Initial aggregate conditions and heterogeneity in firm-level markups *

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Abstract

I explore the role of aggregate fluctuations as a persistent determinant of heterogeneity in firm-level markups. To analyze how business cycles generate dispersion in markups, I estimate the effects of aggregate conditions at key moments of firms' lives on the age profiles of markups for a sample of U.S. listed companies. Using the estimated markups, I calibrate a general equilibrium model that features heterogeneous product markets, customer base accumulation and firm dynamics. A novel feature of the model is that, in addition to making direct investments in customer acquisition, firms can accumulate customers by increasing sales, which is important to match the empirical findings. As the value of operating in each product market fluctuates endogenously with business cycles, aggregate conditions generate a selection on the product-market composition of firm cohorts that results in time-varying heterogeneity in product-market characteristics across active companies. This heterogeneity is persistent and can significantly affect both the response of the economy to future aggregate shocks and the co-movements of aggregate markups with output.

JEL Codes: D21, D22, E32, L11. **Keywords:** markups, business cycles, heterogeneous firms, cohort effects.

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1 Introduction

This paper combines micro-level empirical analysis and a structural general equilibrium model to estimate the persistence of business cycle effects on firm-level markups and to document the importance of compositional effects in the cross-section of firms for the behavior of markups along business cycles.

The slow recoveries that characterized the years after the Great Recession have renewed the interest in the persistent effects of aggregate fluctuations on both workers and firms. For the latter, recent studies show that businesses' starting conditions matter significantly for their future size and dynamics. In particular, when looking at their sizes as measured by employment, firms that are born in a recession suffer from a persistent gap compared to those that start in expansions. Notably, these differences can be traced back to different ease and need of new firms to acquire customers depending on the state of the business cycle.[1]

I explore a novel channel linking customer-base accumulation, business cycle conditions early in firms' lives, and markup dynamics. As aggregate conditions influence other aspects of firms' operations besides their employment decisions, business cycles could potentially also affect how firms manage variables like prices and markups that are commonly considered more volatile and less history-dependent than their sizes. The prominence of price-cost markups as one of the critical variables for the transmission of aggregate shocks is well established in macroeconomics. Until recently, however, estimates of their co-movement with business cycles have relied mostly on aggregate or industry-level measures.[2] The increased availability of firm-level data and the development of methods that allow estimating markups at finer levels of aggregation have spurred an increase in the number of papers studying the dynamics of markups at the micro-level and their repercussions on aggregate economies. However, as most of these studies focus on long-run dynamics,[3] shedding more light on possible determinants of markups' heterogeneity and their interplay with business cycles is a promising avenue to uncover new channels for the transmission of aggregate shocks.

This paper has two parts: empirical and theoretical. In the empirical part, I present novel facts on firm-level markups, their relationship with aggregate fluctuations at crucial junctures of firms' lives, and an analysis of their contemporaneous correlation with business cycles. In the theoretical part, I develop a firm-dynamics model that links the heterogeneity in the markups of active firms to the aggregate state of the economy at their listing times. I then use the model as a laboratory to assess the importance of business cycles as a source of heterogeneity for firm-level markups. I show that firm heterogeneity significantly affects the transmission of aggregate shocks and the co-movements of aggregate markups with output.

[1] See for example Sedláček and Sterk [2017] and Moreira [2015].

[2] See, among others, Rotemberg and Woodford [1999] and Bils [1987]. For a recent analysis of how aggregate macroeconomic shocks transmit to aggregate and industry-level markups, see Nekarda and Ramey [2013].

[3] Relevant contributions in this field are, among others, De Loecker, Eeckhout and Unger [2020], Traina [2018], Autor, Dorn, Katz, Patterson and Van Reenen [2017], Hall [1988]. Recently, Hong [2017] and Burstein, Carvalho and Grassi [2020], instead, estimate the cyclicality of markups at the firm level.

Empirically, I document three main facts. First, I show that markups slightly increase as firms' age. Second, I estimate the elasticity of markups with aggregate conditions at two key moments in firms' lives: i) the time of listing and ii) the time of firm creation. In both cases, I find that firm-level markups exhibit a negative correlation with business cycles. In other words, markups tend to be higher when the creation or listing event occurs in a recession. In addition, I show that these effects are persistent but not permanent, resulting in flatter age profiles for firms that are founded or become listed in recessions. Third, I show that firm-level markups are countercyclical for young firms while they are mildly procyclical for old firms.

There are two reasons why these facts are important from a macroeconomic perspective. First, in a world where the composition of firms in the economy is not fixed, the macroeconomic response to aggregate shocks is bound to be shaped by the characteristics of active firms. Second, the firm cross-sectional compositions can be persistently affected by previous business cycles because aggregate conditions early in firms' lives or at the time of key decisions, like listing, have persistent effects on how firms manage their markups. In light of these results, a model that is able to characterize both the firm-level and the aggregate behavior of markups and, at the same time, incorporate firms' life cycles can serve two purposes: first, it helps to gauge which characteristics of the cross-section of active firms are at the root of the observed heterogeneity in markups and, second, it allows us quantify the relevance of these characteristics to the transmission of aggregate shocks.

In the theoretical part of the paper, therefore, I rationalize the empirical facts by building a firm-dynamics model with product-market heterogeneity and customer-base accumulation. Notably, firms' demands are assumed to be constrained by the size of their customer bases. However, firms have the ability to relax these constraints by managing their customer-base size. The novelty of the mechanism proposed in this paper relies on the fact that firms use two complementary channels to accumulate customers: i) increasing sales and ii) direct demand investments, a shorthand for the range of activities specifically designed to increase the reach of firms' products in order to, in turn, increase demand. The literature on firm dynamics and pricing has mostly looked at these two effects in isolation, while I show that considering them *jointly* is important to replicate the empirical findings.[4]

The mechanism that I explore in the paper relies on an implicit selection of the cohorts of newly listed firms operated by business cycles. Specifically, in the model, firm profitability depends on the product market in which firms operate, and this, in turn, affects whether it is more important to list during a recession or an expansion. This is because, across product markets, firms face different trade-offs between using prices and directly investing in their customer bases to relax demand constraints. For example, lowering prices could be an effective strategy to increase demand for goods, such as mass goods, whose production

[4] The importance of pricing in models with customer markets dates back to Phelps and Winter [1970], Bils [1989], and more recently Nakamura and Steinsson [2011]. The link between markups and customer bases is highlighted in Perla [2019], Gilbukh and Roldan [2018], Gourio and Rudanko [2014], Argente, Lee and Moreira [2018], Fitzgerald and Priolo [2018], Bilbiie, Ghironi and Melitz [2019], Edmond, Xu and Midrigan [2018], Kueng, Yang and Hong [2014], Foster, Haltiwanger and Syverson [2016]. In this literature, the papers that are the closest in spirit to this one are Sedláček and Sterk [2017] and Hong [2017].

can be easily scaled to satisfy large markets and where customers are more focused on prices rather than other characteristics of the products. In these markets, therefore, the price lever could be particularly convenient as it enables firms to quickly increase their output and, at the same time, lock more customers in their specific product varieties. For firms that instead operate in less scalable markets where customers are highly sensitive to other product characteristics besides prices, the expansion of their customer bases has to rely more heavily on investments in product characteristics like marketing and brand value or other forms of quality improvements. Importantly, in these cases, investment in demand necessarily requires a diversion of resources from production to direct customer-base acquisition.

Aggregate conditions affect the relative efficacy of these two investments and persistently impact the aggregate economy by skewing the composition of firms' cohorts towards product markets that rely more heavily on either one of the two channels. This is because firms face demands with different sensitivities to customer bases and the acquisition of repeat customers relies more heavily on firms' sales in some product markets than in others.

Firms that operate in product markets where sales are an important channel for the acquisition of new customers will find it more profitable to expand their operations in periods where productivity is high and the cost of expanding is low. On the other hand, firms that have to devote a larger share of resources away from production to acquire customers will find it more profitable to start or expand their operations in periods of high aggregate demand when the effectiveness of these targeted investments is higher. Finally, as I assume a common cost of listing across product markets, fluctuations in firms' expected profitability create the premises for an endogenous selection of newly listed firms due to business cycles.

I calibrate the parameters determining product-market characteristics so that moments implied by the stationary solution of the model are consistent with the firm-level estimates of markup age profiles and the distribution of firm sizes. Interestingly, the parametrization that delivers the best fit implies a negative relationship, across product markets, between the sensitivity of demands to customer bases and the relevance of firms' sales in attracting customers. Under this baseline parametrization, I show that the model is able to replicate the cyclicality profile of markups, delivering countercyclical markups for young firms and mildly procyclical ones for older firms. In addition, I show that the model is able to correctly generate an age profile for markups that is higher and flatter for firms that begin their listing process in recessions, a feature consistent with the empirical findings. Moreover, in the context of the model, I show that it is necessary to allow for investments in the customer base to depend on firms' sales in order to generate a realistic life-cycle profile for firm-level markups.

Finally, simulating the model with a fixed firm composition and under different parametrization of product-market characteristics, I show that the firms' product-market composition and the heterogeneity of customer-acquisition processes are key determinants of the correlation of aggregate markups with output. Thanks to the endogenous selection operated by business cycles, the model is able to replicate both the cyclical behavior of markups at the firm level and the correlations of aggregate measures of markups with output.

Relation to the literature. The paper relates to two literatures in macroeconomics: one that studies the role of firms for the macroeconomy and their interactions with aggregate fluctuations and the other that focuses on understanding how markups behave during business cycles.

Building upon the seminal works of Hopenhayn [1992] and Khan and Thomas [2008] various papers have explicitly studied the importance of firm dynamics for the behaviour of output, investment and employment along business cycles (e.g. Ottonello and Winberry [2019], Carvalho and Grassi [2019], Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry [2018], Clementi and Palazzo [2016], Fort, Haltiwanger, Jarmin and Miranda [2013] , Moscarini and Postel-Vinay [2012], Bilbiie, Ghironi and Melitz [2012]). Within this literature, some papers have recently started to connect aggregate fluctuations with firm heterogeneity.[5] A compelling thesis of these studies is that firms are endowed with ex-ante heterogeneous characteristics that play a significant role in determining their economic outcomes (Pugsley, Sedláček and Sterk [2019], Hottman, Redding and Weinstein [2016], Foster *et al.* [2016], Foster, Haltiwanger and Syverson [2008]). Sedláček and Sterk [2017] in particular, show that aggregate fluctuations can have persistent effects on firm sizes by affecting the composition of new firms, whereby different realizations of the business cycle select different types of start-ups based on ex-ante characteristics that then result in persistent size differences across active firms. Similarly, Moreira [2015] shows that firms that start their activities during bad economic times suffer from a persistent size gap compared to similar types of businesses that start during booms. Notably, rather than firm-level productivity, whereby more productive firms are born during booms, aggregate conditions at the time of inception and the resulting easiness of expanding their customer bases appear to drive these differences. My contribution to this literature is to expand the focus of the analysis on markups and show that the endogenous selection operated by business cycles on firm characteristics and the ensuing compositional effects are crucial for the behavior of price-cost markups at the aggregate level.

The importance of product selection to match many business cycle features of firms balance sheets, such as procyclical profits and countercyclical markups, has been investigated by Dhingra and Morrow [2019] and Bilbiie *et al.* [2012] among others. More recently, Bilbiie *et al.* [2019] focus on the importance of product selection as a determinant of market power. In this paper, I follow the spirit of the analysis carried out in these studies, but I consider firms' product market decisions not as a continuous choices but something that can be done infrequently and only at particular junctures of firms' lives.[6]

The second strand of literature this paper contributes to is the broad literature that studies the co-movements of markups with business cycles (Hall [1986], Bils [1987], Rotemberg and Woodford [1999], Nekarda and Ramey [2013, 2020], Anderson, Rebelo and Wong [2018],

[5] Notable examples in this literature are also Luttmer [2007] and more recently Arkolakis [2016], Bernard, Dhye, Magerman, Manova and Moxnes [2019], Hoffmann [2017] and Alp [2019].

[6] See Stoughton, Wong and Zechner [2001], Wies and Moorman [2015], Bernstein [2015] and Chemmanur, He, He and Nandy [2018], Chemmanur and He [2011] to see how the decision of going public, for example, is not only linked to funding motives but also to product-market considerations. González [2020] studies how aggregate conditions affect the selection firms that decide to go public and how, in turn, these are linked to recent changes in IPO patterns.

Bils, Klenow and Malin [2018] and others). In the context of this paper, this literature has highlighted how different levels of aggregation can yield different answers on the cyclicality of markups, with firm-level estimates pointing towards a negative correlation between markups and output while more aggregate measures of markups tend to indicate mildly positive correlations with output. In this paper, I show that compositional effects in firm product market characteristics help in reconciling this apparent divergence.

In addition, it is useful to distinguish between two theoretical approaches that the literature has used to rationalized the time-series behavior of markups. The first one is linked to changes to the market structure in which firms operate (e.g. Jaimovich and Floetotto [2008], Burstein *et al.* [2020]) while the second is linked to the role of customer acquisition in firm dynamics models (e.g. Neiman and Vavra [2019], Bornstein [2018], Perla [2019], Paciello, Pozzi and Trachter [2019]) and the role of customer markets in determining the cyclical behavior of markups (e.g. Bils [1989], Ravn, Schmitt-Grohé and Uribe [2006], Gourio and Rudanko [2014], Gilchrist, Schoenle, Sim and Zakrajsek [2017], Hong [2017], Gilbukh and Roldan [2018]). In particular, in a recent working paper Burstein *et al.* [2020] show that, in a model with oligopolistic competition, the sign of markup cyclicality depends on the level of aggregation considered, the market structure and the set of shocked firms. Gilbukh and Roldan [2018], instead, develop a directed-search model of the U.S. retail sector in which firm pricing strategies are used to increase the probability of matches with perspective customers. Similar to this paper, they find that compositional changes to the distribution of firms can have a significant impact on the behavior of markups. In this paper, I contribute to this literature exploring more in depth how the relationship between customer acquisition and markups changes when firms use both prices and direct investments to manage their customer base.

Structure of the paper. The paper is structured as follows: Section 2 discusses new evidence on the correlations between aggregate conditions early in firms lives and markups. Section 3 develops a theoretical model that incorporates incentives to accumulate customer base together with the documented effects of aggregate conditions on markups. Section 4 discusses the calibration of the model and the main functional form assumptions while Sections 5 and 6 present the main model implications. Finally, Section 7 concludes.

2 Empirical evidence

In this section, I summarize the empirical facts on firm-level markups regarding the effects of business cycles at key moments of firms' lives and the contemporaneous correlations with business cycles along firms' life cycles. I briefly summarize the procedure I use to recover markups at the firm-level and then I describe the empirical methodology used to estimate the persistency of past business cycles on the age profile of markups. More details on the empirical analysis, its limitations and various robustness checks are discussed in Appendix C.

2.1 Measuring markups at the firm-level

The estimation strategy I use in this paper exploits firms' optimal behavior to back-out an estimate of markups at the firm level without the need to specify an explicit demand system. The methodology I follow here has been developed by De Loecker and Warzynski [2012] and De Loecker *et al.* [2020] and it is based on the production function approach pioneered by Hall [1986, 1988] on industry-level data.

Consider a firm j that produces using the following technology, $Q_{j,t} = Q(\mathbf{X}_{j,t}, K_{j,t}, \omega_{j,t})$, where $\mathbf{X}$ is a vector of variable inputs and ω is firm specific productivity and K are predetermined inputs. The cost minimization problem for each producer therefore is

$$\min_{\mathbf{X}_{j,t}, K_{i,t}} \left\{ \mathbf{X}'_{j,t}\mathbf{P}_{j,t} + R_{j,t}K_{j,t} + \lambda_{j,t}(Q_{j,t} - Q(\cdot)) \right\}.$$

The first-order condition, for a generic variable input $X^{\nu} \in \boldsymbol{X}$, is

$$\frac{\partial \mathcal{L}(\cdot)}{\partial X^{\nu}_{j,t}} = P^{\nu}_{j,t} - \lambda_{j,t}\frac{\partial Q(\cdot)}{\partial X^{\nu}_{j,t}} = 0, \tag{1}$$

where $\lambda_{j,t}$ can be interpreted as the marginal cost of producing at a given level of output. Equation (1) can be rearranged as

$$\frac{\partial Q(\cdot)}{\partial X^{\nu}_{j,t}}\frac{X^{\nu}_{j,t}}{Q_{j,t}} = \frac{1}{\lambda_{j,t}}\frac{P^{\nu}_{j,t}X^{\nu}_{j,t}}{Q_{j,t}}. \tag{2}$$

Defining the markup as price over marginal costs, $\mu_{j,t} \equiv \frac{P_{j,t}}{\lambda_{j,t}}$, it is possible to rewrite Equation (2) so that

$$\mu_{j,t} = \theta^{\nu}_{j,t}\frac{P_{j,t}Q_{j,t}}{P^{\nu}_{j,t}X^{\nu}_{j,t}}, \tag{3}$$

where $\theta^{\nu}_{j,t}$ is the elasticity of output with respect to the variable input X^{ν}.

Obtaining consistent estimates of markups in this setting requires assuming that firms are free to adjust their prices without incurring in any costs. The use of yearly data to estimate the production function mitigates the potential drawbacks of abstracting from price rigidities at the firm level.[7]

2.2 Persistent effects of business cycles on markups

In this section, I extend the existing analysis on the persistence of business cycles on firm sizes to firm-level markups. I rely on a standard age-period-cohort model where I break the well-known multicollinearity between age, time and cohort fixed effects by using a proxy variable for cohort effects (Moreira [2015], Heckman and Robb [1985]). Notably, I assign firms to cohorts using two alternative definitions. As I am relying on public firms, the baseline definition is based on firms' listing years. Hence, for the main specification, I proxy cohort effects with a measure of aggregate conditions close to listing (first year of

[7] The implicit estimation of markups as wedges between output elasticities of variable inputs and their expenditure shares arise naturally in any market structure different than perfect competition. However, these wedges can reflect other distortions if the structural assumption of no adjustment cost is violated.

available accounting data).[8] Alternatively, for a subsample of firms for which I am able to retrieve the founding date I also consider a definition of firm cohorts based on the founding year. Hence, for this alternative subsample, I proxy the set of cohort-level fixed effects with aggregate conditions at firms' founding years.[9] Both definitions consider key moments in firms lives that are linked to structural transformations that are likely to persistently influence firms performances, making aggregate conditions at both moments a valid proxy for the set of cohort fixed effects in the age-period-cohort model. This estimation strategy allows studying how business cycle realizations at the time of relevant structural decisions about firms operations and organization, such as listing and founding, correlate with firms' markups and allows me to the estimate the persistency of this correlation along firms' lives.

The main firm-level specification that I bring to the data is the following:

$$\log(\mu_{j,c,t}) = \alpha + \phi_a + \phi_t + \beta \widetilde{Y}_c + \mathbf{X}_{j,t}\gamma + u_{j,c,t}, \tag{4}$$

where $\mu_{j,c,t}$ is the markup charged by a firm j belonging to cohort c at time t; ϕ_a and ϕ_t are respectively age and time fixed effects; $\widetilde{Y}_c$ is a measure of initial aggregate conditions for firms belonging to cohort c and $\mathbf{X}_{j,t}$ is a vector of controls that includes sector fixed effects and a second-order polynomial in firm j's sale share in her three digits sector.

The main coefficient of interest is β, that measures the percent change in average markup associated with a one percent variation in the initial business cycle conditions, after controlling for aggregate conditions faced by firms throughout their lives and the aging process. The age fixed effects, instead, are estimates of an age profile of markups that reflects only the dynamics that can be attributed to the aging process, controlling for cohort and time effects.

The specification in Equation (4) allows one to measure the average effect on yearly markups but does not allow to verify if the effects of business cycles are persistent or tend to vanish as firms age. To assess these features of aggregate conditions around the time of listing, I rely on the following specification,

$$\log(\mu_{j,c,t}) = \alpha + \phi_a + \phi_t + \beta_0 \widetilde{Y}_c + \beta_1 \widetilde{Y}_c \times a_{j,t} + \mathbf{X}_{j,t}\gamma + u_{j,c,t}. \tag{5}$$

The coefficients of interest in this case are β_0, that captures the effect on aggregate conditions in the first year, and β_1, that estimates the supplementary effect of business cycle realizations for each additional year firm j is observed.

[8] In Compustat, firms report their balance sheets also a few years before the first trading date. I use their first reporting year as the main definition of firm cohorts for two reasons: the first one is that this is the time in which companies are most likely starting the process for the IPO; the second one is that this variable is much more populated than the IPO date or the founding date. However, the first reporting year and the IPO year are very close to each other. For the companies that report the IPO date (approximately 39% of firms in the sample), the average time between the first available year and the reported IPO year is 1.8 years while the median is one year (90th percentile three years). Restricting the sample to firms that have the first year of available accounting data at the IPO year does not qualitatively affect the results. Figure C.1 plots the relationship between the first year of available accounting data, IPO and founding dates for the subsample of firms for which I am able to retrieve all the information.

[9] Founding dates are retrieved from the Field-Ritter Database of Founding Dates and I merge it to my baseline Compustat sample via CRSP.

2.3 Cyclicality of firm level markups along the age profile

It is reasonable to think that the elasticity of markups with respect to aggregate conditions is not constant over firm life cycles. For example, Gilchrist *et al.* [2017] show that for reasons linked to the availability of internal funds, smaller firms tend to react more strongly to aggregate shocks.[10]

To quantify how the elasticity to current business cycle conditions changes along firms' age profiles I am considering the following specification:

$$\log(\mu_{j,t}) = \alpha + \beta_0 Y_t + \beta_1 Y_t \times a_{j,t} + \beta_2 Y_t \times a_{j,t}^2 + \phi_a + \phi_j + \mathbf{X}_{j,t}\boldsymbol{\gamma} + u_{j,t}, \tag{6}$$

where ϕ_a are age fixed effects and $a_{j,t}$ is firm j's age at time t measured in years since the first available year of balance sheet data and where Y_t is a contemporaneous measure for the business cycle.

The object of interest is the set of β_i with $i = \{0, 1, 2\}$ as they estimate the strength of the contemporaneous elasticity between the business cycle and the firm-level markup at different stages of firms' life-cycle, depicting the evolution of the correlation between output and markups as firms age. I am not including time fixed effects as I am interested in recovering the baseline coefficient β_0. However, I include controls for a cubic time trend. For the estimation of cohort effects and the age profile, instead, I use time fixed effects rather than contemporaneous GDP realizations as they allow to simultaneously control for all time-dependent characteristics that could otherwise bias the estimation of markups' age profile.

2.4 Data overview

The main data source is the annual segment of Compustat. In this section, I discuss briefly the strengths and limitations of the data. I provide more details on the data cleaning process and the construction of the sample of analysis in Appendix C.

The dataset includes detailed balance sheet information on US listed firms. Unlike other firm-level databases, these data allow following a large number of cohorts for many years giving the possibility to use the full time series variation in the proxy variables to approximate cohort effects and allowing to trace longer age profiles for firm-level markups that are useful in the calibration of the model developed in Section 3.

However, the data present three main limitations: i) the difficulty of recovering a consistent measure of variable costs from *Cost of Goods Sold* (COGS) due to some flexibility in reporting standards for the firms included in the dataset;[11] ii) the fact that it is impossible to distinguish quantity and prices, which bears consequences on the measurement of the production function elasticity to variable inputs and iii) the possible selection issues arising from using listing dates instead of firm creation to define firm cohorts.

[10] Hong [2017] and Burstein *et al.* [2020] find similar results using a sample of European data and administrative French data.

[11] See Traina [2018] for an analysis of the long-run trends in markups and how these are affected by using *Operating Expenditure* as a measure for variable cost measure.

Table 1: Cohort effects on firm-level markups, cohorts defined on listing years

Dep.Variable: Log-Markup	(1)	(2)	(3)
Cohort-level GDP	-0.249***	-0.672***	-0.960***
	(0.064)	(0.135)	(0.214)
Cohort-level GDP × Age		0.043***	0.127***
		(0.010)	(0.040)
Cohort-level GDP × Age^2			-0.004**
			(0.002)
Controls	Yes	Yes	Yes
Age FE	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes
Initial Sale Decile FE	Yes	Yes	Yes
R^2	0.14	0.14	0.14
N	91,317	91,317	91,317

Notes: Robust standard errors in parentheses, p :$^{***} < 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports the main estimates of the elasticities between firm-level markups and aggregate conditions at the time the firm is first observed. Cohort effects are proxied using quadratically detrended log real GDP in the year the firm appears in the sample. Columns (1) and (2) report estimates using the specification in Equation (4) and (5), respectively. Column (3) instead reports the persistence of the cohort effects based on a second-order polynomial in age.

To address the concerns linked to possible mis-measurements of variable costs, I estimate markups using two measures frequently used in the literature, *Cost of Goods Sold* and *Operating Expenditure*. The cyclical behavior of markups is robust to this variable choice. Having to rely on the *revenue* elasticity to variable inputs can be problematic if firm-specific demand shocks can influence firm prices. If unaccounted for, this pass-through would bias the estimate of productions parameters. Relatedly, Bond, Hashemi, Kaplan and Zoch [2020] show that markups measured using the revenue elasticity could only reflect noise. I address these concerns adding controls in the estimation of the production function and by replicating the baseline analysis under different production function specifications and for firms net operating profit margins. The baseline results are robust to these different variations. I discuss these robustness checks more in detail in Appendix C. To address the last possible concern, I estimate the cohort effects specifications also for a subset of firms for which I am able to obtain the founding year.

2.5 Persistent effects of business cycles on firm-level markups

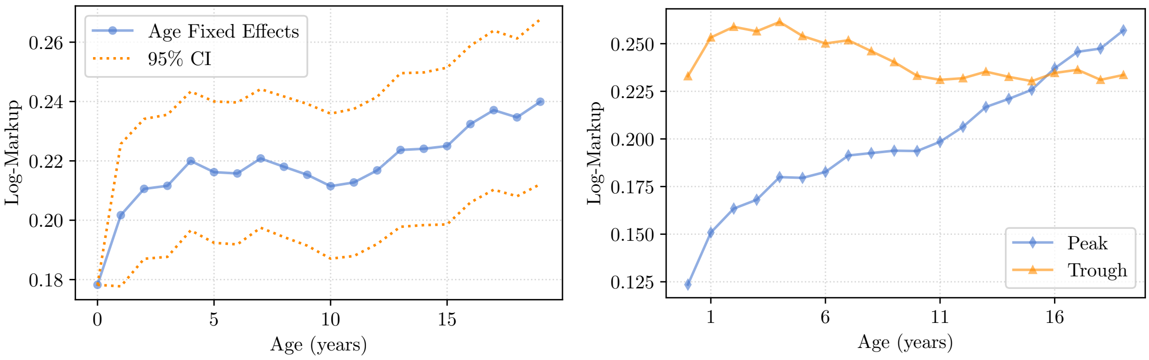
In this section, I discuss the reduced-form estimates of the elasticities of firm-level markups to aggregate conditions at the time of listing and at the founding date.

2.5.1 Cohorts based on listing years

Markups' age profile. The age fixed effects estimated in Equation (4) show the dynamics of markups along firms' life cycles that can be attributed to aging, taking into account aggregate conditions and cohort effects. Figure 1a plots the fixed effect coefficients together

Figure 1: Age profile and cohort effects for firm-level markups

(a) Markups' age profile **(b)** Cohort effects on markups' age profile



Note: Panel 1a plots the estimated age profile for markups from Equation (4) together with the 95% confidence interval. Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a$, where $\hat{\mu}_0$ is the average log-markup in the first available year and $\hat{\phi}_a$ are the estimated age fixed effects. Panel 1b plots the age profile for markups estimated from Equation (5). Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{\mu}_0$ is the average log-markup in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the first available year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of quadratically detrended real GDP. The coefficients on which this plot is based are reported in Table 1, Column (2).

with a 95% confidence interval for the first 20 years of firms' lives.[12] The figure shows how the average markup steadily increases up to 15 years after the firsts available data.[13]

Cohort effects. The main correlations between aggregate conditions and firm-level markups are reported in Table 1. The main results is that the correlation between markups and aggregate conditions at the time of the first available balance sheet are significantly negative indicating that firms that start their listing process in periods of below-trend output tend to charge an higher markup on average. The age profiles estimated from Equation (5) are plotted in Figure 1b. The figure shows how for firms that experience positive aggregate conditions, defined as periods when the cyclical component of GDP is two-standard deviations above trend, close to their listing date tend to charge an initial markup that is approximately 8% lower that similar firms that instead face a negative realization of the business cycle. In addition, besides the magnitude of the initial effect, it is worth noticing that firms that are first observed during booms exhibit a steeper age profile of markups.

Considering a two-standard deviation negative change in the cyclical component of log-real GDP, approximately -6%, the average effect reported in Column (1) implies that firms will charge a markup $(-0.06)(-0.249) \approx 1.4\%$ higher every year, only due to these worse aggregate conditions they faced when starting the listing process. To put it in perspective, this effect implies that firms that were close to listing at the height of the Great Recession,

[12] As the estimates of the age fixed effects stabilize after approximately 20 years but the number of firms decline substantially do to attrition, the point estimates after 20 years are more volatile but do not add any additional information to the estimates.

[13] Argente *et al.* [2018], estimate the marginal cost and price profiles at the product level and they show that firms increase the markups on their products over time but constantly introduce new products that command an higher share of overall firm revenues. In their structural estimation they also find that markups increase, almost linearly at the product level for the first 16 quarters from their introduction with newer products having markups close to 0 at the time of their introduction.

