Real Effect

Thursday 03/11/2022

Contents

1 Introduction

$$\max u(c_1, c_2) = \log c_1 + \beta \log c_2 \tag{1}$$
$$c_1 = y_1 - t_1 \text{ +d1} \tag{2}$$
$$c_2 = y_2 - t_2 - (1+r)d_1 \tag{3}$$
$$d_1 \leq 0 \tag{4}$$

We can change (2) as $d_1 = c_1 - y_1 + t_1$ and using it into (3), thus:

$$c_1 + (1+r)^{-1}c_2 = (y_1 - t_1) + (1+r)^{-1}(y_2 - t_2)$$

This is the **intertemporal budget constraint**. The left side is called present discounted value of consumption, whereas the right side is the presented discounted value of disposable income. The intertemporal budget constraint is equivalent to the borrowing constraint (4) only if agents are allowed to borrow and save as much as they like: this constraint was derived without the borrowing constraint.

Lagrangian Using the Lagrangian method:

$$\begin{aligned}\mathcal{L} &= \log c_1 + \beta \log c_2 \\ &+ \lambda_1[y_1 - t_1 - c_1 + d_1] \\ &+ \lambda_2[y_2 - t_2 - c_2 - (1+r)d_1] \\ &- \psi d_1\end{aligned}$$

we can solve for $\nabla_{x,y,\lambda}\mathcal{L}(x, y, \lambda) = 0$

$$\nabla\mathcal{L} = \begin{bmatrix}\delta\mathcal{L}/\delta c_1 \\ \delta\mathcal{L}/\delta c_2 \\ \delta\mathcal{L}/\delta d_1\end{bmatrix} = \begin{bmatrix}1/c_1 - \lambda_1 \\ \beta/c_2 - \lambda_2 \\ \lambda_1 - \lambda_2(1+r) - \psi\end{bmatrix} = 0$$

We also have the **complementary slackness**[1] condition $\psi d_1 = 0$. As all Lagrangian multipliers are non negative, if $d_1 \leq 0$ and $\psi d_1 = 0$, then $\psi = 0$. When $d_1 < 0$ the agent wants to save, which is always allowed, then $\psi = 0$. However, if the agent want to borrow $d_1 > 0$, which is never allowed, then $\psi > 0$ and the borrowing constraint is binding and $d_1 = 0$. In this case the borrowing constraint becomes active and the agent is forced to choose the point where hse neither borrows nor saves.

Table 1: sump up

agent wants	but	necessarily	so that
$d < 0$	$\psi = 0$ borrowing constraint disappears	$d < 0$	$\psi d = 0$
$d > 0$	$\psi > 0$ borrowing constraint binding	$d = 0$	$\psi d = 0$

From FOCs we get the **Euler equation**

$$\frac{1}{c_1} = \frac{\beta(1+r)}{c_2} + \psi \tag{5}$$

where $1/c_1$ is the marginal utility of consumption in period 1, and $\beta(1+r)/c_2$ is the marginal utility of saving.

[1] non ho capito cosa cazzo è questa.

1.1 Absent borrowing constraint

The case where $\psi = 0$, thus the intertemporal budget constraint and the euler equation are, respectively:

$$c_2 = [(1+r)(y_1 - t_1) + (y_2 - t_2)] - (1+r)c_1 \qquad \text{[intertemporal budget constraint]}$$
$$\frac{1/c_1}{\beta/c_2} = 1 + r \qquad \text{[Euler equation]}$$

In the Euler equation, the left side is MRS_{c_1,c_2}, whereas the right side $(1+r)$ is the relative price of period 1 consumption. Depending on the level of the real interest rate and on the path of income in the two periods, the consumer may choose to be a saver or a borrower in period 1. The agent is a borrower (saver) in period 1 if she chooses to consume more (less) than her available income.

