Andrea Tongsah MTH 254

1. $\vec{r}(t) = \langle -1+3t, 2-4t, 6+5t \rangle$

$$2(-1+3t) - 3(2-4t) + 4(6+5t) = 2$$

$$-2 + 6t - 6 + 12t + 24 + 20t = 2$$

$$38t - 8 + 24 = 2$$

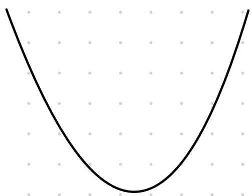
$$38t + 14 = 0$$

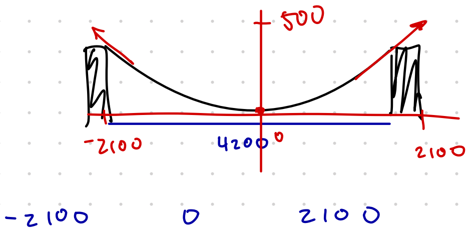
$$t = -\frac{14}{38}$$

$$r(t) = \left\langle -1 + 3\left(-\frac{14}{38}\right), 2 - 4\left(-\frac{14}{38}\right), 6 + 5\left(-\frac{14}{38}\right) \right\rangle$$

$$= \left\langle -\frac{40}{19}, \frac{66}{19}, \frac{79}{19} \right\rangle$$

2.
a.





$$\vec{r}(t) = \left\langle t, \frac{500}{(2100)^2} t^2 \right\rangle$$

$$\vec{r}(-2100) = \left\langle -2100, \frac{500}{\cancel{(2100)^2}} \cancel{(-2100)^2} \right\rangle$$

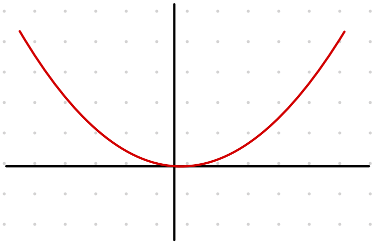
$$= \langle -2100, 500 \rangle$$

$$\vec{r}(0) = 0$$

yes, it makes sense

$$\vec{r}(2100) = \langle 2100, 500 \rangle$$

2b. $r(t) = \langle t, \frac{500}{(2100)^2} t^2 \rangle$



$\vec{r}'(t) = \langle 1, \frac{1000}{(2100)^2} t \rangle$

$\vec{r}'(t) = \langle 1, \frac{1}{4410} t \rangle$

arc length $= \int_a^b \|\vec{r}'(t)\| \, dt$

$= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt$

$= \int_{-2100}^{2100} \sqrt{1 + (\frac{1}{4100} t)^2} \, dt$

change of variables "u-sub" $\begin{cases} t = 4410 \tan\theta \\ dt = 4410 \sec^2\theta \\ dt = 4410(1 + \tan^2\theta) d\theta \end{cases}$

$= 2 \int_0^{2100} \sqrt{1 + (\frac{1}{4100} t)^2} \, dt$

$= 2 \int_{t=0}^{t=2100} \sqrt{1 + (\cancel{\frac{1}{4100}} (\cancel{4410} \tan\theta))^2 \, 4410(1 + \tan^2\theta)} \cdot 4410 \sec^2\theta \, d\theta$

$= 2 \int_{t=0}^{t=2100} \sqrt{1 + (\tan\theta)^2 \, 4410(1 + \tan^2\theta)}$

$= 2 \cdot 4410 \int_{t=0}^{t=2100} \sqrt{1 + \tan^2\theta} \, \sec^2\theta \, d\theta$

$= 2 \cdot 4410 \int_{t=0}^{t=2100} \sec\theta \sec^2\theta \, d\theta$

$t = 0 \Rightarrow 4410 \tan\theta = 0$
$\theta = 0$

$t = 2100 \Rightarrow 4410 \tan\theta = 2100$
$\theta = 0.44$

$= 8820 \int_0^{0.44} \sec^3\theta \, d\theta$

$= 4304.16$ m

3.

a. $4x - 2y + 3z = 2$ $\quad$ $12x - 6y + 9z = 9$

$4x - 2y + 3z = 2$ $\quad$ $4x - 3y + 3z = 3$

dot product is orthogonal

parallel

b.
$$\overset{u}{x - 3y + 2z = -4} \qquad \overset{v}{x - 3z = 0}$$

$$\begin{matrix} 1 & -3 & 2 \\ 1 & 0 & -3 \end{matrix}$$

$= \langle 9, -5, 3 \rangle$

$$|u \times v| = |u||v| \sin\theta$$

$$\frac{|u \times v|}{|u||v|} = \sin\theta$$

$$\frac{\sqrt{9^2 + (-5)^2 + 3^2}}{\sqrt{((1)^2 + (-3)^2 + 2^2)}\sqrt{(1^2 + (-3)^2)}} = \sin\theta$$