Table 2: Cohort effects on firm-level markups, cohorts defined on founding years

Dep.Variable:Log-Markup	(1)	(2)	(3)
Cohort-level GDP	-1.231***	-2.151***	-2.679**
	(0.196)	(0.556)	(1.113)
Cohort-level GDP $\times$ Age		0.077*	0.188
		(0.041)	(0.198)
Cohort-level GDP $\times$ Age^2			-0.005
			(0.008)
Controls	Yes	Yes	Yes
Age FE	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
R^2	0.17	0.17	0.17
N	21,336	21,336	21,336

Notes: Robust standard errors in parentheses, p :$^{***} < 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports the main estimates of the elasticities between firm-level markups and aggregate conditions at the time the firm is first observed. Cohort effects are proxied using quadratically detrended log real GDP in the year the firm is founded. Founding dates are based on the *Field-Ritter Database of Founding Dates.* Columns (1) and (2) report estimates using the specification in Equation (4) and (5), respectively. Column (3) instead reports the persistence of the cohort effects based on a second-order polynomial in age.

in 2009, charged an average markup approximately 2% higher than similar firms that were close to listing in 2007.

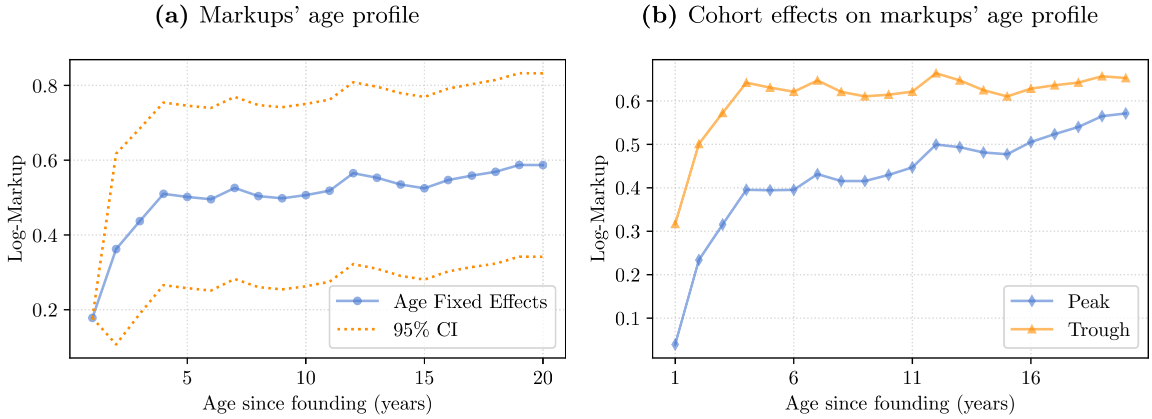
Decomposing this effect to control for its persistence, Column (2) reports the main coefficients of interest from Equation (5). The estimates reveal that the relevance of aggregate conditions close to listing is stronger in the initial years of firms' lives and then progressively vanishes within approximately ten to fifteen years, as also shown in Figure 1b.[14] When the age effect is restricted to be linear, as in Column (2), the effects on markups in the initial years, is larger in magnitude than the average effect. A two-standard deviations drop of the cycle component of GDP, approximately 6%, implies a markup in the initial year close to 4% higher than a similar firm that gets listed when GDP is on trend. Similarly, this implies that, for firms that were close to listing in 2009, the Great Recession is associated with a 5.1% increase in the first-year markup compared to similar firms that were close to listing in 2007.

2.5.2 Cohorts based on founding years

In this section, I discuss the results of estimating the age-period-cohort model in Equation (4) and (5) for the subsample of firms for which I have the founding date. Thus, for these results firms' ages are relative to their founding years and cohort effects are proxied using aggregate conditions in the *founding year* rather than close to the listing year as in the previous section.

[14] This result is at odds with what the firm dynamics literature has found regarding the effects on business cycles on firm sizes that tends to be permanent, see Moreira [2015], Sedláček and Sterk [2017] and more in line to the labor literature documenting the scarring effects of recessions on workers' earnings.

Figure 2: Age profile and cohort effects for firm-level markups, sample from founding dates



Note: Panel 1a plots the estimated age profile for markups from Equation (4) together with the 95% confidence interval. Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a$, where $\hat{\mu}_0$ is the average log-markup in the first available year and $\hat{\phi}_a$ are the estimated age fixed effects. Panel 1b plots the age profile for markups estimated from Equation (5). Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{\mu}_0$ is the average log-markup in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the founding year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of quadratically detrended real GDP. The coefficients on which this plot is based are reported in Table 2, Column (2). Sample with founding dates.

Markups' age profile. Figure 2a plots the age profile of markups for observed since their founding years. For these firms, the age profile of markups is strongly increasing, growing by approximately 40% in the first 20 years of firms lives.

Cohort effects. Table 2 reports the estimated elasticities between aggregate conditions in the founding year and firm-level markups. The effects estimated in this subsample indicate of a strong negative relationship between aggregate conditions and the level of markups charged by firms during their life cycles. Even if the magnitude of the coefficients is much larger than the ones estimated in the baseline sample, the qualitative direction of the results it very similar: firms that start their operations in recessions are associated with a markup higher than similar firms that are founded in expansions, this effect however is not persistent and slowly fading over time.

The point estimates imply that a two-standard deviations negative change in cyclical output translates to an average yearly markup approximately 7% higher than firms that instead are founded when output is on trend. This effect is six times larger than the one estimated on the sample with cohorts defined on listing years but, despite the difference in magnitude it draws a very similar picture: negative aggregate conditions, early in firms lives, are associated with higher markups. Similarly for the results with cohorts defined on listing years, Columns (2) and (3) indicate that cohort-effects estimated on this sample have a similar persistence than the ones estimated on the baseline one, albeit being barely significant. Figure 2b plots the estimated cohort effects on the age profiles for firms that are founded at different points of the business cycle, showing how the initial difference between the two profiles gets smaller as firms age.

Table 3: Markup co-movements with business cycles

Dep.Variable: Log-Markup	(1)	(2)
Current GDP	-0.165***	-0.359***
	(0.041)	(0.124)
Current GDP×Age		0.051**
		(0.02)
Current GDP×Age^2		-0.0014**
		(0.001)
Controls	Yes	Yes
Firm FE	Yes	Yes
R^2	0.62	0.62
N	123,997	123,997

Note: Robust standard errors in parentheses, p :$^{***} < 0.01$,$^{**} < 0.05$,$^{*} < 0.1$. The table reports the coefficients of interest from estimating Equation 6 on the main sample of firms followed up to 25 years; controls include firm fixed effects, sector shares, HHI index in three digit NAICS, cash holdings and log-employment and a second-oder time trend; the measure of business cycle is quadratically detrended log-real GDP.

2.6 Markups co-movements with the business cycle

In this section, I discuss the elasticities of markups to contemporaneous business cycles and their evolution over firms' life cycles.

Table 3, Column (2) plots the main coefficients of interest from the regression specified in Equation (6), while Column (1) reports the coefficient on contemporaneous cycle realizations in a version of Equation (6) without the interaction of business cycle measures with firm age. Column (1) shows the average correlation between markups and contemporaneous measure of business cycles. The coefficient implies that a one-percent variation of output from its non-linear trend translates to a 0.16 percent decline in firm-level markups. Controlling how the elasticity of markups evolves as firms age, as reported in Column (2), highlights how the co-movements with the business cycles are not stable over firms life cycles. In fact, as firms grow, the co-movements of markups and output becomes positive (after approximately ten years), indicating a mild procyclicality for older firms. The main takeaway from this simple analysis, therefore, is that firm-level markups are characterized by a negative correlation with business cycles that declines as firms age and eventually becomes positive.

2.7 Taking stock

The empirical analysis unveils three main facts on the relationship of firm-level markups with business cycles:

1. Markups increase as firms age, see Figure 1a;
2. Aggregate conditions at the time of listing exhibit a negative correlation with the average markup firms are able to charge in their lives. Moreover, firms that are first observed during bad realization of GDP report higher initial markups but flatter age profiles, see Figure 1b;

3. The contemporaneous correlation between firm-level markups and business cycles is negative for young and small firms and mildly positive for older and bigger firms, see Table 3, Column(2).

In the next sections of the paper, I explore how the combination of sales and direct demand investment in a model where firms are required to accumulated customers, can account for these empirical facts and help reconciling the fact that aggregate measures of markups tend to be acyclical or even procyclical, with firm-level evidence.

3 A model with product market selection and customer base accumulation

I consider an economy where firms operate in different product markets and where demands are constrained by the size of firms' customer bases. Firms can expand their customer bases in two ways: i) either by increasing sales, something easily achievable by cutting prices or ii) by devoting some resources specifically to the accumulation of customer base.[15]

3.1 Demographics and preferences

The economy comprises two sides: a production side and a consumption side. Time is discrete and denoted by t.

The production side of the economy is populated by listed and unlisted firms. Listed firms are big players whose accumulation of customer base is explicitly modelled. Unlisted firms are not modelled explicitly, however, when a firms goes public its initial customer base is drawn from a fixed distribution. Listed firms, indexed by j, produce a set of differentiated varieties and operate in distinct product markets, indexed by i. These firms have the incentive to accumulate customers using both sales and direct marketing investments. The degree to which firms' demands are responsive to the size of customer bases and how relevant sales are in the accumulation of repeat customers depends on the product market in which firms operate. Importantly, firm types are fixed and are not allowed to change also at time of listing. Nonetheless, the composition of listing firms will fluctuate endogenously. This is because the profitability of listing responds endogenously to aggregate shocks and its response is heterogeneous across product markets. Exit is exogenous but determined by each firm's age after listing, denoted by a.

The consumption side of the economy is populated by a representative household that makes standard consumption, saving and labor supply choices.

[15]In Sedláček and Sterk [2017] customer bases are managed only through firms' direct investments that, in that context, are assimilated to marketing expenditure. Other papers in the literature (see among others Moreira [2015], Hong [2017] and Gourio and Rudanko [2014] as recent examples), instead, assume that firms' customer capital is accumulated exclusively through firms' output. In this paper I focus on the combination of both channels.

3.2 Consumption side

The economy is populated by a representative household whose members derive utility from consuming an aggregate index of all varieties produced by listed firm. Notably, households form habits over each variety produced by listed firms.[16]

I assume that households aggregate varieties produced by listed firms using the following aggregator

$$C_t = \left(\sum_i \int_{j(i) \in \mathcal{J}_t(i)} k_i(b_{j(i),t})^{\frac{1}{\eta}} c_{j(i),t}^{\frac{\eta-1}{\eta}} \mathrm{d}j(i) \right)^{\frac{\eta}{\eta-1}} \tag{7}$$

where, with a slight abuse of notation, $\mathcal{J}_t(i)$ is the set of listed firms operating in product market i, $b_{j(i),t}$ is the size of the customer base for firm j in product market i and $c_{j(i),t}$ is consumption of firm $j(i)$'s variety.

Importantly, the utility weight for the household is an increasing function of the customer base served by each variety. This formulation captures the notion that households attach more weight to consumption of varieties that are perceived to be more attractive.[17] Throughout the paper I am assuming that households take the size of the customer base for each variety, $b_{j,t}$, as given when making their consumption choices. I discuss how $b_{j(i),t}$ is determined later in the paper when I describe the existing firms' problem.

The function $k_i(\cdot)$ acts as a demand shifter for each firm's demand and its parameterization is allowed to vary across different product markets.[18] Hence, the elasticity of the function $k_i(\cdot)$ tells by how much firms' demands increase (in percentage terms) when their individual customer bases increase.

To capture the idea that higher customer bases translate to higher demand I rely on the following reduced form specification for function $k_i(\cdot)$:[19]

$$k_i(b_{j(i),t}) = \kappa_i b_{j(i),t}^{\varepsilon_b^i}. \tag{8}$$

This is a useful formulation as it guarantees a constant elasticity of the utility weights to the size of firm's customer base, a feature that is going to be helpful in the calibration of the model and to introduce and additional channel of heterogeneity across product markets that will differ primarily by the values assigned to ε_b^i.

[16]See Ravn *et al.* [2006] for a discussion of the introduction of deep habits in macroeconomic models.

[17]Product appeal has been recently found to explain a large share of sales' variation both across products (Argente *et al.* [2018]) and firms (Hottman *et al.* [2016]). Similarly, Bernard *et al.* [2019] link these differences to the ability of firms to acquire customers.

[18]For customer base I mean the set of potential customer of a company, i.e. those that are aware and willing to buy a certain good/service.

[19]I take this functional form for the utility weights from Sedláček and Sterk [2017]. As they show, it is possible to micro-found this particular formulation of the utility weights by considering the customer base of each firm as a measure of awareness of their products among household members. On one hand, goods in which an increase in awareness leads to large increases in demand can be thought of as *mass* goods. On the other hand, demands of *niche* goods are not very sensitive to this awareness measure.

3.2.1 Household problem

Consumer choices are modelled thought a representative household that every period maximizes its utility choosing the level of consumption and labor supply:

$$\max_{C_t, H_t, B_{t+1}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t U(C_t, H_t) \tag{9}$$

subject to

$$P_t C_t + z_t B_{t+1} = w_t H_t + B_t + \Pi_t,$$

where C_t is habit-adjusted aggregate consumption as defined in Equation (7); w_t is the wage; B_t is the stock of risk-free bonds; H_t is aggregate labor and Π_t denotes aggregate profits from the production side of the economy while P_t is the habit-adjusted price index.[20]

3.3 Production side

The production side of the economy models focuses on the activities of listed firms.[21] For tractability reasons, I do not model explicitly pre-listing years but I assume that the economy is populated by a mass of smaller firms that, given their limited sizes and resources, do not compete for customers with listed firms until they are given the opportunity to go public. This assumption, therefore, implies that listing, in the model as in reality, does not coincide with firm creation. The initial level of customer base, however, is drawn from fixed distributions for each product market.

In the model, firms differ between each other along two margins: i) the type of products they sell and ii) their age since listing. Firm types are fixed and are not affected by the listing process.

Every period, a fraction of the pre-existing firms has the opportunity to go public upon payment of a fixed listing cost. The attractiveness to compete to become listed and having the opportunity to scale up and grow depends on the product market in which each firm operates and on the aggregate state of the economy. This implies that for some firm types it is more attractive to go public during expansions, while for others during recessions.[22] For example, firms that sell products that rely heavily on marketing and similar investments to attract customers might find the option of going public more appealing in expansions rather than in recessions. This interpretation is useful in this context as it allows for a correlation between the state of the business cycle at the time of listing and ex-ante firm characteristics.[23]

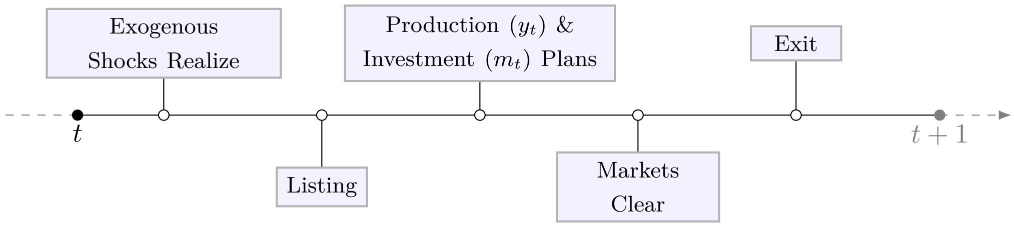
[20]From household expenditure minimization $P_t = \left(\sum_i \int_{j(i)\in\mathcal{J}(i)} p_{j(i),t}^{1-\eta} k_i(b_{j(i),t}) dj(i)\right)^{\frac{1}{1-\eta}}$.

[21]Despite being a small fraction of total firms in the US, public firms are highly relevant. On average, firms included in Compustat account for approximately 30% of non-farm business employment and approximately 49% of corporate profits before tax.

[22]See, for example, Stoughton *et al.* [2001], Chemmanur and He [2011], Chemmanur *et al.* [2018] for a review on how product market characteristics can influence firms' listing decisions.

[23]It is also reasonable to consider that, as firms decide go public, they can use the extra funding and publicity to redirect their activities towards markets that appear more profitable when making the listing choice. It is difficult to empirically identify these two motives, but, in the context of the theory developed in this paper, it is important that the correlations summarized in Section 2 are informative of either ex-ante

Figure 3: Timeline



Note: Decisions timeline for firms in each product market.

Hence, product types are going to affect two aspects of firms' production. First, given the presence of deep habits on household preferences, firms in each product market will face demands that are sensitive to the size of their customer bases and this sensitivity varies across products. Second, the relative importance that firms' output has in the acquisition of customer is also a characteristics that is specific to the product type in which each firm operates. This margin of heterogeneity, therefore, links the optimal strategy to accumulate customers to the product market faced by each firm.

3.3.1 Existing firms

In this section, I describe the behavior of incumbent firms. They are active in a finite number of product markets, indexed by i, and their demands are constrained by the size of their customer bases. Hence, at any point of their life-cycle they face a trade-off between accumulating new customers and harvesting the maximum possible profits from their current customer base.[24]

Firms' demands. As customers are able to form habits on each variety, firm demands are constrained by the size of the customers they are able to reach. Given the consumption index for listed firms as in Equation (7), a firm operating in product market i faces the following demand function:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} k_i(b_{j,t})Y_t. \tag{10}$$

Details on the derivation of firms' demand functions are reported in Appendix B.

Accumulation process for customer base. To model the fact that firms can use multiple channels to relax their demand constraints I am assuming that firms' customer bases evolve according to the following rule,

$$b_{j,t} = (1-\delta)b_{j,t-1} + Q_t F_i(y_{j,t}, m_{j,t}), \tag{11}$$

firms' types or their choices upon listing. In both cases, there exists a link between business cycles at the time of listing and the cross-section of firms' characteristics that can be used to discipline the model and study the effects of this dimension of firm heterogeneity on the macroeconomy.

[24] The invest/harvest trade-off is common in models of customer markets and it is also documented empirically in Galenianos and Gavazza [2017]. From now on, when there is no risk of confusion, I drop the dependence on the product market in firm subscripts, so that a firm in product market i is denoted simply by j instead of $j(i)$.

where $m_{j,t}$ is the expenditure on direct demand investments (e.g. marketing, product quality improvements, sale strategies etc) expressed in units of labor and $y_{j,t}$ are firm j's sales, and Q_t is an aggregate stochastic shock that influences the ability of firms to accumulate customers and is meant to capture, in reduced form, the effects of aggregate demand shocks.[25]

Previous studies have mostly focused on formulations of the law of motion for customer base where the acquisition of new customers depended only on either m or y.[26] The novelty of this formulation, therefore, relies on the *joint* role of firms' sales and direct investments in the definition of firms' investment in customer base. In the next sections, after having described the rest of the model, I discuss more in detail how the dependence of customer accumulation to both channels of investment deeply affects the economy's behavior over the business cycle thanks to the selection effect embedded in this framework.

The function that governs the acquisition of customers by firms is indexed by the product market in which each firm operates as the dependence of investments in customer base to firms' output varies across product markets. Therefore, product markets differ in the sensitivities of demand functions to customer bases and in how strongly the customer accumulation process in each product market depends on sales. To keep the interaction between these two investment channels as tractable as possible I am modeling firms' investment in customer base with the following CES function:

$$F_i(y_{j,t}, m_{j,t}) = [\psi_i y_{j,t}^{\sigma} + (1-\psi_i) m_{j,t}^{\sigma}]^{\frac{1}{\sigma}}. \tag{12}$$

The advantage of this formulation is that it allows to capture both the relevance of each channel in the acquisition of customers, through the dependence parameters ψ_i, that are also allowed to vary across product markets, and the complementarity between the two channels, captured by the elasticity of substitution, $\frac{1}{1-\sigma}$.

3.3.2 Existing firms' problem

An incumbent in product market i chooses how much to produce and invest in customer base taking into account the household's demand for its specific variety and the law of motion of its customer base.

Each incumbent internalizes that the size of its customer base can be used to relax the demand constraint, hence, in any period, firms will optimally adjust their prices and direct demand investments to balance their contrasting incentives to increase their customers bases and maximize revenues in every period.

[25] Mechanically Q increases the ability of firms to recruit new customers along all channels. This effect is comparable to aggregate demand shocks if periods of higher than usual aggregate demand are also periods when it is easier to acquire customers. Albeit in very reduced form, this shock is meant to capture this effect.

[26] Sedláček and Sterk [2017], Perla [2019] construct models where firms accumulate customer bases by investing resources in the accumulation of new customers but their sales are irrelevant for the determination of the pool of potential customers. Moreira [2015], Hong [2017], Gilbukh and Roldan [2018], among others, assume that firms build customer bases by locking customers to their products but the only possibility for them to acquire new customers is only through sales. Bornstein [2018] uses taste shocks to lock-in customers in relationships with specific firms and prices are a tool that increases the probability of a good match.

Formally, the optimal choice of the incumbent is the solution to the following problem:

$$V_i(b_{j,t-1}, a_{j,t}; S_t) = \max_{\substack{p_{j,t}, y_{j,t}, h_{j,t},\\ m_{j,t}, b_{j,t}}} \left\{ \begin{array}{c} p_{j,t} y_{j,t} - w_t \left(h_{j,t} + \zeta(m_{j,t})\right) + \\ +(1-\rho(a_{j,t}))\mathbb{E}_t \left[q_{t,t+1} V_i(b_{j,t}, a_{j,t+1}, S_{t+1})\right] \end{array} \right\}, \tag{13}$$

subject to the relevant constraints

$$b_{j,t} = (1-\delta) b_{j,t-1} + Q_t F_i(y_{j,t}, m_{j,t}), \tag{14}$$

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} k_i(b_{j,t}) Y_t, \tag{15}$$

$$y_{j,t} = A_t h_{j,t}^{\alpha}, \tag{16}$$

$$a_{j,t+1} = a_{j,t} + 1, \tag{17}$$

where $q_{t,t+1}$ is the household stochastic discount factor between t and $t+1$, Equation (14) is the law of motion of customer base, Equation (15) is the demand constraint, Equation (16) is the technology and Equation (17) is the evolution of firm j's age.

3.3.3 Listing process

I use the listing process and its strict link to product markets as a modeling shortcut to account for a broader range of business decisions that firms take in consideration when going public and that are linked to the way they manage their products.[27] In the finance and marketing literature there are numerous studies that show how the decision of going public is a transforming event for a firm that is influenced not only by funding needs but also by product market considerations.[28]

The economy is divided into a fixed number of product markets, $i = 1, ..., I$, and in each of them, I assume that there is a positive mass of firms in every period. The mass of successful new listings in every product market, therefore, can be described by the following matching function in which firms compete with each other to get listed in each product market. Competition among perspective public firms generates congestion so that the success probability of going public is decreasing in the mass of competitors, $e_{i,t}$ (i.e. $\phi < 1$).[29]

This process can be succinctly described by the following matching function, where $\Gamma_{i,0,t}$ is the mass of successful listings in product market i and ω_i a product-market specific parameter

[27] This framework could be easily modified to consider firm creation by making newly active firms start with a zero customer base. All else equal, this change would increase firms incentives to accumulate customer base making markups more countercyclical for small firms and stronger selection effects of business cycle.

[28] For example, Wies and Moorman [2015] founds that after going public firms tend to introduce more varieties of the same product in order to increase sales but holding back on the development of new products. On a related note, Bernstein [2015], finds that going public reduces firms' patent quality. Chemmanur and Yan [2009] established another link between product markets and the decision of going public studying how IPO outcomes can be affected by product market advertising. Chod and Lyandres [2011], instead, highlights how the different ownership structure between private and public firms allows the latter to take on riskier product market strategies thanks to higher risk sharing among shareholders while Grullon, Kanatas and Kumar [2006] shows how firms capital structure affects their advertising strategies.

[29] Competition among firms in their listing process might ensue for a variety of reasons. For example, firms might compete for investors and markets' attention. As the strategic choice of listing, albeit very interesting, is not the main focus of this paper, this margin not explicitly modelled in the paper and is left for future research. For a recent exploration of these choices see González [2020].

scaling the mass of competitors in each product market:

$$\Gamma_{i,0,t} = \omega_i e_{i,t}^{\phi}. \tag{18}$$

To participate in the listing process, firms have to pay a common cost denominated in units of aggregate consumption, X_t. Firms that successfully list in a product market, then, will start off with a positive level of customer base and face the same problem as other incumbents henceforth. To keep the model tractable I do not model explicitly the pre-listing years but in simulating the model I allow for different initial conditions by drawing values for the starting level of customer bases in each product market from distinct uniform distributions.[30]

3.4 Aggregation and market clearing

From Equation (23) it is possible to see that incumbents with the same age, a, operating in the same product market, i, face the same problem and therefore will make the same decisions. This observation makes the aggregation very easy as each cohort of listed firms operating in a specific product market behaves as a mass of homogeneous firms.

The resource constraint for the economy is

$$C_t + \sum_{i}^{I} e_{i,t} X_t = Y_t, \tag{19}$$

Denoting, $\Gamma_{i,a,t}$ the mass of firm with age a in product market i, labor market clearing requires that

$$\sum_{i}^{I} \sum_{a}^{\bar{a}} \Gamma_{i,a,t} \left(h_{i,a,t} + \frac{m_{i,a,t}^{\chi}}{\chi} \right) = H_t, \tag{20}$$

while the law of motion for the mass of listed firms in each cohort-product cell is

$$\Gamma_{i,a,t} = (1 - \rho(a-1))\Gamma_{i,a-1,t-1}. \tag{21}$$

By definition, aggregate output produced by listed firms has to be such that

$$\sum_{i}^{I} \sum_{a}^{\bar{a}} \Gamma_{i,a,t} p_{i,a,t} y_{i,a,t} = P_t Y_t. \tag{22}$$

In this economy, the household is the unique owner of firms, hence, firms' discount their future values using the stochastic discount factor of the representative household that in turn depends on the distribution of listed firms in each product market-cohort cell. As a consequence, the aggregate state of the economy, S_t, includes the distribution of active firms, $\Gamma_{i,a,t}$, in each product market-cohort cell (i, a), plus the set of exogenous processes for aggregate productivity, A_t, the listing cost, X_t and aggregate demand conditions, Q_t.

[30] It is possible to easily extend the framework developed in this paper to allow for a mechanical non-listed sector without deeply affecting the results, as model simulations with exogenous processes for the initial customer base highlight that, in this framework, the average sizes of initial conditions are more relevant that their variances. On a similar note, Vardishvili [2018] and Carvalho and Grassi [2019] relax the assumption of a continuum of homogeneous potential entrants in models with firm entry and exit showing that this has a profound effect on the aggregate economy.

3.5 Model properties

It is possible to characterize the key aspects of the model by looking at two objects: the optimal markup management by existing firms and the equilibrium condition for the listing process described in Section 3.3.3. They characterize the behavior of markups at the firm-level and the endogenous selection effect generated by the model, respectively, and can be formally stated in the following propositions.

Markups behavior. The behavior of markups over the cycle can be described by the following expression, derived from the first-order conditions of the problem in Equation (13). It has the nice property of condensing all the incentives an existing firm face both over its life cycle and over the business cycle.

Proposition 1. *The optimal markup management of listed firms can be described by the following condition:*

$$\mu_{j,t}^{-1} - \bar{\mu}^{-1} = Q_t F_y(\cdot|_t) \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t}}{b_{j,t}} \left[1 + \mathbb{E} \sum_{\tau=1}^{\infty} \prod_{s=0}^{\tau-1} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau} q_{t,t+\tau} \frac{p_{j,t+\tau}}{p_{j,t}} \frac{y_{j,t+\tau}}{y_{j,t}} \frac{b_{j,t}}{b_{j,t+\tau}} \right] \tag{23}$$

where $\widetilde{\rho}(a_{j,t}) \equiv (1 - \rho(a_{j,t}))$ is the surviving probability up to t, $\mu_{j,t} = \frac{w_t h_{j,t}}{\alpha y_{j,t}} \frac{P_t}{p_{j,t}}$ is the firm-level markup and $\bar{\mu}$ is the markup that would prevail under standard monopolistic competition and without dynamic incentives in pricing.

Proof. Details on the proof and derivations in Appendix B. □

Looking at the right hand side of Equation (23) is helpful to uncover the main element of the incentive structure faced by firms over their lives. First, it is always positive as composed by only positive terms. Therefore it implies a positive difference between the inverse markup charged by firm j and the inverse markup under standard monopolistic competition. The fact that the process for the acquisition of new customers induces some dependence on firms' output creates the incentive to set markups *below* their standard monopolistic competition value. This is because firms internalize the positive effects that higher levels of output have on their current and future demands. As firms grow and get closer to their optimal production scales, their investment motive diminishes, pushing markups closer to the monopolistic competition ceiling. This incentive structure therefore is responsible for the increasing age profile of firm-level markups.

Equation (23) is also useful to gain some intuition on the cyclical behavior of markups. First, it is important to note that firms discount profits using the household stochastic discount factor. Given a procyclical stochastic discount factor, on the margin, firms value customers more in booms than in recessions.[31] As firms can use sales to attract customers, this implies that they are more willing to charge lower markups in expansions rather than

[31] As noted in Beaudry and Guay [1996], Cooper and Willis [2014] for RBC and Clementi and Palazzo [2016], Winberry [2020] for state-of-the-art heterogeneous firms models, including habit-formation in the utility function is sufficient to generate a procyclical stochastic discount factor. This feature makes the model consistent with recent empirical evidence in finance that finds countercyclical risk-free rates at various time horizons.

in recessions, because in doing so they can stimulate sales and expand their future customer base. This mechanism therefore is a strong force in generating countercyclicality in firm-level markups.[32] However, even if all firms discount the future with the same discount factor, the model generates some heterogeneity in the cyclicality of markups thanks to the age-dependent survival probability. As young firms are less likely to survive to the next period, they discount the future more and thus react more strongly to business cycle fluctuations. For example, in a downturn a young firm will have a stronger incentive to exploit its current customers as its future prospects are worse than those of an older firms in the same product market.

The presence of both sales and direct investments in the determination of firms' investments in customer base is a key element for markup dynamics along the age profile of firms. A model in which firms were able to attract new customers without changing markups, would not generate an increasing age profile of markups as the one observed in the data.

