CS 331: Artificial Intelligence
Propositional Logic I

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Outline

1. Knowledge-based Agents
2. The Wumpus World
3. Logic

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Knowledge-based Agents

- Can represent knowledge
- And reason with this knowledge
- How is this different from the knowledge used by problem-specific agents?
  - More general
  - More flexible

Knowledge-based Agents

- Knowledge of problem solving agents is specific and inflexible
- Knowledge-based agents can benefit from knowledge expressed in very general forms, combining information in different ways to suit different purposes
- Knowledge-based agents can combine general knowledge with current percepts to infer hidden aspects of the current state

Knowledge-based Agents

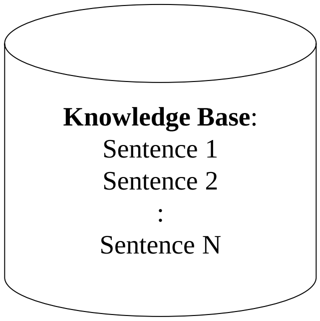
Flexibility of knowledge-based agents:

- Accept new tasks in the form of explicitly described goals
- Achieve competence quickly by being told or learning new knowledge about the environment
- Adapt to changes in the environment by updating the relevant knowledge

Knowledge is definite

- Knowledge of logical agents is always definite
- That is, each proposition is entirely true or entirely false
- Agent may be agnostic about some propositions
- Logic doesn’t handle uncertainty well

The Knowledge Base (KB)



- A knowledge base is a set of “sentences”
- Each sentence is expressed in a knowledge representation language and represents some assertion about the world

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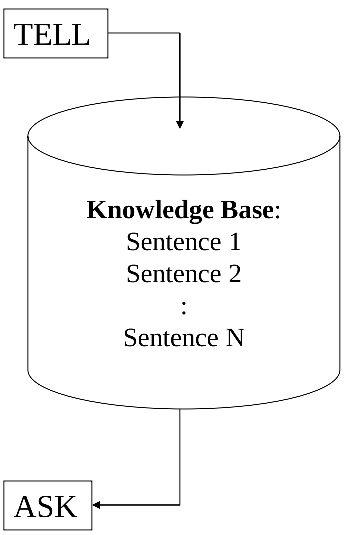
The Knowledge Base (KB)



- Need to add new sentences to the knowledge base (this task is called TELL)
- Need to query what is known (this task is called ASK)

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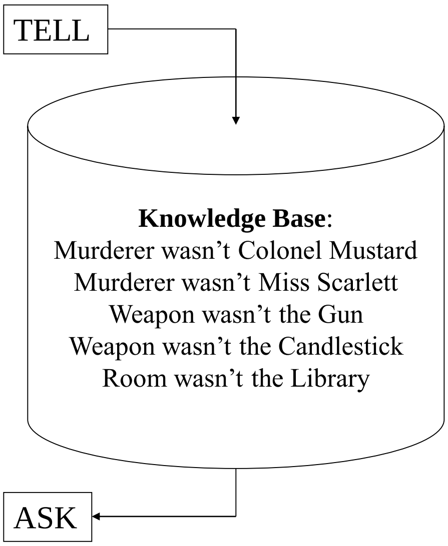
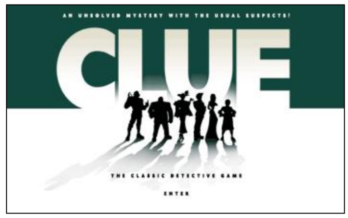
Knowledge Base Example





When you discover a new fact like "The murder room wasn't the study", you would TELL the KB

You can then ASK the KB what to ask next

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Inference

- Inference: deriving new sentences from old ones
- Must obey fundamental requirement: when one ASKs a question of the knowledge base, answer should follow from what has been TELLed to the KB previously

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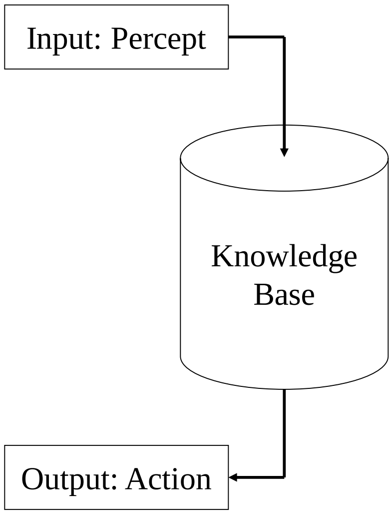
A Generic Knowledge-based Agent



