CS 331: Artificial Intelligence
Informed Search

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Informed Search

- How can we make search smarter?
- Use problem-specific knowledge beyond the definition of the problem itself
- Specifically, incorporate knowledge of how good a non-goal state is

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Best-First Search

- Node selected for expansion based on an evaluation function $f(n)$. I.e., expand the node that *appears* to be the best
- Node with lowest evaluation is selected for expansion
- Uses a priority queue
- We'll talk about Greedy Best-First Search and A* Search

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Heuristic Function

- $h(n)$ = estimated cost of the cheapest path from node n to a goal node
- Non-negative
- h(goal node) = 0
- Contains additional knowledge of the problem

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Greedy Best-First Search

- Expands the node that is closest to the goal
- $f(n) = h(n)$

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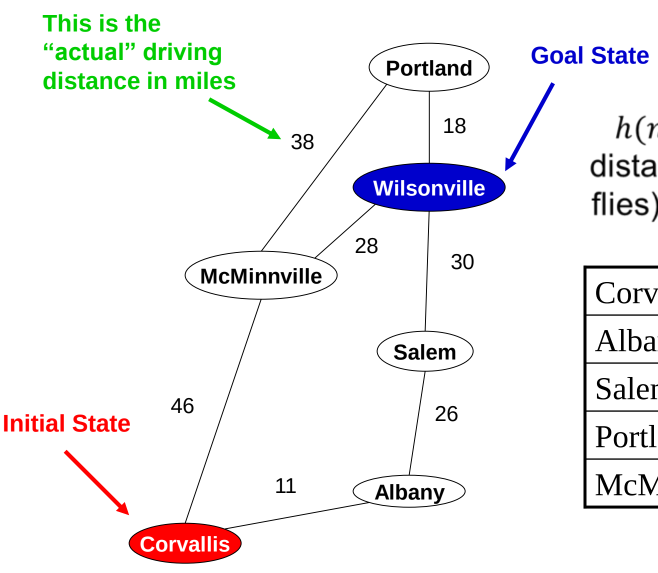
Greedy Best-First Search Example



$h(n)$: Straight line distance (as the crow flies) to Wilsonville in miles

Corvallis	56
Albany	49
Salem	28
Portland	17
McMinnville	18

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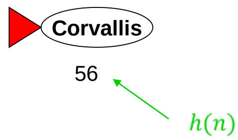
Greedy Best-First Search Example



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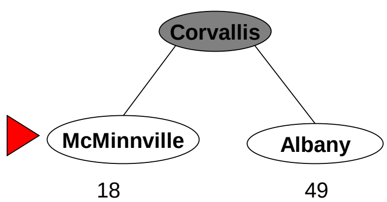
Greedy Best-First Search Example



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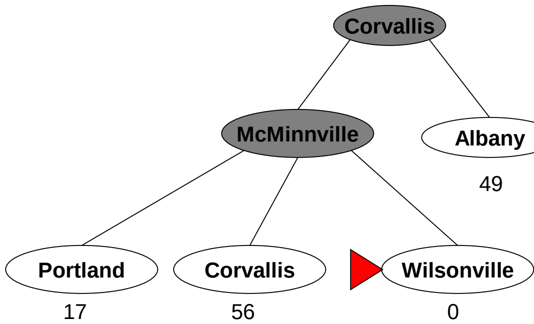

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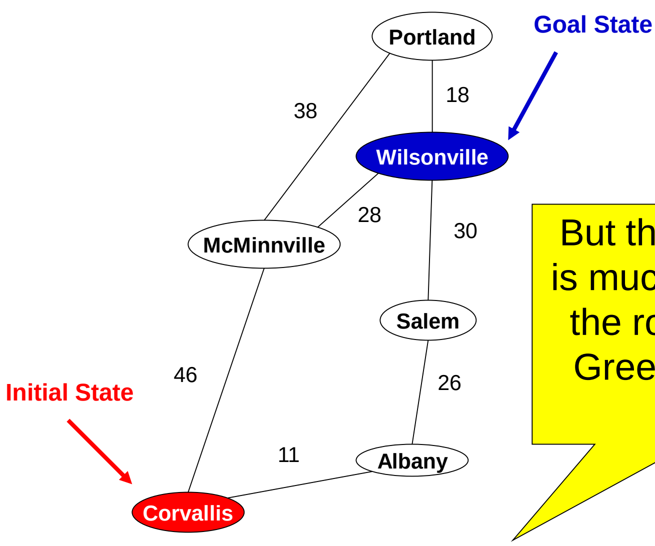
Corvallis →McMinnville→ Wilsonville = 74 miles

Greedy Best-First Search Example



But the route below is much shorter than the route found by Greedy Best-First Search!