Selection mechanism. Every period, firms in each product market compete to get listed. However, as this process is costly, the ex-ante value of becoming a listed firm has to be at least greater or equal than the cost of participating in the listing process, regardless of the product market in which firms operate. Competition among firms that try to go public implies that, in equilibrium, the expected value of becoming a public firm has to be equal to the cost of listing. The following proposition, therefore, states a key characteristic of the equilibrium in each product market.

Proposition 2. *In equilibrium, the expected value of becoming a public firm in each product market with initial customer base* $b_{i,0,t-1} > 0$*, is equal to the cost of listing, hence each product market* i*:*

$$\frac{\Gamma_{i,0,t}}{e_{i,t}} V_i(b_{i,0,t-1}, 0, S_t) = X_t. \tag{24}$$

Proof. Free-entry in each product market implies that the ex-ante value of becoming a public firm, $\frac{\Gamma_{i,0,t}}{e_{i,t}} V_i(b_{i,0,t-1}, 0, S_t)$, equals the cost of attempting the listing process, X_t. □

It is important to note that, while attempting to go public has a common cost across all product markets, the expected value of becoming a listed firm depends on the demand characteristics in each product market because, depending on the aggregate state of the economy, the value of being listed strongly depends on the need to acquire customers, governed by the elasticity of the demand shifter to customer base, and the channel that firms rely more on to expand their customer bases, sales or direct demand investments.

This detachment between a common cost of listing and heterogeneous firm values across product markets is the main driver of the selection effect operated by aggregate conditions at the moment of listing. Given that the value of operating in a specific product market, $V_i(b_{i,0,t-1}, 0, S_t)$, is a function of the aggregate state, S_t, it will respond differently to the same aggregate shock across product markets. However given that potential listing firms face the same cost, regardless of the product market in which they operate, free entry guarantees that the expected value of listing is equalized across product markets. Thus, success probabilities

[32] Ravn *et al.* [2006] are the first to highlight this channel in models with deep habits.

in each product market, $\frac{\Gamma_{i,0,t}}{e_{i,t}}$, have to adjust to guarantee that the expected value of listing is equalized across product markets. As more firms compete to go public ($e_{i,t}$ increase) more firms get listed but the chance of succeeding in the process decreases. Thus, any change in success probabilities has to be due to a change in the mass of firms attempting and ultimately succeeding to go public in each product market.

3.6 Equilibrium definition and solution method

Definition (Equilibrium)**.** *A recursive equilibrium in this economy is a set of policy and value functions, for each product market and cohort pair, such that the household and the incumbents' problems, Equation* (9) *and Equation* (13) *are satisfied.*

The free entry condition, Equation (24)*, is respected and markets clear following equations* (19) *and* (20).

In addition, the distribution of listed firms over age and product markets follows Equation (21) *and aggregate output is defined as in Equation* (22) *while the exogenous states* (A_t, X_t, Q_t) *follow standard AR(1) processes.*

Solution method. The aggregate state is an infinite dimensional object due to the fact that firms are potentially infinitely lived. As an approximation, I assume that firms have finite lives, as in Sedláček and Sterk [2017]. Paired with the assumption of finite number of product markets, this approximation makes it possible to keep track of each point of the firms' distribution and to solve the model using standard perturbation techniques. The accuracy of the solution increases the longer the life-span of firms. I discuss the model and solution method's details more in depth in Appendix B.

4 Calibration and model fit

One period in the model corresponds to one year in the data. The calibration is geared towards capturing the correct size and shape for the age profile of markups estimated using the definition of firm cohorts based on listing. The main reason for opting for this sample is that those estimates are based on a larger set of firms, and particularly at low ages are more likely to be representative than the sample of firms observed since their founding dates.

4.1 Functional forms

The production of customer base is governed by the CES function in Equation (12). Importantly, I am allowing the weight on current output in accumulating customer base to vary across product markets. Direct investments in customer base are subject to the following cost function: $\zeta(m_{i,a,t}) = \frac{m^{\chi}_{i,a,t}}{\chi}$. The exit probabilities at each age are set according to the following rule: $\rho(a) = \rho_0 + \rho_1 a$, with ρ_0 and ρ_1 measured using a simple regression on Compustat exit rates. With these specified functional forms the model economy is governed by the following parameters: $[\alpha, \beta, \delta, \nu, \chi, X, \phi, \eta, \sigma, \{\varepsilon^i_b\}^I_{i=1}, \{\psi_i\}^I_{i=1}, \{\kappa_i\}^I_{i=1}, \{b_{0,i}\}^I_{i=1}, \{\omega_i\}^I_{i=1},$ $, \rho_0, \rho_1]$.

I choose the number of product types, I, to match the dispersion in firm sizes conditional on age. Specifically, to allow the model to have firms of the same age but with different sizes. The number of product markets needs to be sufficiently high to have realistic dispersion but not too high because of computational constraints. Therefore, I set $I = 10$ and target the average sizes in each size decile in the data. I choose to match the dispersion of firm sizes with product markets in the model because firm sizes can be informative of firms' product types. Firms that operate in large, scalable markets, for example, where the ability to reach a large number of customer is an important driver of demand, will also be bigger in sizes while firms that operate in more niche markets will tend to remain relatively smaller as their ability to serve their customers does not depend heavily of their size.

The maximum age, $\bar{a}$, is set at 60, which is approximately the 99th percentile of the age distribution in the sample used in the empirical section of the paper.

The utility function of the household is chosen to keep the consumption side of the model as tractable as possible, hence $U(C_t, H_t) = \log(C_t) - \nu H_t$. Labor disutility, ν, is set so that the habit-adjusted real wage, $\frac{w}{P}$, is equal to the inverse of the monopolistic competition markup, as this particular value is very convenient when solving for the stationary solution of the model.

4.2 General parameters

As labor is the only input in production, I set α to 1 so that firms operate with constant returns to scale technologies. The discount factor for the representative household, β, is set to 0.96 for consistency with the macro literature that uses yearly data.

The elasticity of the cost function of direct investments in customer base, χ, is set to 2 and the elasticity of the matching function for potential entrants, ϕ, is 0.844 as in Sedláček and Sterk [2017].

The price elasticity of firms' demands, η, is set to 3.857 to ensure that the markup under monopolistic competition is equal to the average markup of older firms (25 years and more). This parametrization results in a monopolistic competition markup of 1.35 that is higher than the usual values used in the literature. However, the dynamic incentives of the model ensure that the resulting cost-weighted average markup in the economy is approximately 1.3, which is a value closer to the the range usually seen in the literature.

I set the depreciation rate of the customer base, δ, equal to 0.2 which is on the high range of the documented customer capital depreciation rates but it is still much lower than depreciation rates estimated for advertising and marketing expenditures or customer turnover rates documented in the literature.[33]

The elasticity of substitution between sales and direct demand investments for the creation of customer base, $\frac{1}{1-\sigma}$, is set approximately to 0.67 ($\sigma = -0.5$) to capture the fact that current output and direct investments are not perfect substitutes in their ability to attract

[33] For a discussion on recent estimates of customer turnover rates and marketing investments see Gourio and Rudanko [2014] and Foster *et al.* [2016].

new customers. I pick this value to ensure that the model is able to match the markups profile of variable inputs without requiring implausibly high sensitivities to customer base, and still preserving an empirically consistent correlation between sales and advertising expenditure.[34]

The model features three exogenous processes: TFP (A_t); Aggregate Demand (Q_t) and Listing costs (X_t).

Each of these processes affects the relative efficacy of prices and direct demand investments as customer acquisition tools as well as aggregate output. As the relative importance of the two channels changes during firms' lives co-movements of cohort level markups with output carry some information of the relative importance of each shock. Therefore, I use the cyclicality profile for cohort-level markups to calibrate the volatility of the exogenous processes innovations so that the model profile matches the one estimated in the data.[35]

More specifically, I use the following specification at the cohort level to trace out an age-profile of markup cyclicality, estimating how the correlation with output changes across cohorts of different ages:

$$\log(\mu_{a,t}) = \alpha + \sum_{a=1}^{10} \beta_a Y_t \times D_a + \phi_a + \phi_c + u_{a,t}, \tag{25}$$

where ϕ_a and ϕ_c are age and cohort fixed effects, Y_t a measure of business cycle and D_a a set of dummy variables for each age.[36] This particular specification is also particularly convenient as it can be replicated exactly both in the data and in model simulated samples. Therefore, the values for the volatilities of the three exogenous processes in the model are chosen to minimize the sum of squared errors between the set of β_a coefficients from Equation (25) estimated in the model and the ones in the data.

4.3 Product markets and demand parameters

The product market specific parameters that have to be calibrated are: $\kappa_i, \psi_i, \omega_i$, ε_b^i and $b_{i,0}$. These are respectively: the scale of firms' demands in each product market, the relevance of sales in customer acquisition, the mass of available listing opportunities, the elasticity of firms' demands to their customer base and the average size of initial customer base upon listing.

The number of listing opportunities in each product market, ω_i, are calibrated to match the share of firms in each size decile and are scaled so that aggregate demand at the stationary

[34] Including the calibration of σ in the procedure used to calibrate the parameters specific to product markets delivers a more negative value of sigma, approximately -0.7, at the cost of generating elasticity parameters for the demand shifters that are outside the values usually found in the literature, see Sethuraman, Tellis and Briesch [2011]. I used that value as a guide to pick the baseline one before recalibrating the product market parameters.

[35] In this calibration I set the persistencies of the exogenous processes to the same values and I leave a more formal estimation of the exogenous processes for future research.

[36] Note that each coefficient, β_a, using the time-series variation in the average markups by age, captures the co-movements of the average markup of firms with age a with output. The collection of these coefficients, then, allows to trace out an age-profile of markup cyclicality, estimating how the correlation with output changes across cohorts of firms.

solution of the model is equal to one. While, the average sizes of initial customer bases are calibrated so that the stationary solution of the model exhibits a convex age profile for sales growth in line with the data.[37]

I calibrate ψ_i and ε_b^i so that the average age profile of markups implied by the stationary solution of the model matches the profile estimated by the age fixed effects in Equation (4). The intuition behind this choice is linked to the fact that the elasticity of demand with respect to the customer base is a key determinant of the growth rate of firms as it regulates how much firms benefit from an extra unit of customer base. See below for details.

The values of κ_i, instead, are chosen to match the size distribution of old firms in the data. In practice, for each product market, I choose the value of κ_i such that the size of 25 years old firms in the model equals the average size of firms between 25 and 30 years in the data. While the elasticity of the demand shifter $k_i(b_{j,t})$, ε_b^i is informative of the growth path of markups as firms age, its scale, κ_i, is tightly linked to the optimal size of firms, that is more likely to affect the size distribution when firms are mature.[38] The full baseline calibration of the model is reported in Table 4.

Details on moments matching procedure. In order to pin down the values of ψ_i and ε_b^i I am using the following procedure. Starting from guesses for each ψ_i and ε_b^i, I solve for the stationary distribution of the model and I measure the age profiles for markups as

$$\mathcal{P}_a^{\mu}(\tilde{\psi}^{(0)}, \tilde{\varepsilon}^{(0)}) = \log\left(\frac{\mu_a(\tilde{\psi}^{(0)}, \tilde{\varepsilon}^{(0)})}{\mu_0(\tilde{\psi}^{(0)}, \tilde{\varepsilon}^{(0)})}\right),$$

where $\mu_a = \sum_i \Gamma_{i,a}\mu_{i,a}$ is the average markup at each age across product types. The empirical counterparts for these age profiles are given from the age fixed effects estimated using the model in Equation (4). Then, the condition I use to pin down the values for $\{\psi_i, \varepsilon_b^i\}$ is:

$$(\tilde{\psi}^*, \tilde{\varepsilon}^*) = \underset{\psi_i, \varepsilon_b^i}{\arg\min} \left\{ \left(\mathcal{P}_a^{\mu}(\tilde{\psi}, \tilde{\varepsilon}) - \hat{\phi}_a\right)' \left(\mathcal{P}_a^{\mu}(\tilde{\psi}, \tilde{\varepsilon}) - \hat{\phi}_a\right) \right\}.$$

In practice, I include the first forty age fixed effects from Equation (4) and the growth rates relative to age-zero at the stationary solution to pin down twenty parameters. The model, however, does not perfectly replicate the age profile of markups due to the non-linearities in the mapping between the age profile of markups at the stationary solution and the estimated age fixed-effects.[39]

[37]In the model simulations, initial customer bases in each product market are allowed to fluctuate between $[0, 2b_{0,i}]$ by drawing i.i.d. shocks from a set of uniform distributions between $(-1, 1)$. Varying these parameters does not significantly affect the overall behavior of the model. I provide more details in Appendix B.

[38]For example, the size of 25 years old firms in product market one is matched to the average size of firms in the first decile of the size distribution of 25-30 years old firms in the data.

[39]A procedure similar in spirit to the one described here is adopted in Christiano, Eichenbaum and Evans [2005] and Hong [2017].

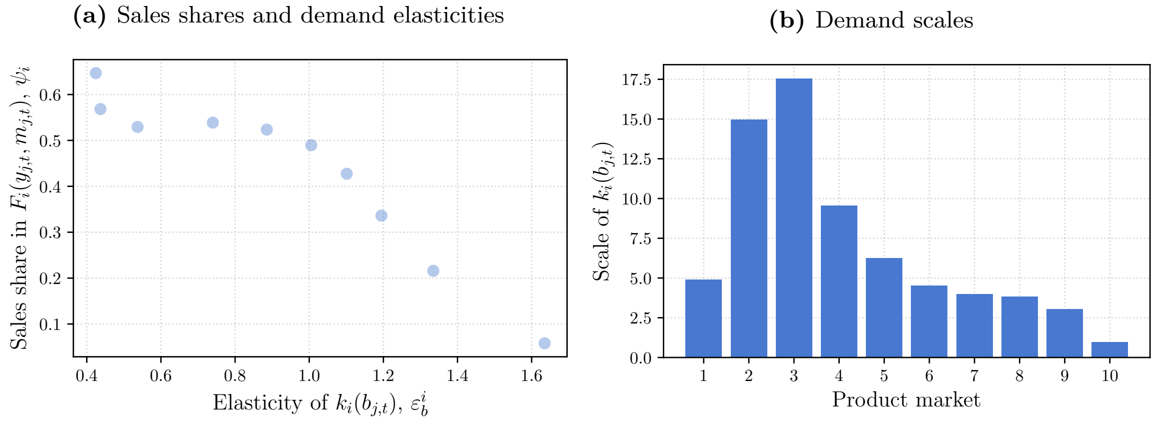
Table 4: Calibration

Parameter	**Value**	**Target**
α Technology	1	CRS technolgy
β Discount factor	0.96	Annual discount rate in macro-literature
δ Customer base depreciation	0.20	Customer turnover ratio [Gourio and Rudanko, 2014]
η Price elasticity of demand	3.86	Monopolistic markup equal to 1.35
ν Labor disutility	0.766	Habit-adjusted real wage equal to $\frac{\eta-1}{\eta}$
χ Curvature of cost function for customer base investments	2.0	Quadratic cost [Sedláček and Sterk, 2017]
ϕ Matching function elasticity	0.84	[Sedláček and Sterk, 2017]
$\bar{X}$ Average listing cost	33.60	Average success in product market (1) equal to 0.99
σ Substitution parameter	-0.5	Correlation of sales and advertising expenditure $\approx$ 0.87 (Compustat)
ρ_0, ρ_1 Exit rates	0.08, -0.001	Exit rates by age (Compustat)
ρ_A, σ_A Productivity process	0.95, 0.15	Markup ciclicality profile, see Figure 6
ρ_Q, σ_Q Demand process	0.95, 0.4	" "
ρ_X, σ_X Listing cost process	0.95, 0.6	" "
Ω Measure of listing opportunities	$1.105e^{-5}$	Normalization so that aggregate output equals 1 at the stationary solution

Product Market Parameters	**Target**	**Values**									
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ψ_i Share parameter in $F_i(y_{j(i),t}, m_{j(i),t})$	Age profile of markups	0.647	0.568	0.529	0.539	0.524	0.490	0.427	0.336	0.216	0.058
ε_b^i Elasticity of $k_i(b_{j(i),t})$	Age profile of markups	0.424	0.437	0.537	0.740	0.886	1.005	1.101	1.195	1.335	1.635
κ_i Scale of $k_i(b_{j(i),t})$	Average size of mature firms	4.875	14.947	17.543	9.560	6.243	4.529	3.979	3.832	3.026	0.954
$b_{0,i}$ Avg. initial customer base	Sales' growth profile	2.779	6.009	9.194	15.064	21.301	27.313	32.367	36.831	43.284	69.350
$\frac{\Gamma_{i,0}}{e_i}$ Success Probability in i	Share of firms in i when old	0.999	0.231	0.116	0.071	0.045	0.029	0.018	0.01	0.006	0.002

Note: The table reports the main calibration of the model. Note that instead of the values for the masses of listing opportunities in each product market, ω_i, that are of small significance given the normalization, the table reports the implied success probabilities.

Figure 4: Product market characteristics in baseline calibration



Note: The figure reports the relationship between the parameters determining the heterogeneity across product markets in the model. Panel 4a plots the relationship between the elasticity of demand to customer base versus the share of current sales in direct demand investments creation. Panel 4b plots the scale parameter in the demand shifter $k_i(b_{j(i),t})$, product markets in increasing order on demand shifter elasticity.

4.4 Stationary solution and model's fit

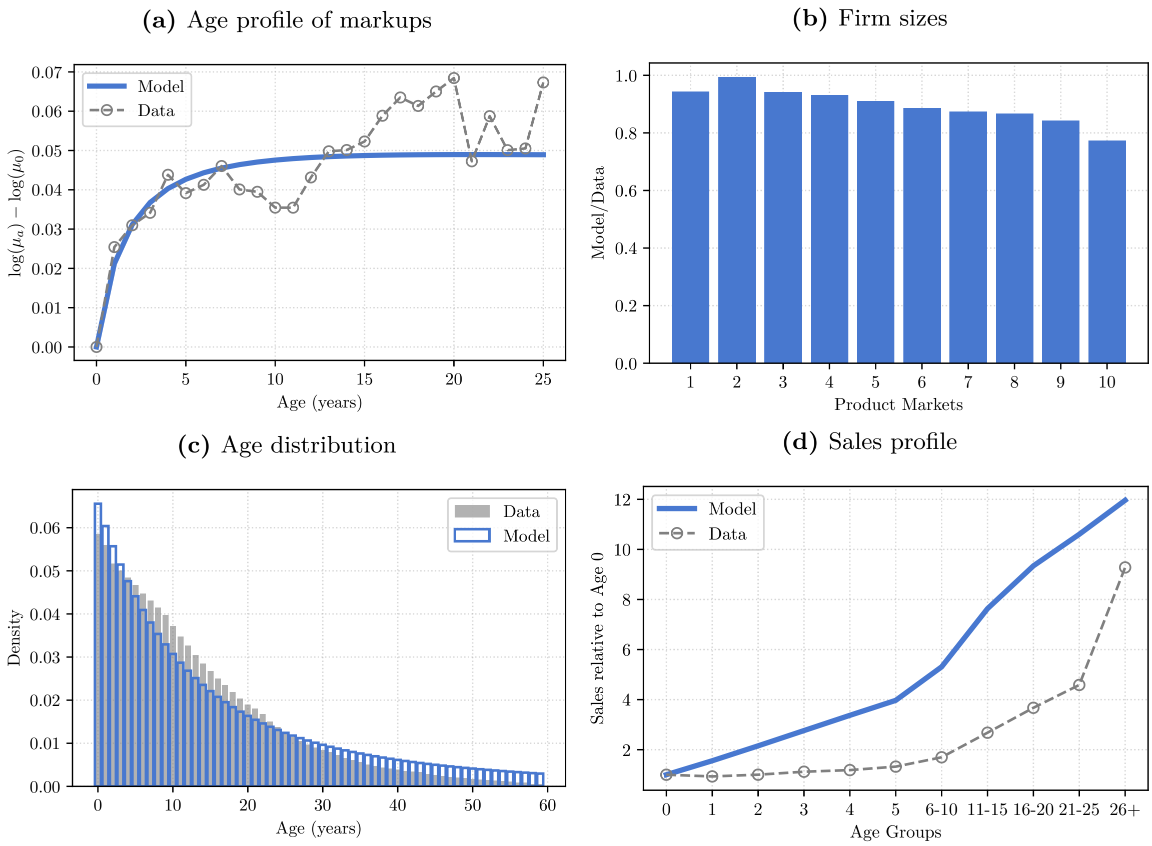
As discussed in the previous section, I calibrate the model using more target moments than parameters. In this section, I examine the fit of the model under the baseline calibration

Product market characteristics. Figure 4, reports the values for the product market characteristics under the baseline calibration. In particular, the model implies a negative relationship between the share parameters of firms' sales for customer acquisition and the elasticity of demand to customer base. This stark negative relationship between the two implies that firms operating in product markets with high demand sensitivity to customer base are also characterized by a low incentive to use the price lever to attract customers. These characteristics of the demand properties in each product market turn out to be largely responsible for the dynamic response of the model to aggregate shocks and for the transmission of aggregate shocks as they directly control the way firms use both prices and employment for the acquisition of new customers.

Calibration targets. Figure 5 reports the main calibration targets: a) growth profile of markups; b) the size distribution of firms; c) the age distribution and d) the sales profile.

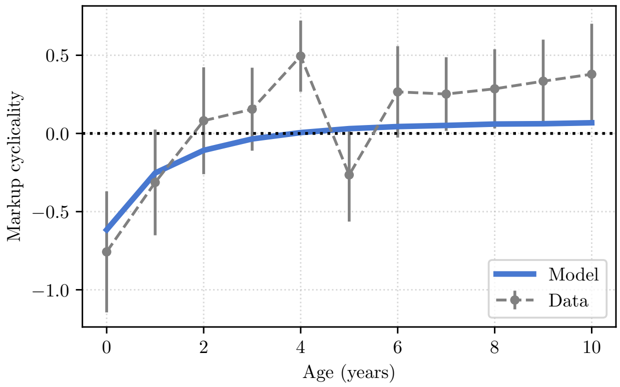
The growth profile of markups is the main calibration target for the parameters defining the product market characteristics and it is constructed deriving the growth rates of markups relative to the initial year for both the model and the data. The non-linearities in the mapping between the model-based growth profiles and the age fixed-effects prevent the model of achieving a perfect fit. However, the model correctly captures the incentive structure of firms that start their operations with a low markup to build their customer base and then increase it as they age and their harvest-invest motives tilt towards extracting more monopoly rents from their existing customers rather than acquiring new ones. Looking at the ratio

Figure 5: Stationary solution targets



Note: The figure reports the main calibration targets. Panel 5a reports the age profile of markups in the model and in the data up to 25 years, from estimating age fixed effects in Equation (4) in the main sample and at the stationary solution. Panel 5b reports the average size of firms in each product market relative to the average size of firms in each size decile in the data. The calibration targets firms age 25 years old in the model, the panel reports the average for all ages. Panel 5c reports the age distribution of firms in the the data and at the stationary solution. Panel 5d reports the growth profile of sales in the model and in the data.

Figure 6: Markup cyclicality profile



Note: The figure plots the cyclicality profile estimated at the cohort-level in a sample of cohorts followed up to 10 years and in a sample of model generated data obtained from simulating the model for 2500 periods. More specifically the figure plots the coefficients of the interaction between age dummies and contemporaneous realizations of the cycle component of GDP, i.e. $\hat{\beta_a}$ from $\log(\mu_{c,a,t}) = \alpha + \phi_a + \phi_c + \sum_a^{10} \beta_a D_a \times Z_t + u_{c,a,t}$ where ϕ_a, ϕ_c are respectively age and cohort fixed effects, Z_t is a measure of contemporaneous business cycles and D_a is a set of dummies for each age. *Data* reports the point estimate and a one standard deviation error band. The estimates are based from aggregating the main sample at the cohort level and following cohorts for up to 10 years. *Model* is based on following cohorts for 10 years from a dataset based on model simulations of 2500 periods. In the model the measure of business cycles are deviations of output from the stationary solution.

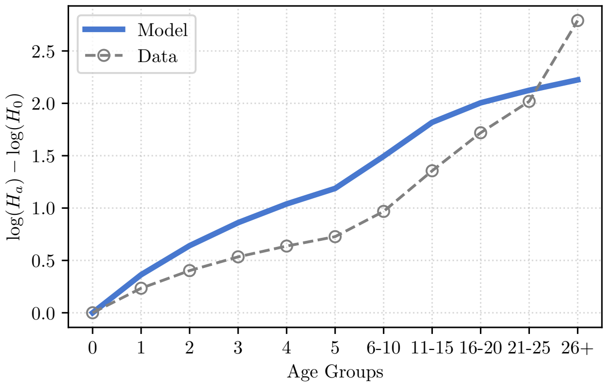
of average size of firms by product markets with the average size of firms by size decile, the model is also able to generate a realistic size distribution.[40] Despite the very mechanic evolution of firm sizes, due to the law of motion for the distribution of active firms, the model replicates also the age distribution of firms quite well. In addition, as shown in Figure 5d, the average sales by age groups in the model exhibits a convex profile similar to the data. This feature stems from the fact that, especially for later age groups, the growth dynamic is driven by firms that operate in product markets that are highly responsive to customer bases and hence provide incentives for sustained growth also later in firm lives. However, at the stationary solution the profile of sales growth is steeper than the one estimated in the data. This is mainly due to the strong incentives that firms have a the beginning of their lives to accumulate customers which leads them to grow fast in their initial years and then slowing down as they reach maturity.[41]

The calibration of the volatility of the exogenous processes is targeted to replicate the cohort-level correlations between markups and output. Figure 6 plots the cyclicality profile, estimated following Equation (25), for cohort-level markups in the data and the model. The model is able to replicate the ciclicality pattern observed in the data, where the correlations of cohort-level markups with output are increasing as we consider cohorts of older firms. This profile tightly reflects the fact that younger firms are more cyclical than older ones as their incentive to accumulate customers are stronger plus they discount the future more than their older counterparts, given their higher exit probability.

[40]The model achieves a perfect fit for firms of age 25 in the model with the average of mature (25 to 30 years) firms in the data. The figure plots the ratio between the averages in the model and in the data across all ages

[41]Recall that for the solution of the model I am truncating firms lives at sixty years.

Figure 7: Employment growth by age groups - model versus data



Note: The figure plots the growth profiles for the average firm size, measured by employment, for different age groups.

Additional moments. Even if the model is geared toward replicating the age profile of markups it is important to check how it performs in replicating some non-targeted moments in the data. In particular, it is important to check how the model performs to generate a growth profile of firms that is in line with the data. A reasonable growth profile in firm sizes is particularly relevant, especially when looking at aggregate measures of markups, as these are usually obtained by weighting firm-level estimates using a measure of firm size. To this end, Figure 7 compares the growth profile by age groups firms size, measured by employment. The employment profile is not targeted and the model is able to generate a growth profile that follows the data well for the first years of firms lives and exhibits strong growth firms that reach maturity.

Nonetheless, the baseline calibration of the model is not able to generate the faster employment growth for older firms observed in the data. As shown also in Figure 5d, firms in the model tend to grow relatively fast, thus the model slightly over-represents the growth rates in early years of firms' lives and then gradually slows down at later years. The main reason for this last effect is mechanical and linked to the solution approach I use. In fact, as I solve the model by truncating firms' lives at sixty years, firms start actively shrinking in size when they get closer to the end of their lives. This dynamic puts a downward pressure on mature firms sizes, preventing the model to fully capture the higher growth of older age groups.

A possible solution to this limitation comes at the cost of greater computational complexity. Pushing the end of firms' lives further in time, thereby extending the number of years firms are allowed to live is likely to improve the solution and address this issue better. However, as the main focus of this paper is the behavior of markups and given that the size profile implied by the model, albeit not perfect, follows the one estimated in the data reasonably well, I opt for the most tractable solution and keep the maximum allowed age close the one observed in the data.

5 Main results

In this section, I describe the main results. In particular, I document the ability of the model to generate age profiles and cohort effects that are in line with the empirical evidence presented in Section 2. In addition, I show that the model replicates the observed correlations between markups and output both at the firm and aggregate level. Moreover, I test the ability of the model to capture relevant incentives at the firm-level by estimating cohort effects and age profiles for advertising expenditure, a close empirical counterpart for direct demand investments. I then discuss the role of aggregate shocks and product market heterogeneity in generating these results in Section 6.

5.1 Persistent effects of business cycles

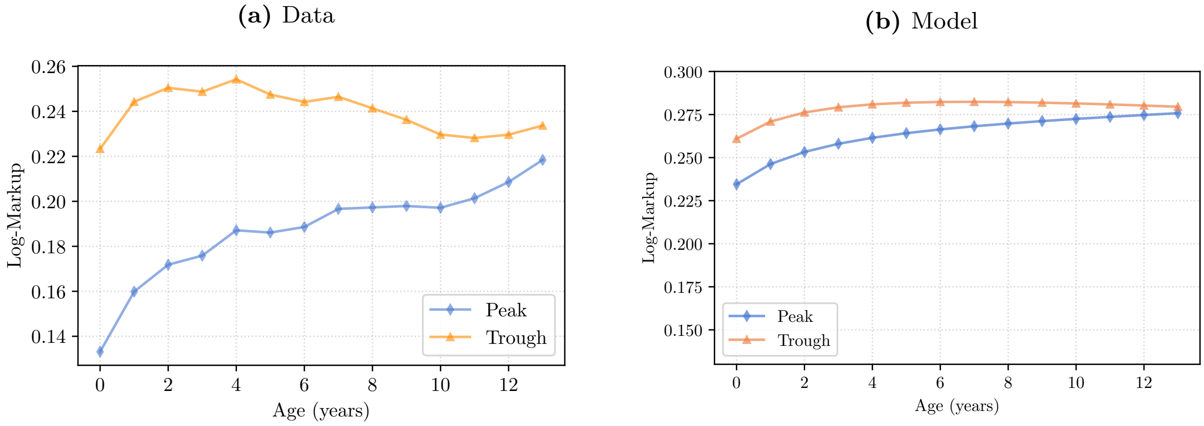
Cohort effects on firm-level markups. The selection mechanism embedded in the listing phase of the model allows business cycles to have persistent effects on the life-cycle behavior of firm-level markups.

