Universal Portfolios & GAs

A brief comparison between the performance of Thomas Cover universal portfolios and GAs' ones in investment decision

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Table of Contents

Table of Contents

Introduction

A bankroll is described as such

$$X_n = X_0(1+f)^S(1-f)^F$$

$$\left(\frac{X_n}{X_0}\right)^{1/n} = (1+f)^{S/n}(1-f)^{F/n}$$

The term on the left is the growth rate per bet on average and we want to find an f that maximizes this quantity via the right-hand side. In log-world this is equivalent to

$$\frac{1}{n}(\log X_n - \log X_0) = \frac{S}{n}\log(1+f) + \frac{F}{n}\log(1-f),$$

or in the limit for n large we obtain:

$$\mathbb{E}[g(f)] = p\log(1+f) + (1-p)\log(1-f)$$

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Introduction

Suppose we have n assets, with random return vector $x \in \mathbb{R}^n$, where the x_i are of the form $x_i = \frac{p_i(\text{ new})}{p_i\text{ (old)}}$, i.e., relative price changes. In the spirit of Kelly's approach, we allocate fractions across these assets, so that the logarithmic growth rate is given as $\log f^\top x$.

$$f \in \Delta(n) \doteq \left\{ f \in \mathbb{R}^n \mid \sum_i f_i = 1, f \geq 0 \right\}$$

Introduction

Thus, We would have the relative price change realizations x_t in time t and allocations f_t, so that the logarithmic portfolio growth over time is given by

$$\sum_{t=1}^{T} \log f_t^T x_t$$

Equivalently the average logarithmic growth rate is given by:

$$\frac{1}{T} \sum_{t=1}^{T} \log f_t^T x_t$$

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Table of Contents

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A Constant Rebalancing Portfolio (CRP) is one where the $f_t = f$ are constant over time with the implicit assumption that the return distribution is somewhat stationary and hence from a sequential decision perspective we maximize the expected logarithmic growth rate by picking the expectation maximizer in each step assuming i.i.d. returns.

The algorithm constructs a portfolio over time, so that when $T \to \infty$, for any sequence of relative price changes $x_1, \ldots, x_T$, then

$$\frac{1}{T}\sum_{t=1}^{T} \log f_t^T x_t \to \max_{f \in \mathbb{R}^n} \frac{1}{T}\sum_{t=1}^{T} \log f^T x_t$$

Universal Portfolios

The total return after n days is $T_n = \prod_{i=1}^{n} \mathbf{f}_i^T \mathbf{x}_i$. With Universal Portfolios, we can approximate the return of optimal Constant Rebalancing Portfolio, as we defined it as a fixed wealth allocation throughout all days. Given a fixed wealth allocation $\mathbf{f} \in \Delta^m$, the return of CRP on day n is $S_n(\mathbf{f}) = \prod_{i=1}^{n} \mathbf{f}^T \mathbf{x}_i$. With Universal Portfolio algorithm, we can achieve

$$\frac{T_n}{S_n(\mathbf{f}^*)} \geq \frac{1}{(n+1)^{m-1}}$$

Thus we get an average daily return comparable with optimal CRP. Such implementations of Cover's algorithm are exponential in the number of stocks with worst-case run times of $\Theta\left(n^{m-1}\right)$.

Universal Portfolios

Here $\mathbf{f}^*$ is the optimal CRP. Considering the average return per day, we have

$$\left(\frac{T_n}{S_n(\mathbf{f}^*)}\right)^{\frac{1}{n}} \geq \frac{1}{(n+1)^{\frac{m-1}{n}}}$$

As $n \to \infty$, the right hand side approach 1 . Thus we get an average daily return comparable with optimal CRP.