Corvallis →Albany→ Salem → Wilsonville = 67 miles

Evaluating Greedy Best-First Search

Complete?	No (could start down an infinite path)
Optimal?	
Time Complexity	
Space Complexity	

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Evaluating Greedy Best-First Search

Complete?	No (could start down an infinite path)
Optimal?	No
Time Complexity	
Space Complexity	

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Evaluating Greedy Best-First Search

Complete?	No (could start down an infinite path)
Optimal?	No
Time Complexity	$O(b^m)$
Space Complexity	

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Evaluating Greedy Best-First Search

Complete?	No (could start down an infinite path)
Optimal?	No
Time Complexity	$O(b^m)$
Space Complexity	$O(b^m)$

Greedy Best-First search results in lots of dead ends which leads to unnecessary nodes being expanded

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A* Search

- A much better alternative to greedy best-first search
- Evaluation function for A* is:

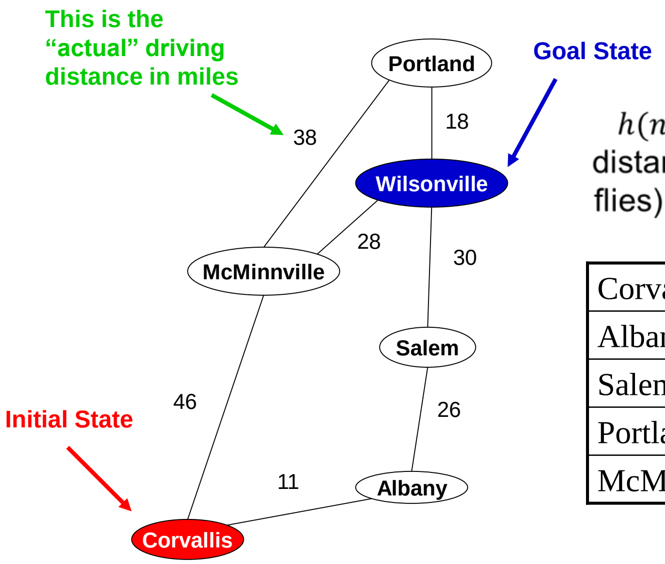
  $f(n) = g(n) + h(n)$

  where $g(n)$ = path cost from the start node to n
- If $h(n)$ satisfies certain conditions, A* search is optimal and complete!

Admissible Heuristics

- A* is optimal if $h(n)$ is an admissible heuristic
- An admissible heuristic is one that never overestimates the cost to reach the goal
- Admissible heuristic = optimistic
- Straight line distance was an admissible heuristic

Greedy Best-First Search Example



$h(n)$: Straight line distance (as the crow flies) to Wilsonville in miles

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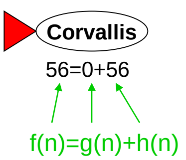
A* Search Example



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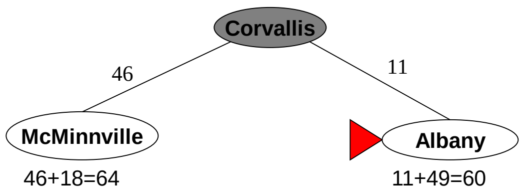

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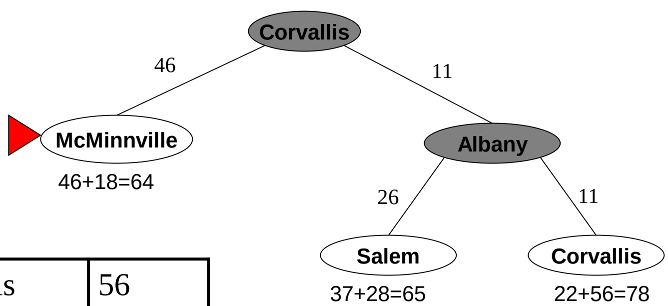
A* Search Example



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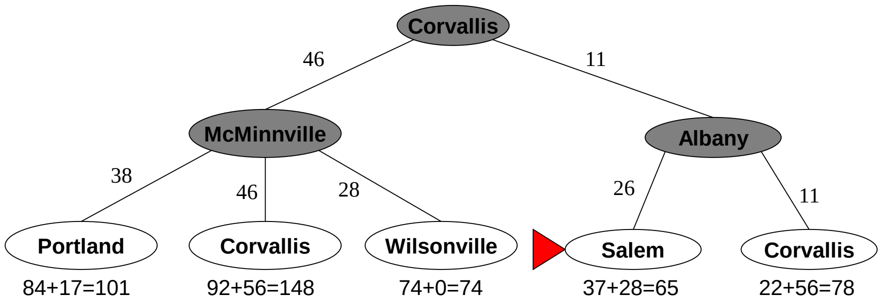
A* Search Example



