Midterm 1

$\text{proj}_{\vec{v}}\,\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{u}}\right)\vec{v}$

$|\vec{u}\times\vec{v}| = |\vec{u}|\,|\vec{v}|\sin(\theta)$

line parallel to a vector $\vec{v} = \langle a, b, c\rangle$ and through $P = (x_0, y_0, z_0)$

$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

$\dfrac{\langle 0, 3, 3\rangle}{\sqrt{3^2+3^2}}$

$\left\langle 0, \dfrac{3}{\sqrt{18}}, \dfrac{3}{\sqrt{18}}\right\rangle$

9 2

$\left\langle 0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right\rangle$

$\vec{p} = \langle 0, 3, 3\rangle$

$\vec{w} = \langle 2, 0, 2\rangle$

$|\vec{p}\times\vec{w}| = \quad 6 + 6 + 6$

$\sqrt{6^2+6^2+6^2}$

$36 + 36 + 36$

$72 + 36 \quad \sqrt{108}$

$\sqrt{108} = \sqrt{9+9}\,\sqrt{4+4}\,\sin\theta$

2 $\cancel{18}$

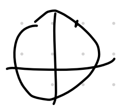
$\sqrt{108} = \sqrt{18}\,\sqrt{8}\,\sin\theta$

3 36

$\dfrac{\sqrt{108}}{\cancel{\sqrt{18}}\,\sqrt{8}} = \dfrac{\sqrt{6}}{\sqrt{8}} = \sin\theta$

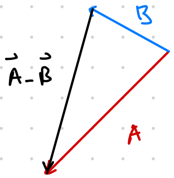
$\dfrac{\sqrt{\cancel{6}}\,3}{2\sqrt{\cancel{2}}} = \sin\theta$

$\theta = \dfrac{\pi}{3}$



$\dfrac{\sqrt{3}}{2}$

$\vec{A} - \vec{B}$



$\overset{A}{(1,2,3)}$ $\overset{B}{(2,4,5)}$ $\overset{C}{(0,5,3)}$

$\overrightarrow{AB}$

$\overrightarrow{BC}$

$|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{\quad}$

divide by 2

$\vec{r}(t) = \langle 2-3t,\ 5+5t,\ 1 \rangle$

$\vec{r}(0) = \langle 2,\ 5,\ 1 \rangle$

$\vec{r}(1) = \langle -1,\ 10,\ 1 \rangle$

$\langle 3, -5, 0 \rangle \cdot \langle ? \rangle = 0$

$3(x) + (-5)(y) + (0)(z) = 0$

magnitude $\sqrt{a^2+b^2+c^2}$

$\text{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v}$ $\quad \overset{\vec{u}}{\langle 8,1 \rangle}$ $\overset{\vec{v}}{\langle 1,2 \rangle}$