Table of Contents

Results from the paper

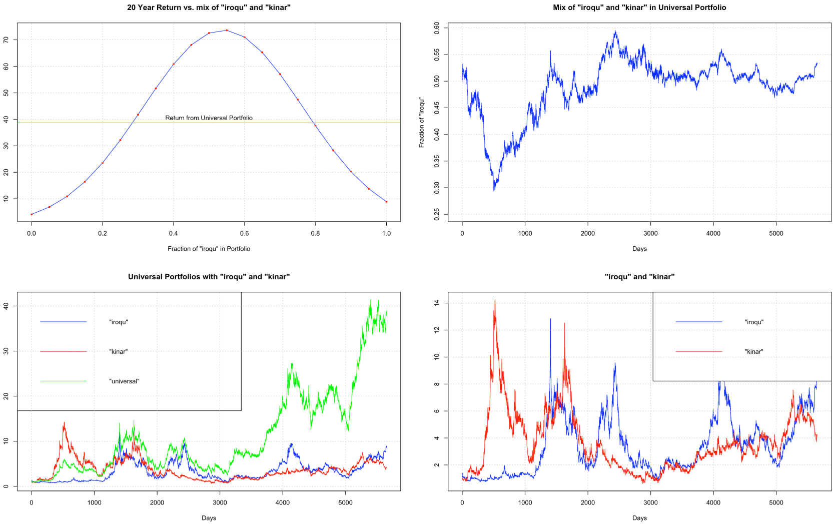
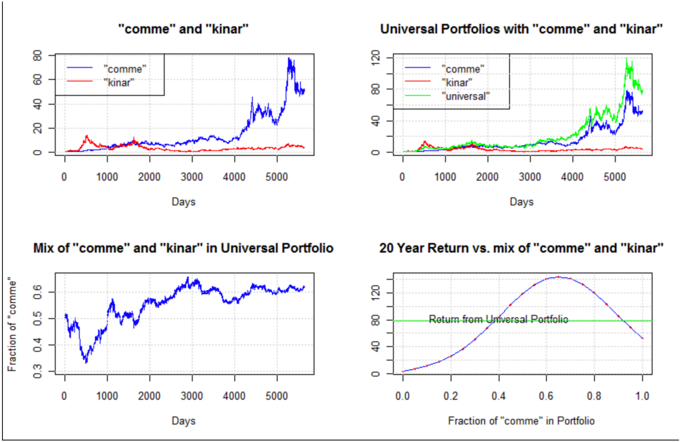


Figure: Results of Universal Portfolio for 2 stocks

Results from the paper



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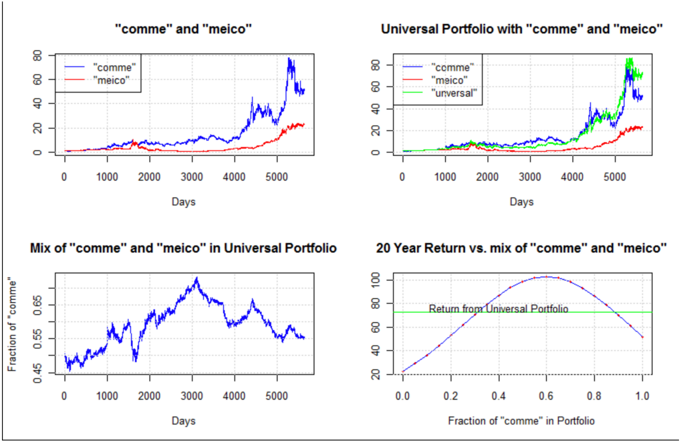
Results from the paper



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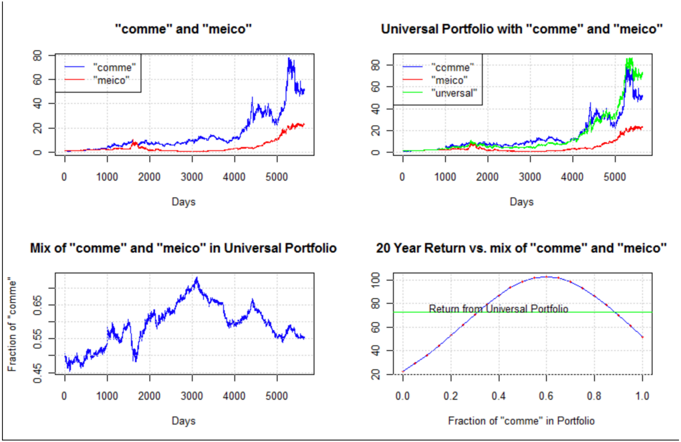