Note: Don’t stop when you put a goal state on the priority queue (otherwise you get a suboptimal solution)

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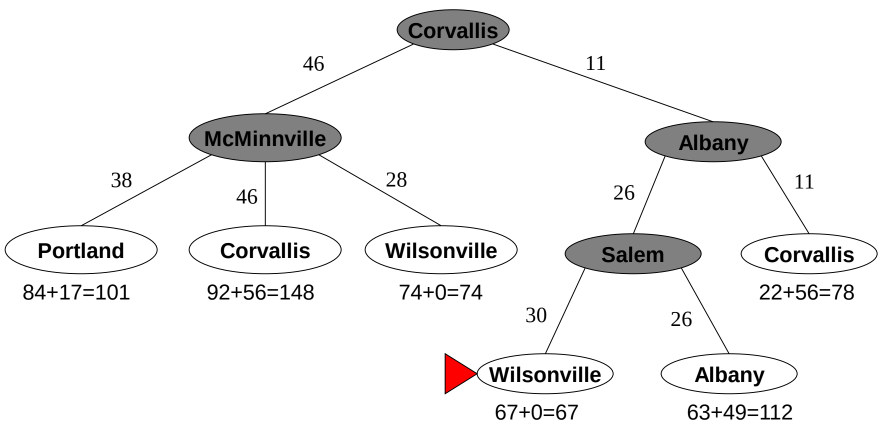
A* Search Example

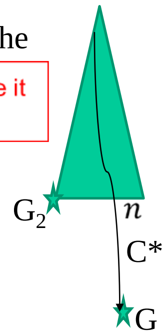


Proper termination: Stop when you pop a goal state from the priority queue

Proof that A* using TREE-SEARCH is optimal if $h(n)$ is admissible

- Suppose A* returns a suboptimal goal node G_2.
- G_2 must be the least cost node in the frontier. Let the cost of the optimal solution be C*.
- Because G_2 is suboptimal:
  $f(G_2) = g(G_2) + h(G_2) = g(G_2) > C^*$

  $h(G_2) = 0$ because it is a goal node
- Now consider a frontier node n on an optimal solution path to the goal G.
- If $h(n)$ is admissible, then:
  $f(n) = g(n) + h(n) \leq C^*$
- We have shown that $f(n) \leq C^* < f(G_2)$, so G_2 will not get expanded before n. Hence A* must return an optimal solution.



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What about search graphs (more than one path to a node)?

- What if we expand a state we've already seen?
- Suppose we use the GRAPH-SEARCH solution and not expand repeated nodes
- Could discard the optimal path if it's not the first one generated
- One simple solution: ensure optimal path to any repeated state is always the first one followed (like in Uniform-cost search)
- Requires an extra requirement on $h(n)$ called consistency (or monotonicity)

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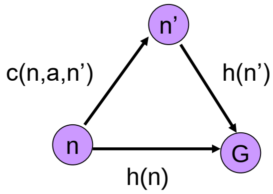
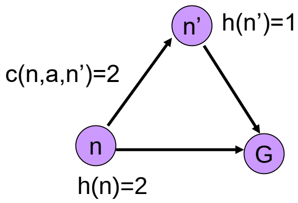
Consistency

- A heuristic is consistent if, for every node n and every successor n' of n generated by any action a:

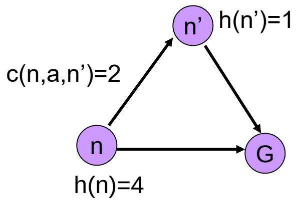
$$h(n) \le \underbrace{c(n, a, n')}_{\text{Step cost of going from } n \text{ to } n' \text{ by doing action } a} + h(n')$$

- A form of the triangle inequality – each side of the triangle cannot be longer than the sum of the two sides





CONSISTENT



INCONSISTENT

Consistency

- Every consistent heuristic is also admissible
- A* using GRAPH-SEARCH is optimal if $h(n)$ is consistent
- Most admissible heuristics are also consistent

Consistency

- Claim: If $h(n)$ is consistent, then the values of $f(n)$ along any path are non-decreasing
- Proof:

Suppose n' is a successor of n. Want to show $f(n') \geq f(n)$

$$
\begin{aligned}
g(n') &= g(n) + c(n,a,n') \text{ for some } a \\
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

From defn of consistency:
$c(n,a,n') + h(n') \geq h(n)$

- Thus, the sequence of nodes expanded by A* is in non-decreasing order of $f(n)$
- First goal selected for expansion must be an optimal solution since all later nodes will be at least as expensive

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A* is Optimally Efficient

- Among optimal algorithms that expand search paths from the root, A* is **optimally efficient** for any given consistent heuristic function
- Optimally efficient: no other optimal algorithm is guaranteed to expand fewer nodes than A*
  - Fine print: except A* might possibly expand more nodes with $f(n) = C^*$ where C^* is the cost of the optimal path – tie-breaking issues
- Any algorithm that does not expand all nodes with $f(n) < C^*$ runs the risk of missing the optimal solution

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Evaluating A* Search

With a consistent heuristic, A* is complete, optimal and optimally efficient. Could this be the answer to our searching problems?