Figure 8b shows the age profiles estimated using Equation (5) on a dataset of model-generated data. The figure shows that the model can qualitatively reproduce the age profile and the cohort effects estimated in real data and reported in Figure 8a. Firms that start their operations during a recession end up charging higher markups and operate on a flatter age profile. Through the lenses of the model, these effects are highly tied to differences in the demand characteristics of the different product markets. As recessions are periods in which it is difficult to attract customers, new listings are skewed towards product markets that guarantee higher up-front profits. These are specifically those that do not require big investment in demand acquisition and where the optimal size is reached earlier in firms lives. These product markets are characterized by a relatively low dependence of demand on customer base, and this implies that firms operating in these markets have a low need to expand their customer base. This lack of incentives to attract new customers makes them particularly appealing during bad economic conditions when the value of expanding the scale of operations is smaller. Moreover, as firms that operate in these markets have a smaller optimal size than firms operating in other product markets, they optimally charge higher markups from the start of their activities as they have weaker incentives to accumulate customers and hence are not harmed by keeping prices relatively higher at the beginning of their careers.

Quantitatively, running the same regression as in Equation (5) on model-generated data, reveals that a two-standard deviations positive increase from the stationary solution value of aggregate output induces a change in the markups charged in the initial year of approximately -1.41%. Therefore, the initial effect of aggregate conditions in the model is approximately 31% of the one estimated in the data for similarly sized movements in the cycle component of GDP.

Alternatively, the persistence of business cycles can be checked by comparing the correlation of cohort-level markups at different ages with their initial level. Figure 9 shows a comparison of these correlations for the data and the model. As business cycles persistently affect the

Figure 8: Cohort effects and age profile of markups



Note: The figure plots the age profile for markups estimated from Equation (5) for the main sample (reported only up to age 13) and for a panel dataset constructed from simulating the model for 2500 periods. Specifically, at each age a, I am plotting $\hat{m}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{m}_0$ is the average markup in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the first available year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of listed firm output. In the model the measure of business cycles are deviations of output from the stationary solution.

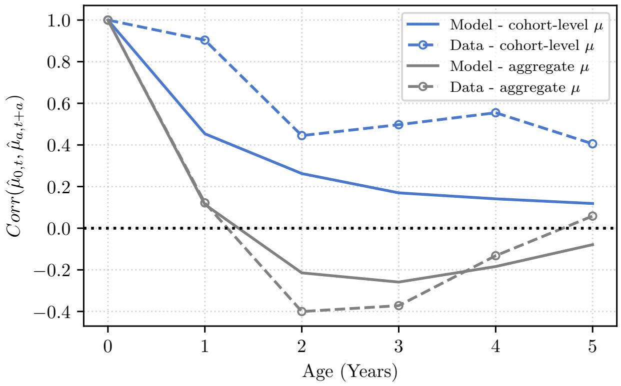
cohort of newly listed firms along their product market characteristics, the autocorrelation of markups across cohorts - i.e. the correlation of the average markup by cohort with the average markup of the same cohort some years in the future - exhibits an higher persistence than the aggregate measure of markup, remaining higher than zero up to age five. This result indicates that cyclical variations in markups across cohorts persist into later years without mean-reverting. This particular feature is not shared by the measure of aggregate markups that does not show any persistent autocorrelation beyond one year, both in the model and in the data.[42]

5.2 Markups' co-movements with aggregate conditions

Firm-level co-movements. A well-known fact in models with customer markets where firms' sales affect the number of customers they can acquire is that markups are strongly countercyclical. The intuition hinges on the fact that, due to deep-habits and procyclical stochastic discount factors, firms value extra customers more in booms than in recessions. Hence, when a recession hits, firms will find it profitable to exploit their current customer base rather than keep markups slightly lower to benefit from an higher customer base in the future. These incentives are at play also in the model presented in this paper with the addition that age is also an important force in determining the co-movements of markups with aggregate conditions. Table 5 reports the coefficients for a version of Equation (6) estimated on model simulated data, showing that the model can replicate the age profile for the cyclicality of firm level markups quite well, with young firm exhibiting countercyclical markups and older firms procyclical ones.

[42] The cohort-level autocorrelation in the model can be made even stronger allowing for initial conditions in customer base accumulation to be correlated with the aggregate state of the economy.

Figure 9: Autocorrelation of aggregate markups in the data and in the model



Note: The figure plots the cohort-level and aggregate autocorrelation of markups in the model and in the data. *Cohort-level* refers to correlations of average markup by cohorts of firms with the average markup of the same cohort a years into the future, i.e. $Corr(\hat{\mu}_{0,t}, \hat{\mu}_{a,t+a})$, where ˆ indicates deviations from an Hodrick-Prescott trend taken *across* cohorts of the same age. *Aggregate* refers to autocorrelations of cost-weighted markup in the economy between t and $t + a$, i.e. $Corr(\hat{\mu}_t, \hat{\mu}_{t+a})$ where ˆ indicates deviations from an Hodrick-Prescott trend.

Table 5: Markups' comovements with the cycle

Dep.Variable: Log-Markup	Data	Model
Cycle Measure	-0.356***	-0.352
	(0.124)	
Cycle Measure × Age	0.051**	0.102
	(0.020)	
Cycle Measure × Age^2	-0.002**	-0.003
	(0.001)	
Controls	Yes	Yes
R^2	0.62	0.67
N	123,997	445,500

Note: Robust standard errors in parentheses, $p :^{***} < 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports estimates of the co-movements between firm-level markups and a measure of business cycle. *Data* reports the coefficients of interest from estimating Equation 6 on the main sample of firms followed up to 25 years; controls include firm fixed effects, sector shares, HHI index in three digit NAICS, cash holdings and log-employment; the measure of business cycle is quadratically detrended log-real GDP. *Model* shows the coefficients of running a regression as in Equation 6 on a model simulated dataset; controls include age fixed effects, firm sizes and cohort fixed effects; the measure of business cycle is the log-deviation of output from the stationary solution value. The data are constructed by simulating the model for 2500 periods and then keeping firms up to 25 years to maintain consistency with the empirical counterpart.

Aggregate co-movements. A direct consequence of the fact that markups become less countercyclical as firms grow is that weighted and unweighted measures of aggregate markups in the model exhibit different cyclicality.[43]

In particular, under the baseline calibration, the cost-weighted measure of markups in the model features a 0.11 correlation with aggregate output (0.36 in the data), while the unweighted average shows a -0.15 correlation with aggregate output (-0.18 in the data).[44] The root of this difference is once again linked to the nature of firm heterogeneity in the model. As firms age and grow bigger, their incentives to adjust markups in reaction to business cycles diminish as they get closer to the monopolistic competition limit and reach their optimal scale of operations. This intuition is also shown more formally in Proposition 1.

Moreover, the firms that are destined to grow in size and command a larger share of the overall economy are also those that operate in product markets with a large demand elasticity to customer base. Under the baseline calibration, these product markets are associated with relatively low sales relevance in the acquisition of new customers. This implies that the largest firms in the model do not respond strongly to business cycles not only because they are larger, hence closer to the monopolistic competition limit, but also because they operate in product markets where the price lever is not very effective in managing the size of their customer bases.

The age and product market heterogeneity present in the model, therefore, is important to generate dynamics of aggregate markups that speak both to the literature on firm-level estimates, that, as documented also in this paper, founds strong negative co-movements between markups and aggregate conditions and to the broader and older literature on markup cyclicality that instead estimates lower countercyclicality if not acyclical or even procyclical markups.[45]

5.3 Testing the model's predictions using *Advertising Expenditure*

The model presented in the paper is geared towards explaining the behavior of markups using the heterogeneity in customer accumulation motives that firms operating in different product market have. However, the fact that firms in the model can explicitly devote resources to the acquisition of customers allows them to compare the behavior of direct investment in demand with a similar empirical counterpart. To check if the model delivers sensible results on this margin, which is completely untargeted, I estimate the cohort effects on a measure of advertising intensity in a small sub-sample of firms that report their expenditure on advertising. I define advertising intensity as the ratio on advertising expenditure to total operating expenditure. As direct demand investments in the model are denoted in units of labor, the model counterpart for this measure is the ratio of firms' wage bill that directly

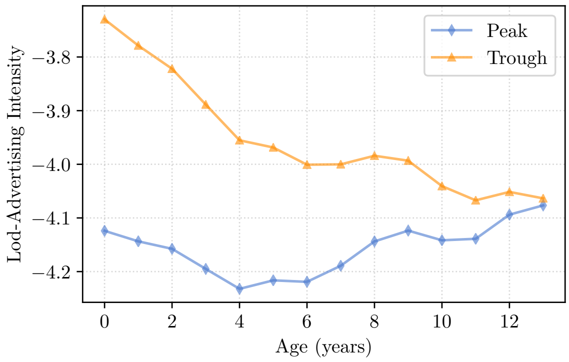
[43] Note that unweighted measures of markups overrepresent the behavior of small firms relatively to their contribution to total output.

[44] As noted by Edmond *et al.* [2018] in a large class of models the correct model based measure of aggregate markups is a cost-weighted average rather than a sale-weighted average. This is true also in the model developed here as I show in Appendix B

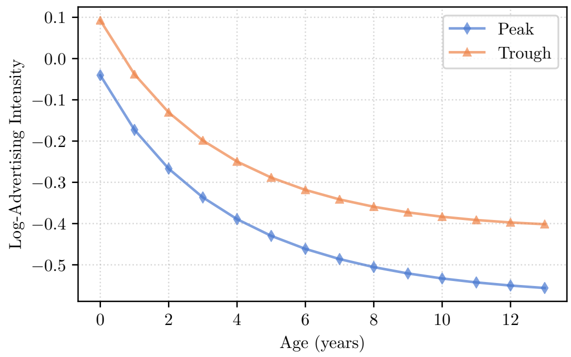
[45] See among others, Nekarda and Ramey [2013].

Figure 10: Cohort effects and age profiles for intensity of advertising expenditure

(a) Data



(b) Model



Note: The figure plots the age profile for the intensity of advertising expenditure (*Advertising expenditure/Operating Expenditure*) from Equation (5) for the main sample and for a panel dataset constructed from simulating the model for 2500 periods. Specifically, at each age a, I am plotting $\hat{m}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{m}_0$ is the average advertising expenditure in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the first available year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of listed firm output. In the model the measure of business cycles are deviations of output from the stationary solution.

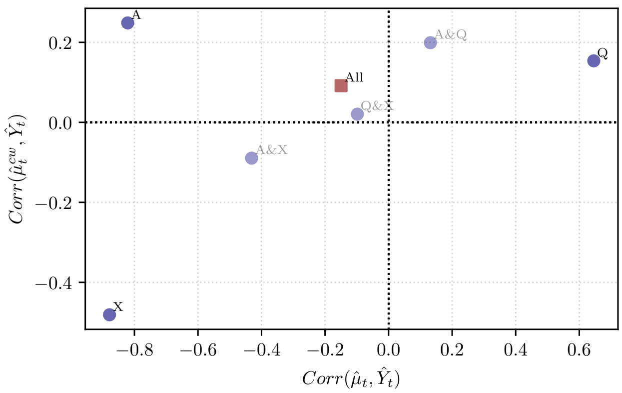
depends on these direct investments, i.e. $\zeta(m_{j(i),t})$ over $[\zeta(m_{j(i),t}) + h_{j(i),t}]$. I then estimate the effects of initial aggregate conditions on this measure using Equation (5) in the data and in a model simulated dataset.

The comparisons of the age profiles is reported in Figure 10. Figure 10a shows the estimated age profiles for firms that are first observed in periods of above trend GDP in the data while Figure 10b replicates the exercise in the model. While the size of the initial effects of aggregate conditions is very similar, a recession of two-standard deviations in the model generates an increase in the intensity of advertising intensity that is approximately 90% of the one estimated in the data. However, the overall impact on the age profiles is very different, especially for firms that are first observed in booms.

The main reason for this detachment between the model prediction and the data is likely due to the very stark assumption in how the model deals with direct demand investments. As firms' age, they expand both their customer base and their production capabilities. As sales is a channel through which firms can accumulate customers the need to diver resources from production to customer acquisition decreases mechanically as firms age in the model. The additional mechanism that makes firms that start their operations more reliant on direct demand investments is the fact the these firms are self-selected in product markets where customers are less responsive to sales and more sensitive to the direct investments (low ψ product markets).

A potential explanation for the very different age profiles in the data instead can be linked to the fact that firms that start their operations in booms are facing also higher competition and therefore need to keep up spending on advertising as they age. These competition effects

Figure 11: Correlation coefficients with output of cost-weighted and average markup for different shock mixes



Note: The figure plots the correlation cofficients between different shock mixes of the model. ˆ variables denote deviations from a Hamilton filter trend (one lag, two leads); μ_t^{cw} and μ_t are respectively cost-weighted and average markup. A: TFP shocks, Q: Aggregate demand shocks and X: Listing cost shocks.

are not present in the model hence the product market choice and the aging structure of the economy fully determine the age profiles of direct demand investments.[46]

6 Aggregate shocks, product markets and firm composition

In this section, I describe how the correlation of aggregate measures of markup is affected by each exogenous process included in the model and by the heterogeneity across different product markets. In addition, I examine the importance of the endogenous changes to the product market composition of existing firms for the ability of the model to generate realistic correlations of aggregate markups with output.

6.1 The role of aggregate shocks

The model developed in this paper features three sources of uncertainty: aggregate productivity shocks, aggregate demand shocks, and listing cost shocks. In this section, I discuss the role of different shock mixes in shaping the correlations between output and aggregate measures of markups.

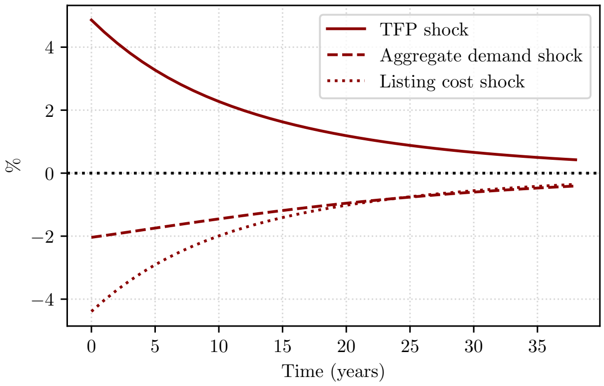
To this end, Figure 11 summarizes the relationships between correlation coefficients of the average and the cost-weighted average markup obtained from simulating the model with different shock mixes. Looking at these two measures of aggregate markups is helpful as they allow to build some intuition on the role of changes to the composition of active firms for markup cyclicality.[47] Two interesting patterns emerge.

First, when aggregate productivity shocks are the unique source of business cycles in the economy, cost-weighted and average markup exhibit a stark distinction in their cyclical behavior. While the former measure is strongly procyclical, the latter is mildly countercyclical.

[46]Explicitly modeling the interaction of higher competition and product market characteristics with business cycles is left for future research.

[47]For mode details on the IRFs to each shock for markups and firm masses by age and product market see Figures A.6, A.7 and A.8.

Figure 12: Impulse responses for the total mass of newly listed firms



Note: The figure plots the IRFs for the total mass of newly listed firms to one-percent shocks to aggregate productivity, aggregate demand and listing costs.

This pattern is consistent with the observation that, as productivity improves, incumbent firms increase their production as well as markups. However, higher volumes due to higher productivity more than compensate the customer loss caused by increase in markups. As noted in the previous section, the average markup in the economy, which reflects the extensive margin more strongly, is countercyclical as positive productivity realizations are associated with a spur in new listings that increase the share of young firms in the economy putting downward pressure on the average markup, see Figure 12.[48]

Second, when listing cost shocks are the only sources of aggregate fluctuations, both cost-weighted and average markup exhibit countercyclicality. In contrast, aggregate demand shocks make both measures of markups procyclical, albeit the average exhibits a higher correlation with output than the cost weighted one.

The explanation of these patterns lies again in the changes at the extensive margin that these shocks generate. In fact, in the model, periods with high aggregate demand are not associated with a generalized increase in firms' profitability across product market. As shocks to the aggregate demand process affects the efficacy of customer base investments, their influence on the ex-ante value of becoming public varies for firms operating in different markets. In particular, under the baseline calibration, positive aggregate demand shocks improve the profitability of firms only in the product market with the highest sensitivity to customer base, as firms in these markets are the ones that rely more on direct demand investments and therefore are more equipped to exploit the most the improved efficacy of these investments. On aggregate, however, this implies that fewer firms are successfully completing their listing process compared to the stationary solution, lowering the total share of young firms in the economy and skewing the population of active firms toward older and larger businesses.

When output is driven exclusively by aggregate demand shocks, high-output periods are also times when investment in customer base is more efficient. This creates incentives for incumbent firms to lower their markups regardless of their product market. As these firms

[48]Figure A.2b reports the IRFs of newly listed firms for each product market under different calibrations.

are also relatively larger, at the intensive margin, cost-weighted markups tend to decrease as output increases following a positive demand shock. At the extensive margin, however, positive aggregate demand shocks reduce the total mass of firms in the economy, as more firms attempt listing in product market that are highly sensitive to customer base acquisition and have smaller success probabilities.[49] This behavior skews the cross-section of active firms towards older and larger firms. Overall, as fewer young firms get listed, periods of high demand and high output are associated with a pool of active firms overpopulated by old incumbents. This change in the composition leads to an increase in the average markup and a positive correlation with total output. This is because despite the incentive to acquire customers, incumbent firms have higher markups compared to newly listed firms. The resulting correlation is the combination of these two forces. The higher procyclicality of average markup can be explained by noting that, by construction, the simple average is a measure more sensitive to changes at the extensive margin, thus is more sensitive to the fact that periods of high-demand will reduce the total number of listings.

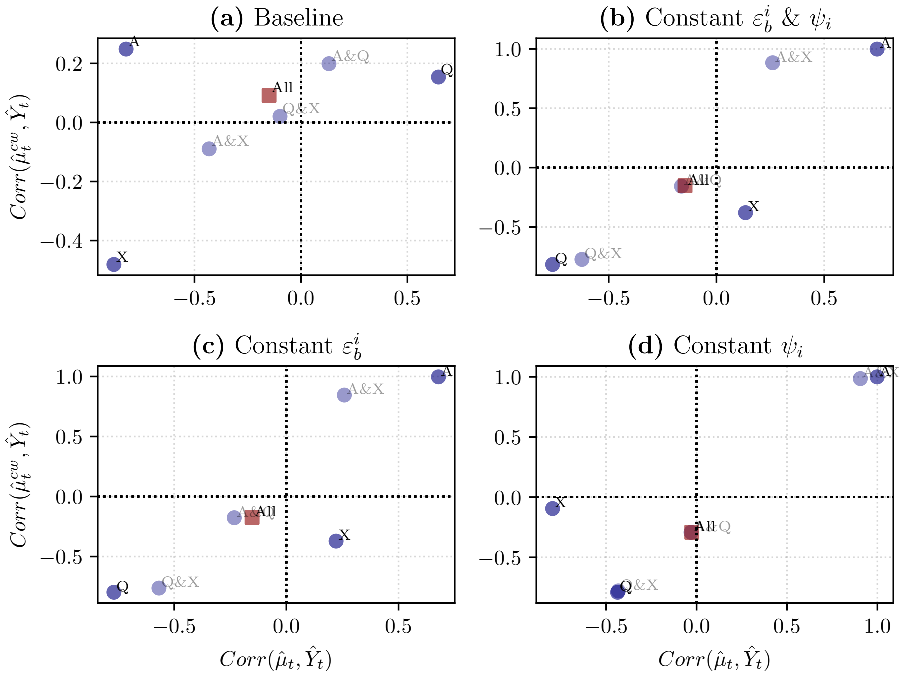
The first-order consequence of an increase in listing costs, instead, is that as fewer firms are listed in all product markets output declines. Then at the extensive margin response increases the relative weight of older and larger firms in the economy, putting an upward pressure on average markup delivering the countercyclical behavior shown in Figure 11.In addition, this effect is stronger for firms operating in product markets with low demand sensitivity to customer base that are also less willing to use prices to manage their customer base. The intuition for this stronger selection effect is that, in these markets, firms reach their optimal size earlier in their life-cycle, hence, their present discounted value is more affected by aggregate conditions at the time of listing, given that business cycles at the beginning of their listed lives have a greater scope to persistently affect their values. All these effects compound to generate a negative correlation between output and the average markup in the economy.

The recessions caused by an increase in listing cost shocks, instead, force especially young firms to lower markups, attempting to counteract the effects of lower sales with lower prices. Older firms, instead, being less sensible to the loss of customers caused by the initial reduction in output are able to recoup their losses in customers faster than young firms and therefore can benefit from higher demand and increase their markups, leading to a lower countercicality of cost-weighted markups compared to that of average markup when listing cost shocks are the only source of business cycles in the economy.

As shown by Figure 11, combining the three shocks allows the economy to generate both countercyclical average markup, and acyclical aggregate (cost-weighted) markups. Under the baseline calibration, increasing the strength of productivity shocks would push the economy towards generating more procyclical aggregate markups while increasing the role of aggregate demand shocks would make aggregate markups more countercyclical and, at the same time, induce stronger compositional effects of business cycles.

[49] See Figure A.3b. Notice that as customers are easier to acquire, operating in product markets that are highly sensitive to customer base becomes more attractive as these are markets that are ex-ante more profitable.

Figure 13: Correlation coefficients with output of cost-weighted and average markup for different shock mixes and alternative calibration of product markets paramaters



Note: The figure plots the correlation coefficients between different shock mixes of the model and different degrees of product market heterogeneity. ^ variables denote deviations from a Hamilton filter trend (one lag, two leads); μ_t^{cw} and μ_t are respectively cost-weighted and average markup. A: TFP shocks, Q: Aggregate demand shocks and X: Listing cost shocks.

6.2 The role of product market heterogeneity

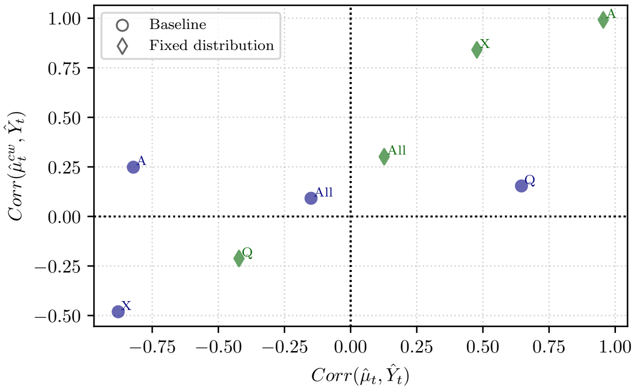
The model features a high degree of product market heterogeneity. For the dynamics of the model, the most relevant parameters are the elasticity to customer base of the demand shifters $\varepsilon_b^i \equiv \frac{k_i'(b_{j(i),t})}{k_i(b_{j(i),t})} b_{j(i),t}$, and the relevance of sales in the customer acquisition process ψ_i.

In this section, I show that it is necessary to allow for both margins of heterogeneity to generate correlations of aggregate markups that are consistent with the empirical ones. To do so, I compute the correlations betwwen aggregate output under three alternative calibrations of the model: i) one where ε_b^i and ψ_i are equal across markets (homogeneous product markets); ii) one in which product markets differ only along the sales relevance margin (constant ε_b^i) and iii) one in which they differ only along the demand elasticities to customer base (constant ψ_i).[50]

As discuss previously, empirically the correlation of average markups with output is negative while the correlation of cost-weighted markups with output is positive. In other words, in a scatter plot, their relationship would have to be in the second quadrant. Figure 13 plots the relationship between the correlations with output of cost-weighted markups and average markups under different shock mixes and for different calibrations of product market heterogeneity in the model. Besides the baseline one, reported in panel (a), none of the other

[50]In each of the alternative calibrations, I set the constant parameters to the average value implied by the baseline one.

Figure 14: Correlation coefficients with output of cost-weighted and average markup for different shock mixes



Note: The figure plots the correlation coefficients between different shock mixes of the baseline model and a model where average and cost-weighted markups are constructed keeping a the distribution of firms at the stationary-solution values at each simulation period. ˆvariables denote deviations from a Hamilton filter trend (one lag, two leads); μ_t^{cw} and μ_t are respectively the cost-weighted and average markup. A: TFP shocks, Q: Aggregate demand shocks and X: Listing cost shocks.

calibration of product market heterogeneity is able to generate correlations that fall in the second quadrant.

6.3 The role of firm composition

The response of aggregate markups to aggregate shocks is driven by the combination of the change in incumbents' responses and the change in the product market composition of cohorts of newly listed firms. To disentangle how the correlation of aggregate markups with output is affected by changes to the mass of firms in the economy (extensive margin) and by changes to existing firms' behaviors (intensive margin), I recompute aggregate variables from model simulation fixing the distribution of active firm in each product market-age cell at its stationary solution value and then I calculate the correlations of aggregate markups with output under different shock mixes for these new aggregate measures. Specifically, I construct average ($\tilde{\mu}_t$) and cost-weighted ($\tilde{\mu}_t^{cw}$) markups with a constant mass of firms as follows:

$$\tilde{\mu}_t = \sum_i \sum_a \bar{\Gamma}_{i,a,t} \mu_{i,a,t} / \sum_i \sum_a \bar{\Gamma}_{i,a,t} \tag{26}$$

$$\tilde{\mu}_t^{cw} = \sum_i \sum_a \bar{\Gamma}_{i,a,t} h_{i,a,t} \mu_{i,a,t} / \sum_i \sum_a \bar{\Gamma}_{i,a,t} h_{i,a,t}, \tag{27}$$

where $\bar{\Gamma}_{i,a}$ indicates the mass of firms with age a that populate product market i absent any aggregate shocks.

Comparing average and cost-weighted markups helps understanding how compositional changes in the cross-section of active firms influence the response of the economy to aggregate shocks. As noted before, the average measure of markups is more responsive to changes to the composition of firm population in the economy. Intuitively, this is due to the fact that differences in product markets and the implicit selection operated by business cycles have more bite on the average that is more sensitive to changes in the overall number of firms.[51]

The differences in the propagation of aggregate shocks caused by the missing compositional channel has deep repercussions also on how different shocks impact the correlations between output and markups, as shown in Figure 14.

Notably, keeping the distribution of firms at the stationary solution levels, radically affects the correlation between output and markups, especially for productivity and listing cost shocks. For simulations in which productivity is the only source of business cycles the model with a constant distribution generates highly procyclical markups, both average and cost-weighted. To see why, consider that a positive productivity shock pushes all firms to increase markups. This is because the potential loss from an increase in prices is compensated from the gain in customers derived from increased sales. This also implies that profitability of firms in the economy increases after a productivity shock. Thus, while in the baseline model the increase in profitability causes an increase in the number of young firms that go public, by keeping the mass of firms in the economy fixed, both the average and the cost-weighted markups move in the same direction resulting in a highly positive correlation with output.

7 Conclusions

In this paper, I study how aggregate conditions at relevant junctures of firms lives can have persistent effects on firm choices and how these choices affects the behavior of both firm-level and aggregate markups.

In the empirical part of the paper, I provide evidence on the persistence of cohort effects measured by aggregate conditions close to firms' listing years and founding years. For both definitions of firm cohorts, I find that firms facing adverse aggregate conditions at crucial moments of their lives charge higher markups.

In the theoretical part of the paper, I show that the empirical results are consistent with a model of firms dynamics that features heterogeneous product markets and realistic firms life cycles. A novel feature of the model, needed to match the empirical evidence, is that firms rely on both sales and direct demand investments to acquire customers and relax their demand constraints. Notably, product markets differ among each other along two main margins: i) how sensitive firms demands are to the size of customer base and ii) the relevance of firms' sales in the acquisition of new customers. The model generates countercyclical markups for small young firms and mildly procyclical markups for large old ones, in line with recent empirical evidence. In addition, the model is able to generate realistic age profiles for markups and replicate the cohort effects estimated in the data for both markups, albeit with a smaller magnitude, and advertising expenditure. A key feature of the model is that the product market composition of active firms is endogenously affected by business cycle realizations. These changes to the composition of active firms, together with product

[51] See Figure A.9 for IRFs to different aggregate shocks.

market heterogeneity, are essential in generating empirically consistent correlations between aggregate measures of markups and output, making the model able to replicate both the firm-level and the aggregate behavior of markups over the business cycle.

Finally, there are a number of extensions to the structural framework developed in this paper that I plan to explore in future research. In fact, within this structure, one can study more in depth the role of firm heterogeneity for the transmission of aggregate shocks in the macroeconomy. Using a global solution method, for example, would allow the responses of the model to be state-depended and therefore explicitly characterize the influence of changes to the firm distribution on the transitional dynamics of the economy in response to different histories of aggregate shocks. Similarly, pricing incentives can have non-trivial interactions with monetary policy and its transmission to the economy.

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A Additional tables and figures

A.1 A reduced form approach: autocorrelation of cohort-level markups

To estimate the autocorrelation of cohort level markups I am exploiting the following specification at the cohort level

$$\log(\mu_{a,t}) = \alpha + \beta_0 \log(\mu_{0,t-a}) + \beta_1 \log(\mu_{0,t-a}) \times a + \beta_2 \log(\mu_{0,t}) + \beta_3 a + \beta_4 a^2 + u_{a,t}, \quad (A.1)$$

where $\mu_{a,t}$ is the average markup of cohort a in year t; $\mu_{0,t-a}$ is the average markup of cohort a in the year of birth; $\mu_{0,t}$ is the average markup of entering firms in year t; and a is age. The elasticity of each cohort markups to their initial conditions is therefore given by β_0 and the elasticity at each subsequent age is $\beta_0 + \beta_1 \times a$. The comparison of the coefficients of interest for Equation (A.1) estimated in the main sample and in a dataset of model generated data is reported in Table A.1.