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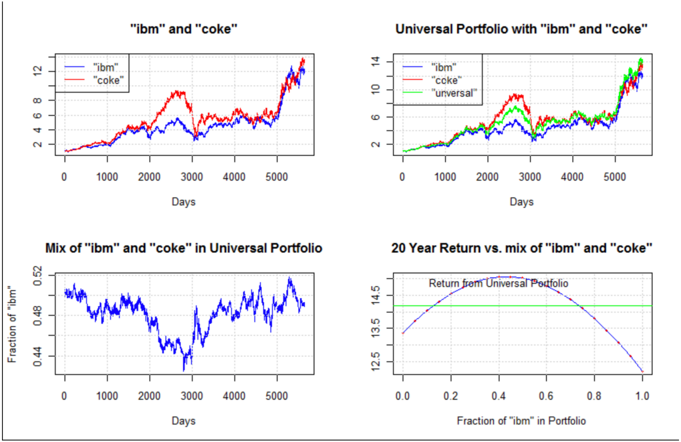

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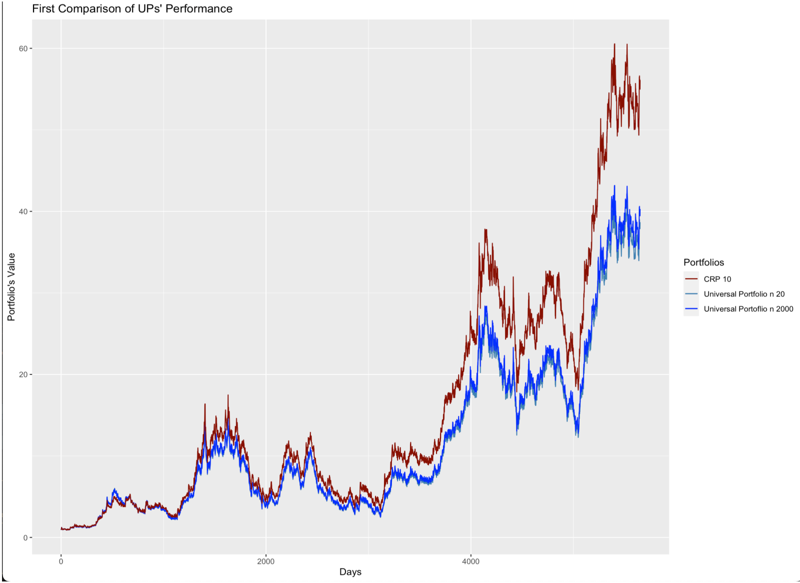

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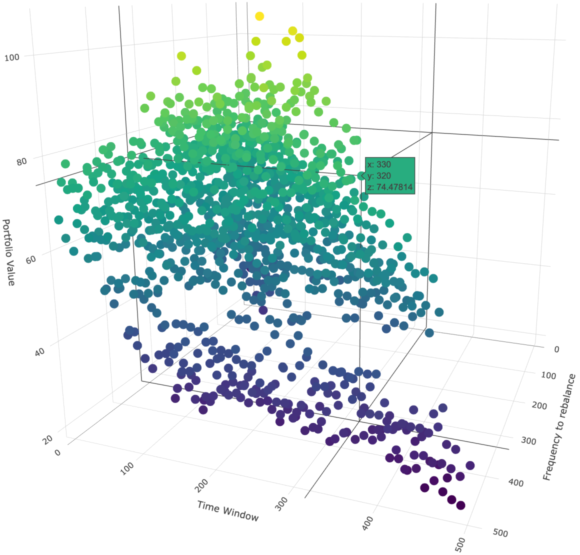

