1. $\lim_{(x,y)\to(0,0)} f(x,y) = 1$

a. $f(x,y) = \dfrac{\cancel{x}(x^2y+1)}{\cancel{x}(1-xy^2)} = \dfrac{1}{1} = 1$

b. $\lim_{(x,y)\to(0,0)} \dfrac{\cancel{x}(x^2y+1)}{\cancel{x}(1-xy^2)} = \dfrac{\lim (x^2y+1)}{\lim (1-xy^2)} = 1$

2. $f(x,y) = \dfrac{x^2-xy}{x^2+y^2}$ $\quad \lim_{(x,y)\to(0,0)} f(x,y)$ is DNE

a. $f(x,y) = \dfrac{x(x-y)}{x^2+y^2} = \dfrac{0}{0}$ DNE

3. $\lim_{(x,y)\to(1,1)} \dfrac{1-\sqrt{x}-\sqrt{y}+\sqrt{xy}}{1-x-y+xy} = \lim_{(x,y)\to(1,1)} \dfrac{(1-\sqrt{x})(1-\sqrt{y})}{(1-x)(1+y)}$

$= \lim_{(x,y)\to(1,1)} \dfrac{\cancel{(1-\sqrt{x})}\cancel{(1-\sqrt{y})}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})\cancel{(1-\sqrt{y})}(1+\sqrt{y})}$

$= \lim_{(x,y)\to(1,1)} \dfrac{1}{(1+\sqrt{x})(1+\sqrt{y})} = \dfrac{1}{4}$

a. $f(x,y) = \dfrac{1}{2\cdot 2} = \dfrac{1}{4}$

b. $\lim_{(x,y)\to(1,1)} f(x,y) = \dfrac{\lim 1}{\lim (1+\sqrt{x})(1+\sqrt{y})} = \dfrac{1}{4}$

4. $\lim_{(x,y)\to(0,-1)} \dfrac{x^2y+x^2}{x^4+y^2+2y+1} = \lim_{(x,y)\to(0,-1)} \dfrac{\cancel{x^2}\cancel{(y+1)}}{\cancel{x^4}(y+1)^{\cancel{2}}}$ x^2

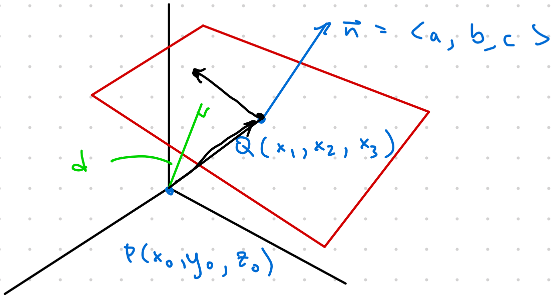
a. $f(x,y) =$ DNE

$= \lim_{(x,y)\to(0,-1)} \dfrac{1}{(y+1)(x^2)}$

b. DNE

DNE.

5.



distance from point to line

$$\vec{d} = \text{proj}_{\vec{n}} \overrightarrow{PQ}$$

$$d = \left| \text{proj}_{\vec{n}} \overrightarrow{PQ} \right|$$

$$= \left| \text{comp}_{\vec{n}} \overrightarrow{PQ} \right|$$

Distance from $P = (x_0, y_0, z_0)$ to a plane w/ $\vec{n} = \langle a, b, c \rangle$ and pt $Q = (x_1, x_2, x_3)$

$$\left| \text{comp}_{\vec{n}} \overrightarrow{PQ} \right| = \left| \frac{\overrightarrow{PQ} \cdot \vec{n}}{\|\vec{n}\|} \right|$$

$\langle 1, 1, -2 \rangle$

$Q = (1, 2, 2)$

$P = (0, 0, 0)$

$\overrightarrow{PQ} = \langle 1, 2, 2 \rangle$

Find $\vec{n}$ $\quad \langle 0,$

cross product

$t = 1 \quad (1, 2, 2)$

$t = 0 \quad (1, 0, 5)$

$\langle 0, 2, -3 \rangle$

$0, 2, -3$

$1, 1, -2$

$(-4 + 3)i - (0 + 3)j + (0 - 2)k$

$\vec{n} = \langle -1, -3, -2 \rangle$

$$\left| \frac{-1 + -6 + -4}{\sqrt{1 + 9 + 4}} \right| = \left| \frac{-11}{\sqrt{14}} \right|$$

$$= \frac{11}{\sqrt{14}}$$

$t \geq 0$ $\qquad r(t) = \left\langle \frac{t}{\sqrt{2}}, \frac{t}{\sqrt{2}}, \frac{4}{3} t^{3/2} \right\rangle$

6. Find the unit tangent vector $r(t)$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$

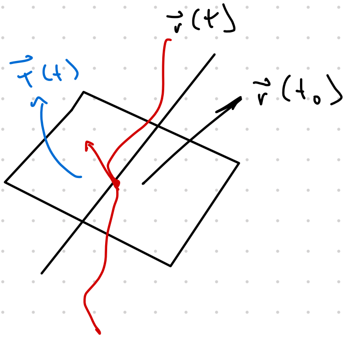
$$\vec{r}'(t) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{4}{\cancel{3}} \cdot \frac{\cancel{3}}{2} t^{1/2} \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2t^{1/2} \right\rangle$$

$$\|\vec{r}'(t)\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(2t^{1/2}\right)^2}$$

$$\vec{T}(t) = \frac{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2t^{1/2} \right\rangle}{\sqrt{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + 4t}}$$

$$= \frac{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2t^{1/2} \right\rangle}{\sqrt{\frac{1}{2} + \frac{1}{2} + 4t}} = \frac{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2t^{\frac{1}{2}} \right\rangle}{\sqrt{1+4t}}$$



$x - y + 6z = 64$

$\cancel{\frac{t}{\sqrt{2}} - \frac{t}{\sqrt{2}}} + 6\left(\frac{4}{3}t^{3/2}\right) = 64$

Solve for t . . .