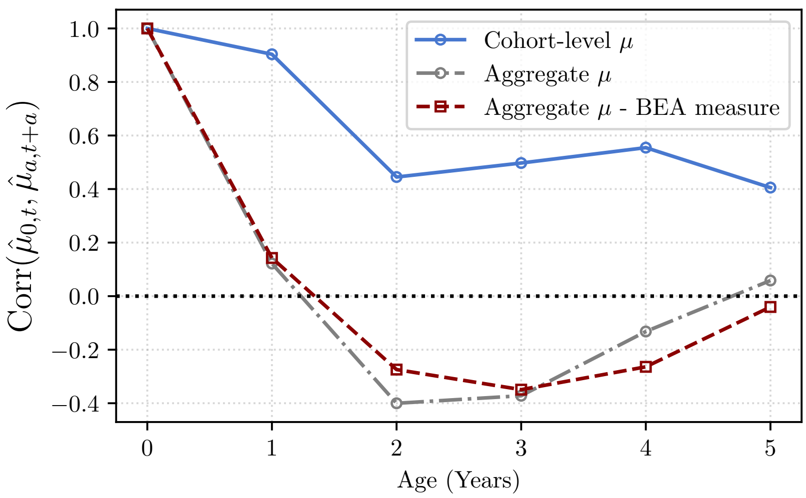
The model slightly overestimates the correlation with the markup charged in the initial year and the decline in the correlation induced by age. However, both coefficients are similar in magnitude and indicate that each cohort of firms in the data and in the model shares some common feature that only slowly fades away with time. In the model, this feature is the product market composition of different cohorts of firms.

Table A.1: Auto correlation of firm-level markup

Dep.Variable: Log-Markup	Model	Data
Log-Markup at A_0	0.535	0.503***
		(0.072)
Log-Markup at A_0 × Age	-0.060	-0.022*
		(0.012)
Controls	Yes	Yes
R^2	0.91	0.34
N	1,850	370

Note: The table reports the elasticity of cohort-level markups with the markup charged in the first year a cohort is observed. In the data each cohort of firms starting from 1970 to 2006 is followed up to 10 years of age (37 cohorts followed for 10 years), robustness checks for 5 and 20 years give qualitatively similar results. For the model simulated data I run a simulation of 2500 period for each cohort and then I keep 37×5 cohorts of firms for 10 periods to have a comparable number of cohorts in the model simulation and in the data. Robustness checks for cohorts followed for 5 and 20 years deliver qualitatively similar results.

Figure A.1: Empirical autocorrelations for aggregate markups



Note: The figure plots the cohort-level and aggregate autocorrelation of markups in the model and in the data. *Cohort-level* refers to correlations of average markup by cohorts of firms with the average markup of the same cohort a years into the future, i.e. $Corr(\hat{\mu}_{0,t}, \hat{\mu}_{a,t+a})$, where $\hat{}$ indicates deviations from an Hodrick-Prescott trend taken *across* cohorts of the same age. *Aggregate* refers to autocorrelations of cost-weighted average markup in the economy between t and $t + a$, i.e. $Corr(\hat{\mu}_t, \hat{\mu}_{t+a})$ where $\hat{}$ indicates deviations from an Hodrick-Prescott trend. The BEA autocorrelation is constructed on a measure of aggregate markup that does not require the estimation of the production function, as in Kaplan and Zoch [2020]. Specifically, it is obtained from the ratio of the final producer price index (BEA code: `WPSFD49207`) to the intermediate producer price index (BEA code: `WPSID61`).

A.2 Impulse response analysis

In this section, I assess the role of differences across product markets by discussing the model's response to aggregate shocks under different calibrations of the model. Specifically, I progressively eliminate the main channels of heterogeneity across product markets: the sales share in customer base investments and the sensitivity of demand to customer base. I compare the baseline calibration to two alternatives: a first one, in which investment in customer base accumulation have the same sales sensitivity in each product market (constant ψ_i case), and a second one, in which firms' demands exhibit the same elasticity to customer base (constant ε_b^i case).[52]

To assess the role of product market heterogeneity for the transmission of aggregate shocks, I compare the economy's responses under different calibrations of the parameters governing product market heterogeneity. First, I analyze the transmission of productivity shocks, then I look at aggregate demand shocks and finally to listing costs shocks.[53]

A.2.1 Aggregate productivity shocks

Figure A.2 shows the impulse responses of the economy to a positive one-percent aggregate productivity shock for the baseline calibration, the constant sale share case, and the constant demand elasticity case. I discuss these two alternatives in turn.[54]

Common sales share, ψ_i. Under this calibration, the only difference across product markets is how sensitive firms' demands are to the size of their customer bases. Interestingly, as shown in the first panel of Figure A.2a, the response of output is practically identical to the baseline case. The intuition for this result follows from noting that, compared to the baseline calibration, the composition of newly listed firms, shown in Figure A.2b, is not highly affected by the change in TFP when ψ_i is constant. This result indicates that incumbents' profitability is not affected by the relevance of sales in the acquisition of customers. However, as shown from the last two panels of Figure A.2a, the responses of the cost-weighted and the simple average markup are quite different from the baseline calibration. In particular, setting all product markets to the same average value for ψ induces a large procyclical response of aggregate markups. To grasp some intuition, consider that the aggregate productivity shock makes production more efficient, lowering marginal costs. Given that sales are still a method to acquire customers, firms can trade-off the boost in productivity from the aggregate shock with an increase in markups.

This mechanism is particularly strong for bigger and older firms, contributing to the high response of cost-weighted markups with constant ψ_i. In the baseline calibration, instead, firms with high demand sensitivities operate in product markets where sales are relatively ineffective in accumulating customers. This implies that to exploit the higher profitability generated by higher TFP, firms are incentivized to invest in relaxing their demand constraints via direct demand investments. However, as sales and direct investments are complements in the investment function for customer base, higher TFP strengthens these large firms' incentive to accumulate customers, putting downward pressure on aggregate markups.

In addition, given higher productivity, the profitability of firms in all product markets increases, boosting listing. Consequently, the share of young firms increases, lowering the

[52]Figure A.5 plots the two different calibrations together with the baseline. To preserve the size differences across product markets and maintain some consistency with the baseline economy, I recompute the scale parameters in firms' demands, κ_i, to match the average size of old firms in both alternative calibrations.

[53]Appendix A reports the impulse responses to each shock for markups and the mass of firms for each product market and age cohort.

[54]Figure A.6 reports the IRFs to aggregate productivity shocks for markups and masses of active firms by cohort and product market.

simple average markup in the economy below its steady-state level for some periods. This is because the increase in the share of new firms outweighs the fact that active firms can charge higher markups. When the sale share in customer acquisition, ψ_i, is constant across product markets, the average markup's initial response is less negative than in the baseline calibration. In this case, fewer firms have to rely on direct investment in customer acquisition and can exploit the benefits of being more productive. As a larger fraction of newly listed firms chooses product markets where demand is not very sensitive to customer base accumulation, the average markup overshoots its steady-state value before converging back to it as more firms exploit the productivity increase to charge higher markups.

Common demand elasticity, ε_b^i. Under this calibration, the only element that differentiates product markets is the role of sales in the acquisition of new customers.

The main effect caused by this alternative calibration is a significant change in the composition of newly listed firms, as shown in the third panel of Figure A.2b.

Compared to the baseline case and the calibration with differences in ε_b^i, now a positive TFP shock induces a similar increase in listing across product markets. This implies that cohorts of newly listed firms are more homogeneous. This, in turn, generates a response of the average markup that is smaller than under the previous calibration. The intuition hinges on the fact that, in this case, the economy does not feature firms that are very eager to accumulate customers, and hence the initial value of markups that they choose to set is higher compared to the baseline for newly listed firms.

The initial response of the cost-weighted measure of markups, instead, is more muted under this calibration because firms that were previously operating in product markets with high-ψ and low-ε are now facing stronger incentives to accumulate customers. Hence, as productivity increases, the incentive to extract rents from the current customer base is counterbalanced as higher production can lead to larger future profits thanks to the role of sales in building customer base. These two contrasting forces cancel out in the aggregate contributing to the minimal response of cost-weighted markups to TFP shocks. Through the same reasoning, we can rationalize the larger response of output. In this calibration, more firms are eager to use prices to expand their customer bases, which implies that as productivity improves, incumbent firms do not increase markups as much and more output is produced.

It is worth mentioning that the transmission of TFP shocks on markups in this economy is very different from the standard procyclical results induced by price rigidities. In this model, markups' cyclical behavior, both in the aggregate and at the firm level, is dictated more by the dynamics of the incentives to accumulate customers than by the response of marginal costs.

A.2.2 Aggregate demand shocks

Figure A.3 plots the impulse responses to a positive one-percent aggregate demand shock for the baseline economy, the economy with constant demand elasticities and the one with constant sales share in customer base investments.[55]

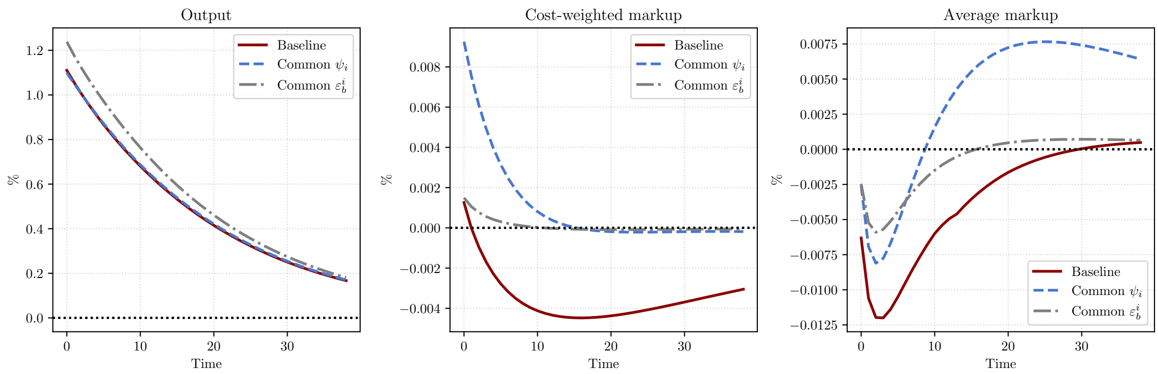
Common sales share, ψ_i. As discussed in the previous section, when the only difference across product markets is the elasticity of demand to customer base, the composition of newly listed firms does not change dramatically compared to the baseline case.

Nonetheless, the response of markups to aggregate demand shocks presents stark differences across the three calibrations. This is due to the deep changes in the composition of newly

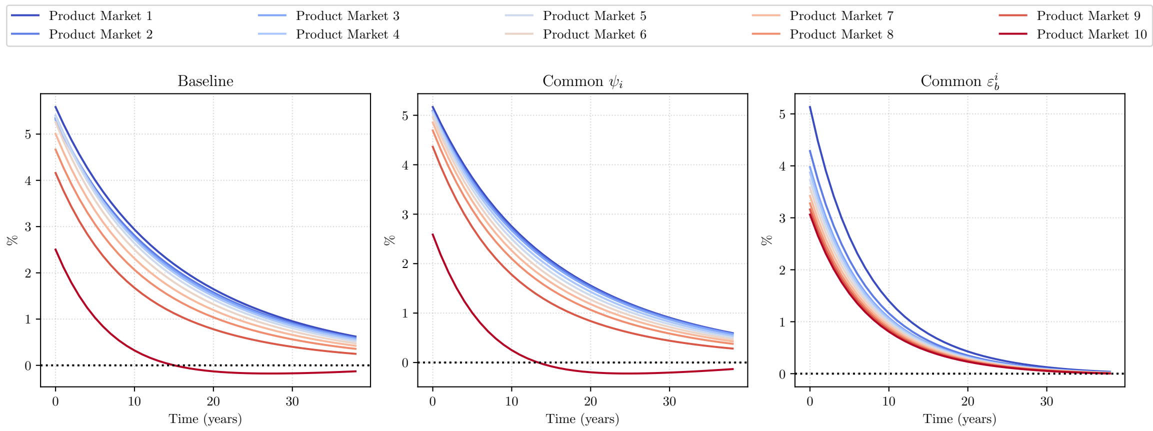
[55]Figure A.7 reports the IRFs to aggregate demand shocks for markups and masses of active firms by cohort and product market.

Figure A.2: Impulse responses to aggregate productivity shock

(a) Aggregates



(b) New firms in each product market



Note: The figure plots the impulse responses to a 1% aggregate productivity shock for selected aggregate variables under three different calibration of product market parameters ψ_i, ε_b^i. *Baseline* shows the IRFs to the baseline calibration of the mode where both ψ_i and ε_b^i vary across product markets. *Common* ψ_i shows the IRFs for a calibration of the model where product markets differ only across the demand elasticity ε_b^i and ψ_i is set at the average value for all i. *Common* ε_b^i, instead reports the IRFs for a calibration of the model where product markets differ only across ψ_i and ε_b^i is set at the average value for all i.

listed firms caused by fluctuations in Q. In fact, a positive shock to Q triggers a period in which the efficiency of customer base investment is high, which improves the profitability of firms operating in product markets that depend heavily on customer acquisition. In turn, this improvement induces a shift in the composition of new listings towards product markets characterized by highly sensitive demands and low dependence on sales to accumulate customers, as these product markets are relatively more profitable in periods of high demand. However, incumbent firms in these markets can enjoy the enhanced efficiency of their investment and hence, are incentivized to increase their markups. Consequently, both the cost-weighted and the average markup in the economy overshoot their steady state values, reflecting the fact that after an aggregate demand shock, the composition of active firms in the economy shifts towards product markets that rely a lot on customers but in which acquiring new customers does not depend a lot on sales and therefore prices.

When the sales share in the customer base investment function is fixed at its average value across product markets, instead, more firms are eager to reduce markups to expand their customer bases in periods of favorable aggregate conditions. This is because firms in highly sensitive product markets now rely relatively more on sales compared to the baseline calibration. As sales and direct demand investment are not substitute, these firms respond to their incentive to accumulate customers by aggressively cutting prices to benefit from the favorable economic climate.

The differences between the cost-weighted share of markups and the average markup can be again rationalized by looking at the change in the composition of newly listed firms. As more young firms are going to operate in product markets where they are eager to accumulate customers relative to the steady-state, on impact, the response of cost-weighted average markup is larger than the simple average. Also, old and large firms are more willing to lower their prices to increase their sales and benefit from easier customer acquisition due to the shock, contributing in keeping aggregate markups below their stationary solution levels.

Common demand sensitivity, ε_b^i. Under this calibration, the line of reasoning outlined in the previous paragraph is partially reversed. As all product markets feature the same demand sensitivity aggregate shocks induce a smaller change in composition, confirming the finding that this particular margin of heterogeneity across product markets is highly relevant for the selection effect of business cycles.

The third panel of Figure A.3a shows that the average markup exhibit strong counterciclicality. This is a direct consequence of the combination of two facts. First, contrary to the baseline and the calibration with common ψ_i, with homogeneous demand sensitivities, the economy experiences an overall increase in the number of young firms. Second, not only these firms are relatively more eager to accumulate customers, but they also operate in product markets where this incentive is stronger.[56] This pushes down the average markup more than the cost-weighted one as more young firms enter the market and both incumbents and newly listed firms are eager to use the price lever to build their customer base.

A.2.3 Listing cost shocks

Figure A.4 plots the impulse responses to a one-percent increase to listing cost for the baseline economy, the economy with constant demand elasticities, and the one with constant sales share in customer base investments.

The overall behavior of the economy in response to this shock resembles closely the inverse of aggregate productivity shocks.[57] The intuition relies on noticing that while better productivity pushes firms to go public more in product markets that do not rely heavily on customer acquisition (low ε_b^i), higher listing costs do the opposite, making listing *less* profitable for these firms as they operate in product markets that do not have strong growth prospects. As a consequence, as output falls, both cost-weighted and average markup increase countercyclically.

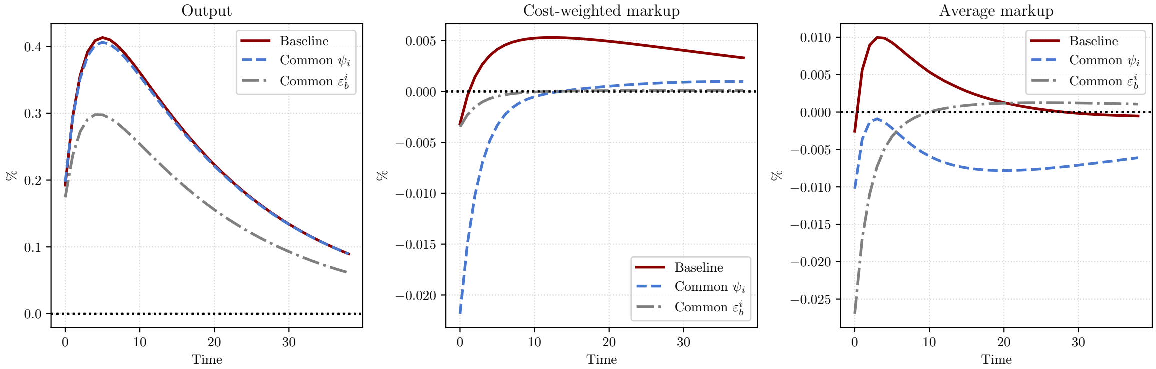
Common sales share, ψ_i. When the model is calibrated to homogenize firms across their incentive to use sales as a tool to acquire customers, the overall response of the economy is very similar to the baseline one. Again, the reason is that having product markets with similar sales share parameters does not dramatically affect the composition of new listings. The transition back to the stationary solution however is different. The average and the cost-weighted markups revert back to their stationary solution levels more quickly, with the average markup overshooting its stationary value. The positive listing cost shock reduces

[56] Notice that high ψ_i product markets, under this calibration, face stronger incentives to accumulate customer as the common ε_b is higher than the one associated in these product markets under the baseline calibration.

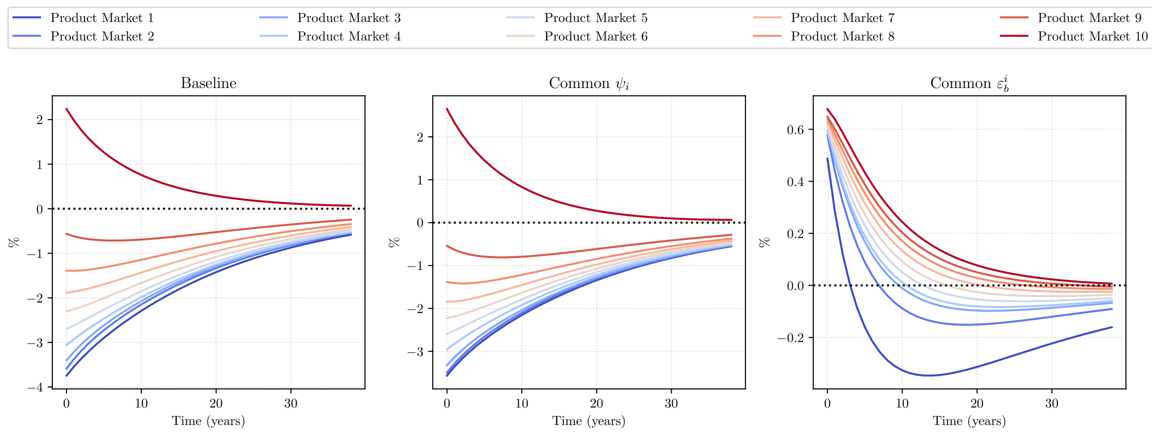
[57] Figure A.8 reports the IRFs to listing cost shocks for markups and masses of active firms by cohort and product market.

Figure A.3: Impulse responses to aggregate demand shock

(a) Aggregates



(b) New firms in each product market



Note: The figure plots the impulse responses to a 1% aggregate demand shock for selected aggregate variables under three different calibration of product market parameters ψ_i, ε_b^i. *Baseline* shows the IRFs to the baseline calibration of the mode where both ψ_i and ε_b^i vary across product markets. *Common* ψ_i shows the IRFs for a calibration of the model where product markets differ only across the demand elasticity ε_b^i and ψ_i is set at the average value for all is. *Common* ε_b^i, instead reports the IRFs for a calibration of the model where product markets differ only across ψ_i and ε_b^i is set at the average value for all i.

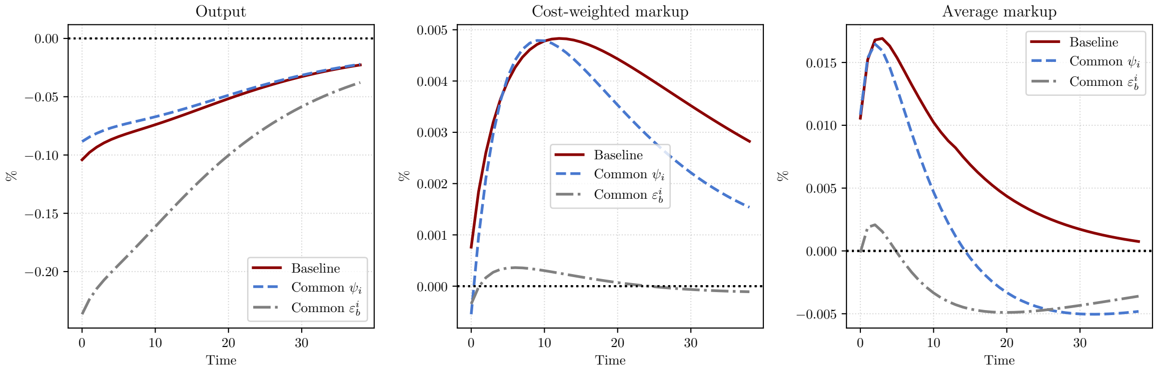
the number of young firms in the economy, and as these firms age, they can increase their markups more quickly than the baseline.

Common demand sensitivity, ε_b^i. The calibration with common demand sensitivity generates very different responses to a one-percent increase in the listing cost. As in the previous section, muting the heterogeneity of demand sensitivities generates a big change in the composition of newly listed firms. In this case, the increase in listing costs, increases the fall in listings of firms that have weaker dependence on customer base accumulation on sales. These firms are also the ones that are bigger and have more growth potential, thus output falls by more than in the baseline cases.

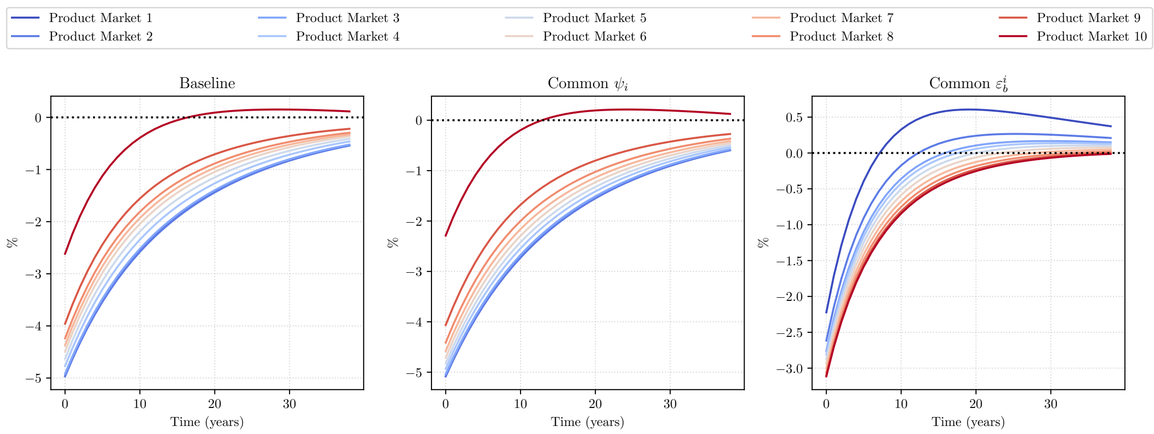
As now all firms use prices to increase their customer base, the responses of both cost-weighted and average markup are muted. In this case, the average markup overshoots its stationary value even sooner than under common sales shares. This is because the early cohorts of newly listed firm age are relatively more populated by firms that use prices to attract customers, while they try to reach their optimal size, pushing the average markup below its stationary level.

Figure A.4: Impulse responses to listing cost shock

(a) Aggregates

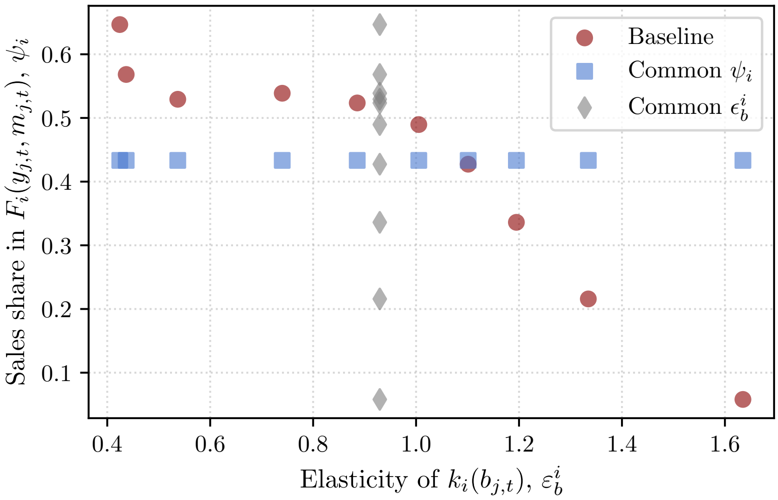


(b) New firms in each product market



Note: The figure plots the impulse responses to a 1% listing cost shock for selected aggregate variables under three different calibration of product market parameters ψ_i, ε_b^i. *Baseline* shows the IRFs to the baseline calibration of the mode where both ψ_i and ε_b^i vary across product markets. *Common* ψ_i shows the IRFs for a calibration of the model where product markets differ only across the demand elasticity ε_b^i and ψ_i is set at the average value for all *i*s. *Common* ε_b^i, instead reports the IRFs for a calibration of the model where product markets differ only across ψ_i and ε_b^i is set at the average value for all i.

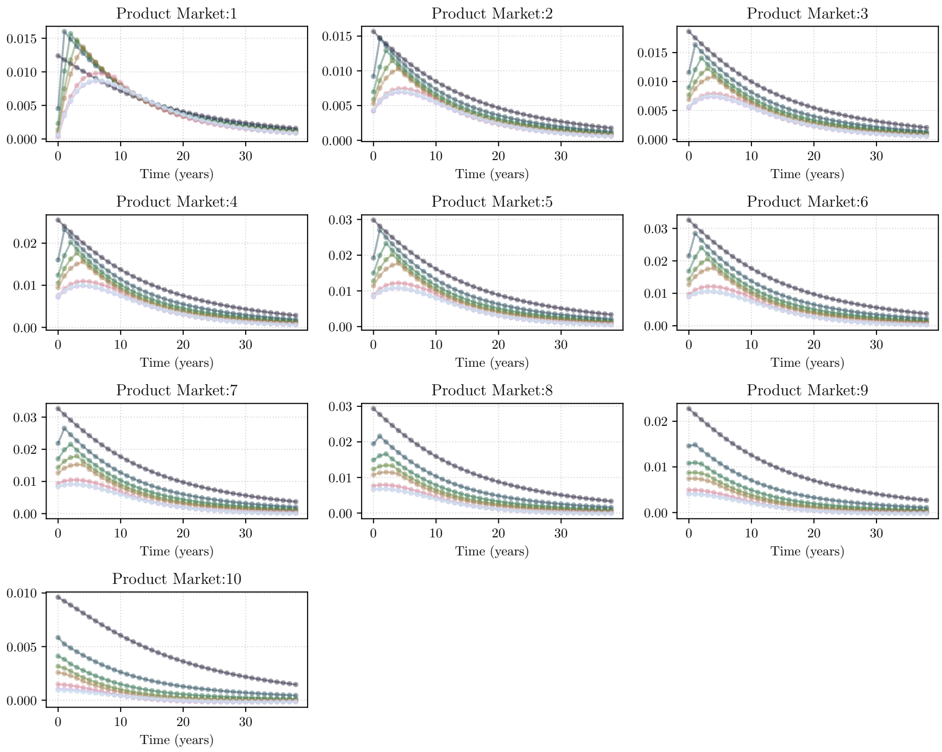
Figure A.5: Product markets characteristics under different calibrations



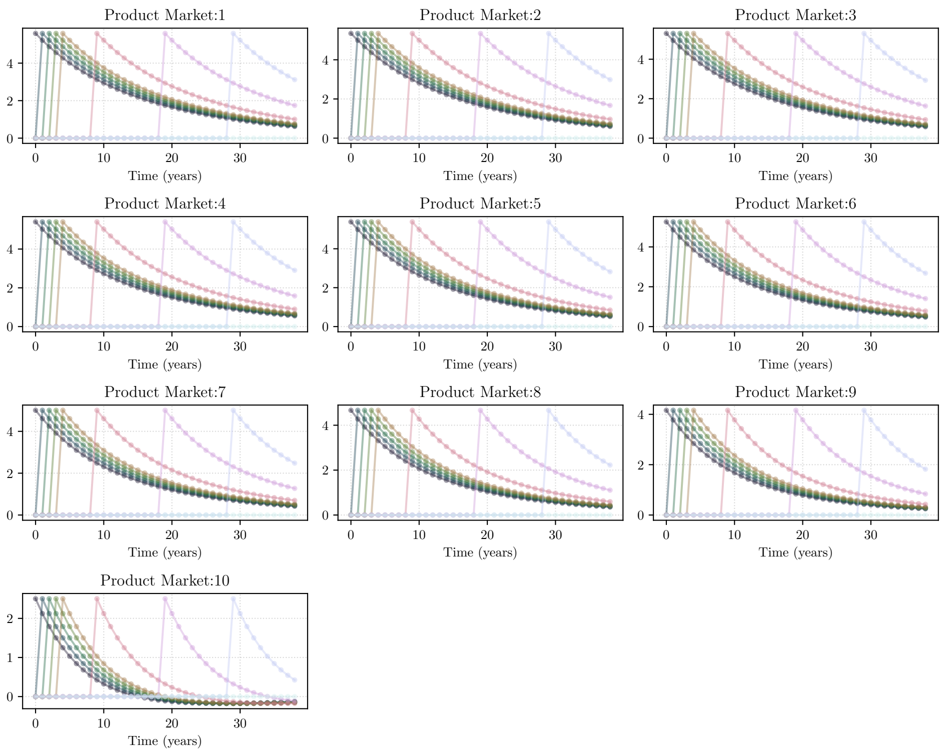
Note: The figure plots the parameters defining product market characteristics under the different calibrations discussed in Section 6.