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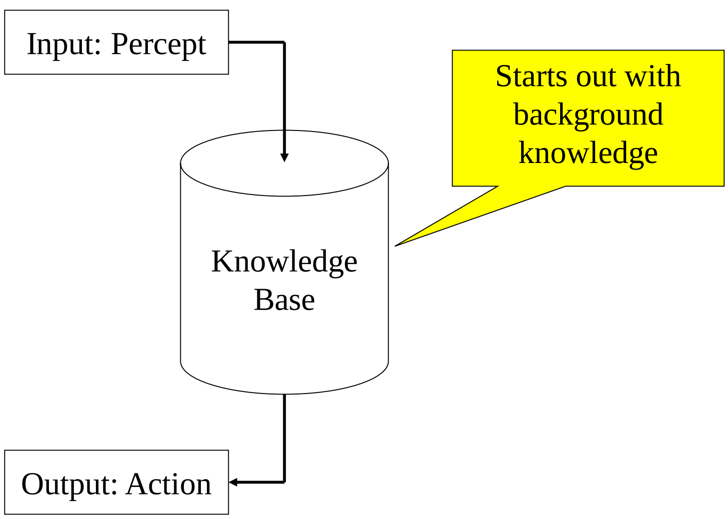
A Generic Knowledge-based Agent



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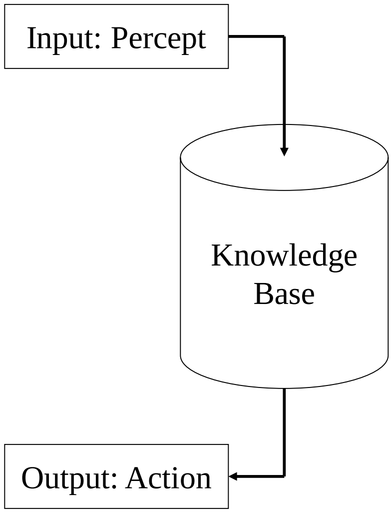
A Generic Knowledge-based Agent



1. TELL the KB what it perceives
2. ASK the KB what action it should perform
3. TELL the KB that the action was executed

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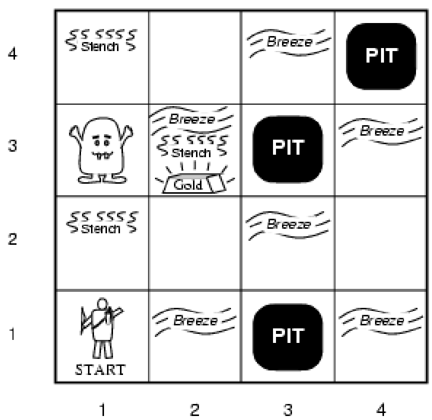
The Wumpus World



- Wumpus eats anyone that enters its room
- Wumpus can be shot by an agent, but agent has one arrow
- Pits trap the agent (but not the wumpus)
- Agent's goal is to pick up the gold

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The Wumpus World

- **Performance measure**:
  - +1000 for picking up gold, -1000 for death (meeting a live wumpus or falling into a pit)
  - -1 for each action taken, -10 for using arrow
- **Environment**:
  - 4x4 grid of rooms
  - Agent starts in (1,1) and faces right
  - Geography determined at the start:
    - Gold and wumpus locations chosen randomly
    - Each square other than start can be a pit with probability 0.2

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The Wumpus World

- **Actuators**:
  - Movement:
    - Agent can move forward
    - Turn 90 degrees left or right
  - Grab: pick up an object in same square
  - Shoot: fire arrow in straight line in the direction agent is facing

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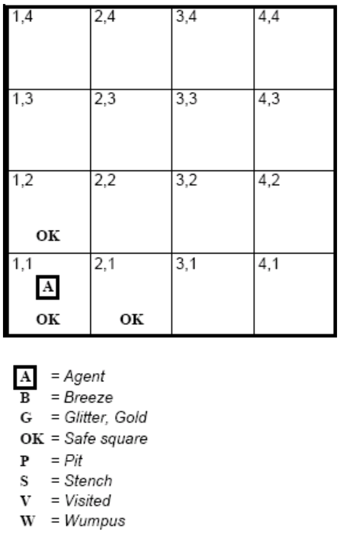
The Wumpus World