$$\theta = 65.00^\circ$$

c.
$$\overset{u}{2x - 2y - 6z = 7} \qquad \overset{v}{5x - 2y - z = 2}$$

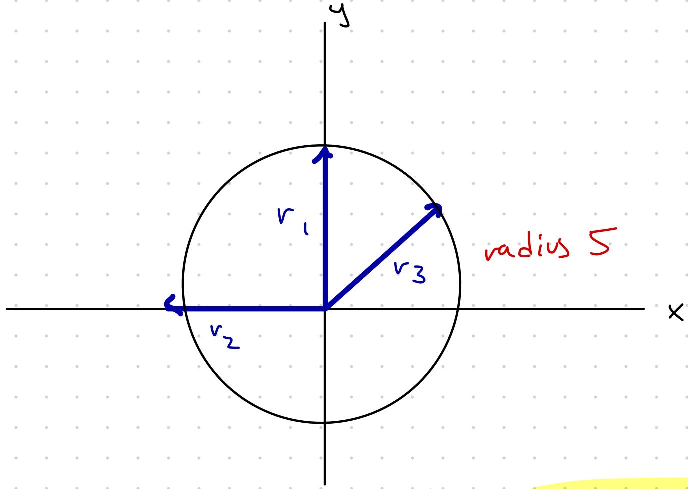
$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

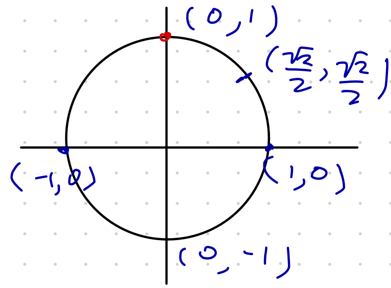
$$\cos\theta = \frac{10 + 4 + 6}{\sqrt{2^2 + (-2)^2 + (-6)^2}\sqrt{5^2 + (-2)^2 + (-1)^2}}$$

$$\cos\theta = \frac{20}{36.331}$$

$$\theta = 56.59^\circ$$

4





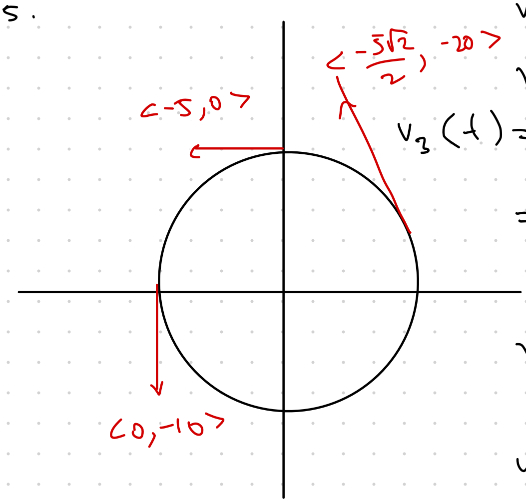
$r_1(\frac{\pi}{2}) = 5 \langle \cos\frac{\pi}{2}, \sin\frac{\pi}{2} \rangle = \langle 0, 5 \rangle$

$r_2(\frac{\pi}{2}) = 5 \langle \cos(\cancel{2}\cdot\frac{\pi}{\cancel{2}}), \sin(\cancel{2}\cdot\frac{\pi}{\cancel{2}}) \rangle = \langle -5, 0 \rangle$

$r_3(\frac{\pi}{2}) = 5 \langle \cos(\frac{1}{\pi}(\frac{\pi}{2})^2), \sin(\frac{1}{\pi}(\frac{\pi}{2})^2 \rangle$

$\langle \cos(\frac{\pi^{\cancel{2}}}{4\cancel{\pi}}), \sin(\frac{\pi}{4\pi}) \rangle = \langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \rangle$

5.



$v_1(t) = 5 \langle -\sin t, \cos t \rangle$

$v_2(t) = 5 \langle -2\sin(2t), 2\cos(2t) \rangle$

$v_3(t) = 5 \langle -\frac{2t\sin(\frac{t^2}{\pi})}{\pi}, -4\sin(2x) \rangle$

$= \langle \frac{-10t\sin(\frac{t^2}{\pi})}{\pi}, -20\sin(2x) \rangle$

$v_1(t) = 5 \langle -\sin\frac{\pi}{2}, \cos\frac{\pi}{2} \rangle$

$= \langle -5, 0 \rangle$

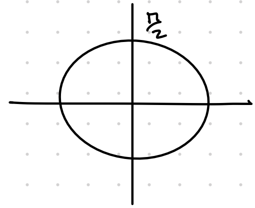
$v_2(t) = 5 \langle -2\sin(\pi), 2\cos(\pi) \rangle$

