1. $z = f(x,y) = \sqrt{50 - x^2 - 4y^2} = (50 - x^2 - 4y^2)^{1/2}$

a. $z = (50 - x^2 - 4y)^{1/2}$

$f_x = \frac{1}{2}(50 - x^2 - 4y^2)^{-1/2} \cdot (-2x)$ $\qquad f(x,2) = \dfrac{-x}{\sqrt{50 - x^2 - 4y^2}}$

$= \dfrac{-x}{\sqrt{50 - x^2 - 4y^2}}$ at $y = 2$

b. $z = (50 - x^2 - 4y^2)^{1/2}$

$f_y = \frac{1}{2}(50 - x^2 - 4y)^{-1/2} \cdot -4$ $\qquad f(-3,y) = \dfrac{-2}{\sqrt{50 - x^2 - 4y}}$

$= \dfrac{-2}{\sqrt{50 - x^2 - 4y}}$ at $x = -3$

c. Find rate of change in positive y-direction

$\dfrac{-2}{\sqrt{50 - x^2 - 4y}} = -2(50 - x^2 - 4y)^{-1/2}$

Find rate of change in positive x direction

$0 + \cancel{2} \cdot \cancel{-\frac{1}{2}}(50 - x^2 - 4y)^{-3/2} \cdot (-2x)$

$f_{yx} = \dfrac{-2x}{(50 - x^2 - 4y)^{3/2}}$ at $(-3, 2)$

$= \dfrac{6}{(50 - 9 - 8)^{3/2}} = 0.0316$

2. a. $\frac{\text{mass in kg}}{(\text{height in m})^2}$

$$B(w,h) = \frac{w}{h^2} = w(h^{-2})$$

b. $\frac{\partial B}{\partial w} = \frac{1}{h^2}$ As mass increases while one's height stays constant, their BMI increases.

c. $\frac{\partial B}{\partial h} = \frac{-2w}{h^3}$ As height increases while one's mass stays constant, their BMI will decrease.

3

a. $\frac{\partial h}{\partial t}$ for $h(t,s) = t^2 s + \sin(ts)$

$$\frac{\partial h}{\partial t} = 2ts + \cos(ts) \cdot s$$

$$\frac{\partial h}{\partial t} = 2ts + s\cos(ts)$$

b. $$\frac{\partial g}{\partial y} = e^{-x^2 - y^2}(-2y)$$

$$= -2ye^{-x^2 - y^2}$$

c. $$\frac{\partial f}{\partial x} = \frac{1}{(x+y)^2+1} + 2xy^2 - \frac{1}{x\cancel{y}} \cdot \cancel{y}$$

$$= \frac{1}{(x+y)^2+1} + 2xy^2 - \frac{1}{x}$$

d. $$\frac{\partial Q}{\partial b} = 2(2b - c) \cdot 2$$

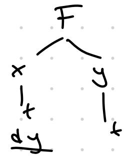
$$= 6b - 4c$$

e. $$\frac{kT}{P} = kT(P)^{-1}$$

$$\frac{\partial V}{\partial P} = \frac{kT}{P^2}$$

f. $$\frac{\partial S}{\partial r} = 2\pi(h+r) + 2\pi r$$

4. $y' = \frac{-F_x}{F_y}$



$x = f(t)$
$y = g(t)$

$$\frac{d}{dx} F = F_x \cancel{\frac{dx}{dx}}^{1} + F_y \frac{dy}{dx} \qquad F(x, y(x))$$

$$0 = F_x + F_y y'$$

$$y' = \frac{-F_x}{F_y}$$

$$x^2y^3 + xy - e^{2x-y} = F$$

$$2xy^3 + y - e^{2x-y} \cdot 2 = F_x$$

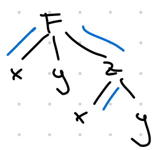
$$2xy^3 + y - 2e^{2x-y} = F_x$$

$$3x^2y^2 + x + e^{2x-y} = F_y$$

$$y' = \frac{-(2xy^3 + y - 2e^{2x-y})}{(3x^2y^2 + x + e^{2x-y})}$$

5. $F(x, y, z) = k$

$$\frac{\partial}{\partial x} F(x,y,z)$$



$$F_x + F_z z_x$$

$$z_x = -\frac{F_x}{F_z}$$

$$F: \quad z^2 + xyz - \frac{1}{x^2+y^2+z^2} = 1$$

$$F_x = \quad yz + \frac{1}{(x^2+y^2+z^2)^2} \cdot 2x$$

$$F_z = 2z + xy + \frac{1}{(x^2+y^2+z^2)^2} \cdot 2z$$

$$z_x = \frac{-\left(yz + \frac{2x}{(x^2+y^2+z^2)^2}\right)}{\frac{2z}{(x^2+y^2+z^2)^2} + 2z + xy}$$

6.

a. $P = (200, 800, 3665)$

b. <u>partial of y</u>

$h_y = -0.002(y-1000)$

at 800 $= 0.4$

c. $\nabla h(200,800) = \langle h_x(200,800), h_y(200,800) \rangle$

$= \langle 3, 0.4 \rangle$

<u>partial of x</u>

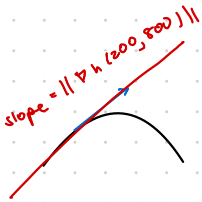
$h_x = -0.01(x-500) = 3$

at $x = 200$

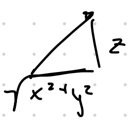
$\hat{u} = \dfrac{\langle 3, 0.4 \rangle}{\sqrt{3^2 + 0.4^2}}$

$= \left\langle \dfrac{3}{\sqrt{1.44}}, \dfrac{0.4}{\sqrt{1.44}} \right\rangle \approx \left\langle \dfrac{15}{\sqrt{229}}, \dfrac{2}{\sqrt{229}} \right\rangle$

d



magnitude of the gradient



slope $= \dfrac{z}{1}$

$z = \|\nabla h(200,800)\|$

$= \dfrac{\sqrt{229}}{5}$

$\vec{v} = 0.2 \dfrac{\vec{T}}{\|\vec{T}\|}$

$\left\langle \dfrac{0.6}{\sqrt{1.44}}, \dfrac{0.08}{\sqrt{1.44}}, 0.5044 \right\rangle$