$= \dfrac{8+2}{1+4}\langle 1,2 \rangle$

$= \dfrac{10}{5}\langle 1,2 \rangle = 2\langle 1,2 \rangle$

$= \langle 2,4 \rangle$

11. $w = \vec{F} \cdot \vec{d}$

$= 200 \cdot 5 \cos(60 - 30°)$

$= 200 \cdot 5 \cos(30°)$

$= \cancel{200}^{1} \cdot 5 \frac{\sqrt{3}}{\cancel{2}}$

$= 500\sqrt{3}$

Midterm 2

orthogonal to $z = 2x + 3y + 6$

$-6 = 2x + 3y - z$

$\langle 2, 3, -1 \rangle$

where does the line

A $(1, 2, 0)$ B $(2, 3, 1)$ C $(3, 5, 2)$

find normal vector from 3 pts

$\overrightarrow{CB}$ $\langle 1, 2, 1 \rangle$

$\overrightarrow{AB}$ $\langle -1, -1, -1 \rangle$

$\vec{n} = \langle -1, 0, 1 \rangle$ or $\langle 1, 0, -1 \rangle$

$-x + z = d$ $\quad -1 + 0 = -1$

$-(1-t) + (3-2t) = -1$

$-1 + t + 3 - 2t = -1$

$-t = -3$

$t = 3$

$l(t) = \langle 1-3, 2+3(3), 3-2(3) \rangle$

$\langle -2, 11, -3 \rangle$

$f(x,y) = x + 4y^2 + 11$ is parabola

intersection

$x + y + z = 1$

$2x + 2y + 2z = 2$

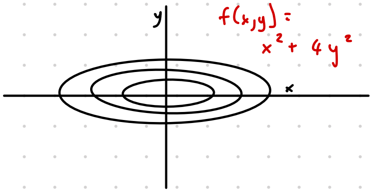
$2x + y - z = 2$

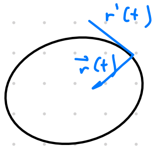
$\cancel{2x} + 2y + 2z = \cancel{2x} + y - z$

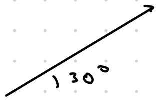
$3z = -y$

$y = -3z$

$\langle 1, 1, 1 \rangle = a \quad \langle 2, 1, -1 \rangle = b$







$$v_0 = \langle 8\cos 30°, \ 8\sin 30° \rangle$$
$$= \langle \frac{8\sqrt{3}}{2}, \ 8 \cdot \frac{1}{2} \rangle$$
$$= \langle 4\sqrt{3}, \ 4 \rangle$$

$$\vec{a} = \langle 0, -10 \rangle$$
$$\int \vec{a} = \langle c_1, -10t + c_2 \rangle$$
$$\vec{v} = \langle 4\sqrt{3}, -10t + 4 \rangle$$

0 since start is (0,0)

$$\vec{r} = \int \vec{v}\,dt = \langle 4\sqrt{3}t + c_1, \ -5t^2 + 4t + c_2 \rangle$$
$$= \langle 4\sqrt{3}t, -5t^2 + 4t \rangle$$

find points at which the position and ground intersect

$$\langle 0, 0 \rangle$$
$$4\sqrt{3}t = 0$$

$$\cancel{4\sqrt{3}t} = -5t^2 + 4t$$
$$0 = -5t^2 + (4 \cancel{- 4\sqrt{3}})t$$
$$= t(-5t + 4 \cancel{- 4\sqrt{3}})$$
$$5t = 4$$
$$t = \frac{4}{5} = 0.8$$

unit tangent vector

$T(t) = \frac{v'(t)}{|v'(t)|}$

$\vec{v}'(t) = \langle -1, 2t, 2t^2 \rangle$

$\sqrt{1^2 + (2t)^2 + (2t^2)^2}$

$\frac{\langle -1, 2, 2 \rangle}{\sqrt{1^2 + 2^2 + 2^2}}$

$= \langle \frac{-1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

$L = \int_a^b |\vec{r}'(t)|$

$\langle -2\sin(2t), 2\cos(2t), 2t \rangle$

squ. $4 + 4t^2$

$\int_0^{\pi} \sqrt{4(1 + t^2)}$

$v(t) = \langle t, \frac{t^2}{2}, 4\sqrt{t} \rangle$

$v'(t) = \langle 1, t, \frac{2}{\sqrt{t}} \rangle$

$|\vec{u}| = \frac{\langle 1, t, \frac{2}{\sqrt{t}} \rangle}{\sqrt{1^2 + t^2 + \frac{4}{t}}}$

$l(t) = c(t_0) + (t - t_0)c'(t$

$\langle \underbrace{4 + 1(t), 8 + 4t, 8 + t} \rangle$

$t = 4 \quad (4, 8, 8)$

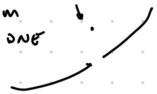
so

$v'(4) = \langle 1, 4, \frac{\cancel{2}^{1}}{\cancel{\sqrt{4}}} \rangle$

Midterm Exam 3

1. $\frac{2}{1}$

lim
ONE



not continuous at (0,10)

rate of change

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$z = \frac{x^2+y^2}{xy}$$

$$= \frac{x^2}{xy} + \frac{y^2}{xy}$$

$$\frac{\partial z}{\partial x} = \frac{1}{y} - \frac{y}{x^2} = \frac{1}{-1} - \frac{-1}{4} = -1+\frac{1}{4}$$

$$= -\frac{3}{4}$$ ✗ 2

in the x direction

$$\frac{dx}{dt}$$

$$\frac{\partial z}{\partial y} = \frac{1}{x} - \frac{x}{y^2} = \frac{1}{2} - \frac{2}{1} = \frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$$