Results from the paper

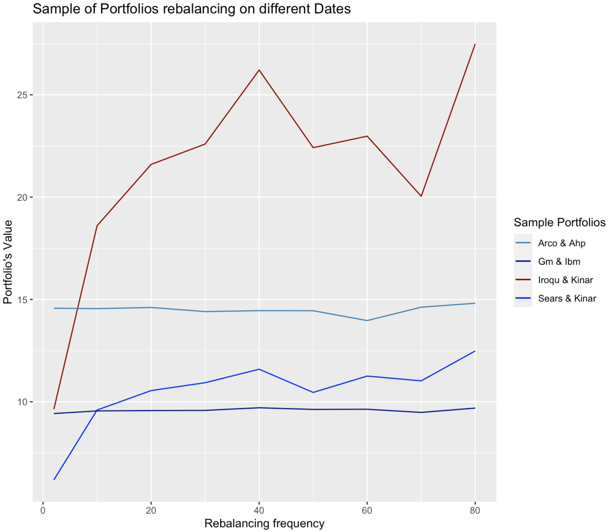

Table of Contents

Knapsack Problem

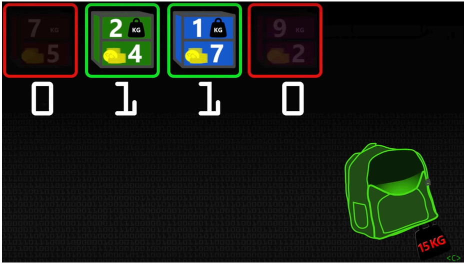

Figure: The Knapsack problem is a well-known optimization problem in computer science and mathematics. It can be formulated as follows: given a set of items, each with a weight and a value, determine the subset of items that maximizes the total value while staying within a given weight limit.

Table of Contents

Genetic Algorithm

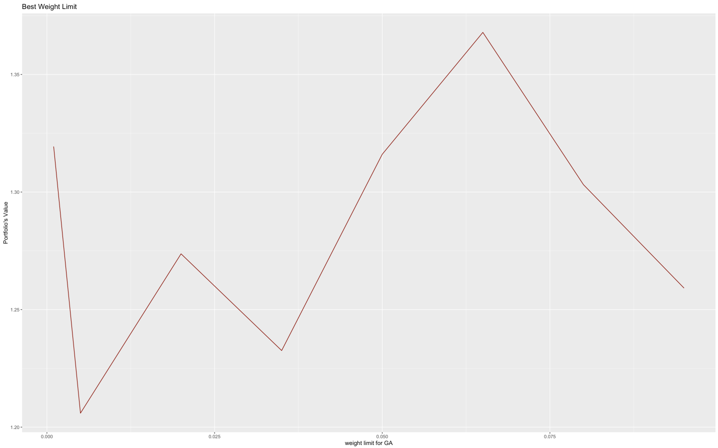

Figure: Performance under different weight liimits for the GA

Genetic Algorithm

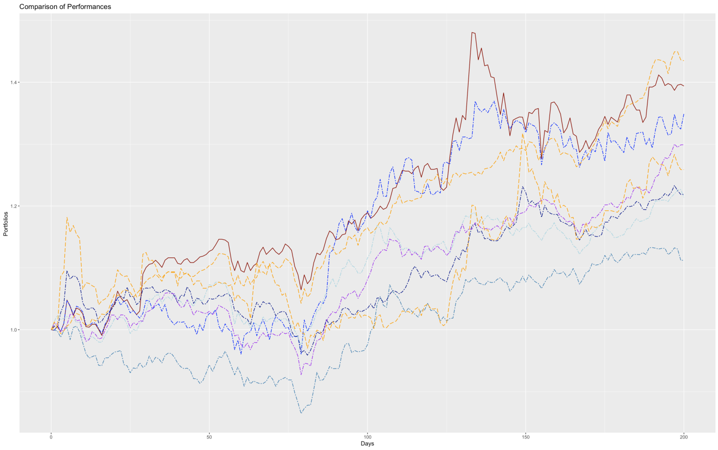

Figure: Performance of GA's portfolio compared to other strategies

End of the Presentation

Thanks for the attention.
All the scripts used can be found by this github directory.

Table of Contents

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