Figure A.6: Impulse response functions to aggregate productivity shocks for markups and mass of active firms by cohort and product market

(a) Markups



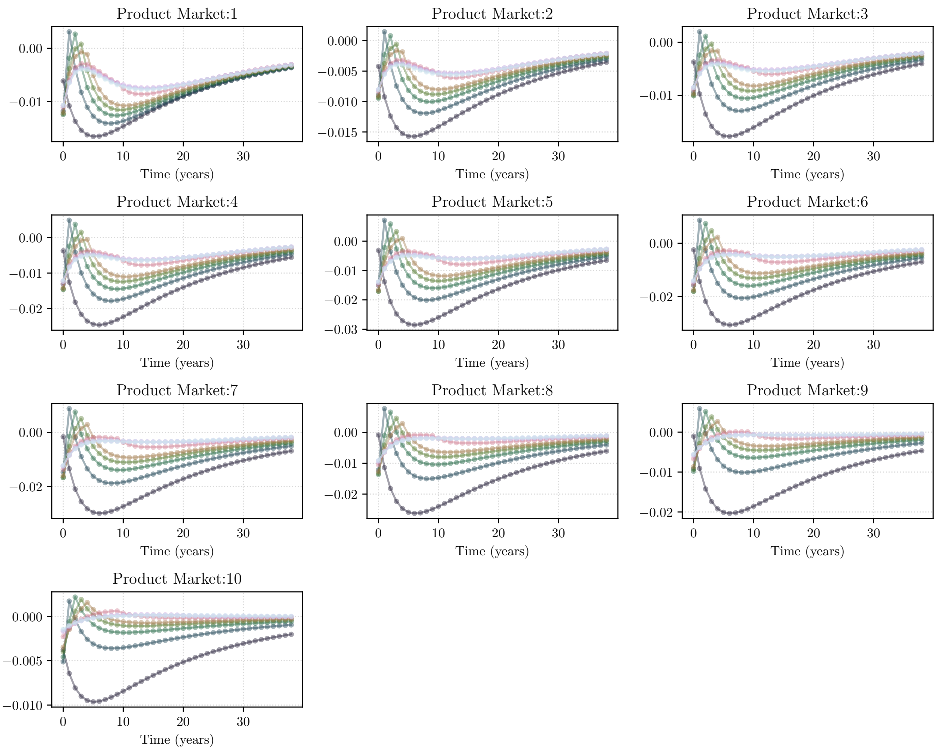
(b) Firm masses



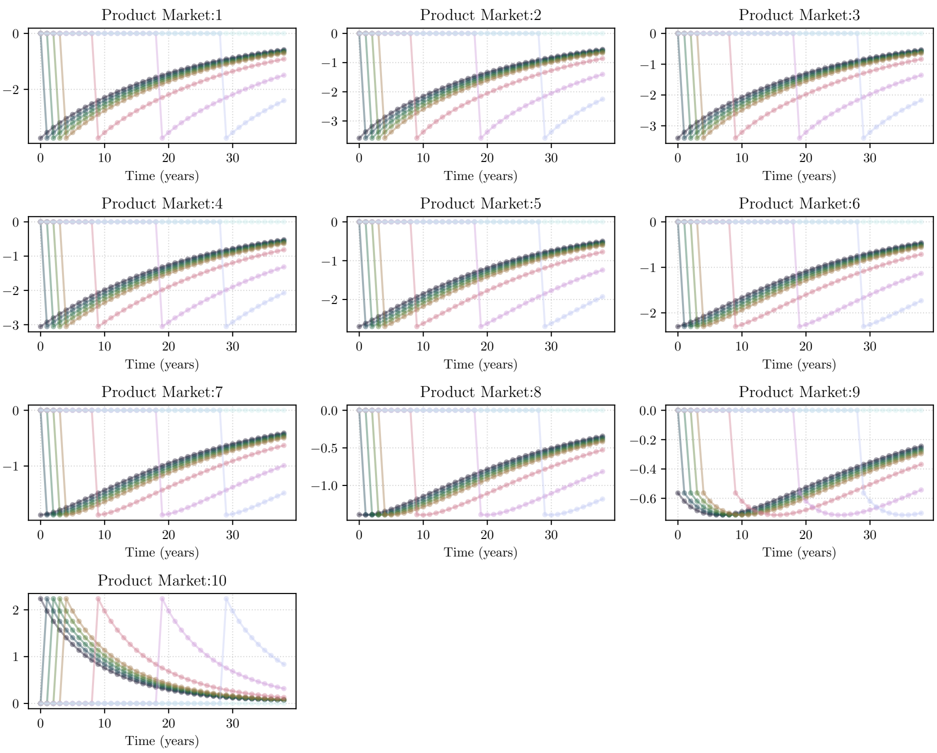
Note: The figure plots impulse response functions and percent deviations from the stationary solution to a one-percent productivity shocks. Each panel reports the IRFs for selected cohorts in a given product market. Darker colors are younger cohorts and selected cohorts are firms aged $a \in \{0, 1, 2, 3, 4, 5, 10, 20, 30, 40\}$. The time periods are years since the shock realization. Therefore, each dot shows the response of firms aged a years old t periods after the shock.

Figure A.7: Impulse response functions to aggregate demand shocks of markups and masses of active firms by cohort and product market

(a) Markups



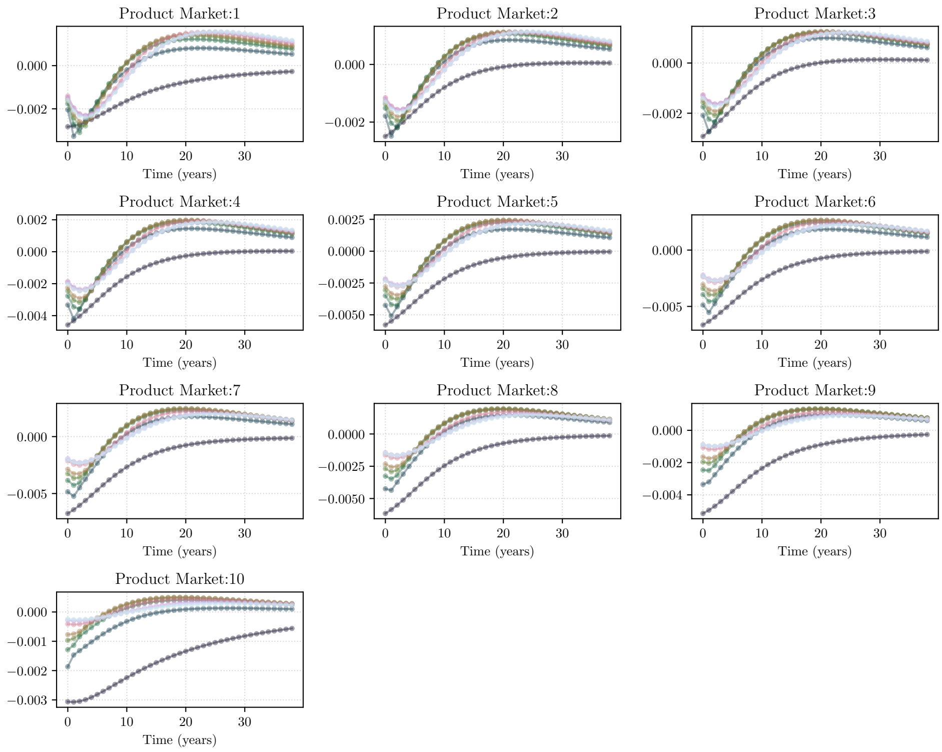
(b) Firm masses



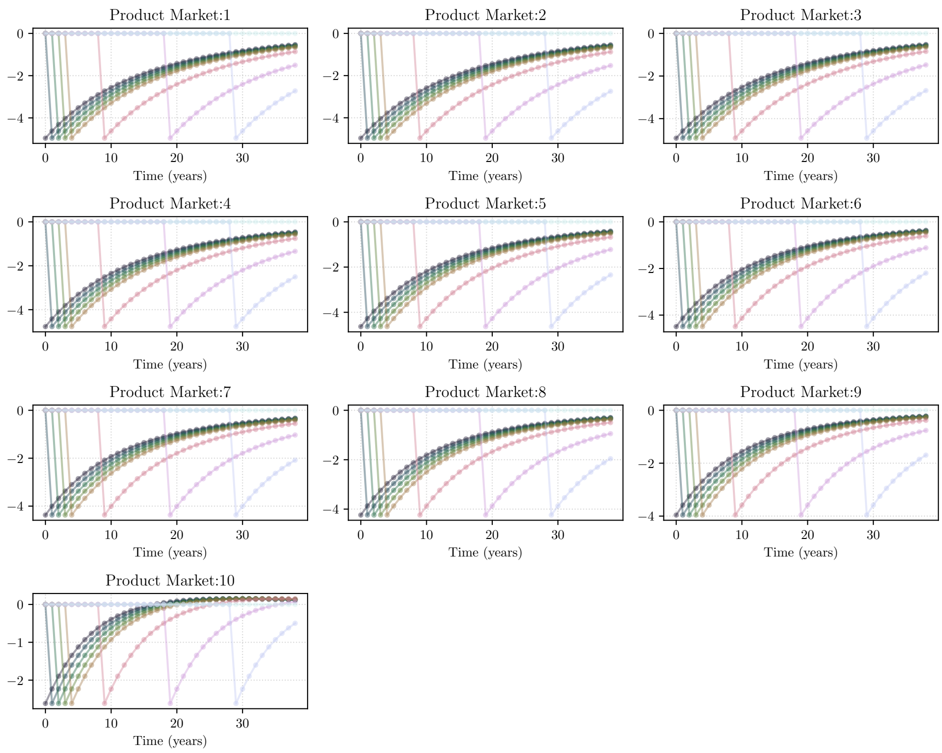
Note: The figure plots impulse response functions and percent deviations from the stationary solution to a one-percent aggregate demand shocks.Each panel reports the IRFs for selected cohorts in a given product market. Darker colors are younger cohorts and selected cohorts are firms aged $a \in \{0, 1, 2, 3, 4, 5, 10, 20, 30, 40\}$. The time periods are years since the shock realization. Therefore, each dot shows the response of firms aged a years old t periods after the shock.

Figure A.8: Impulse response functions to listing cost shocks for markups and masses of active firms by cohort and product market

(a) Markups

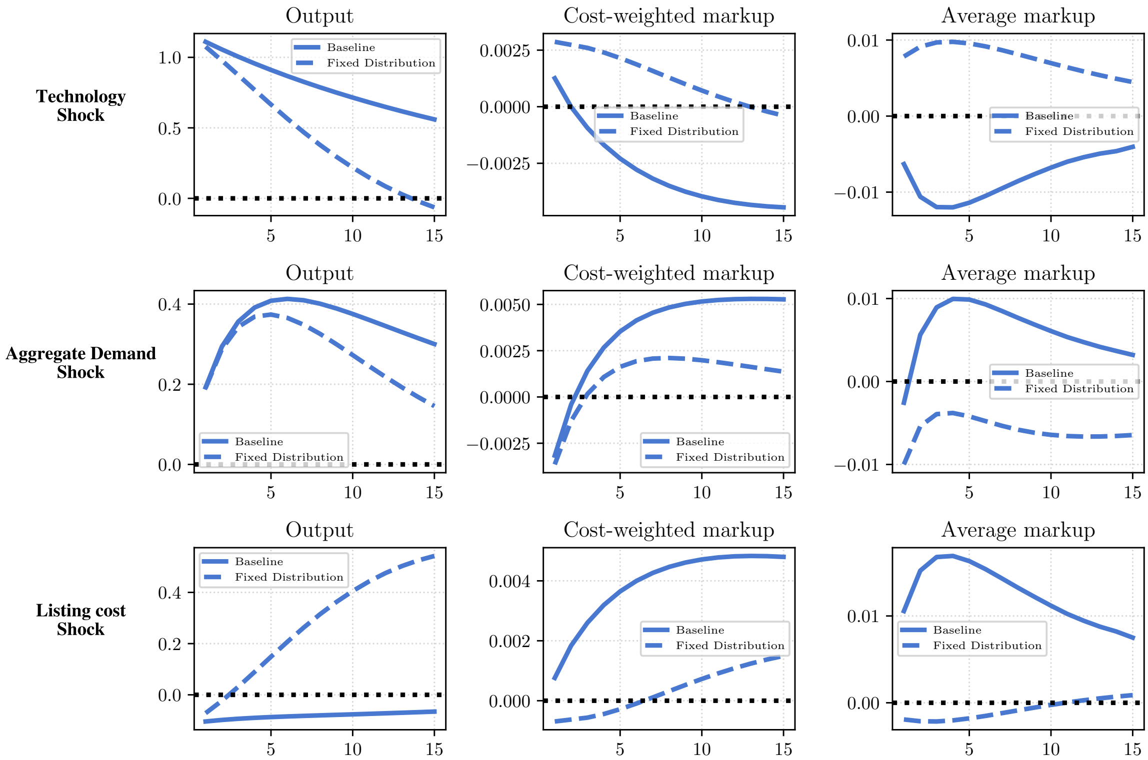


(b) Firm masses



Note: The figure plots impulse response functions and percent deviations from the stationary solution to a one-percent listing cost shock. Each panel reports the IRFs for selected cohorts in a given product market. Darker colors are younger cohorts and selected cohorts are firms aged $a \in \{0, 1, 2, 3, 4, 5, 10, 20, 30, 40\}$. The time periods are years since the shock realization. Therefore, each dot shows the response of firms aged a years old t periods after the shock.

Figure A.9: Role of firm composition for the transmission of aggregate shocks



Note: The figure plots the impulse responses to a one-percent change in the exogenous variables for the average and cost-weighted measures of markups in two versions of the economy: i) the baseline one (solid) and ii) one in which the distribution of firms in the economy is kept constant at its stationary solution throughout the simulations (dashed). Time is years and IRFs express percentage deviations from stationary solution levels.

B Model and solution details

In this section I provide additional details about the model results and derivation discussed in the main body of the paper.

B.1 Rescaling of incumbent's value function

I solve the model with the habit-adjusted consumption good as the numeraire. The main consequence of this choice is that it is convenient to rescale the incumbent value function $V_i(b_{j,t-1}, a_{j,t}; S_t)$ to take the change into account. In particular, consider the original problem

$$V_i(b_{j,t-1}, a_{j,t}; S_t) = \max_{\substack{p_{j,t}, y_{j,t}, h_{j,t}, \\ m_{j,t}, b_{j,t}}} \left\{ \begin{array}{c} p_{j,t} y_{j,t} - w_t \left(h_{j,t} + \zeta(m_{j,t})\right) + \\ +(1-\rho(a_{j,t}))\mathbb{E}_t \left[q_{t,t+1} V_i(b_{j,t}, a_{j,t+1}, S_{t+1})\right] \end{array} \right\} \tag{B.1}$$

subject to the relevant constraints

$$\begin{aligned} b_{j,t} &= (1-\delta) b_{j,t-1} + Q_t F_i(y_{j,t}, m_{j,t}) \\ y_{j,t} &= \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} k_i(b_{j,t}) Y_t \\ y_{j,t} &= A_t h_{j,t}^{\alpha} \\ a_{j,t+1} &= a_{j,t} + 1. \end{aligned}$$

Now divide both left hand side and the right hand side of the Bellman equation in (B.1) by the aggregate habit-adjusted price index P_t and define $\tilde{V}(\cdot) \equiv V(\cdot)/P_t$.

Then, (B.1) becomes

$$\tilde{V}_i(b_{j,t-1}, a_{j,t}; S_t) = \max_{\substack{p_{j,t}, y_{j,t}, h_{j,t}, \\ m_{j,t}, b_{j,t}}} \left\{ \begin{array}{c} \frac{p_{j,t}}{P_t} y_{j,t} - \frac{w_t}{P_t} \left(h_{j,t} + \zeta(m_{j,t})\right) + \\ +(1-\rho(a_{j,t}))\mathbb{E}_t \left[\frac{q_{t,t+1}}{P_t} V_i(b_{j,t}, a_{j,t+1}, S_{t+1})\right] \end{array} \right\}. \tag{B.2}$$

From the household optimality conditions, however, we know that $q_{t,t+1} = \beta \frac{u'(C_{t+1}) P_t}{u(C_t) P_{t+1}}$, hence

$$\tilde{V}_i(b_{j,t-1}, a_{j,t}; S_t) = \max_{\substack{p_{j,t}, y_{j,t}, h_{j,t}, \\ m_{j,t}, b_{j,t}}} \left\{ \begin{array}{c} \frac{p_{j,t}}{P_t} y_{j,t} - \frac{w_t}{P_t} \left(h_{j,t} + \zeta(m_{j,t})\right) + \\ +(1-\rho(a_{j,t}))\mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u(C_t)} \tilde{V}_i(b_{j,t}, a_{j,t+1}, S_{t+1})\right] \end{array} \right\},$$

so that, defining $\Lambda_{t,t+1} \equiv \beta \frac{u'(C_{t+1})}{u(C_t)}$ and substituting the price ratio and production technology from the constraints the incumbents' problem simplifies to

$$\tilde{V}_i(b_{j,t-1}, a_{j,t}; S_t) = \max_{h_{j,t}, m_{j,t}, b_{j,t}} \left\{ \begin{array}{c} (k_i(b_{j,t}) Y_t)^{\frac{1}{\eta}} (A_t h_{j,t}^{\alpha}) - \frac{w_t}{P_t} \left(h_{j,t} + \zeta(m_{j,t})\right) + \\ +(1-\rho(a_{j,t}))\mathbb{E}_t \left[\Lambda_{t,t+1} \tilde{V}_i(b_{j,t}, a_{j,t}+1, S_{t+1})\right] \end{array} \right\} \tag{B.3}$$

subject to

$$b_{j,t} = (1-\delta) b_{j,t-1} + Q_t F_i(A_t h_{j,t}^{\alpha}, m_{j,t}).$$

B.2 Derivation of firm demands

For varieties produced by listed firms, household expenditure minimization requires that the agent solves the following problem:

$$\max_{\{c_{j,t}\}_i} \left(\sum_i \int_{j \in \mathcal{J}_i} k_i(b_{j,t})^{\frac{1}{\eta}} c_{j,t}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} - \lambda \left[\sum_i \int_{j \in \mathcal{J}_i} p_{j,t} c_{j,t} - E_t \right].$$

Combining the optimal choices for each variety delivers the following household demand

$$c_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\eta} k_i(b_{j,t}) C_t,$$

where $P_t := \left(\sum_i \int_{j \in \mathcal{J}_i} k_i(b_{j,t}) p_{j,t}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$. Now, note that entry requires X_t units of the habit-adjusted consumption good from listed firms , hence each entrant will demand a fraction

$$X_t \left(\frac{p_{j,t}}{P_t} \right)^{-\eta} k_i(b_{j,t})$$

of firm j's output. Therefore the total amount of firm j's output used for entry is:

$$x_{j,t} = \sum_{i=1}^{I} e_{i,t} X_t \left(\frac{p_{j,t}}{P_t} \right)^{-\eta} k_i(b_{j,t}),$$

then market clearing for each variety implies that

$$y_{j,t} = c_{j,t} + x_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\eta} k_i(b_{j,t}) \left[C_t + \sum_{i=1}^{I} e_{i,t} X_t \right].$$

As $C_t + \sum_{i=1}^{I} e_{i,t} X_t$ is the total expenditure in listed firms for this economy, then market clearing for the listed goods sector requires that

$$C_t + \sum_{i=1}^{I} e_{i,t} X_t = Y_t,$$

and therefore

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\eta} k_i(b_{j,t}) Y_t. \tag{B.4}$$

B.3 Equilibrium conditions

The incumbent problem for a firm in market i is

$$V_i(b_{j,t-1}, a; \boldsymbol{S}_t) = \max_{\left\{ \substack{p_{j,t}, y_{j,t}, h_{j,t}, \\ m_{j,t}, b_{j,t}} \right\}} \left\{ \begin{array}{c} \frac{p_{j,t}}{P_t} y_{j,t} - w_t(h_{j,t} + \zeta(m_{j,t})) + \\ (1-\rho(a)) \mathbb{E}[\Lambda_{t+1} V_i(b_{j,t}, a+1; \boldsymbol{S}_{t+1})] \end{array} \right\} \tag{B.5}$$

subject to

$$\begin{aligned} y_{j,t} &= \left(\tfrac{p_{j,t}}{P_t} \right)^{-\eta} k(b_{j,t}) Y_t \\ b_{j,t} &= (1-\delta) b_{j,t-1} + Q_t F(y_{j,t}, m_{j,t})) \\ y_{j,t} &= A_t h_{j,t}^{\alpha} \end{aligned}$$

Using the individual firm demands to express the price ratio as a function of y and b, the technology to express y as a function of h then the incumbents problem can be simplified as follows[58]

$$V_i(b_{j,t-1}, a; \boldsymbol{S}_t) = \max_{\{h_{j,t}, m_{j,t}, b_{j,t}\}} \left\{ \begin{array}{l} \left(A_t h_{j,t}^{\alpha}\right)^{1-\frac{1}{\eta}} (k_i(b_{j,t})Y_t)^{\frac{1}{\eta}} - w_t(h_{j,t} + \zeta(m_{j,t})) \\ +(1-\rho(a))\mathbb{E}[\Lambda_{t+1} V_i(b_{j,t}, a+1; \boldsymbol{S}_{t+1})] \end{array} \right\} \tag{B.6}$$

subject to

$$b_{j,t} = (1-\delta)b_{j,t-1} + Q_t F(A_t h_{j,t}^{\alpha}, m_{j,t}))$$

The first order conditions for incumbents are:

$$\gamma_t = \frac{w_t \zeta'(m_{j,t})}{Q_t F_m(\cdot|_t)}, \tag{B.7}$$

$$\frac{w_t}{\alpha A_t h_{j,t}^{\alpha-1}} = \left(1 - \frac{1}{\eta}\right) \left(\frac{k_i(b_{j,t})Y_t)}{A_t h_{j,t}^{\alpha}}\right)^{\frac{1}{\eta}} + \frac{F_h(\cdot|_t)}{F_m(\cdot|_t)} \zeta'(m_{j,t}) w_t, \tag{B.8}$$

$$\frac{w_t \zeta'(m_{j,t})}{Q_t F_{m,t}(\cdot)} = \frac{1}{\eta} k_i'(b_{j,t}) Y_t \left(\frac{k_i(b_{j,t} Y_t)}{A_t h_{j,t}^{\alpha}}\right)^{\frac{1}{\eta}-1} + (1-\rho(a))(1-\delta)\mathbb{E}\left[\Lambda_{t,t+1} \frac{w_{t+1}\zeta'(m_{j,t+1})}{Q_{t+1} F_m(\cdot|_{t+1})}\right]. \tag{B.9}$$

The same set of optimality conditions holds for all firms producing the same products and facing the same exit probability (i.e. firms in the same cohort), therefore the full equilibrium of the model can be described by the following conditions:

- For each cohort $a \in \{0, ..., \bar{a}\}$ and product type $i \in \{1, ..., I\}$:

$$\begin{aligned}
&\frac{w_t}{\alpha A_t h_{a,i,t}^{\alpha-1}} = \left(1 - \frac{1}{\eta}\right) \left(\frac{k_i(b_{a,i,t})Y_t)}{A_t h_{a,i,t}^{\alpha}}\right)^{\frac{1}{\eta}} + \frac{F_h(\cdot|_t)}{F_m(\cdot|_t)} \zeta'(m_{a,i,t}) w_t \\
&\frac{w_t \zeta'(m_{a,i,t})}{Q_t F_m(\cdot|_t)} = \frac{1}{\eta} k_i'(b_{a,i,t}) Y_t \left(\frac{k_i(b_{a,i,t}) Y_t}{A_t h_{a,i,t}^{\alpha}}\right)^{\frac{1}{\eta}-1} + (1-\rho(a))(1-\delta)\mathbb{E}\left[\Lambda_{t,t+1} \frac{w_{t+1}\zeta'(m_{a+1,i,t+1})}{Q_{t+1} F_m(\cdot|_{t+1})}\right] \\
&b_{a,i,t} = (1-\delta)b_{a-1,i,t-1} + F(A_t h_{a,i,t}^{\alpha}, m_{a,i,t}) \\
&V_{a,i,t} = \left(A_t h_{a,i,t}^{\alpha}\right)^{1-\frac{1}{\eta}} (k_i(b_{a,i,t})Y_t)^{\frac{1}{\eta}} - w_t(h_{a,i,t} + \zeta(m_{a,i,t})) + (1-\rho(a))\mathbb{E}[\Lambda_{t+1} V_{a+1,i,t+1})] \\
&\Gamma_{a,i,t} = (1-\rho(a-1))\Gamma_{a-1,i,t-1} \\
&\frac{p_{i,a,t}}{P_t} = \left(\frac{k_i(b_{a,i,t})Y_t)}{A_t h_{a,i,t}^{\alpha}}\right)^{\frac{1}{\eta}}
\end{aligned}$$

  with initial conditions for customer bases in each product market $b_{0,i,t-1} > 0$, calibrated to match sales profiles in the data. In practice, to simulate the model for each product market i, I draw idiosyncratic shocks $u_{i,t-1} \sim U(-1,1)$ and set $b_{0,i,t-1} = b_{0,i} \cdot (1 + u_{i,t-1})$.

- Free entry in each sector $i \in \{1, ..., I\}$:

$$\begin{aligned}
\frac{\Gamma_{0,i,t}}{e_{i,t}} &= \frac{X_t}{V_{0,i,t}} \\
\Gamma_{0,i,t} &= (\omega_i)^{\phi} e_{i,t}^{1-\phi}
\end{aligned}$$

  where ω_i are scaled by a common factor calibrated to ensure that aggregate output is equal to 1 in the stationary solution with no aggregate uncertainty.

[58] Recall that as I am using the rescaled version of the incumbent problem and I am solving everything in terms of the habit-adjusted price level, P_t, w_t is indicating the habit-adjusted real wage.

- Household optimality condition, market clearing and exogenous processes:

$$
\begin{aligned}
&\tfrac{w_t}{C_t} = \nu \\
&\Lambda_{t+1,t} = \beta \tfrac{u'(C_{t+1})}{u'(C_t)} \\
&H_t = \textstyle\sum_{i=1}^{I} \sum_{a=0}^{\bar{a}} (h_{a,i,t} + \zeta(m_{a,i,t}))\Gamma_{a,i,t} \\
&\textstyle\sum_{i=1}^{I} \sum_{a=0}^{\bar{a}} \left(\frac{k_i(b_{a,i,t})Y_t}{A_t h_{a,i,t}^{\alpha}} \right)^{\frac{1}{\eta}} A_t h_{a,i,t}^{\alpha} \Gamma_{a,i,t} = Y_t \\
&C_t + X_t \textstyle\sum_{i=1}^{I} e_{i,t} = Y_t \\
&A_t = \rho_a A_{t-1} + \epsilon_t^A \\
&X_t = \rho_X X_{t-1} + \epsilon_t^X \\
&Q_t = \rho_Q X_{t-1} + \epsilon_t^Q \\
&\epsilon_t^j \sim N(0, \sigma_j), j = \{A, X, Q\}
\end{aligned}
$$

B.4 Aggregation

As noted by Edmond *et al.* [2018], aggregate markups in a model economy like the one presented in this paper are equal to the inverse of the labor share.

Therefore, it is possible to define

$$\frac{1}{\mathcal{M}_t} = \frac{w_t H_t^g}{Y_t},$$

as the aggregate markup, where H_t^g denotes the amount of labor inputs dedicated to the production of the output good and not to the accumulation of customer base.

Firms' optimal behavior implies a firm-specific markup over marginal costs so that

$$\frac{1}{\mu_{j,t}} = \frac{P_t w_t h_{j,t}}{p_{j,t} y_{j,t}},$$

therefore

$$\frac{\frac{p_{j,t}}{P_t} y_{j,t}}{Y_t} = \mu_{j,t} \frac{h_{j,t}}{H_t^g} \frac{w_t H_t^g}{Y_t}. \tag{B.10}$$

As $Y_t = \sum_i \int_j \frac{p_{j,t}}{P_t} y_{j,t} dj$ then I can integrate both sides of the previous equation and, using the definition of aggregate markups for this economy, get

$$\mathcal{M}_t = \sum_i \int_j \mu_{j,t} \frac{h_{j,t}}{H_t^g} dj, \tag{B.11}$$

so that the aggregate markup is the cost-weighted average of firm-level markups.

Alternatively, it is possible to rewrite Equation (B.10) as

$$\frac{\frac{p_{j,t}}{P_t} y_{j,t}}{Y_t} \frac{1}{\mu_{j,t}} = \frac{h_{j,t}}{H_t^g} \frac{1}{\mathcal{M}_t},$$

and, after integrating both sides over the distribution of active firms, get

$$\mathcal{M}_t = \left(\sum_i \int_j \frac{\frac{p_{j,t}}{P_t} y_{j,t}}{Y_t} \frac{1}{\mu_{j,t}} dj \right)^{-1}. \tag{B.12}$$

Therefore, consistently with Edmond *et al.* [2018], also in the model presented in this paper aggregate markups can be derived as the cost-weighted average of firm level markups or as the harmonic weighted average of firm markups with firms' sales shares over GDP as weights.

B.5 Proof of Proposition 1.