$$c_1^* > y_1 - t_1$$

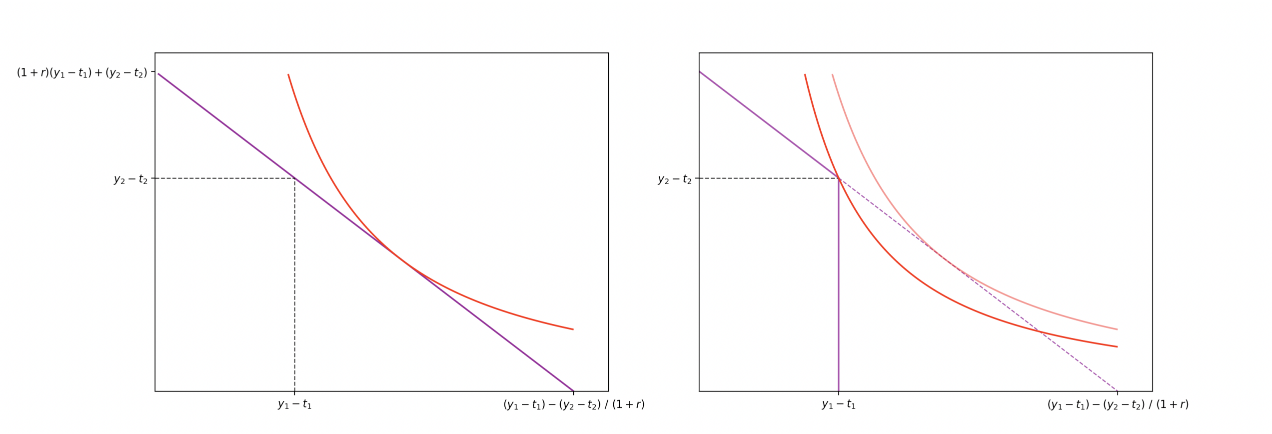
where $y_1 - t_1$ is the hand to mouth point in $t = 1$

1.2 Binding borrowing constraint

The case where $\psi > 0$, we know that $d_1 = 0$. From the borrowing constraint (4), we have

$$c_1 = y_1 - t_1$$
$$c_2 = y_2 - t_2$$

The optimal choice is not feasible; agents behave as hand to mouth consumer (consumer entire disposable income in each period).



2 Credit Market

We suppose now there are two types of agents in the economy: patients/savers and impatient/borrowers

$$u(c_1^b, c_2^b) = \log c_1^b + \beta^b \log c_2^b$$
$$u(c_1^s, c_2^s) = \log c_1^s + \beta^s \log c_2^s$$

With the assumption that $\beta^s > \beta^b$

equilibrium condition In equilibrium, the amount borrowed must equal to the amount saved. Borrowers borrow d_1^b while savers save $-d_1^s$ (remember that in the model saving corresponds to negative borrowing). Therefore

$$d_1^b = -d_1^s \equiv d$$

2.1 Demand of Debt

The budget constraints are

$$c_1^b = y_1^b - qk + d_1^b \tag{6}$$
$$c_2^b = y_2^b + q'k - (1+r)d_1^b \tag{7}$$

With k durable asset, whose price is q, and q' is the resale value of the house. Besides the budget constraints, we also have the **borrowers' collateral constraint**

$$d_1^b \leq \chi q'k$$

where current borrowing cannot exceed a fraction $\chi < 1$ of the resale value of the durable asset. The parameter χ can be interpreted as a loan to value (LTV) ratio. Changes in χ will be interpreted as financial disturbances, i.e. exogenous variation sin the agent's ability to borrow that are independent of the future value of the collateral at the time or repayment. We now proceed considering the collateral constraint not binding. In this case, the MU of consumption equals the MU of saving, and the Euler equation comes from the same Lagrangian method.

$$\frac{1}{c_1^b} = \frac{\beta^b(1+r)}{c_2^b}$$

Similarly, we can change (6) as $d_1^b = c_1^b - y_1^b + qk$ and using it into (7), thus the **intertemporal budget constraint for borrowers**.

$$c_1^b + \frac{c_2^b}{1+r} = y_1^b + \frac{y_2^b + [q' - q(1+r)]k}{1+r}$$

Now we can write the Euler equation as $c_2^b = \beta^b(1+r)c_1^b$, and using this composition of c_2^b in (7), and in the same equation, we change c_1^b with (6), always in (7).

