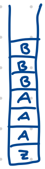
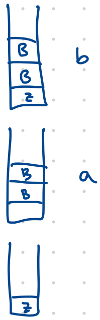
CS 321 Thur March 3, 2022

HW #6

1. a) $L_1 = \{a^n b^m c^{n+m} \mid n \geq 0, m \geq 0\}$



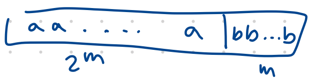
b) $L_2 = \{w : n_a(w) = 2 n_b(w)\}$



2 $B \rightarrow bBB \mid A$ substitute Gremech's NF

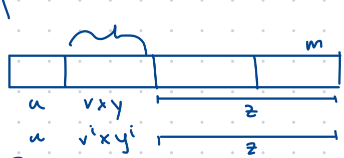
3. Pumping lemma (CFL)

(a) $L = \{a^n b^m \mid n = 2^m\}$

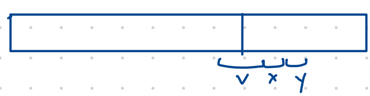


uvxyz

Case 1



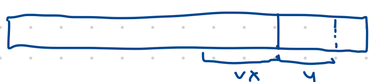
Case 3



v contains a's and b's

Case 2

Case 4



(b)

(c) $L_3 = \{w : w \in \{a,b,c\}^* \mid n_a(w) < n_b(w) < n_c(w)\}$

$a^m b^{m+1} c^{m+2}$

$uvxyz$

$\Rightarrow uv^i xy^i z \in L$

$i = 0, 1, 2, \ldots$

when you eliminate one, then the proof will follow.

Set Theory

1) Finite set

$\{1, 4, 8, 15, 25\}$

# of elements in finite

2) Infinite Set

a) Countable infinite

b) Uncountable infinite

countable infinite : there exists a 1-1 function from the set to the positive integers $\{1, 2, 3, \ldots \infty\}$

$S: \{-\infty \ldots -4, -3, -2, -1, 0, 1, 2 \ldots \infty\}$

$f: S \rightarrow Z$

$f(i) = 2(i+1)$ if $i \geq 0$

$f(i) = -2i - 1$

$f(4) = 2(4) = 8$

$f(-3) = -2 \times -3 - 1 = 5$

EX.

even

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \ldots$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$

$$
\begin{array}{cccccccccc}
-1 & 0 & -2 & 1 & -3 & 2 & & & \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \ldots
\end{array}
$$

proper enumeration

$\{a, b, c\}^*$ countable infinite

a
aa
aaa
aaab

What is the proper enumeration?

a	1	ba	7
b	2	bb	8
c	3	bc	9
aa	4	ca	10
ab	5	cb	11
ac	6	cc	12

aaa → 13
aab – 14
aac – 15
aba – 16
abb – 17
abc – 18

Radix - 3
number system
a → 1
b → 2
c → 3

$$abb = 122$$
$$= 1 \times 3^2 + 2 \times 3 + 2 = 9 + 6 + 2$$
$$= 17$$

$$babc \to 2123$$
$$= 2 \times 2^3 + 1 \times 3^2 + 2 \times 3^1 + 3$$
$$= 54 + 9 + 6 + 3$$
$$= 72$$

3 ⌊72
3 ⌊23 – 3
3 ⌊7 – 2
2 – 1

remainder can be
1, 2, 3

2123 → babc

binary
0 → 1
1 → 2
00 → 3
01 → 4
10 → 5
11 → 6
000 → 7
001 → 8

0 → 1
1 → 2

000 → 111

$$1 \times 2^2 + 1 \times 2^1 + 1$$
$$4 + 2 + 1 = 7$$

turing machine

$\{1, 2, 3 \ldots \omega\}$

↳ countable infinite

powerset is uncountable.

↳ infinite