Andrea Tongsak MTH 254 Recitation Activity Week 2.

1.

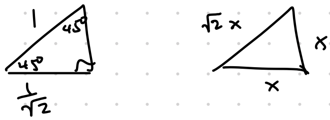
a. $r(0) = \langle 50e^{-0}\cos(0),\ 50e^{-t}\sin(0),\ 5(1-e^{-0}) \rangle$

$= \langle 50(1)(1),\ 50(1)(0),\ 0 \rangle$

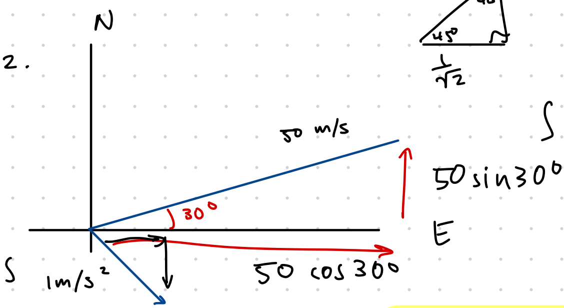
$= \langle 50,\ 0,\ 0 \rangle$

b. $\lim_{t\to\infty} r(t) = \langle 0,\ 0,\ 5 \rangle$

As e approaches infinity, it gets closer to zero



2.



$x^2 + y^2 = 1^2$

a. Acceleration: $\vec{a}(t) = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -9.8 \rangle$

b. initial velocity: aka v_0

$v_0 = 50\cos 30°\, i + 50\sin 30°\, j$

c. vector valued function:

$\vec{v}(t) = \int \vec{a}(t)\,dt$

$= \langle \left(\frac{1}{\sqrt{2}}t + C_1\right) + \left(\frac{1}{\sqrt{2}}t + C_2\right) + \left(-9.8t + C_3\right) \rangle$

$v_0 = \langle C_1,\ C_2,\ C_3 \rangle$

$C_1 = 50\cos 30°$ $\quad C_2 = 50\sin 30°$ $\quad C_3 = 0$

$\vec{v}(t) = \langle \frac{1}{\sqrt{2}}t + 50\cos 30°,\ \frac{1}{\sqrt{2}}t + 50\sin 30°,\ -9.8t \rangle$

d. Take integral of position

$$v(t) = \langle \frac{1}{\sqrt{2}}t + 50\cos 30°, \ \frac{1}{\sqrt{2}}t + 50\sin 30°, \ -9.8t \rangle$$

$$s(t) = \int v(t) = \langle \frac{1}{2\sqrt{2}}t^2 + 50\cos 30° t, \ \frac{1}{2\sqrt{2}}t^2 + 50\sin 30° t, \ -\frac{9.8t^2}{2} \rangle$$

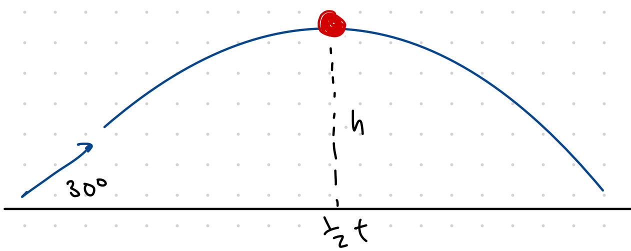
e. When would the ball hit the ground? Where?

$t = ?$ distance

use position vector

$t = 0, \ t = \frac{250}{49}$

f. Maximum height the ball reached?



When the velocity is zero, the ball is at its maximum height.

$$\frac{250}{49} \cdot \frac{1}{2} = \frac{250}{98} \text{ seconds}$$

plug into y for height

$$\frac{1}{2\sqrt{2}}\left(\frac{250}{98}\right)^2 + 50\sin 30°\left(\frac{250}{98}\right)$$

3. $\langle 4, c, 12 \rangle$

$$\sqrt{4^2 + c^2 + 12^2} = 13$$

$$\sqrt{16 + c^2 + 144} = 13$$

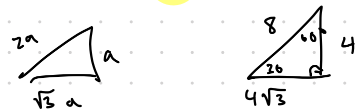
$$\sqrt{160 + c^2} = 13$$

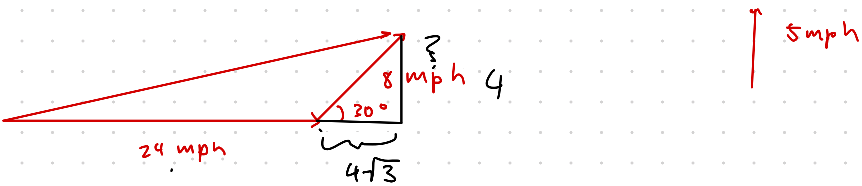
$$160 + c^2 = 169$$

$$c^2 = 9$$

$$c = 3$$

4.