$= \langle 0, -10 \rangle$

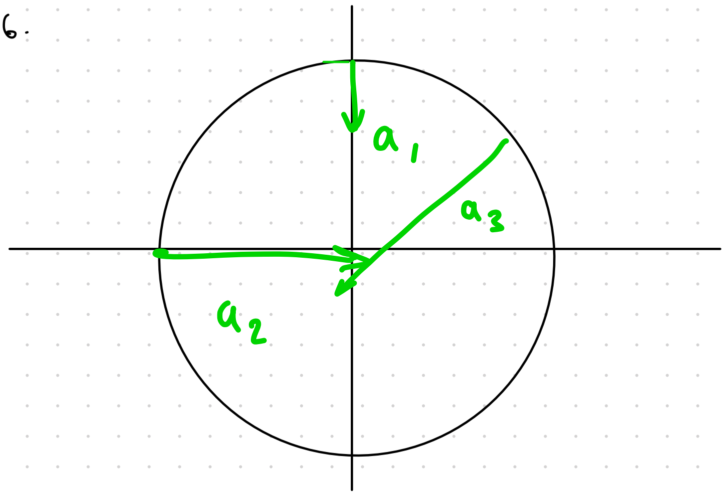
$v_3(t) = \langle \frac{\overset{5}{\cancel{-10}}\,\frac{\pi}{\cancel{2}}\sin(\frac{\pi^{\cancel{2}}}{4}\cdot\frac{1}{\cancel{\pi}})}{\pi}, -20\sin(\cancel{2}\cdot\frac{\pi}{\cancel{2}}) \rangle$

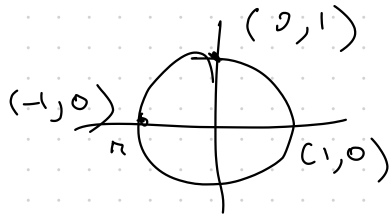
$$v_3(t) = \left\langle \frac{-5\cancel{\pi}\sin\left(\frac{\pi}{4}\right)}{\cancel{\pi}}, -20\sin\left(\frac{\pi}{2}\right) \right\rangle$$

$$= \left\langle -\frac{5\sqrt{2}}{2}, -20 \right\rangle$$



6.





$$a_1(t) = 5\langle -\cos t, -\sin t \rangle$$

$$a_1\left(\frac{\pi}{2}\right) = \langle 0, -1 \rangle$$

$$a_2(t) = \langle -20\cos(2t), -20\sin(2t) \rangle$$

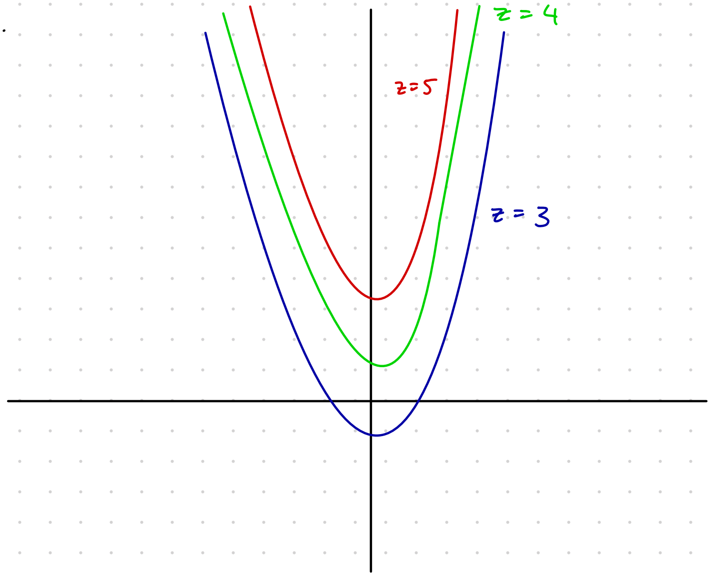
$$a_2\left(\frac{\pi}{2}\right) = \langle 20, 0 \rangle$$

$$a_3(t) = \left\langle \frac{-2\sin\left(\frac{t^2}{\pi}\right)}{\pi} - 4x^2\cos\left(\frac{x^2}{\pi}\right), -40\cos(2x) \right\rangle$$

$$a_3\left(\frac{\pi}{2}\right) = \left\langle \frac{-2\sin\left(\frac{\pi^{\cancel{2}}}{4}\cdot\frac{1}{\cancel{\pi}}\right)}{\pi} - 4\left(\frac{\pi}{2}\right)^2\cos\left(\frac{\left(\frac{\pi}{2}\right)^2}{\pi}\right), -40\cos(\pi) \right\rangle$$

$$= \left\langle -\frac{\sqrt{2}}{\pi} - \frac{\pi^2}{\sqrt{2}}, 40 \right\rangle$$

7.



8. a. $$\kappa(t) = \frac{|v'(t) \times v''(t)|}{|v'(t)|^3}$$

$$v'(t) = \langle a, b, c \rangle$$

$$v''(t) = \langle 1, 1, 1 \rangle$$

$$\kappa(t) = \frac{\sqrt{0}}{\sqrt{a^2+b^2+c^2}} = \boxed{0}$$

$$\begin{array}{ccc} a & b & c \\ 1 & 1 & 1 \end{array}$$

$$(b-c) - (a-c) + (a-b)$$

$$\cancel{b - c - a + c + a - b}$$

the cross product is zero

b. $$v(x) = \langle x, f(x), 0 \rangle$$

$$v'(x) = \langle 1, f'(x), 0 \rangle$$

$$v''(x) = \langle 0, f''(x), 0 \rangle$$

$$\kappa(x) = \frac{\|\vec{v}'(x) \times \vec{v}''(x)\|}{\|\vec{v}'(x)\|}$$

$$= \frac{|f''(x)|}{\sqrt{(1^2 + (f'(x))^2)}}$$