Proposition 1. The optimal markup management of listed firms can be summarized by the following condition:

$$\mu_{j,t}^{-1} - \bar{\mu}^{-1} = Q_t F_y(\cdot|_t) \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t}}{b_{j,t}} \left[1 + \mathbb{E} \sum_{\tau=1}^{\infty} \prod_{s=0}^{\tau-1} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau} q_{t,t+\tau} \frac{p_{j,t+\tau}}{p_{j,t}} \frac{y_{j,t+\tau}}{y_{j,t}} \frac{b_{j,t}}{b_{j,t+\tau}} \right]$$

where $\widetilde{\rho}(a_{j,t}) = (1-\rho(a_{j,t})$ is the surviving probability up to t, $\mu_{j,t} = \frac{w_t h_{j,t}}{\alpha y_{j,t}} \frac{P_t}{p_{j,t}}$ is the firm-level markup and $\bar{\mu}$ is the markup under standard monopolistic competition that would prevail without dynamic incentives in pricing.

Proof. From the first order conditions of an incumbent, equations (B.7),(B.8) and (B.9) we have that the Euler equation for the marginal value of customer base is

$$\gamma_t = \frac{1}{\eta} k_i'(b_{j,t}) Y_t \left(\frac{k_i(b_{j,t}) Y_t}{y_{j,t}} \right)^{\frac{1}{\eta}-1} + (1-\rho(a_{j,t}))(1-\delta)\mathbb{E}\left[\Lambda_{t,t+1}\gamma_{t+1}\right].$$

given that $k_i(b_{j,t}) = \kappa_i b_{j,t}^{\varepsilon_b^i}$, we can multiply and divide by $k_i(b_{j,t})$ and $b_{j,t}$ to get

$$\gamma_t = \frac{\varepsilon_b^i}{\eta} \left(\frac{k_i(b_{j,t}) Y_t}{y_{j,t}} \right)^{\frac{1}{\eta}} \frac{y_{j,t}}{b_{j,t}} + (1-\rho(a_{j,t}))(1-\delta)\mathbb{E}\left[\Lambda_{t,t+1}\gamma_{t+1}\right].$$

Denote the survival probability to the current period for a firm of age a as $\widetilde{\rho}(a_{j,t}) = (1 - \rho(a_{j,t})$ and consider that $\left(\frac{k_i(b_{j,t})Y_t}{y_{j,t}}\right)^{\frac{1}{\eta}} = \frac{p_{j,t}}{P_t}$ from the demand constraint. Therefore iterating forward this equation for T period we get:

$$\gamma_t = \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t}}{b_{j,t}} \frac{p_{j,t}}{P_t} + \mathbb{E} \sum_{\tau=1}^{T} \prod_{s=0}^{\tau-1} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau} \Lambda_{t,t+\tau} \frac{p_{j,t+\tau}}{P_{t+\tau}} \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t+\tau}}{b_{j,t+\tau}} + \prod_{s=0}^{T} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{T+1} \gamma_{t+T+1} \Lambda_{t,t+T+1},$$

taking the limit for $T \to \infty$:

$$\gamma_t = \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t}}{b_{j,t}} \frac{p_{j,t}}{P_t} + \mathbb{E} \sum_{\tau=1}^{+\infty} \prod_{s=0}^{\tau-1} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau} \Lambda_{t,t+\tau} \frac{p_{j,t+\tau}}{P_{t+\tau}} \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t+\tau}}{b_{j,t+\tau}} + \lim_{T\to\infty} \mathbb{E} \prod_{s=0}^{T} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{T+1} \gamma_{t+T+1} \Lambda_{t,t+T+1}.$$

The term $\mathbb{E} \prod_{s=0}^{T} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{T+1} \gamma_{t+T+1} \Lambda_{t,t+T+1}$ represents the discounted value, in utility terms, of an extra unit of customer base at infinity. Using an argument similar to the transversality condition in consumer problems, as profits are increasing in the size of customer base, at the optimal plan, an incumbent firm has to choose a path for its customer base such that this value is zero in the limit, thus

$$\gamma_t = \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t}}{b_{j,t}} \frac{p_{j,t}}{P_t} + \mathbb{E} \sum_{\tau=1}^{+\infty} \prod_{s=0}^{\tau-1} \widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau} \Lambda_{t,t+\tau} \frac{p_{j,t+\tau}}{P_{t+\tau}} \frac{\varepsilon_b^i}{\eta} \frac{y_{j,t+\tau}}{b_{j,t+\tau}}.$$

Now, using Equation (B.8), we can express the value of γ as deviations of firm-level markups from the monopolistic competition level. Using the demand constraint, we can rewrite Equation (B.8) as follows:

$$\gamma_t Q_t F_y(\cdot) = \frac{p_{j,t}}{P_t}\left[\frac{P_t}{p_{j,t}}\frac{w_t}{\alpha A_t h_{j,t}^{\alpha-1}} - \left(1-\frac{1}{\eta}\right)\right],$$

and given that by definition $1-\frac{1}{\eta} = \bar{\mu}^{-1}$ and $\frac{P_t}{p_{j,t}}\frac{w_t}{\alpha A_t h_{j,t}^{\alpha-1}}$ is the inverse of the ratio of firm level prices to marginal costs, hence a measure of firm-level markups $\mu_{j,t}^{-1}$, we can rewrite the equation as

$$\frac{(\mu_{j,t}^{-1} - \bar{\mu}^{-1})}{Q_t F_y(\cdot)}\frac{p_{j,t}}{P_t} = \gamma_t.$$

Combining the definition of γ_t from the forward iteration with the expression above then implies that

$$\frac{(\mu_{j,t}^{-1} - \bar{\mu}^{-1})}{Q_t F_y(\cdot)}\frac{p_{j,t}}{P_t} = \frac{\varepsilon_b^i}{\eta}\frac{y_{j,t}}{b_{j,t}}\frac{p_{j,t}}{P_t} + \mathbb{E}\sum_{\tau=1}^{+\infty}\prod_{s=0}^{\tau-1}\widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau}\Lambda_{t,t+\tau}\frac{p_{j,t+\tau}}{P_{t+\tau}}\frac{\varepsilon_b^i}{\eta}\frac{y_{j,t+\tau}}{b_{j,t+\tau}},$$

which in turn can be rearranged as the condition in the proposition

$$\mu_{j,t}^{-1} - \bar{\mu}^{-1} = Q_t F_y(\cdot|_t)\frac{\varepsilon_b^i}{\eta}\frac{y_{j,t}}{b_{j,t}}\left[1 + \mathbb{E}\sum_{\tau=1}^{\infty}\prod_{s=0}^{\tau-1}\widetilde{\rho}(a_{j,t+s})(1-\delta)^{\tau} q_{t,t+\tau}\frac{p_{j,t+\tau}}{p_{j,t}}\frac{y_{j,t+\tau}}{y_{j,t}}\frac{b_{j,t}}{b_{j,t+\tau}}\right]$$

using the fact that $q_{t,\tau} = \mathbb{E}\Lambda_{t,t+\tau}\frac{P_t}{P_{t+\tau}}$ from household's optimality conditions. □

C Details on empirical methodology and robustness checks

C.1 Sample selection and data cleaning

The main data source used in the paper is the annual version of North-America COMPUSTAT accessed through WRDS on April, 2020. For the sub-sample of firms with founding dates, I rely on the *Field-Ritter Database of Founding Dates.* I merge it to the main sample using the links between firm identifiers in COMPUSTAT and firm identifiers in the CRSP Database (accessed using WRDS) and the Field-Ritter dataset. The aggregate time series at annual frequency for US real GDP (series ID: `GDPCA`) and unemployment rate (series ID: `UNRATE`) are from FRED.

- I consider only domestic, consolidated statements of industrial firms incorporated in the United States and that report their balance sheet in U.S. dollars;
- I exclude financial firms (SIC codes: 6900-6999) and utilities (SIC codes: 4900-4999);
- The capital stock is computed using the perpetual inventory method (PIM), iterating forward on the capital accumulation equation, $k_{j,t} = (1 - \delta_{j,t})k_{j,t-1} + i_{j,t}$.

  I initialize the value of the capital stock, $k_{j,0}$, with the first non-missing observation for the gross value of property, plant and equipment (`PPEGT`). For each firm, I then construct a series of net investments, $i_{j,t} - \delta_{j,t}k_{j,t-1}$, by taking the first difference of the reported net value of property plant and equipment (`PPENT`), i.e $i_{j,t} - \delta_{j,t}k_{j,t-1} = \texttt{PPENT}_{j,t} - \texttt{PPENT}_{j,t-1}$. In the few cases in which the value of `PPENT` is missing, I take a liner interpolation within each reporting spell using neighboring values before constructing the investment series. I deflate the series of net investment using the deflator for private, non-residential fixed investments (BEA code: `B008RG`);
- I drop observations with negative sales, capital or employment;
- For the main empirical results presented in the paper, I restrict the analysis to firms that report their first accounting data between 1960 and 2017. The main reason of focusing on this restricted sample is to avoid considering the set of very old firms that, by construction, report their first balance sheet in 1950. The results for the whole sample of firms are qualitatively very similar.

Table C.1: Summary statistics

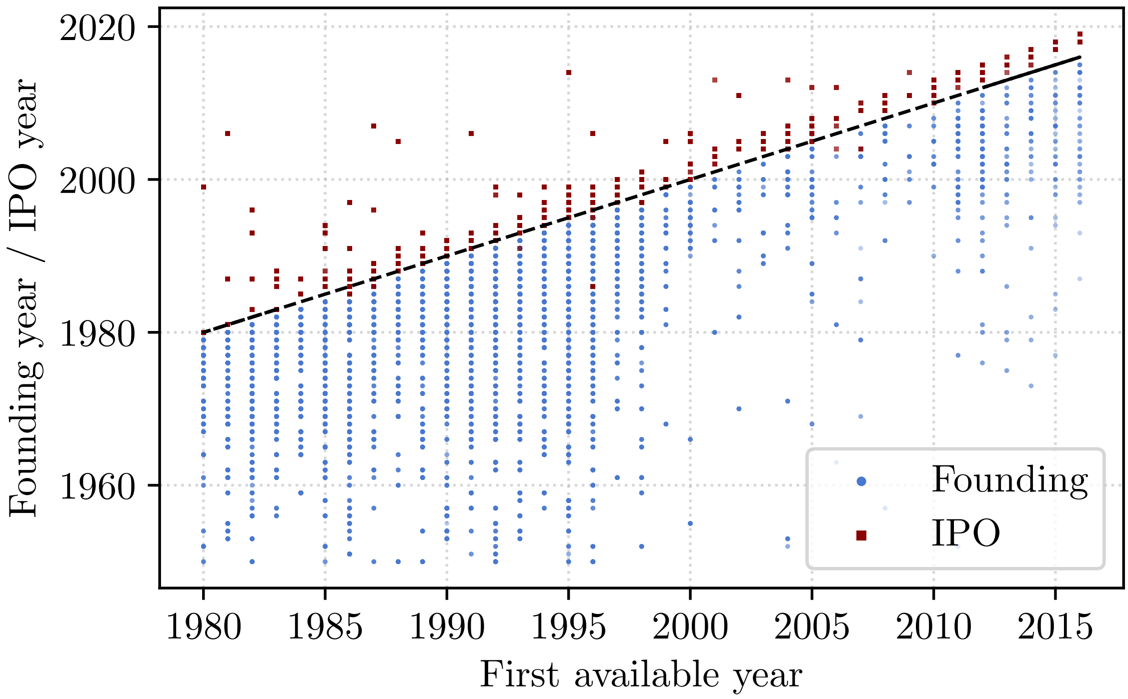
	Sales	Cost of Goods Sold	Employ-ment	Capital Stock *(book value)*	Capital Stock *(PIM)*	Age
Mean	1,896.29	1,309.84	7.41	1,660.52	1,217.25	13.16
Standard Dev.	10,706.65	8,162.24	35.50	11,423.76	7,865.71	10.62
25^{th} Percentile	26.814	16.47	0.15	9.46	7.04	5.00
Median	139.10	87.62	0.80	53.87	39.63	10.00
75^{th} Percentile	659.27	430.44	3.54	318.22	227.79	19.00
N	181,173	181,173	165,657	179,910	178,603	183,381

Note: Main variables used for the estimation of the production function and the measurement of markups. Financial variables are deflated using GDP deflator (base year 2010). COMPUSTAT sample 1960-2017, firms observed from initial year of available accounting data.

C.2 Estimation of production function elasticities

The measurement of markups developed by Decker, Haltiwanger, Jarmin and Miranda [2017] relies on the estimation of the output elasticity to variable inputs.

Figure C.1: Joint distribution of *First Available Year-IPO Year* and *First Avail- able Year-Founding Year*



Note: The figure plots the joint distribution of *First Available Year-IPO Date* and *First Available Year-Founding Year* for the subset of Compustat firms for which I have the date of the IPO. Each dot is an observation in the sample and darker colors indicate higher mass. Founding dates are from the *Field-Ritter Database.*

Table C.2: Summary statistics, markups

	Markup			
	Cobb-Douglas *(time varying)*	Cobb-Douglas	TransLog	Cost Share
Mean	1.52	1.53	1.50	1.57
Standard Dev.	0.99	1.04	1.01	1.10
25^{th} Percentile	1.09	1.10	1.07	1.12
Median	1.30	1.30	1.30	1.34
75^{th} Percentile	1.64	1.65	1.64	1.69
N	174,581	183,381	177,386	181,173

Note: Markups measures. COMPUSTAT sample 1960-2017, firms observed from initial year of available accounting data.

For the baseline results presented in the paper, I estimate the following revenue trans-log production function for each two-digits NAICS code:

$$y_{j,t} = \theta_k k_{j,t} + \theta_v v_{j,t} + \theta_{kk} k_{j,t}^2 + \theta_{vv} v_{j,t}^2 + \theta_{kv} k_{j,t} v_{j,t} + \omega_{j,t} + u_{j,t}, \tag{C.1}$$

where $y_{j,t}$ are firm j's log-revenue at time t; $k_{j,t}$ the logarithm of its capital stock and $v_{j,t}$ the log-value of a bundle of variable inputs.

As usual, the main identification challenge in production function estimation is the simultaneity bias induced by the unobserved time-varying firm level productivity, $\omega_{j,t}$.

I follow the proxy variable literature,[59] and in particular De Loecker and Eeckhout [2017], to estimate the production function in (C.1) using a two step approach based on the use of a control function for the productivity process. The identification relies on the observation that the optimal choice of firms about a variable input follows a policy function like $v_{j,t} = v(k_{j,t}, \omega_{j,t})$. Then, providing that the policy function is invertible[60], the productivity process can be proxied by a control function so that $\omega_{j,t} = \omega(k_{j,t}, v_{j,t})$ with $\omega(\cdot) = v^{-1}(\cdot)$.

First step. In the first step I clean the output value from measurement errors and unanticipated productivity shocks using a second order polynomial of capital and variable inputs

$$y_{j,t} = P(k_{j,t}, v_{j,t}; \phi) + u_{j,t},$$

where $P(\cdot)$ is a composite function of the productivity and the unknown control function.[61]

Second step. Using the estimates of ϕ from the previous stage I can construct a measure of productivity that does not depend on the measurement error $u_{j,t}$. That is,

$$\omega_{j,t} = P(k_{j,t}, v_{j,t}; \hat{\phi}) - [\theta_k k_{j,t} + \theta_v v_{j,t} + \theta_{kk} k_{j,t}^2 + \theta_{vv} v_{j,t}^2 + \theta_{kv} k_{j,t} v_{j,t}].$$

[59] The proxy variable literature, pioneered by Pakes [1994] relies on adopting a control function to estimate the production function. An alternative approach is given by the literature on Dynamic Panel data, for more detais see Blundell and Bond [2000].

[60] Pakes [1994] proves the invertibility of policy functions associated to a wide class of production functions.

[61] While it is possible to estimate the coefficient on variable inputs directly at this step, as noted by Ackerberg, Caves and Frazer [2015], it is more efficient to use the first stage only to clean the output variable from potential measurement errors and estimate all the production function coefficients in the second stage.

Then, exploiting the assumption that productivity follows an AR(1) process, is it is possible to construct a measure of productivity innovations, $\xi_{j,t}(\Theta)$ projecting $\omega_{j,t}$ on $\omega_{j,t-1}$. Under the assumption that firms react to unanticipated productivity shocks contemporaneously so that the lagged values of variable inputs can be used as valid instruments, the production function coefficients in Θ can be identified using the following moment conditions:

$$\mathbb{E}\left[\xi_{j,t}(\Theta)\begin{pmatrix} k_{j,t} \\ v_{j,t-1} \\ k_{j,t}^2 \\ v_{j,t-1}^2 \\ k_{j,t}v_{j,t-1} \end{pmatrix}\right] = 0. \tag{C.2}$$

The output elasticity to variable inputs that is relevant for the measure of markups in Equation (3) therefore is:

$$\theta_{j,t}^v = \theta_v + 2\theta_{vv}v_{j,t} + \theta_{kv}k_{j,t}. \tag{C.3}$$

With a Cobb-Douglas specification of the production function the relevant measure for the output elasticity to variable inputs, $\theta_{j,t}^v$, is given by Equation (C.3) without the cross-products.

C.2.1 Discussion on limitations

Due to data availability I am not able to separate between prices and quantities for my measures of firm-level output. As also noted by Bond *et al.* [2020], this is problematic as variations in output and input prices could bias the estimates of the output elasticities and hence the measurement of markups.

To see how lack of price data can make the estimation problematic, consider the Cobb-Douglas version of (C.1). As discussed by De Loecker *et al.* [2020], $y_{j,t} = q_{j,t} + p_{j,t}$ so that

$$q_{j,t} + p_{j,t} = \theta_v\tilde{v}_{j,t} + \theta_l\tilde{k}_{j,t} + p_{j,t} - \sum_{i\in\{k,v\}}\theta_i p_{j,t}^i + \omega_{j,t} + u_{j,t}$$

with $\tilde{x}_{j,t} = x_{j,t} + p_{j,t}^x$ being the deflated values of input x, and $p_{j,t}^x$ the input price paid by firm j. Note that if we consider the standard specification when coefficients do not change over time, then the bias caused by not observing prices would only affect the level of markups, but not their time-series behavior.

However, there is still the concern that firm-specific shocks could be reflected in input and output prices. This pass-through is not an issue in the unlikely case that variations in output prices are completely offset by variations in input prices. In more realistic settings, in which the pass-trough of shocks between input and output prices is not perfect, firms will be able to create a wedge between the input price bundle and the output price. Yet, assuming a constant returns to scale production function we can link the size of the bias induced by the incomplete pass-through exactly to marginal costs, $\lambda_{j,t} = \sum_{i\in\{k,v\}}\theta_i p_{j,t}^i - \omega_{j,t}$.

Given that the optimal pricing strategy implies a markup over marginal costs we can rewrite the price as, $p_{j,t} = \lambda_{j,t} + \mu_{j,t}$ and substituting it back in measurement equation for the production function and get

$$q_{j,t} + p_{j,t} = \theta_v\tilde{v}_{j,t} + \theta_l\tilde{k}_{j,t} + \mu_{j,t} + u_{j,t}.$$

Hence, as long as it possible to control for the markup that a firm is allowed to charge we can correctly estimate the output elasticities following the procedure outlined in the previous section. As the markup is unobserved, it is possible to approximate them with a function whose arguments are relevant determinants of markups.

In my empirical application I follow De Loecker *et al.* [2020] and I include a sector-year linear function in firm's market share and productivity as an approximation for markups. This is clearly a second-best solution to an important issue in the estimation of the production function and therefore of markups' levels. I verify that the Θ estimates are robust to various specification of the production function, but without more detailed data on output quantities, it is not possible to go beyond these second-best solutions to the problem.

C.3 Measuring markups in a model with customer base accumulation and dynamic pricing

The estimation strategy for firm-level markups outlined in the main text relies on the assumption that pricing decisions are static. In other words, firms reset their prices every period without being subject to price rigidities (like menu costs or other adjustment costs).

However when firms' customer base accumulation is influenced by the level of their demand, even without price rigidities, their pricing choices are going to be dynamic as their current production contributes to the determination of their customer base and hence to the level of future demand.

To see how, consider a general setting where a firm j produces output using only labor and faces a demand function $D(p_{j,t}, b_{j,t})$ that depends on prices and the level of customer base b. Firms accumulate customer base through sales, so that customer base investments are a function of firm j's sales $F(y_{j,t})$.

The Lagrangian of a firm that has incentives to use prices to expand production and accumulate customer takes the following form:

$$\mathcal{L} = \max_{p_{j,t}, y_{j,t}, b_{j,t+1}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \Lambda_{t,t+1} \left\{ \begin{array}{c} p_{j,t}y_{j,t} - P_t w_t h_{j,t} \\ -\lambda_{j,t}[y_{j,t} - f(h_{j,t})] \\ -\gamma_{j,t}[y_{j,t} - D(p_{j,t}, b_{j,t})] \\ -\phi_{j,t}[b_{j,t+1} - (1-\delta)b_{j,t} - F(y_{j,t})] \end{array} \right\} \tag{C.4}$$

where the λ constraint is the technology the firms uses, the γ constraint is the demand faced by firm j and the ϕ constraint is the law of motion of firm j's customer base. The optimal choice of the firm is described by the following first order conditions:

$$[p_{j,t}]: \quad y_{j,t} + \gamma_{j,t} D_p(\cdot) = 0 \tag{C.5}$$
$$[y_{j,t}]: \quad p_{j,t} - \lambda_{j,t} - \gamma_{j,t} + \phi_{j,t} F_y(\cdot) = 0 \tag{C.6}$$
$$[h_{j,t}]: \quad -P_t w_t + \lambda_{j,t} f_h(h_{j,t}) = 0 \tag{C.7}$$
$$[b_{j,t+1}]: \quad \phi_{j,t} = \mathbb{E}_t \left[\Lambda_{t,t+1}((1-\delta)\phi_{j,t+1} + \gamma_{j,t+1} D_b(\cdot|_{t+1}))\right] \tag{C.8}$$

Using equations (C.5), (C.6) and (C.7), keeping in mind that $\frac{D_p(\cdot)p_{j,y}}{y_{j,t}} \equiv \eta_{j,t}$ and $\frac{f_h(\cdot)h_{j,t}}{y_{j,t}} \equiv \theta_t^h$, I obtain the following condition that links firm-level markups and the expenditure share on variable inputs

$$\theta_t^h \left(\frac{P_t w_t h_{j,t}}{p_{j,t} y_{j,t}}\right)^{-1} = \frac{\eta_{j,t}}{\eta_{j,t} - 1} \left(1 - F_y(\cdot) \frac{\phi_{j,t}}{\lambda_{j,t}}\right). \tag{C.9}$$

The left hand side of equation (C.9) is the De Loecker and Warzynski [2012] measurement equation. Note that if current output does not play any role in creating customer base (i.e. $F_y(\cdot) = 0$) then this condition collapses back to the standard condition $\theta_t^h \left(\frac{P_t w_t h_{j,t}}{p_{j,t} y_{j,t}}\right)^{-1} = \frac{\eta_{j,t}}{\eta_{j,t}-1}$. The willingness of firms to use prices to expand their customer bases however, introduces a negative bias to the standard measure of markups used in the literature. The size of the bias depends on the relative magnitude of the multiplier on the customer base accumulation process to the one on the technology used in production. In the model the size of the Lagrange multipliers depend on the state variable, hence we can think of the bias as being a function of firms' customer bases. I exploit this fact and I include a second order polynomial in firms' sector shares in three digits sectors to try to mitigate this problem as much as possible.

C.4 Robustness checks

The quality of the data, the difficulty in measuring firm age and the intrinsic difficulties in the estimation of markups at the firm level are all possible causes of concern for the results discussed so far. In this section, I analyze a battery of robustness checks that address the most important issues affecting the measurement of markups and firm ages.

C.4.1 Robustness checks on the baseline specification

As robustness checks, I estimate the cohort effects using also different proxy variables for aggregate conditions. The results are reported in Table C.3, I am able to preserve the negative effect of aggregate conditions on markups for most of the proxy variables considered. The one that completely fails is the simple indicator of recessions. This results should not be particularly worrying as the recession indicator is a poor proxy variable for aggregate conditions as it pools firms of very different recessions together. In addition, as I use yearly data, the variation in the recession in the recession indicator is very low as it is assigns a disproportionate amount of firms to booms.

As the main sample of analysis is based on firms that are already mature, I include fixed effects for deciles of initial firm sizes and deciles of initial sales to account for differences on markups that could affect markups through sizes and not only aggregate conditions close to the time of listing.

Table C.4 reports the coefficients of interest for the baseline specifications in Equation (4) and (5) with either initial size or initial sales controls. As expected, not including controls for firms sizes the average effect becomes insignificant indicating that some of the effects of business cycles on markups are mediated from an effect on size. Decomposing the effect of aggregate condition close to the time of listing along the age profile of firms, in both cases we recover the baseline result that these effects are stronger at the beginning of firms' listed lives, albeit of a smaller magnitude compared with the baseline specification highlighting once again the importance of controlling for initial firm sizes.

Table C.3: Cohort effects estimates with different proxy variables

	HP Filtered GDP		Hamilton-Filtered GDP		Unemployment rate		Recession Indicator	
Dep.Variable: Log-Markup	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cycle Measure	-0.092	-0.480**	0.015	-0.292***	0.006***	0.008***	-0.022***	0.011
	(0.108)	(0.195)	(0.049)	(0.092)	(0.001)	(0.003)	(0.004)	(0.008)
Cycle Measure × Age		0.040***		0.032***		-0.0002		-0.003***
		(0.015)		(0.007)		(0.0002)		(0.001)
Age FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Initial Sale Decile FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
N	91,317	91,317	91,317	91,317	91,317	91,317	91,317	91,317

Notes: Robust standard errors in parentheses, p :$^{***} < 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports estimates from Equation (4) and (5) using different proxy variables for initial aggregate conditions. In particular, I report the main coefficient of interest for Hodrick-Prescott filtered log real GDP (smoothing equal to 6.25); Hamilton filtered log real GDP (one lag and two leads); the unemployment rate (in percent) and a dummy indicator for when the first available year in the data is indicated as an NBER recession.

Table C.4: Cohort effects with either size or sale controls

Dep.Variable: Log-Markup	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Cycle measure	-0.206***	-0.653***		0.002	-0.275**		0.083	-0.275**	
	(0.065)	(0.137)		(0.055)	(0.109)		(0.055)	(0.112)	
Cycle measure × Age		0.045***			0.028***			0.037***	
		(0.010)			(0.008)			(0.008)	
Cycle measure × Age_0			-0.882**			-0.592**			-0.531*
			(0.393)			(0.284)			(0.297)
Cycle measure × Age_1			-0.827**			-0.399			-0.345
			(0.353)			(0.266)			(0.275)
Cycle measure × Age_2			-0.375			-0.066			-0.013
			(0.298)			(0.242)			(0.249)
Cycle measure × Age_3			-0.549*			-0.275			-0.231
			(0.315)			(0.262)			(0.267)
Cycle measure × Age_4			-0.679**			-0.184			-0.147
			(0.303)			(0.257)			(0.262)
Cycle measure × Age_5			-0.945***			-0.180			-0.189
			(0.324)			(0.266)			(0.270)
Cycle measure × Age_{6-10}			-0.180			-0.077			-0.088
			(0.134)			(0.115)			(0.116)
Cycle measure × Age_{11-15}			0.224			0.360***			0.484***
			(0.137)			(0.114)			(0.115)
Cycle measure × Age_{16-20}			0.220			0.437***			0.642***
			(0.136)			(0.123)			(0.122)
Cycle measure × Age_{21-25}			-0.009			-0.086			0.084
			(0.140)			(0.125)			(0.126)
Age FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes	No	No	No	No	No	No
Initial Sale Decile FE	No	No	No	Yes	Yes	Yes	No	No	No
R^2	0.12	0.13	0.13	0.12	0.12	0.12	0.10	0.10	0.10
N	91,317	91,317	91,317	153,255	153,255	153,255	153,255	153,255	153,255

Notes: Robust standard errors in parentheses, p :$^{***}< 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports the main estimates of the elasticities between firm-level markups and aggregate conditions at the time the firm is first observed. Cohort effects are proxied using quadratically detrended log real GDP. Columns (1-2), (4-5) and (7-8) report estimates using the specification in Equation (4) and (5) with either initial size or initial sale controls. Columns (3), (6) and (9), instead, report the result of the same exercise for different age groups. Increasing the number of bins to control for initial firm sizes and sales does not affect the qualitative results reported in this table. The measure of business cycle is quadratic detrended real log-GDP.

C.4.2 Alternative production function: Cobb-Douglas

The ratio estimator of firm-level markups in Equation (3) is highly dependent on correctly estimating the elasticity of firm-level output to variable inputs. An incorrect measure of this elasticity would introduce a bias in the scale of markups. While this bias may not be too problematic for the analysis of changes in markups and their long-run trends, this parameter is particularly relevant in the context of the exercise developed in this paper as I estimate the level effect of business cycles on the average markup charged by different cohorts of firms.

Besides the common issues related to the identification of production functions' parameters, the estimation of a production function in a long panel, like the one I use in this paper, poses an additional conceptual problem linked to the time-frame used for the estimation of the technology used by firms. Pooling all years together and estimating the production function by sector implicitly assumes that the technology available to firms at the beginning of the sample is exactly the same as the one available to more modern enterprises. This is obviously an unrealistic strong assumption that would have relevant consequences for the measurement of markups if not only the technology but also the output elasticities with respect to inputs were constant over time.

The choice of the Translog as the baseline specification for the production function is in fact dictated by a compromise between stability of the estimation across sectors and allowing a time-varying elasticity of output to variable inputs. In fact, even if the baseline production function is constant over time, the relevant elasticity for the measurement of markups is allowed to vary, as shown in Equation (C.3). Nonetheless, the estimated production function is still assumed constant throughout the sample.

