Notes on Macroeconomic Model with Habit Formation

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May 14, 2025

1 Framework

1.1 Core Question and Strategy

The model seeks to explain the observed regime shift in the correlation between bond and stock returns:

Why did the correlation between bond and stock returns switch from positive (pre-2000) to negative (post-2000)?

To address this, the authors build a macro-finance model that:

- Generates **time-varying risk premia**
- Links these premia to **macroeconomic fundamentals** (output gap, inflation, interest rates)
- Derives all pricing equations from **microeconomic preferences**, ensuring internal consistency

1.2 Step 1: Preferences and the Asset Pricing Kernel

The model uses external habit formation preferences:

$$U(C_t, H_t) = \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma}$$

This implies a stochastic discount factor (SDF):

$$M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)} = \beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\gamma}$$

Key implications:

- Asset prices depend on consumption growth and the surplus consumption ratio S_t
- Risk aversion is time-varying, driven by economic conditions

1.3 Step 2: Linking Macro Dynamics via the Euler Equation

The model assumes that the output gap x_t is detrended consumption:

$$x_t = \hat{c}_t = c_t - (1-\phi)\sum_{i=0}^{\infty}\phi^i c_{t-1-i}$$

This yields the log-linear Euler equation:

$$x_t = f_x \mathbb{E}_t x_{t+1} + \rho_x x_{t-1} - \psi r_t$$

1.4 Step 3: Inflation and Interest Rate Dynamics

To complete the macro block, the authors model inflation and nominal interest rate gaps:

$$\hat{\pi}_t = \pi_t - \pi_t^*, \quad \hat{i}_t = i_t - \pi_t^*$$

$$\begin{aligned}
\hat{\pi}_t &= b_{\pi x} x_{t-1} + b_{\pi\pi}\hat{\pi}_{t-1} + b_{\pi i}\hat{i}_{t-1} + v_{\pi,t} \\
\hat{i}_t &= b_{ix} x_{t-1} + b_{i\pi}\hat{\pi}_{t-1} + b_{ii}\hat{i}_{t-1} + v_{i,t} \\
\pi_t^* &= \pi_{t-1}^* + v_t^*
\end{aligned}$$

These allow for persistent inflation dynamics critical to nominal bond pricing.

1.5 Step 4: Recursive Asset Pricing

The recursive pricing formula is:

$$P_t = \mathbb{E}_t[M_{t+1}D_{t+1} + M_{t+1}P_{t+1}]$$

Bonds:

$$P_{n,t}^{\$} = \mathbb{E}_t[M_{t+1}e^{-\pi_{t+1}}P_{n-1,t+1}^{\$}]$$

Stocks:

$$R_{t+1}^{\delta} = \frac{1}{\delta}R_{t+1}^{c} - \frac{1-\delta}{\delta}e^{r_t}$$

1.6 Step 5: Risk Premia and Comovement

Expected excess returns are:

Equity:

$$\mathbb{E}_t[r_{t+1}^{\text{stock}} - r_t] \propto \gamma(1+\lambda(s_t))\sigma_x^2$$

Bond:

$$\mathbb{E}_t[r_{t+1}^{\text{bond}} - r_t] \propto \gamma(1+\lambda(s_t))\text{Cov}_t(x_{t+1}, -i_{t+1} - \pi_{t+1})$$

1.7 Step 6: Empirical Implementation

- Identify a structural break in the inflation-output relationship (circa 2001)
- Calibrate macro parameters using simulated method of moments (SMM)
- Compare model-implied vs. empirical bond-stock correlations

Result: The model reproduces the shift from positive (pre-2001) to negative (post-2001) bond-stock return correlation.

1.8 Conclusion

$$\text{Preferences} \rightarrow \text{SDF} \rightarrow \text{Euler} + \text{Inflation System} \rightarrow \text{Asset Prices} \rightarrow \text{Risk Premia}$$

$$\text{Macro shifts} \Rightarrow \text{Risk premium shifts} \Rightarrow \text{Return comovement shifts}$$