT).

Figure 61 shows the spreadsheet for finding the value of the equation of time. We have also provided a spreadsheet function called EqOfTime which calculates this value in hours for the Greenwich date specified in the three arguments as day, month and year. Using this function, you could delete rows 7–10 of the spreadsheet of Figure 61 and insert the following two spreadsheet formulas into cells H3 and H4:

=DHMin(EqOfTime(C3,C4,C5))

=DHSec(EqOfTime(C3,C4,C5)).

Remember to save a copy first in case you need it later.

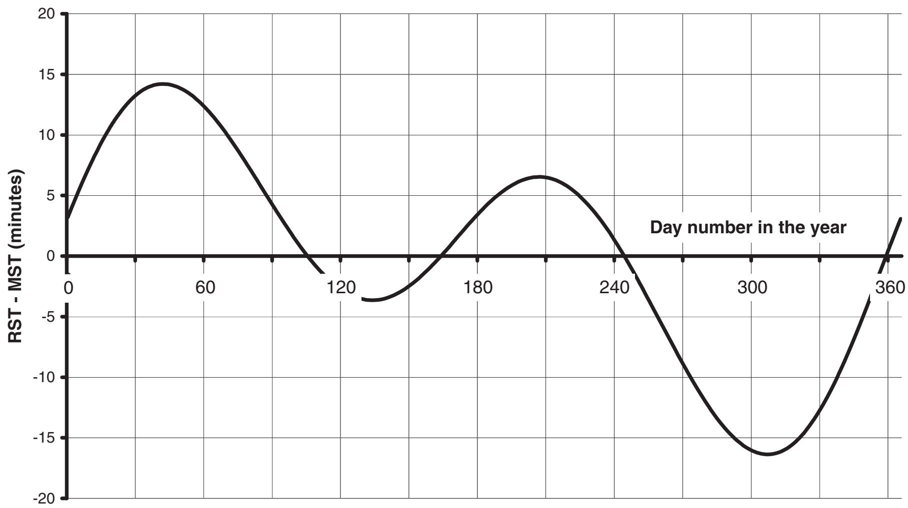


Figure 60. The equation of time. This diagram was made in *Excel* using the functions described in this section.

	A	B	C	D	E	F	G	H	I	J
1	**The equation of time**									
2										
3	*Input*	**Greenwich date (day)**	**27**			*Output*	**equation of time (min)**	**6**	=DHMin(C10)	
4		**Greenwich date (month)**	**7**				**equation of time (sec)**	**31.52**	=DHSec(C10)	
5		**Greenwich date (year)**	**2010**							
6										
7	1	Sun longitude (deg)	124.3626189	=SunLong(12,0,0,0,0,C3,C4,C5)						
8	2	Sun RA (hours)	8.446349638	=DDDH(ECRA(C7,0,0,0,0,0,C3,C4,C5))						
9	3	equivalent UT (hours)	12.10875487	=GSTUT(C8,0,0,C3,C4,C5)						
10	4	equation of time (hours)	0.108754868	=C9-12						

Figure 61. Finding the value of the equation of time.

52 Solar elongations

The **solar elongation** of a planet (or other celestial object) is the angle between the lines of sight from the Earth to the Sun and from the Earth to the planet. It is quite often necessary to find the value of this angle, as it tells us how close to the Sun we should look to see the planet and hence whether it will be visible. The formula for the solar elongation, ε, is