- **Sensors**:
  - Returns a 5-tuple of five symbols e.g., [stench, breeze, glitter, bump, scream] (note that in this 5-tuple, all five things are present. We indicate absence with the value None)
  - In squares adjacent to the wumpus, agent perceives a stench
  - In squares adjacent to a pit, agent perceives a breeze
  - In squares containing gold, agent perceives a glitter
  - When agent walks into a wall, it perceives a bump
  - When wumpus is killed, it emits a woeful scream that is perceived anywhere

The Wumpus World

- Biggest challenge: Agent is ignorant of the configuration of the 4x4 world
- Needs logical reasoning of percepts in order to overcome this ignorance
- Note: retrieving gold may not be possible due to randomly generated location of pits
- Initial knowledge base contains:
  - Agent knows it is in [1,1]
  - Agent knows it is a safe square

Wumpus World Example



1st percept is:

[None, None, None, None, None]

(Corresponding to [Stench, Breeze, Glitter, Bump, Scream])

Agent concludes squares [1,2], [2,1] are safe. We mark them with OK. A cautious agent will move only to a square that it knows is OK.

Agent now moves to [2,1]

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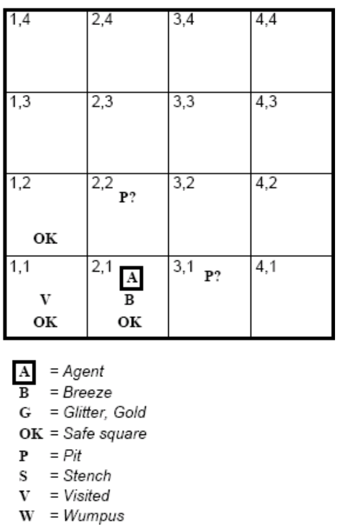
Wumpus World Example



2nd percept is:

[None, Breeze, None, None, None]

Must be a pit at [2,2] or [3,1] or both. We mark this with a P?.

Only one square that is OK, so the agent goes back to [1,1] and then to [1,2]

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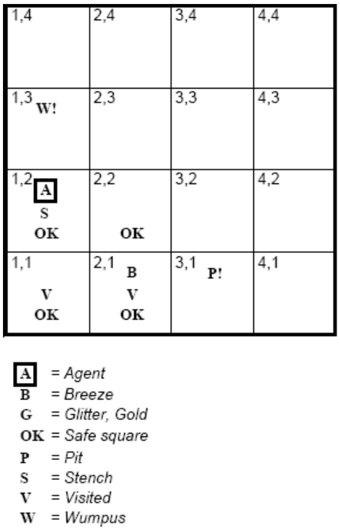
Wumpus World Example



3rd percept is:

[Stench, None, None, None, None]

Wumpus must be nearby. Can't be in [1,1] (by rules of the game) or [2,2] (otherwise agent would have detected a stench at [2,1])

Therefore, Wumpus must be in [1,3]. Indicate this by W!.

Lack of breeze in [1,2] means no pit in [2,2], so pit must be in [3,1].

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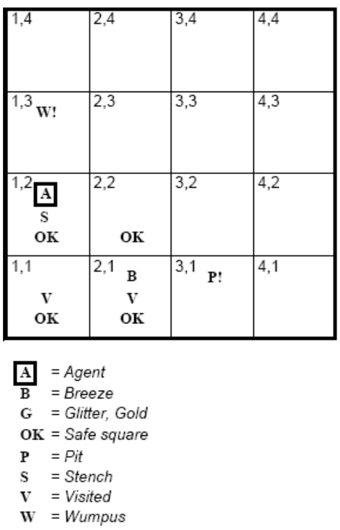
Wumpus World Example



Note the difficulty of this inference:

- Combines knowledge gained at different times and at different places.
- Relies on the lack of a percept to make one crucial step

At this point, the agent moves to [2,2].