$$\frac{dy}{dt} =$$

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Midterm 4

critical points

$f(x,y) = \frac{x^3}{3} + \frac{y^2}{2} - x - y - xy$

$f_{xy} = -1$

$f_x = x^2 - 1 - y = 0$

$f_{xx} = 2x$

$f_y = y - 1 - x = 0$

$f_{yy} = 1$

CP: $\boxed{2}$

$y = x^2 - 1 = (x+1)(x-1)$

$\underline{(1,0)}$ $\underline{(-1,0)}$

$y = x + 1$

$\cancel{(0,1)\ (-1,0)}$

$x = 1$:

saddle

$f_{xx} f_{yy} - (f_{xy})^2$

$(1,0) \rightarrow 2 - 1 = 1$ local min

$(-1,0) \rightarrow -2 - 1 = -3$ saddle

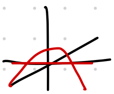
More Review Problems

65. cyl: (r, θ, z)

$z = 0 \qquad z = \sqrt{2} - \sqrt{1+x^2+y^2}$

$x^2+y^2 = r^2$

$z = \sqrt{2} - \sqrt{1+r^2}$



$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} \int_{z=0}^{z=\sqrt{2}-\sqrt{1+r^2}} 1 \quad r\,dz\,dr\,d\theta$$

$$rz \Big|_0^{\sqrt{2}-\sqrt{1+r^2}}$$

$$2\pi \int_0^1 \sqrt{2}r - r\sqrt{1+r^2} \quad dr \qquad u = 1+r^2 \quad du = 2r\,dr$$

$$2\pi \cdot \frac{\sqrt{2}r^2}{2} - \frac{(1+r^2)^{3/2}}{3} \Big|_0^1 \,dr \qquad 2^{3/2}$$

$$\frac{1}{3}(2-\sqrt{2})\pi$$

66.

1. If $2u = \langle 1, 2, -2 \rangle$

$u = \langle \frac{1}{2}, 1, -1 \rangle$

$|u| = \sqrt{\left(\frac{1}{2}\right)^2 + 1^2 + 1^2}$

$= \sqrt{\frac{1}{4} + 2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

2. $u = \langle 1, -3, 2 \rangle$

$b \quad \langle 1, b, b^2 \rangle$

$1 \cdot 1 + -3b + 2b^2 = 0$

$1 - 3b + 2b^2 = 0$

$(2b - 1)(b - 1) = 0$

$b = 1 \quad b = \frac{1}{2}$

3. $\langle 1, -2, 3 \rangle \quad \langle -4, -1, 5 \rangle$

unit vector orthogonal to both vectors

$$\frac{\vec{u} \times \vec{v}}{\|\vec{u} \times \vec{v}\|}$$

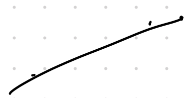
$\langle 1, -2, 3 \rangle$
$\langle -4, -1, 5 \rangle$

$\langle -10 + 3, -(5 + 12), -1 - 8 \rangle$

$\langle -7, -17, -9 \rangle$

$\{ \langle 7, 17, 9 \rangle t \mid t \in \mathbb{R} \}$

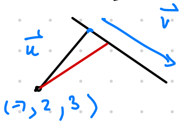
4. $(1, 2, 5) \quad (3, -2, 7)$



$\langle 2, -4, 2 \rangle$

$\langle 1, 2, 5 \rangle + \langle 2, -4, 2 \rangle t$

5. $(-7, 2, 3)$ to $r(t) = \langle 2+t, -3+4t, 8-3t \rangle$



distance:

$\langle 2, -3, 8 \rangle + \langle 1, 4, -3 \rangle t$

$r(0) = (2, -3, 8)$

$r(1) = (3, 1, 5)$

$\vec{v} = \langle 1, 4, -3 \rangle$

$\vec{u} = \langle -9, 5, -5 \rangle$

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|} \vec{v}$$

$$d = |\vec{u} - \vec{p}|$$

$$\frac{-9 + 20 + 15}{\sqrt{81 + 25 + 25}} \langle 1, 4, -3 \rangle$$

$$\frac{35 - 9}{\sqrt{81 + 50}} \langle 1, 4, -3 \rangle$$

$$\langle -9, 5, -5 \rangle - \frac{24}{\sqrt{131}} \langle 1, 4, -3 \rangle$$

$$d = \left| \langle -9, 5, -5 \rangle - \left\langle \frac{24}{\sqrt{131}}, \frac{96}{\sqrt{131}}, \frac{-72}{\sqrt{131}} \right\rangle \right|$$

