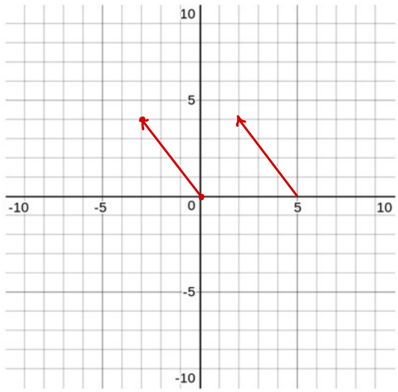
Andrea Tongsak   MTH 254   Recitation Activity wk 1

1. a.



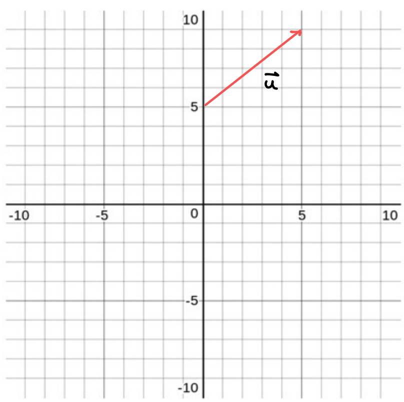
$\vec{v} = \langle -3, 4 \rangle$

b. $|\vec{v}| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16} = 5$

c. $|\vec{v}| = \sqrt{a^2 + b^2}$

d.



$(0,5)$ $(5,10)$

$\vec{u} = \langle 5, 5 \rangle$

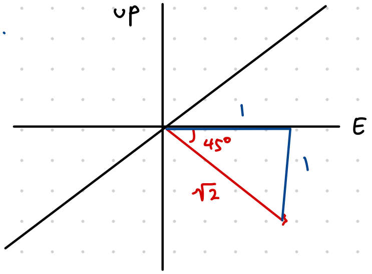
or

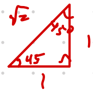
$\vec{u} = 5i + 5j$

e. $\overrightarrow{PQ}$ $P(x_1, y_1)$ $Q(x_2, y_2)$

$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$

2.



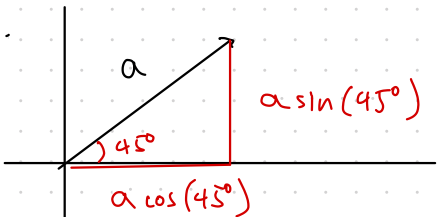


$\sqrt{2}\cos 45° = \sqrt{2} \cdot \frac{1}{\sqrt{2}}$

$\langle 1, 1, 2 \rangle = \sqrt{1^2 + 1^2 + 2^2}$

$= \sqrt{6}$ cm/sec

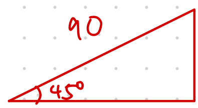
3.



$\cos(45°) = \frac{\sqrt{2}}{2}$

$\sin(45°) = \frac{\sqrt{2}}{2}$

x-component: 45



$z = 90\sin 45°$

y-component: 45

$a = 90\cos 45°$

z-component: $\frac{90\sqrt{2}}{2} = 45\sqrt{2}$

$45i + 45j + 45\sqrt{2}k$

4. Find the midpoint between (0, 0, 10) and (5, 8, 0).

midpoint formula: $\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}$

$$\left(\overset{a}{\dfrac{5+0}{2}}, \overset{b}{\dfrac{8+0}{2}}, \overset{c}{\dfrac{10+0}{2}}\right)$$

$$\left(\frac{5}{2}, 4, 5\right)$$

$$\left(x - \frac{5}{2}\right)^2 + (y-4)^2 + (z-5)^2 = r^2$$

$$\left(0 - \frac{5}{2}\right)^2 + (0-4)^2 + (10-5)^2 = r^2$$

$$\sqrt{\left(-\frac{5}{2}\right)^2 + (-4)^2 + (5)^2} = r$$

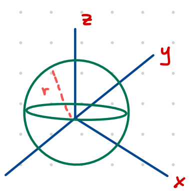
$$\frac{3\sqrt{21}}{2} = r$$

$$\left(x - \frac{5}{2}\right)^2 + (y-4)^2 + (z-5)^2 = \left(\frac{3\sqrt{21}}{2}\right)^2$$

$$\left(x - \frac{5}{2}\right)^2 + (y-4)^2 + (z-5)^2 = 47.25$$

Sphere in 3D

$x^2 + y^2 + z^2 = r^2$



5.

a. $\vec{v} + 2\vec{w} = \langle 2, 2, -1 \rangle + 2\langle 1, -3, 2 \rangle$

$= \langle 2, 2, -1 \rangle + \langle 2, -6, 4 \rangle$

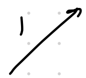
$= \langle 4, -4, -3 \rangle$ angular bracket notation

$4\vec{i} - 4\vec{j} - 3\vec{k}$ coordinate unit vectors

b. $|\vec{w}| = \sqrt{(1)^2 + (-3)^2 + (2)^2}$

$= \sqrt{1 + 9 + 4} = \sqrt{14}$

c. unit vector in direction of $\vec{w}$



$$\hat{u} = \frac{u}{|u|} = \frac{\langle 1, -3, 2 \rangle}{\sqrt{14}}$$

$$= \frac{1}{\sqrt{14}}\hat{i} - \frac{3}{\sqrt{14}}\hat{j} + \frac{2}{\sqrt{14}}\hat{k}$$

$$\hat{u} = \frac{\sqrt{14}}{14}\hat{i} - \frac{3\sqrt{14}}{14}\hat{j} + \frac{2\sqrt{14}}{14}\hat{k}$$

6. P : (1, 2, 3)

Q : (3, 3, 4)

R : (5, 2, 2)

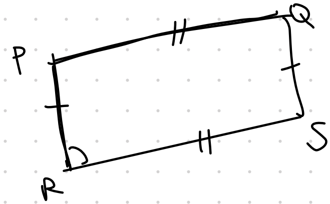
S : (7, 3, 3)

$\vec{u} = \vec{PQ} = \langle 3, 3, 4 \rangle - \langle 1, 2, 3 \rangle$

$= \langle 2, 1, 1 \rangle$

$\vec{v} = \vec{PS} = \langle 7, 3, 3 \rangle - \langle 1, 2, 3 \rangle$

$= \langle 6, 1, 0 \rangle$



Use the cross product

$u \times v = \vec{PQ} \times \vec{PR}$

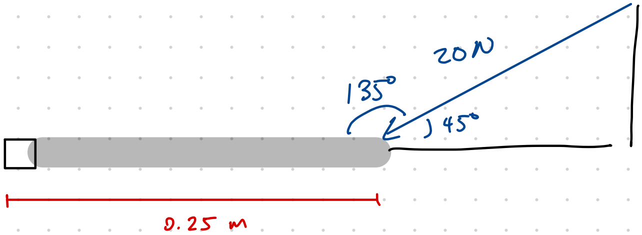
$$= \left\langle +\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, -\det\begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix}, +\det\begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix} \right\rangle$$

$= \langle 0, 0, 4 \rangle$

Magnitude formula to find area:

$$\sqrt{0^2 + 0^2 + 4^2} -$$

7.



$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = |\vec{r}||\vec{F}| \sin\theta$$

$$= 0.25 \cdot 20 \cdot \sin(45°)$$

$$= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \text{ N·m}$$