Therefore, in order to to address this issue thoroughly, in this subsection I describe the effects of changing the assumption on the nature of the technology on the main results of the paper. In particular, I re-estimate output elasticities for a Cobb-Douglas production function in two cases, one in which the technology available to firms is allowed to vary across sectors and time - *Sector-time varying elasticity* - and another one, more basic, where the technology is allowed to vary only across sectors - *Constant elasticity.*[62]

Sector-time varying elasticity. To allow for a sector-time varying technology, I estimate a revenue production function such as:

$$y_{j,t} = \theta_v v_{j,t} + \theta_k k_{j,t} + \omega_{j,t} + u_{j,t}$$

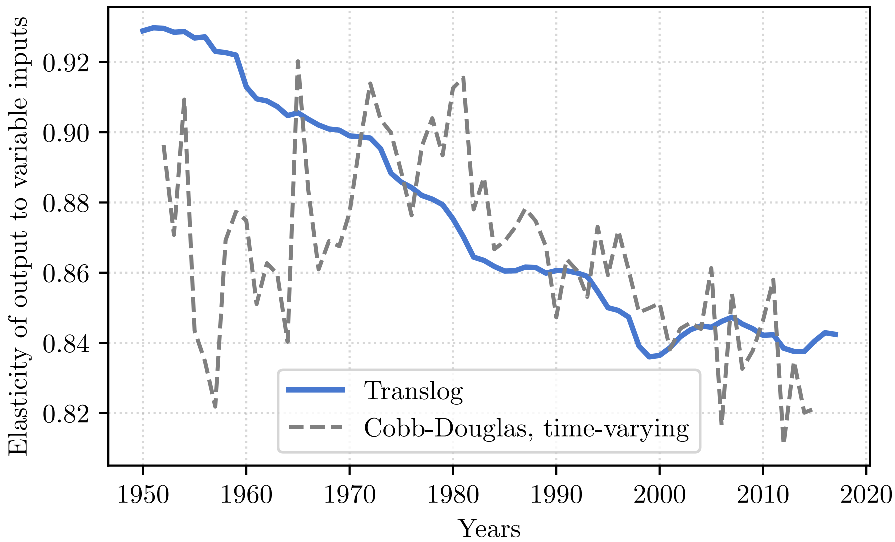
for each two-digits NAICS on a five-year rolling window. This allows to construct a series of sector-time varying elasticities to variable inputs $\{\hat{\theta}_{s,t}\}$ from 1952 to 2015.

Constant elasticity. As a useful benchmark to compare the results to, I estimate the Cobb-Douglas production function across two-digits sectors, pooling all years.

Figure C.2 plots the sector average of this estimated elasticities together with the average elasticity estimated using the baseline Translog production function in Equation (C.3). The figure shows a significant time volatility of output elasticities and a downward trend in both measures of θ_v, especially from 1980. Both estimates show a significant change in the output elasticity over time. The Cobb-Douglas estimate moves from approximately an average of 0.88 between 1950 and 1970 to 0.83 after 2000. A similar decline is captured in the evolution of the elasticity to variable inputs derived from the Translog production function. Therefore,

[62] While the former is a more coherent robustness check as it allows for a time-varying elasticity, I find it useful to report also the constant elasticity case as it provides a useful benchmark, also against other estimates in the literature, in the estimation of the production function.

Figure C.2: Sector-time varying output elasticities to variable inputs



Note: The figure plots the averages across 2 digits sectors of the elasticities of output to variable inputs estimated using a Translog and a Cobb-Douglas production function. The Cobb-Douglas production function is estimated separately for each two-digits NAICS on a five year rolling window to allow changes in the estimated parameters from 1952 to 2015. The elasticity for the Translog function, instead, is estimated for each two-digits sector pooling all years from 1950 to 2017 and then constructed averaging the firm-level elasticities constructed following Equation (C.3).

even if the baseline estimation using the Translog production function is assuming a constant technology over time, the time series behavior of the implied elasticity to variable inputs is in line with a more flexible specification of the production function that takes into account the time-varying nature of the technology available to firms.

Table C.5 reports the estimates of the cohort effects for markups measured using both the constant elasticity and the time-varying specification. The estimated effect of business cycles on markups is larger when time variation in technology are allowed in the estimation. In particular, allowing for time variation in θ_v implies an effect of business cycles on markups 25% higher (the coefficient moves from -0.302 to -0.385). For the time-varying specification, this implies that firms starting the listing process when aggregate output is two-standard deviation below trend report markups that are approximately 2.6% higher than similar firms that instead start the process with aggregate output being on trend. When I use the estimate of the production function with constant elasticity, the same change in output would result in a 2% difference in markups, which is close to the baseline effect estimated using the Translog specification. These estimates mean that firms listed at the height of the Great Recession, in 2009, were able to charge a markup respectively 3% and 2.3% higher than if they decided to go public in 2007. The estimated persistence, instead, is very similar between the two specifications and the baseline estimates.

Table C.5: Cohort effects with Cobb-Douglas production function

	Sector varying θ_v			Sector-Time varying θ_v		
Dep. Variable: Log-Markup	(1)	(2)	(3)	(4)	(5)	(6)
Cycle measure	-0.302***	-0.749***		-0.385***	-0.910***	
	(0.065)	(0.137)		(0.065)	(0.137)	
Cycle measure × Age		0.045***			0.053***	
		(0.010)			(0.010)	
Cycle measure × Age_0			-1.038***			-1.194***
			(0.384)			(0.389)
Cycle measure × Age_1			-0.829**			-0.928***
			(0.342)			(0.346)
Cycle measure × Age_2			-0.489			-0.683**
			(0.301)			(0.299)
Cycle measure × Age_3			-0.656**			-0.941***
			(0.326)			(0.321)
Cycle measure × Age_4			-0.798**			-0.901***
			(0.326)			(0.326)
Cycle measure × Age_5			-1.067***			-1.166***
			(0.335)			(0.339)
Cycle measure × Age_{6-10}			-0.263*			-0.311**
			(0.137)			(0.136)
Cycle measure × Age_{11-15}			0.111			0.085
			(0.136)			(0.135)
Cycle measure × Age_{16-20}			0.134			-0.001
			(0.134)			(0.136)
Cycle measure × Age_{21-25}			-0.100			-0.023
			(0.138)			(0.139)
Age FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes	Yes	Yes	Yes
Initial Sale Decile FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.08	0.08	0.08	0.08	0.08	0.08
N	92,336	92,336	92,336	90,458	90,458	90,458

Note: Robust standard errors in parentheses, $p :^{***} < 0.01, ^{**} < 0.05, ^{*} < 0.1$. The table reports the coefficient of interest for the specifications in Equation (4) and (5) plus age group specific age effects. Markups are measured using the elasticity of output to variable inputs, θ_v, estimated from a Cobb-Douglas production function in two ways: i) pooling all years in the sample - Columns (1) to (3); and ii) estimating the production function coefficients on five-year rolling windows for each two-digits sector - Columns (4) to (6). The measure of business cycle is quadratic detrended real log-GDP.

Figure C.3 shows the estimated cohort effects and the age profiles of markups for firms that are first observed in booms and in recessions for the two Cobb-Douglas estimations of the production function. The resulting age profiles are remarkably similar, indicating a second-order role for the output elasticity of variable inputs in the determination of the cohort effects for markups. Compared to the baseline profiles in Figure 1b, both Cobb-Douglas specifications deliver a higher effect of business cycles on the age profiles. A negative two-standard deviation of the cycle component of GDP from its trend results in a 12% higher initial markup when I consider the sector-time variation in output elasticities, as shown in Figure C.3a. The estimates of the cohort effects with the constant Cobb-Douglas production function instead, shown in Figure C.3b, deliver an initial markup approximately 9% higher for firms that experience the same change in the cycle component of GDP. This last result is closer in magnitude to the baseline results obtained with the Translog specification.

In terms of persistence, the cohort effects estimated with both Cobb-Douglas production functions and the ones estimated using the Translog show the same pattern. Firms that are listed during bad economic times tend to have higher markups for approximately 15 years of their lives as listed firms.

It is important to note that, despite delivering very similar average elasticities, the time varying estimation of the Cobb-Douglas production function, measures a larger effect of initial aggregate conditions on markups. The reason for this discrepancy can be due to the changing nature of technology for different cohorts of firms. The estimation of the production function for each sector across a relatively short horizon, allows to incorporate in the cohort effects on markups the consequences of business cycles on the technology used by firms that get listed at different moments of the cycle. These effects, that are tightly linked to the changing nature of the production process are not fully captured by the Translog specification despite featuring a firm specific elasticity of output to variable inputs.[63]

C.4.3 Alternative measure of output elasticity to variable inputs: cost shares

The measure of markups at the firm-level relies heavily on the estimation of the output elasticity to variable inputs. As the estimation of the production function without price and quantity data risks to be biased, I report the main empirical analysis of the paper for a measure of markups where the elasticity of output to variable inputs is not coming from a direct estimation of the production function but rather from the direct measure of the cost-share in variable inputs.[64] In order to compute total costs I assume a common user cost of capital of 12% that includes an exogenous depreciation rate, the federal fund rate and risk-premia, as in De Loecker *et al.* [2020] and Gutiérrez and Philippon [2019]. I then average the share of expenditure on variable inputs over total costs within each year and 2-digits NAICS to compute a new sector-time varying $\theta_{s,t}$ to measure markups following Equation (3).

Also in this case, the qualitative effects of aggregate conditions at the time of first available balance sheet are very similar to the baseline results show in the main body of the paper. When markups are estimated using cost-shares the effects both on the average effect and its persistence are very similar to the ones estimated using the baseline model, as shown in Table C.6 and Figure C.4.

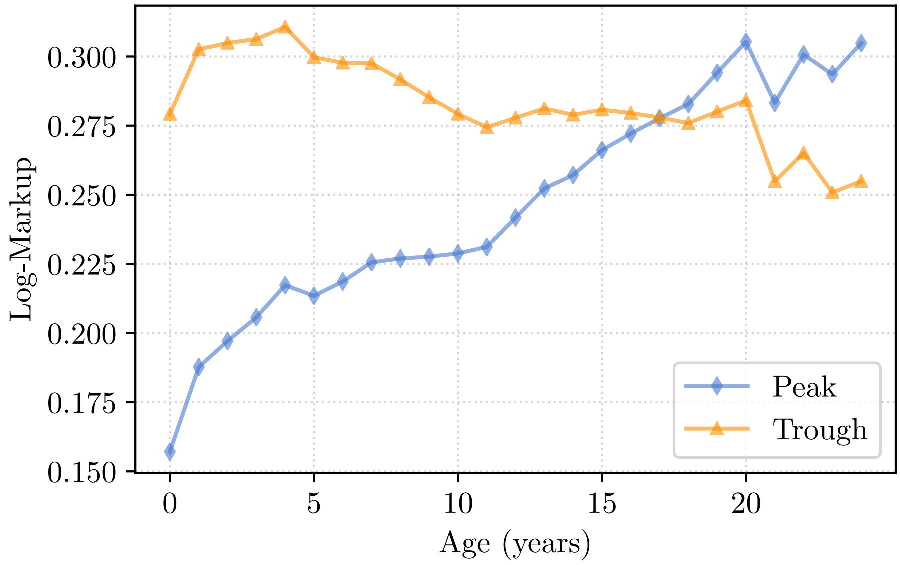
As for the baseline results, even when the output elasticity is estimated with the cost-share of variable inputs a positive two-standard deviation in output (approximately 6%) translates to a 2% drop in the average markup charged by firms. Decomposing this effect along the age

[63]Despite being a relevant aspect for the estimation of markups the investigation of different techniques to estimate production functions are beyond the scope of this project.

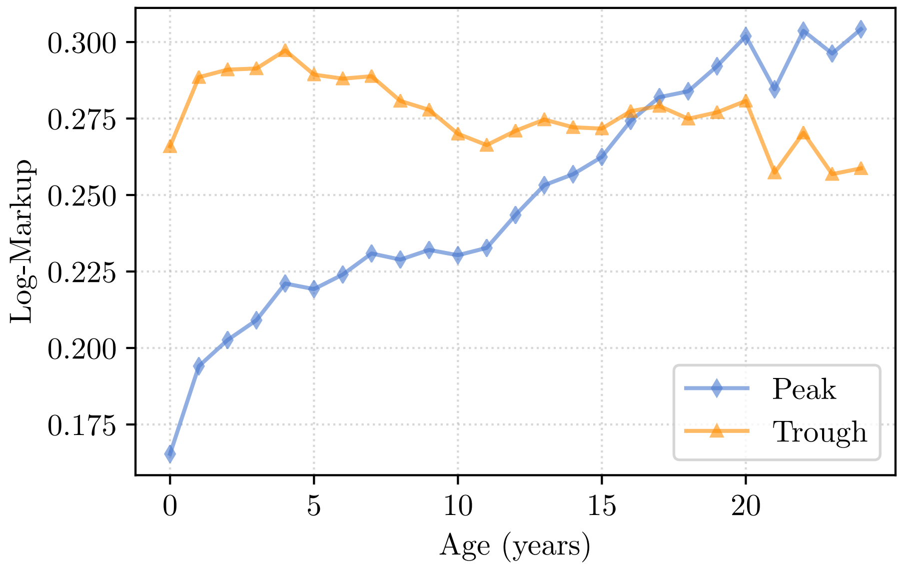
[64]Assuming that the production function follows a Cobb-Douglas and markets for inputs are competitive then the cost-share in variable inputs is a valid measure for the output elasticity.

Figure C.3: Cohort effects and age profiles for Cobb-Douglas production function

(a) Time-varying specification

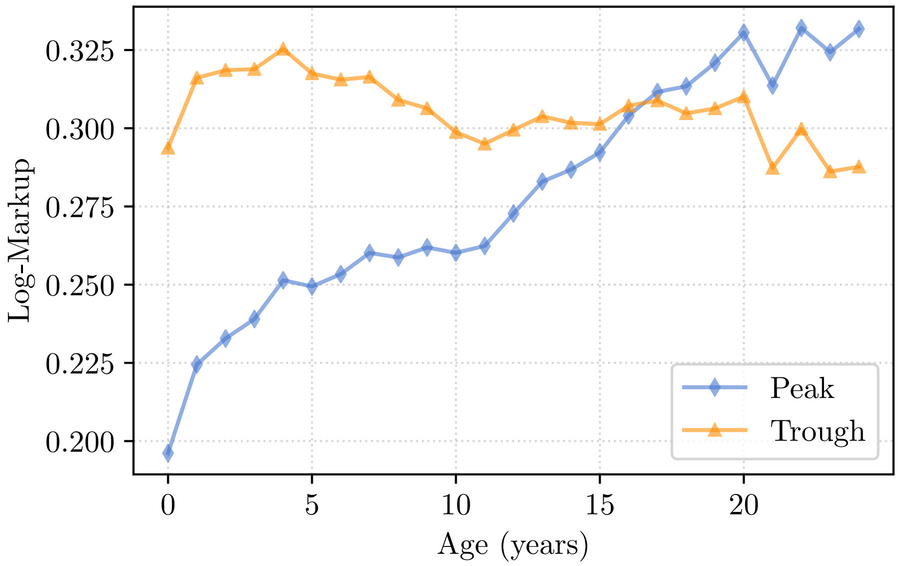


(b) Constant elasticity



Note: The figure plots the age profile for markups estimated from Equation (5) for markups measured using the output elasticity to variable inputs estimated from a Cobb-Douglas production function. Both the sector-time varying specification and the sector varying specification are reported. Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{\mu}_0$ is the average log-markup in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the first available year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of quadratically detrended real GDP. Table C.5, Column (2) reports the coefficients used to construct these graphs.

Figure C.4: Cohort effects on markups' age profiles - Cost shares



Note: The figure plots the age profile for markups estimated from Equation (5). Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{\mu}_0$ is the average log-markup in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the first available year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of quadratically detrended real GDP. The persistence coefficients underlying this plot are reported in Table C.6, Column (2).

profile of firms shows that the effect on the initial markups is more than twice as large as the average one - a positive two-standard deviation change in output generates a 4.9% drop in the markup charged in the first year a firm is observed compared to a similar firm that instead does not face any change in the cyclical component of GDP. The effect of the initial business cycles, as in the baseline case, vanishes approximately after fifteen years of firms lives as public companies. Using the Great Recession as a benchmark, these estimates imply that firms starting the listing process in 2009 have an average markup 2.25% higher than if they went public in 2007. The estimated impact on markups in the first year as listed, instead, is 5.6% higher for firms going public at the height of the Great Recession relative to similar firms going public in 2007.

C.4.4 Alternative measure of markups: Operating Expenditure

One of the main issues with Compustat data is linked to the fact that firms that follow different accounting standards are allowed to assign different expenditures to either *Cost of Goods Sold* (COGS) or to *Selling, General and Administrative Expenditures* (XSGA). As firms are allowed to choose which reporting standard to abide there is the risk that the variation in COGS does not reflect true differences in variable costs but rather differences in reporting standards.[65]

If the choice of the reporting standard was correlated with aggregate conditions at the time of first available data the estimate of the effect could be biased. To ensure that the negative effects of initial aggregate conditions are not due from heterogeneity in reporting standards I re-estimate Equation (4) using *Operating Expenditures* (OPEXP) as the main measure of

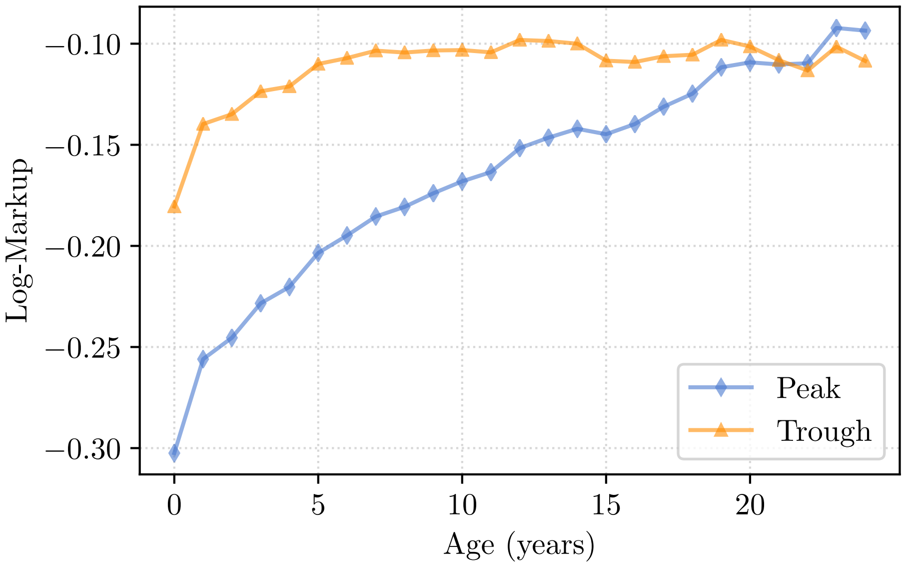
[65] See Traina [2018] and De Loecker *et al.* [2020] for an in-depth discussion of this issue.

Table C.6: Cohort effects for markups measured using cost shares

Dep. Variable: Log-Markup	(1)	(2)	(3)
Cycle measure	-0.294***	-0.727***	
	(0.065)	(0.137)	
Cycle measure × Age		0.044***	
		(0.010)	
Cycle measure × Age_0			-1.041***
			(0.384)
Cycle measure × Age_1			-0.819**
			(0.342)
Cycle measure × Age_2			-0.453
			(0.301)
Cycle measure × Age_3			-0.626*
			(0.326)
Cycle measure × Age_4			-0.781**
			(0.327)
Cycle measure × Age_5			-1.054***
			(0.336)
Cycle measure × Age_{6-10}			-0.247*
			(0.137)
Cycle measure × Age_{11-15}			0.113
			(0.136)
Cycle measure × Age_{16-20}			0.145
			(0.134)
Cycle measure × Age_{21-25}			-0.125
			(0.138)
Age FE	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes
Initial Sale Decile FE	Yes	Yes	Yes
R^2	0.10	0.10	0.10
N	92,336	92,336	92,336

Notes: Robust standard errors in parentheses, $p :^{***} < 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports the coefficient of interest for the specifications in Equation (4) and (5) plus age group specific age effects. Markups are measure using the elasticity of output to variable inputs measured using the cost share of variable inputs in total costs. Capital costs are included with gross user cost of capital of 12%. The measure of business cycle is quadratically detrended real log-GDP.

Figure C.5: Cohort effects on markups' age profiles - Operating Expenditure



Note: The figure plots the age profile for markups estimated from Equation (5). Specifically, at each age a, I am plotting $\hat{\mu}_0 + \hat{\phi}_a + (\hat{\beta}_0 + \hat{\beta}_1 a)Z$, where $\hat{\mu}_0$ is the average log-markup in the first available year, $\hat{\phi}_a$ are the estimated age fixed effects, $(\hat{\beta}_0, \hat{\beta}_1)$ capture the estimated persistence of aggregate conditions in the first available year. Z is a measure of business cycle outcomes that takes value 2σ for *Peak* and -2σ for *Trough*, with σ being the standard deviation of quadratically detrended real GDP. The persistence coefficients underlying this plot are reported in Table C.7, Column (2).

variable costs for the construction of variable costs.[66] Traina [2018] shows that this choice has a profound impact on the long-run trend of aggregate markups, however, for the purpose of this paper the two measures deliver qualitatively very similar results, as shown in Table C.7 and Figure C.5.

In fact, when *Operating Expenditures* is used to measure the the sales to variable cost ratio in Equation (3), the effects of business cycles increase in magnitude compared to the baseline estimates that uses *Cost of Goods Sold* as measure of variable costs. As XSGA includes the fixed costs a firm has to sustain in order to carry-on production, the higher business cycle effects on markups when this measure is considered could be indication of the fact that the selection operated by aggregate conditions reflects on the whole structure of the firms, affecting also the fixed component of its cost structure.

This implies that a positive two-standard deviation in output (approximately 6%) translates to approximately -3.2% change in the average markup charged by firms. Therefore, when measured with operating expenditures, the Great Recession translates to an average increase in markups of approximately 3.7% (firms that start listing in 2009 versus 2007). As for other robustness checks, decomposing the effect along the life cycle of firms highlights how the initial effects are larger (approximately -6.1% in the initial year for a positive two-standard deviation change in the cycle component of GDP) and slowly fading away in twelve to fifteen years.

[66] In Compustat, *Operating Expenditures* = *Cost of Goods Sold* + *Selling, General and Administrative Expenditure*.

Table C.7: Cohort effects from markups measured using *Operating Expenditures*

Dep. Variable: Log-Markup	(1)	(2)	(3)
Cycle measure	-0.487***	-0.909***	
	(0.051)	(0.111)	
Cycle measure × Age		0.043***	
		(0.008)	
Cycle measure × Age_0			-0.971***
			(0.342)
Cycle measure × Age_1			-1.056***
			(0.266)
Cycle measure × Age_2			-1.005***
			(0.264)
Cycle measure × Age_3			-1.080***
			(0.268)
Cycle measure × Age_4			-0.940***
			(0.265)
Cycle measure × Age_5			-0.677***
			(0.230)
Cycle measure × Age_{6-10}			-0.511***
			(0.107)
Cycle measure × Age_{11-15}			0.062
			(0.095)
Cycle measure × Age_{16-20}			-0.225*
			(0.116)
Cycle measure × Age_{21-25}			-0.306***
			(0.106)
Age FE	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes
Initial Sale Decile FE	Yes	Yes	Yes
R^2	0.32	0.32	0.32
N	81,298	81,298	81,298

Notes: Robust standard errors in parentheses, $p :^{***} < 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports the main coefficient of interest for the specifications in Equation (4) and (5) plus age group specific age effects. Markups are constructed using *Operating Expenditures* instead of *Cost of Goods Sold* to mitigate misreporting of variable costs due to different accounting standards. The measure of business cycle is quadratically detrended real log-GDP.

Table C.8: Cohort effects from markups measured using *Net Operating profit margins*

Dep.Variable: Net Oper. Prof. Margins	(1)	(2)	(3)
Cohort-level GDP	-0.282***	-0.409***	-0.527***
	(0.049)	(0.106)	(0.166)
Cohort-level GDP × Age		0.013*	0.047
		(0.008)	(0.030)
Cohort-level GDP × Age^2			-0.002
			(0.001)
Age FE	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Initial Size Decile FE	Yes	Yes	Yes
Initial Sale Decile FE	Yes	Yes	Yes
R^2	0.14	0.14	0.14
N	91,317	91,317	91,317

Notes: Robust standard errors in parentheses, p :$^{***}< 0.01,^{**} < 0.05,^{*} < 0.1$. The table reports the main coefficient of interest for the specifications in Equation (4) and (5). Markups are constructed using *Net operating profit margins*. The measure of business cycle is quadratically detrended real log-GDP.

C.4.5 Alternative measure of markups: Net operating profit margin

Following Anderson *et al.* [2018], I construct a measure of net operating profit margins at the firm level as

$$(\text{Net Operating Profit Margin})_{j,t} = \frac{\text{Sale}_{j,t} - \text{OPEXP}_{j,t}}{\text{Sale}_{j,t}}.$$

The estimation of cohort effects for this variable are reported in Table C.8.

C.4.6 Non-parametric cohort effects

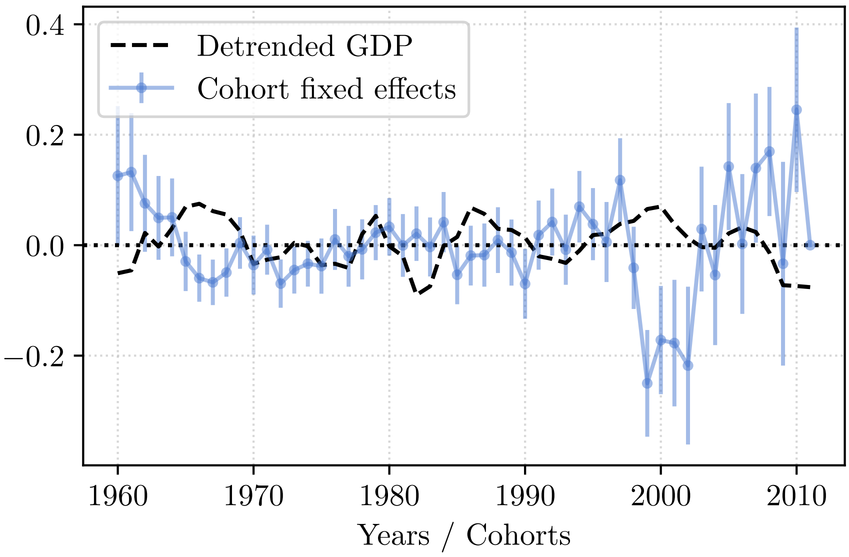
An alternative solution to the identification problem of cohort effects in age-cohort-time models is to choose a normalization for either the cohort effects or the time effects.[67] I follow Moreira [2015] and normalize the cohort effects so that they sum to zero and are orthogonal to a time trend. The implicit identification assumption that this normalization choice implies is that long-run trend are fully captured by the combination of age and time fixed effects.

Therefore, I estimate the effects of business cycle for cohorts of listing firms non-parametrically using the following specification at the firm-level:

$$\log(\mu_{j,t}) = \alpha + \phi_a + \phi_t + \sum_{c=1}^{C} \beta_c \mathbb{1}\{j \in c\} + \mathbf{X}_{j,t}\gamma + u_{j,t} \tag{C.10}$$

[67] Deaton [1997] proposes this normalization in the context of consumers life-cycle problems. See Schulhofer-Wohl [2018] for a discussion of the *time* versus *cohort* normalization in life-cycle models.

Figure C.6: Markups and non-parametric cohort effects



Note: The figure plots the cohort effects estimated using Equation (C.10) and the Hamilton filtered log-real GDP (one lag, two leads) for 51 cohorts of firms (1960-2011) that are observed for at least 5 years. The error bars plot the 95% confidence interval for each coefficient.

subject to

$$\sum_{c=1}^{C} \beta_c = 0,$$
$$\sum_{c=1}^{C} c\beta_c = 0.$$

where ϕ_a and ϕ_t are respectively age and time fixed effects; $\mathbb{1}\{j \in c\}$ is an indicator function that takes value one if firm j belongs to cohort c and $\mathbf{X}_{j,t}$ is a vector of firm-level controls that include size, a second order polynomial in firm j's sector shares and two digits sector fixed effects.

The constraints on Equation (C.10) force the cohort-effects to sum to 0. The identification assumption behind this normalization choice is that any long-run trend in firm-level markups can be controlled for by the age and time effects while the cohort effects, by being orthogonal to a time trend, capture the cyclical effects of markups for each cohort.

As a consequence, to check the robustness of the main estimates I compare the series of cohort-effects coefficients estimated in Equation (C.10) with the baseline measure of business cycles used in the baseline estimates.

Results. Figure C.6 plots the series of normalized coefficients in Equation (C.10) and the detrended GDP in the US. Visually, the time series of estimated cohort effects is very volatile, especially for younger cohorts, and it indicates that periods where aggregate GDP is above trend are also associated with negative cohort effects effects on markup. In fact, the two time series show a correlation coefficient of -0.37, significant at the 1% level, as reported in Table C.9. I interpret these results as supplementary evidence that cohort effects on markups exist and that they are negatively correlated with business cycle conditions.

Table C.9: Correlation between non-parametric cohort effects and measures of aggregate conditions

	Correlation coefficient with cohort effects
Quadratically detrended log-GDP	-0.376
	(0.006)
Hodrick-Prescott filtered log-GDP	-0.075
	(0.596)
Hamilton filtered log-GDP	-0.362
	(0.008)
Unemployment rate	0.277
	(0.047)

Note: p-value in parenthesis. The table reports the Pearson correlation coefficients, and the associated p-values, between cohort effects estimated using Equation C.10 and plotted in Figure C.6, with aggregate measures of aggregate conditions. I restrict the sample to firms that are observed for at least 5 years.