$= \sim$

6. Find a vector valued function $r(t)$

$r'(t) = 3\sqrt{t}\, i + 2e^{2t} j + k$

$3t^{1/2} i + 2e^{2t} j + k$

$r(0) = i - k$

$r(t) = \int r'(t)$

$r(t) = (2t^{3/2} + 1) i + (e^{2t} - 1) j + (t - 1) k$

$r(0) = 1i + 0j + -1k$ ✓

7. length of curve (arc length)

$r(t) = \langle \sin t, \cos t, t^2 \rangle$

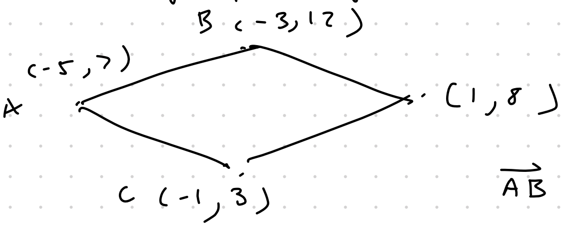
$0, 1, 0$

$$L = \int_0^{\pi} |r'(t)|\, dt$$

$$= \int_0^{\pi} \sqrt{(\cos t)^2 + (-\sin t)^2 + (2t)^2}\, dt$$

$$\int_0^{\pi} \sqrt{1 + 4t^2}\, dt$$

8. Find area of parallelogram



take cross product

$\langle 2, 5 \rangle$

$\vec{AB} = \langle -3+5, 12-7 \rangle$

$\vec{AC} = \langle -1+5, 3-7 \rangle$

$= \langle 4, -4 \rangle$

$\langle 2, 5, 0 \rangle$

$\langle 4, -4, 0 \rangle$

$\langle 5+4, -0, -8-20 \rangle$

$\langle 9, 0, -28 \rangle$

$\sqrt{9^2 + 28^2} = \sqrt{865}$

$= 29.41$

9. tangent line

$\vec{r}'(t) = \frac{1}{t+1} i + 2\pi\cos(2\pi t) j$ at

$\vec{r}(1) = \frac{1}{2} i + 2\pi\cos(2\pi) j$

$\frac{1}{2} i + 2\pi j$

$\ell(t) = r(t_0) + r'(t_0) t$

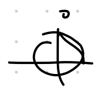
$\ell(t) = \left(\ln(2) i + \sin(2\pi) j\right) + \frac{t}{t+1} i + 2\pi t\cos(2\pi t) j$

$= \ln 2 i + \frac{1}{2} i + 2\pi j$

$= \left(\ln 2 + \frac{1}{2}\right) i + 2\pi j$

10. tangent line $u'v - uv'$

$\vec{\ell}(t) = r(t_0) + t r'(t_0)$



$r(\frac{\pi}{2}) = \langle \frac{\pi}{2}\cos(\frac{\pi}{2}), \frac{\pi}{2}\sin(\frac{\pi}{2}), \frac{\pi}{2} \rangle$

$= \langle 0, \frac{\pi}{2}, \frac{\pi}{2} \rangle$

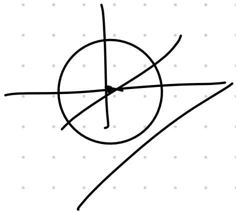
$r'(t) = \langle \cos t - t\sin t, \sin t + t\cos t, 1 \rangle$

$r'(\frac{\pi}{2}) = \langle -\frac{\pi}{2}(1), 1 + 0, 1 \rangle$

$= \langle -\frac{\pi}{2}, 1, 1 \rangle$

$\langle 0, \frac{\pi}{2}, \frac{\pi}{2} \rangle + t \langle -\frac{\pi}{4}, 1, 1 \rangle$

11. distance



12.