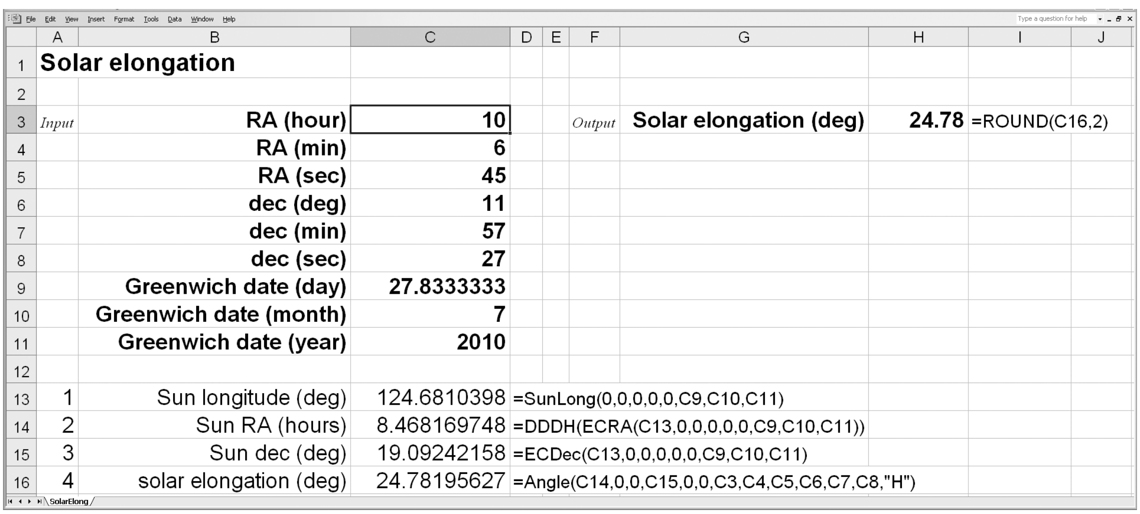
$$\varepsilon = \cos^{-1}\left\{\sin\delta_p \sin\delta_\odot + \cos\left(\alpha_p - \alpha_\odot\right)\cos\delta_p\cos\delta_\odot\right\} \text{ degrees},$$

where $\alpha_\odot$ and $\delta_\odot$ are the right ascension and declination of the Sun, and α_p and δ_p are the right ascension and declination of the planet.

On Greenwich date 27 July 2010 at 8 pm UT, the equatorial coordinates of the planet Mercury were found to be α_p = 10h 6m 45s and $\delta_p = 11°\ 57'\ 27''$. What was the solar elongation?

Method	*Example*
1. Calculate the right ascension and declination of the Sun in decimal degrees. We have used SunPos2 to do this (§47).	$\alpha_\odot$ = 127.022 544 degrees $\delta_\odot$ = 19.092 414 degrees
2. Convert α_p and δ_p into decimal degrees (§§7 and 21).	α_p = 151.687 500 degrees δ_p = 11.957 500 degrees
3. Find $\varepsilon = \cos^{-1}\left\{\sin\delta_p \sin\delta_\odot + \cos\left(\alpha_p - \alpha_\odot\right)\cos\delta_p\cos\delta_\odot\right\}$.	ε = **24.78 degrees**

The spreadsheet for making this calculation is shown in Figure 62. You will see that, in cell C9, we have specified the day part of the date as 27.833 333 to represent 8 pm on the 27th. We have not supplied a spreadsheet function to make the same calculation.



	A	B	C	D	E	F	G	H	I
1	**Solar elongation**								
2									
3	*Input*	**RA (hour)**	**10**			*Output*	**Solar elongation (deg)**	**24.78**	=ROUND(C16,2)
4		**RA (min)**	**6**						
5		**RA (sec)**	**45**						
6		**dec (deg)**	**11**						
7		**dec (min)**	**57**						
8		**dec (sec)**	**27**						
9		**Greenwich date (day)**	**27.8333333**						
10		**Greenwich date (month)**	**7**						
11		**Greenwich date (year)**	**2010**						
12									
13	1	Sun longitude (deg)	124.6810398	=SunLong(0,0,0,0,0,C9,C10,C11)					
14	2	Sun RA (hours)	8.468169748	=DDDH(ECRA(C13,0,0,0,0,0,C9,C10,C11))					
15	3	Sun dec (deg)	19.09242158	=ECDec(C13,0,0,0,0,0,C9,C10,C11)					
16	4	solar elongation (deg)	24.78195627	=Angle(C14,0,0,C15,0,0,C3,C4,C5,C6,C7,C8,"H")					

Figure 62. Finding the solar elongation.