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Evaluating A* Search

With a consistent heuristic, A* is complete, optimal and optimally efficient. Could this be the answer to our searching problems?

The Dark Side of A*…

Time complexity is exponential (although it can be reduced significantly with a good heuristic)

The really bad news: space complexity is exponential (usually need to store all generated states). Typically runs out of space on large-scale problems.

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Summary of A* (tree) Search

Complete?	Yes if h(n) is admissible, b is finite, and all step costs exceed some finite ε
Optimal?	
Time Complexity	
Space Complexity	

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Complete?	Yes if h(n) is admissible, b is finite, and all step costs exceed some finite ε
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Time Complexity	$O(b^d)$ (In the worst case but a good heuristic can reduce this significantly)
Space Complexity	

Summary of A* (tree) Search

Complete?	Yes if h(n) is admissible, b is finite, and all step costs exceed some finite ε
Optimal?	Yes if h(n) is admissible
Time Complexity	$O(b^d)$ (In the worst case but a good heuristic can reduce this significantly)
Space Complexity	$O(b^d)$ – Needs O(number of states), will run out of memory for large search spaces

Summary of A* (graph) Search

Complete?	Yes if h(n) is consistent, b is finite, and all step costs exceed some finite ε [1]
Optimal?	Yes if h(n) is consistent, b is finite, and all step costs exceed some finite ε [1]
Time Complexity	$O(b^d)$ (In the worst case but a good heuristic can reduce this significantly)
Space Complexity	$O(b^d)$ – Needs O(number of states), will run out of memory for large search spaces

[1] Since $f(n)$ is non-decreasing, we must eventually hit an $f(n)$ = cost of the path to a goal state

Iterative Deepening A*

- Use iterative deepening trick to reduce memory requirements for A*
- In each iteration do a "cost-limited" depth first search.
  - Cutoff is based on the f-cost (g+h) rather than the depth
- After each iteration, the new cutoff is the smallest f-cost that exceeded the cutoff in the previous iteration

Complete, Optimal but more costly than A* and can take a while to run with real-valued costs

Examples of heuristic functions

The 8-puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

End State

Heuristic #1: h_1 = number of misplaced tiles e.g., start state has 8 misplaced tiles. This is an admissible heuristic.

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Examples of heuristic functions

The 8-puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

End State

Heuristic #2: h_2 = total Manhattan distance (sum of horizontal and vertical moves, no diagonal moves). Start state is 3+1+2+2+3+2+2+3=18 moves away from the end state. This is also an admissible heuristic.

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Which heuristic is better?

- h_2 dominates h_1. That is, for any node n, $h_2(n) \geq h_1(n)$.
- h_2 never expands more nodes than A* using h_1 (except possibly for some nodes with $f(n) = C^*$)

Proof:

Let h_1 and h_2 be admissible heuristics.

Every node with $f(n) < C^*$ will surely be expanded, since A* is optimal with an admissible heuristic. Since $f(n) = g(n) + h(n)$, every node with $h(n) < C^* - g(n)$ will surely be expanded for either heuristic.

Since h_2 is at least as big as h_1 for all nodes, every node expanded with A* using h_2 will also be expanded with A* using h_1. But h_1 might expand other nodes as well. In other words, we have $h_1(n) \leq h_2(n) < C^* - g(n)$

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Which heuristic is better?

	# nodes expanded		
Depth	**IDS**	**A*(h_1)**	**A*(h_2)**
2	10	6	**6**
4	112	13	**12**
6	680	20	**18**
8	6384	39	**25**
10	47127	93	**39**
12	3644035	227	**73**
14		539	**113**
16		1301	**211**
18		3056	**363**
20		7276	**676**
22		18094	**1219**
24		39135	**1641**

From Russell and Norvig Figure 4.8 (Results averaged over 100 instances of the 8-puzzle for depths 2-24).

Inventing Admissible Heuristics

- Relaxed problem: a problem with fewer restrictions on the actions
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If we relax the rules so that a square can move anywhere, we get heuristic h_1
- If we relax the rules to allow a square to move to any adjacent square, we get heuristic h_2

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What you should know

- Be able to run A* by hand on a simple example
- Why it is important for a heuristic to be admissible and consistent
- Pros and cons of A*
- How do you come up with heuristics
- What it means for a heuristic function to dominate another heuristic function

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