$$24 + 4\sqrt{3}$$

$$\langle 24 + 4\sqrt{3}, 4, 5 \rangle$$

magnitude: $\sqrt{(24 + 4\sqrt{3})^2 + 4^2 + 5^2} = 31.58$ mph

5. $\langle 2t - 1, 5 - t, -2t \rangle$ orthogonal to $\langle -2, 1, 2 \rangle$

Take dot product so its zero

$$(2t-1)(-2) + (5-t)(1) + (-2t)(2) = 0$$

$$(-4t + 2) + 5 - t - 4t = 0$$

$$-8t + 7 - t = 0$$

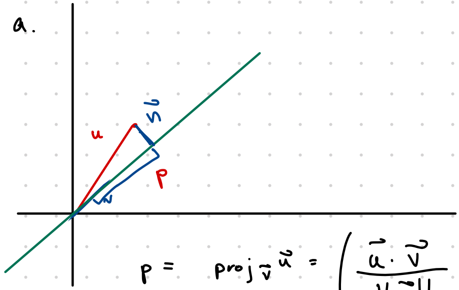
$$-9t + 7 = 0$$

$$-9t = -7$$

$$t = \frac{7}{9}$$

6. $u = p + n$

a.



$$p = \text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|}\right)\left(\frac{\vec{v}}{\|\vec{v}\|}\right)$$

$$= \frac{(3+4)}{1^2+1^2}\langle 1,1\rangle$$

$$p = \frac{7}{2}\langle 1,1\rangle = \frac{\langle 7,7\rangle}{2}$$

$$= \left\langle \frac{7}{2}, \frac{7}{2}\right\rangle$$

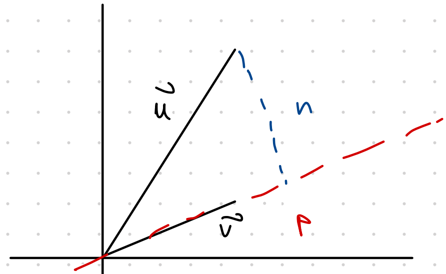
$$\vec{n} = \text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{v}\cdot\vec{u}}{\|\vec{u}\|^2}\right)\vec{u}$$

$$= \frac{(3+4)}{3^2+4^2}\langle 3,4\rangle$$

$$= \frac{7}{25}\langle 3,4\rangle = \left\langle \frac{21}{25}, \frac{28}{25}\right\rangle$$

$$u = p + n = \left\langle \frac{7}{2}, \frac{7}{2}\right\rangle + \left\langle \frac{21}{25}, \frac{28}{25}\right\rangle$$

b. $u = \langle 2, 2, 7 \rangle \qquad v = \langle 1, 1, 2 \rangle$



$$p = \text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|} \right) \frac{\vec{v}}{\|\vec{v}\|}$$

$$p = \frac{(2+2+14)}{\sqrt{1^2+1^2+2^2}} \cdot \frac{\langle 1, 1, 2 \rangle}{\sqrt{1^2+1^2+2^2}}$$

$$p = \frac{18}{6} \cdot \langle 1, 1, 2 \rangle$$

$$p = \langle 3, 3, 6 \rangle$$

$$n = \text{proj}_{\vec{u}} \vec{v} = \left(\frac{\vec{v} \cdot \vec{u}}{\|\vec{u}\|} \right) \frac{\vec{u}}{\|\vec{u}\|}$$

$$n = \frac{(2+2+14)}{2^2+2^2+7^2} \cdot \langle 2, 2, 7 \rangle$$

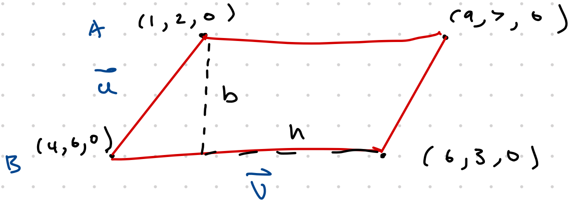
$$n = \frac{18}{4+4+49} \cdot \langle 2, 2, 7 \rangle$$

$$n = \frac{18}{8+49} \langle 2, 2, 7 \rangle = \frac{\overset{6}{\cancel{18}}}{\underset{19}{\cancel{57}}} \langle 2, 2, 7 \rangle$$

$$n = \left\langle \frac{12}{19}, \frac{12}{19}, \frac{42}{19} \right\rangle$$

$$u = p + n = \langle 3, 3, 6 \rangle + \left\langle \frac{12}{19}, \frac{12}{19}, \frac{42}{19} \right\rangle$$