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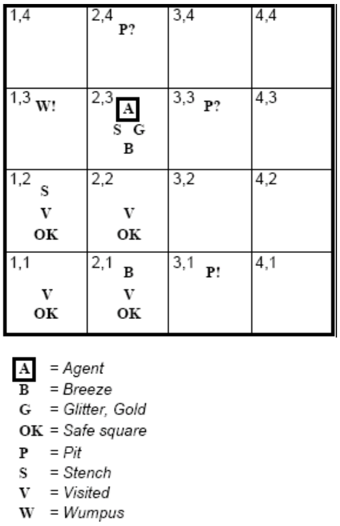
Wumpus World Example



We'll skip the agent's state of knowledge at [2,2] and assume it goes to [2,3].

Agent detects a glitter in [2,3] so it grabs the gold and ends the game

Note: In each case where the agent draws a conclusion from the available information, that conclusion is guaranteed to be correct if the available information is correct

Logic

Logic must define:

1. Syntax of the representation language
   - Symbols, rules, legal configurations
2. Semantics of the representation language
   - Loosely speaking, this is the "meaning" of the sentence
   - Defines the truth of each sentence with respect to each possible world
   - Everything is either true or false, no in between

Models

- We will use the word model instead of "possible world"
- "m is a model of α" means that sentence α is true in model m
- Models are mathematical abstractions which fix the truth or falsehood of every relevant sentence
- Think of it as the possible assignments of values to the variables
  - E.g. the possible models for $x + y = 4$ are all possible assignments of numbers to x and y such that they add up to 4

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Entailment

$\alpha \models \beta$ means α entails β i.e. β follows logically from α, where α and β are sentences

Mathematically, $\alpha \models \beta$ if and only if in every model in which α is true, β is also true.

Another way: if α is true, then β must also be true.

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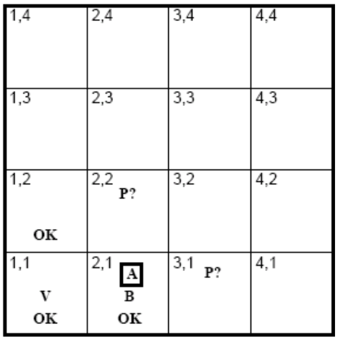
Entailment Applied to the Wumpus World



- Suppose the agent moves to [2,1]
- Agent knows there is nothing in [1,1] and a breeze in [2,1]
- These percepts, along with the agent's knowledge of the rules of the wumpus world constitute the KB
- Given this KB, agent is interested if the adjacent squares [1,2], [2,2] and [3,1] contain pits.

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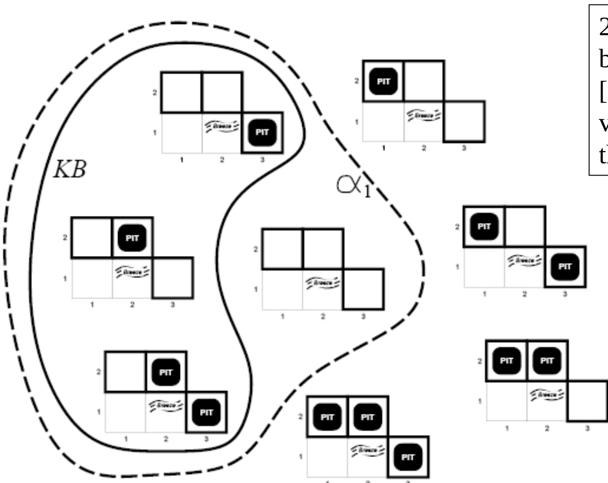
Entailment Applied to the Wumpus World



2^3 = 8 possible models because [1,2], [2,2] and [3,1] can take each take values true or false that there is a pit there

The 3 models inside the line marked KB are those in which the KB is true

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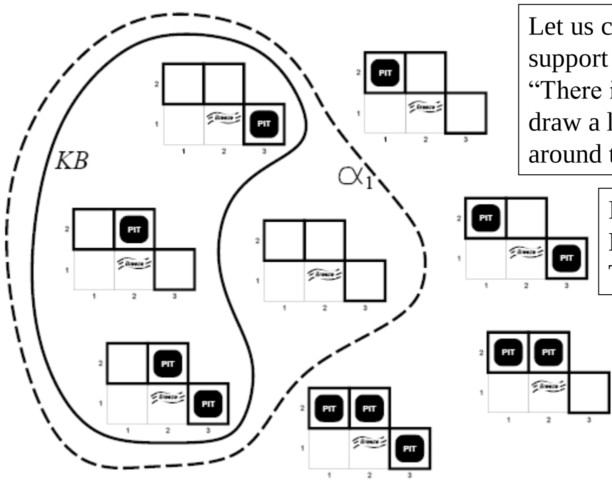
Entailment Applied to the Wumpus World