$t_0 = 4$

$\ell(t) = \vec{r}\,'(t_0)t + \vec{r}(t_0)$

$\ell(t) = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2(4)^{1/2} \rangle t +$

$\langle \frac{4}{\sqrt{2}}, \frac{4}{\sqrt{2}}, \frac{4}{3}(4)^{3/2} \rangle$

$= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 4 \rangle t +$

$\langle \frac{4}{\sqrt{2}}, \frac{4}{\sqrt{2}}, \frac{32}{3} \rangle$

$\vec{\ell}(t) = \vec{r}\,'(4)t + \vec{r}(4)$

$\vec{\ell}(t) = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 4 \rangle t + \langle \frac{4}{\sqrt{2}}, \frac{4}{\sqrt{2}}, \frac{32}{3} \rangle$

7.

a. $\vec{r}\,'(t) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{4}{\cancel{3}} \cdot \frac{\cancel{3}}{2} t^{1/2} \right\rangle$

$= \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2t^{1/2} \right\rangle$

$\vec{r}\,''(t) = \langle 0, 0, t^{-1/2} \rangle$

$\vec{r}\,'(9) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 6 \right\rangle$

$\vec{r}\,''(9) = \left\langle 0, 0, \frac{1}{3} \right\rangle$

b. distance is just traveled by the drone for first 9 sec.

$\int_0^9 \|r'(t)\|\,dt = \int_0^9 \sqrt{(1+4t)}\,dt$ $\qquad u = 1 + 4t$

$du = 4\,dt$

$= \frac{1}{4}\int_1^{37} \sqrt{u}\,du$ $\qquad t = 0 \Rightarrow u = 1$

$t = 9 \Rightarrow u = 37$

$= \frac{1}{4} \cdot \left(\frac{2 \cdot u^{3/2}}{3} - \frac{2 \cdot u^{3/2}}{3} \right)$

$= \frac{1}{4} \cdot \left(\frac{2 \cdot 37^{3/2}}{3} - \frac{2 \cdot 1^{3/2}}{3} \right)$

$= 37.34$

8. $\vec{r}(9) = \left\langle \frac{9}{\sqrt{2}}, \frac{9}{\sqrt{2}}, 36 \right\rangle = p_0$ = initial position

$\vec{r}\,'(9) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 6 \right\rangle = v_0$ = initial velocity

$\vec{a}(t) = \langle 0, 0, -10 \rangle = a_0$ = acceleration

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle c_0, c_1, -10t + c_2 \rangle$$

$$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -10t + 6 \right\rangle$$

$$\vec{p}(t) = \int \vec{v}(t)\,dt = \left\langle \frac{\sqrt{2}}{2}t + c_4, \frac{\sqrt{2}}{2}t + c_5, -5t^2 + 6t + c_6 \right\rangle$$

$$= \left\langle \frac{\sqrt{2}}{2}t + \frac{9}{\sqrt{2}}, \frac{\sqrt{2}}{2}t + \frac{9}{\sqrt{2}}, -5t^2 + 6t + 36 \right\rangle$$

a. hits the ground $p_z = 0$?

$-5t^2 + 6t + 36 = 0 \Rightarrow t = \cancel{-2.14}$ or 3.35 sec

b. where it hits the ground.

$$\vec{p}(3.35) = \left\langle \frac{\sqrt{2}}{2}(3.35) + \frac{9}{\sqrt{2}}, \frac{\sqrt{2}}{2}(3.35) + \frac{9}{\sqrt{2}}, -5(3.35)^2 + 6(3.35) + 36 \right\rangle$$

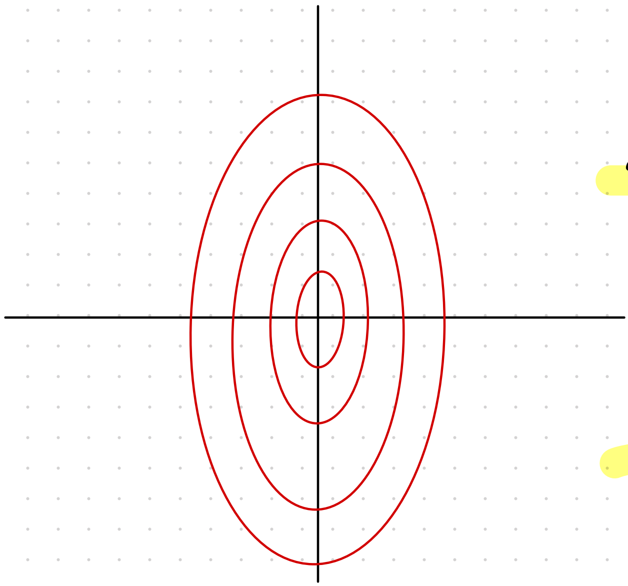
$$= \langle 8.73, 8.73, -0.0125 \rangle$$

c. what speed it hits the ground

$$\|\vec{v}(3.35)\| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + (-10(3.35)+6)^2}$$

$$= 27.52$$

9. Determine the domain and range of $f(x,y)$



$f(x,y) = 16 - \sqrt{25 + x^2 + 4y^2}$

domain:

$\{(x,y) \mid -\infty < x < \infty, -\infty < y < \infty\}$

range: at $(0,0)$
$16 - \sqrt{25} = 11$

$(-\infty, 11]$

10. saddle

ii) $f(x,y) = x^2 - y^2 + 5$