7.



area of parallelogram:

$$\|\vec{u} \times \vec{v}\|$$

$$\vec{AB} = \langle 3, 4, 0 \rangle$$

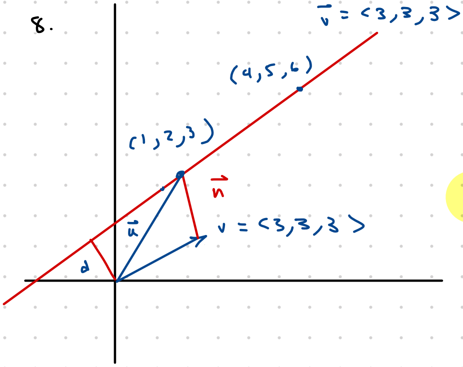
$$\vec{BD} = \langle 2, -3, 0 \rangle$$

$$\vec{u} \times \vec{v} = \langle 0, 0, -9 - 8 \rangle$$

$$= \langle 0, 0, -17 \rangle$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{0 + 0 + (17^2)} = 17$$

8.



$P = (1,2,3)$

$Q = (4,5,6)$

$\overrightarrow{PQ} = \langle 3, 3, 3 \rangle$

$$\ell(t) = \langle 3t+1,\ 3t+2,\ 3t+3 \rangle$$

line through these two points, P and Q

$\|\vec{n}\| = d$

$$d = \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{v}\|}$$

$\vec{u} = \langle 1, 2, 3 \rangle$

$\vec{v} = \langle 3, 3, 3 \rangle$

$\vec{u} \times \vec{v} = -3\hat{i} - 6\hat{j} - 3\hat{k}$

$-3\hat{i} - 6\hat{j} - 3\hat{k}$

$$\frac{\|\vec{u} \times \vec{v}\|}{\|\vec{v}\|} = \frac{\sqrt{(-3)^2 + (-6)^2 + (-3)^2}}{\sqrt{3^2 + 3^2 + 3^2}}$$

$$= \frac{\sqrt{9 + 36 + 9}}{\sqrt{9 + 9 + 9}} = \frac{\sqrt{54}}{\sqrt{27}}$$

$$d = \sqrt{2}$$

9. $v(t) = \langle t\ln(t),\ 2-t^2,\ \cos(\pi t) \rangle$

$x = t\ln(t) = 0 \qquad t = 0, 1$

$y = 2 - t^2 = 1 \qquad t = 1, -1$

$z = \cos(\pi t) = -1 \qquad t = 1, 3, 5$

$t = 1$ for $v(1) = \langle 0, 1, -1 \rangle$

$\vec{v}'(t) = \langle \ln(t),\ -2t,\ -\pi\sin(\pi t) \rangle$

$\vec{v}'(1) = \langle 1,\ -2,\ 0 \rangle$

$\vec{r}(t) = v'(1)t + v'(1)$

$\vec{r}(t) = \langle t,\ -2t+1,\ -1 \rangle$

10. $q(t) = \langle t, 2t, 5+2t \rangle \quad 0 \le t \le 8$
$h(s) = \langle 2+3s, 10-3s, 33-30s \rangle \quad 0 \le s \le 1$

a. $2+3s = t$
so, now replace into the next

$$2t = 10-3s$$

$$2(2+3s) = 10-3s$$

$$4+6s = 10-3s$$

$$9s = 6$$

$$s = \frac{2}{3}$$

when $s = \frac{2}{3}$, $2+\cancel{3}\left(\frac{2}{\cancel{3}}\right) = t$
$t = 4$.

$4 = 2+3\left(\frac{2}{3}\right)$ ✓
$8 = 10-3\left(\frac{2}{3}\right)$ ✓
$13 = 33-30\left(\frac{2}{3}\right)$ ✓

∴ The flight paths of the birds <u>do</u> intersect.

The point of intersection is $(4, 8, 13)$

b. $t = s+5$

$s+5 = 2+3s$?

$$3 = 2s$$

$$\frac{3}{2} = s$$

$$10 - 3\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}+5\right)$$

$$10 - \frac{9}{2} = 3+10$$ False

the hawk will not catch the quail.