Let us consider the models that support the conclusion α_1 = "There is no pit in [1,2]." We draw a line marked with α_1 around these models

In every model in which KB is true, α_1 is also true. Therefore KB $\models \alpha_1$

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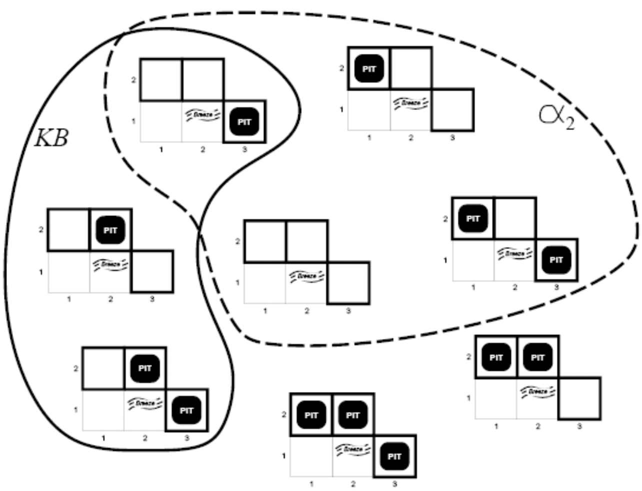
Entailment applied to the Wumpus World



Now let us consider the models that support the conclusion α_2 = "There is no pit in [2,2]." We draw a line marked with α_2 around these models

In some models in which KB is true, α_2 is false. Therefore KB $\not\models \alpha_2$ and the agent cannot conclude that there is no pit in [2,2]

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Logical inference

- Entailment can be applied to derive conclusions (we call this carrying out logical inference)
- Model checking: enumerates all possible models to check that α is true in all models in which KB is true
- If an inference algorithm *i* can derive α from the KB, we write KB $\vdash_i \alpha$
- The above is pronounced "α is derived from KB by *i*" or "*i* derives α from KB"

Soundness

- An inference algorithm that derives only entailed sentences is called sound or truth-preserving
- Soundness is a good thing!
- If an inference algorithm is unsound, you can make things up as it goes along and derive basically anything it wants to

This (unsoundness) is most illogical

Completeness

- An inference algorithm is complete if it can derive any sentence that is entailed
- For some KBs, the number of sentences can be infinite
- Can't exhaustively check all of them, need to rely on proving completeness

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In Summary

- Soundness: *i* is sound if whenever KB $\vdash_i \alpha$, it is also true that KB $\models \alpha$
- Completeness: *i* is complete if whenever *KB* $\models \alpha$, it is also true that *KB* $\vdash_i \alpha$

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Propositional Logic: Syntax and Semantics

Syntax: Backus-Naur Form grammar of sentences in propositional logic

Sentence → AtomicSentence | ComplexSentence

AtomicSentence → **True** | **False** | Symbol

Symbol → **P** | **Q** | **R** | …

ComplexSentence → ¬ Sentence

| (Sentence ∧ Sentence)

| (Sentence ∨ Sentence)

| (Sentence ⇒ Sentence)

| (Sentence ⇔ Sentence)

Atomic Sentences

- The indivisible syntactic elements
- Consist of a single propositional symbol e.g., P, Q, R that stands for a proposition that can be true or false e.g., P=true, Q=false
- We also call an atomic sentence a literal
- 2 special propositional symbols:
  - True (the always true proposition)
  - False (the always false proposition)

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Complex Sentences

- Made up of sentences (either complex or atomic)
- 5 common logical connectives:
  - $\neg$ (not): negates a literal
  - $\wedge$ (and): conjunction e.g. $P \wedge Q$ where P and Q are called the conjuncts
  - $\vee$ (or): disjunction e.g. $P \vee Q$ where P and Q are called the disjuncts
  - $\Rightarrow$ (implies): e.g. $P \Rightarrow Q$ where P is the premise/antecedent and Q is the conclusion/consequent
  - $\Leftrightarrow$ (if and only if): e.g. $P \Leftrightarrow Q$ is a biconditional

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Precedence of Connectives

- In order of precedence, from highest to lowest: $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- E.g., $\neg P \vee Q \wedge R \Rightarrow S$ is equivalent to
  $((\neg P) \vee (Q \wedge R)) \Rightarrow S$
- You can rely on the precedence of the connectives or use parentheses to make the order explicit
- Parentheses are necessary if the meaning is ambiguous

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Semantics (Are sentences true?)