$$\underbrace{\beta^b(1+r)\underbrace{y_1^b - qk + d_1^b}_{c_1^b}}_{c_2^b} = y_2^b + q'k - (1+r)d_1^b$$

Factorizing $(1+r)$

$$(1+r)\Big[d_1^b(1+\beta^b) + \beta^b(y_1^b - qk)\Big] = q'k + y_2^b$$

$$(1+r) = \Big[d_1^b(1+\beta^b) + \beta^b(y_1^b - qk)\Big]^{-1}(q'k + y_2^b)$$

Finally, imposing the equilibrium condition $d_1^b = d$

$$(1+r) = \Big[d(1+\beta^b) + \beta^b(y_1^b - qk)\Big]^{-1}(q'k + y_2^b) \tag{8}$$

However, we derived the **demand of debt** DB under the assumption that the borrowing constraint was not binding.

$$\begin{cases} (1+r) = \left[d(1+\beta^b) + \beta^b(y_1^b - qk)\right]^{-1}(q'k + y_2^b) & \text{if } d < \chi q'k \\ d = \chi q'k & \text{else} \end{cases}$$

2.2 Supply of Credit

The borrowing constraint for savers is irrelevant, thus, the problem for this type of agents is much easier, since the budget constraints are

$$c_1^s = y_1^s + d_1^s \tag{9}$$
$$c_2^s = y_2^s - (1+r)d_1^s \tag{10}$$

Then, solving (9) for d_1^s and substituting it into (10), we obtain the **intertemporal budget constraint for savers**

$$c_1^s + \frac{c_2^s}{1+r} = y_1^s + \frac{y_2^s}{1+r}$$

and the Euler equation

$$\frac{1}{c_1^s} = \frac{\beta^s(1+r)}{c_2^s}$$

Now we can write the Euler equation as $c_2^s = \beta^b(1+r)c_1^s$, and using this composition of c_2^s in (10), and in the same equation, we change c_1^s with (9), always in (10). Same procedure applied for the demand side.

$$\underbrace{\beta^s(1+r)\underbrace{y_1^s + d_1^s}_{c_1^s}}_{c_2^s} = y_2^s - (1+r)d_1^s$$

Factorizing $(1+r)$

$$(1+r)\left[\beta^s y_1^s + d_1^s(1+\beta^s)\right] = y_2^s$$

$$(1+r) = \left[\beta^s y_1^s + d_1^s(1+\beta^s)\right]^{-1} y_2^s$$

Finally, imposing the equilibrium condition $-d_1^s = d$, we obtain the **supply of credit**.

$$(1+r) = \left[\beta^s y_1^s - d(1+\beta^s)\right]^{-1} y_2^s$$

2.3 Conclusions

i **negative credit shock**: A sudden reduction in the LTV parameter χ can be interpreted as borrowers can borrow less for any given value of their collateral. The same value of collateral extracts less liquidity

ii **equilibrium real interest rate**. The vertical part of the demand of debit moves leftward. There are two cases to consider

(a) if the borrowing constraint is not binding and the change in χ is small, then the equilibrium does not change: d and r are the same

(b) if the borrowing constraint is binding to begin with or if the change in χ is sufficiently big, then the equilibrium changes:

$$d \downarrow \quad \Rightarrow \quad \text{borrowers must borrow less}$$
$$1 + r \downarrow \quad \Rightarrow \quad \text{savers must save less}$$

iii **Can the net interest rate go below zero?** When the borrowing constraint is binding, we know that borrowers borrow $d = \chi q' k$. We can substitute the latter in the supply of debt to get

$$1 + r = \left[\beta^s y_1^s - \chi q' k (1 + \beta^s) \right]^{-1} y_2^s$$

To have a negative interest rate we must have $r < 0$, thus $1 + r < 1$. Therefore

$$\left[\beta^s y_1^s - \chi q' k (1 + \beta^s) \right]^{-1} y_2^s < 1$$

or

$$y_2^s < \beta^s y_1^s - \chi q' k (1 + \beta^s)$$

Hence, if χ falls a lot, the right hand side of the inequality increases and this scenario becomes more likely.