Andrea Tongsak MTH 254 Recitation

1. a. $F(x,y,z) = 4x^2 + 9y^2 - z^2 = 0$ at $(2, 1, -5)$

$F_x = 8x$

$F_y = 18y$

$F_z = -2z$

$\langle 16, 18, 10 \rangle = \vec{n}$

$16x + 18y + 10z = 0$

OR

$16(x-2) + 18(y-1) + 10(z+5) = 0$

b. $z = x^2 + y^2$ at $(1, 1, 2)$

$0 = x^2 + y^2 - z$

$F_x = 2x$

$F_y = 2y$

$F_z = -1$

$\langle 2, 2, -1 \rangle = \vec{n}$

$2x + 2y - z = 2$

c. $z = \frac{x}{x+y}$ at $(4, -2, 2)$

$zx + zy = x$

$zx + zy - x = 0$

$F_x = -1$

$F_y = z$

$F_z = x + y$

$\langle -1, 2, 2 \rangle = \vec{n}$

$-x + 2y + 2z = -4$

2. Linear approximation:

Find the tangent equation $h(x,y) = \sqrt{x^4 + y^2}$

$z = \sqrt{x^4 + y^2}$

$(2,3,5)$

$z^2 = x^4 + y^2$

$0 = x^4 + y^2 - z^2$

$z_0 = \sqrt{2^4 + 3^2} = \sqrt{16+9} = 5$

$\vec{\nabla} F = \langle F_x, F_y, F_z \rangle$

$= \langle 4x^3, 2y, -2z \rangle$

$\vec{n} = \langle 32, 6, -10 \rangle$

$32x + 6y - 10z = 32$

Now find linear approx. Solve for z.

$$z = \frac{32 - 32x - 6y}{-10}$$

$$L(x,y) = \frac{32}{-10} + \frac{32x}{10} + \frac{6}{10}y$$

$$L(2.1, 2.9) = -\frac{16}{5} + \frac{16}{5}(2.1) + \frac{3}{5}(2.9) = 5.26$$

3. 2 path test: DNE

$$\lim_{(x,y)\to(0,0)} \frac{x^2+4y^2}{(x+2y)^2} =$$

Path 1: $x=0$

$$\frac{4y^2}{4y^2} = 1$$

Path 2: $y=x$

$$\frac{5x^2}{(3x)^2} = \frac{5\cancel{x^2}}{9\cancel{x^2}}$$

4. $\lim_{(x,y)\to(0,0)} \dfrac{x^2+4x+y}{\sqrt{x^2+4x+y+9}-3}$ Does exist

$$\lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2+4x+y)}(\sqrt{x^2+4x+y+9}+3)}{\cancel{x^2+4x+y+9-9}}$$

$$\lim_{(x,y)\to(0,0)} \sqrt{9}+3 = 6$$

5. $f(x,y) = x\sin\left(\frac{\pi y}{x}\right)$

$f_x = \sin\left(\frac{\pi}{x} y\right) - \cos\left(\frac{\pi y}{x}\right) \frac{\pi y}{x}$

$= \sin(\pi y x^{-1}) - \cos(\pi y x^{-1}) \pi y x^{-1}$

$f_{xx} = \dfrac{-\pi^2 y^2 \sin\left(\frac{\pi y}{x}\right)}{x^3}$

$f_y = \pi \cos\left(\frac{\pi y}{x}\right)$

$f_{yy} = \dfrac{-\pi^2 \sin\left(\frac{\pi y}{x}\right)}{x}$

$f_{xy} = f_{yx} = -\pi \sin\left(\frac{\pi y}{x}\right) \cdot \frac{\pi y}{x} = \frac{-\pi^2 y}{x} \sin\left(\frac{\pi y}{x}\right)$

6. directional derivative

$\vec{\nabla} g \cdot \vec{u}$

$$g(x,y,z) = \frac{x^3yz - z - 1}{x^2}$$

$$(x^3yz - z - 1)\,x^{-2} = xyz - zx^{-2} - x^{-2}$$

$$\vec{u} = \frac{\langle 2, 1, 2 \rangle}{\sqrt{2^2+1^2+2^2}} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$$

$\vec{\nabla} g = \langle F_x, F_y, F_z \rangle$ at $(1,5,9)$

$F_x = yz + 2zx^{-3} + 2x^{-3}$

$F_y = xz$

$F_z = xy - x^{-2}$

$$\left\langle yz + \frac{2z}{x^3} + \frac{2}{x^3},\ xz,\ xy - \frac{1}{x^2} \right\rangle \cdot \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$$

original point $r(1,5,9)$

$$\langle 205, 9, 4 \rangle \cdot \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$$

$$= \left\langle \frac{410}{3}, 3, \frac{8}{3} \right\rangle$$

7. ∇f = direction of max increase

$-\nabla f$ = direction of max decrease

$f = y - x^2 \ln(x-y)$

$(2,1)$

$\|\nabla f\|$ = rate of max increase

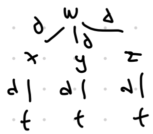
$-\|\nabla f\|$ = rate of max decrease

$f_x = -2x \ln(x-y) + -x^2 \cdot \frac{1}{x-y}$ at $(2,1)$ -4

$f_y = 1 + x^2 \frac{1}{x-y}$ at $(2,1)$ 5

$-\sqrt{(-4)^2 + (5)^2} = -\sqrt{41}$

8.



a.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

b. $g'(t) = f_x\, x'(t) + f_y\, y'(t) + f_z\, z'(t)$

$= 2x(2) + 2y(5) - 4z^3(4) = 12 + -30 - 128$

$= -158 + 12$

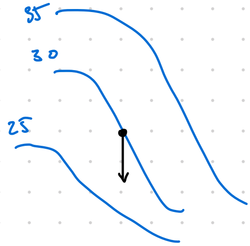
$g(t) = f(\vec{r}(t)) = f(x(t), y(t), z(t))$

$= -146$

9.

a. directional derivative

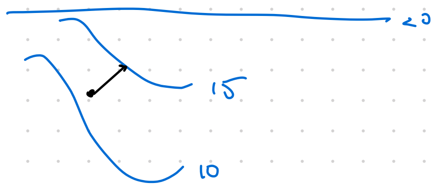
$(1,1)$ in $v = \langle 1, 0 \rangle$



$$\frac{25-30}{0.5} = \frac{-5}{0.5} = -10$$

b. directional derivative

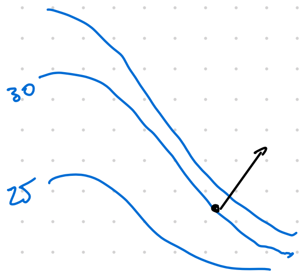
$(-1,-1)$ in direction $v = \langle 2, 1 \rangle$



$$\frac{15-10}{0.4} = \frac{5}{0.4} = \frac{50}{4}$$

$$= 12.5$$

c. $\nabla g(2, 0.5)$



$$\frac{35-30}{0.1} = \frac{5}{0.1} = 50$$

10.

(10) Suppose that below are several level curves of $f(x, y)$. Draw vectors that represent the gradient of f evaluated at points P and Q (with their tails at these respective points). Make sure to get their directions and relative magnitudes correct.