- Defines the rules for determining if a sentence is true with respect to a particular model
- For example, suppose we have the following model: P=true, Q=false, R=true
- Is $(P \wedge Q \wedge R)$ true?

I want the truth!

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Semantics

For atomic sentences:

- True is true, False is false
- A symbol has its value specified in the model

For complex sentences (for any sentence S and model m):

- $\neg S$ is true in m iff S is false in m
- $S_1 \wedge S_2$ is true in m iff S_1 is true in m and S_2 is true in m
- $S_1 \vee S_2$ is true in m iff S_1 is true in m or S_2 is true in m
- $S_1 \Rightarrow S_2$ is true in m iff S_1 is false in m or S_2 is true in m
  i.e., can translate it as $\neg S_1 \vee S_2$
- $S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true in m and $S_2 \Rightarrow S_1$ is true in m

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Note on implication

- $P \Rightarrow Q$ seems weird…doesn't fit intuitive understanding of "if P then Q"
- Propositional logic does not require causation or relevance between P and Q
- Implication is true whenever the antecedent is false (remember $P \Rightarrow Q$ can be translated as $\neg P \vee Q$)
  - Implication says "if P is true, then I am claiming that Q is true. Otherwise I am making no claim"
  - The only way for this to be false is if P is true but Q is false

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Truth Tables for the Connectives

P	¬P
false	true
true	false

P	Q		P ∧ Q	P ∨ Q	P ⇒ Q	P ⇔ Q
false	false		false	false	true	true
false	true		false	true	true	false
true	false		false	true	false	false
true	true		true	true	true	true

With the truth tables, we can compute the truth value of any sentence with a recursive evaluation e.g.,

Suppose the model is P=false, Q=false, R=true

$\neg P \wedge (Q \vee R)$ = true $\wedge$ (false $\vee$ true) = true $\wedge$ true = true

The Wumpus World KB (only dealing with knowledge about pits)

For each i, j:
Let $P_{i,j}$ be true if there is a pit in $[i, j]$
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$

The KB contains the following sentences:

1. There is no pit in [1,1]:
   R_1: $\neg P_{1,1}$
2. A square is breezy iff there is a pit in a neighboring square: (not all sentences are shown)
   R_2: $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$
   R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
   :

The Wumpus World KB

3. We add the percepts for the first two squares ([1,1] and [2,1]) visited in the Wumpus World example:

   R_4: $\neg B_{1,1}$

   R_5: $B_{2,1}$

The KB is now a conjunction of sentences $R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5$ because all of these sentences are asserted to be true.

Inference

- How do we decide if KB |= α?
- Enumerate the models, check that α is true in every model in which KB is true

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
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false	true	false	false	false	false	true	true	true	true	true	true	true
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:	:	:	:	:	:	:	:	:	:	:	:	:
true	true	true	true	true	true	true	false	true	true	false	true	false

Inference

- Suppose we want to know if KB |= $\neg P_{1,2}$?
- In the 3 models in which KB is true, $\neg P_{1,2}$ is also true

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
:	:	:	:	:	:	:	:	:	:	:	:	:
false	true	false	false	false	false	false	true	true	false	true	true	false
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:	:	:	:	:	:	:	:	:	:	:	:	:
true	true	true	true	true	true	true	false	true	true	false	true	false

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Complexity

- If the KB and α contain n symbols in total, the time complexity of the truth table enumeration algorithm is $O(2^n)$
- Space complexity is O(n) because the actual algorithm uses DFS

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The really depressing news

- Every known inference algorithm for propositional logic has a **worst-case** complexity that is exponential in the size of the input

- But some algorithms are more efficient **in practice**

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Things you should know

- Properties of a knowledge-based agent
- What a knowledge-base is
- What entailment and inference mean
- Desirable properties of inference algorithms such as soundness and completeness

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