2014

HAVERFORD COLLEGE

Department of Physics

Physics 309b

Assignment #1

Due: In class, Tuesday, Feb. 4, 2014

Reading: Griffiths, Chapter 1 and Chapter 2, sections 1-4.

1. Griffiths, Chapter 1, Problem 1.12.
2. The flux of a fluid, i.e., the mass flow per unit area per unit time, is given by the mass density of the fluid times the velocity field. Check to make sure this quantity has the appropriate units. Also, under ordinary circumstances, mass is conserved, i.e., mass is neither created nor destroyed.

   a) Use the divergence theorem to show that the divergence of the flux within a fluid flow vanishes if the fluid is incompressible.

   b) Now show that the divergence of the velocity also vanishes.

3. Suppose a fluid rotates about the $\hat{z}$ axis such that velocity of flow at any point is given by (in cylindrical coordinates)

$$\mathrm{v} = \mathrm{v}(s)\hat{\phi}$$

   where the speed v(s) is a function of s only. A model that approximates vortices in viscous fluids is the Rankine vortex for which $\nabla \times \mathrm{v} = \omega\hat{z}$ for $s \leq R$ and $\nabla \times \mathrm{v} = 0$ for $s > R$, where $\omega$ is constant.

   a) What is the radial velocity profile, v(s), for a Rankine vortex? Note: Assume that v(s) is continuous at s = R.

   b) Evaluate the "circulation", $\oint v \cdot d\ell$, for this flow as a function of s.

4. An infinitely long wire, stretching along the z axis, is uniformly charged to $\lambda$ coulombs/meter. Find the electric field intensity outside the wire by: a) using Gauss's Law; b) integrating Coulomb's Law directly, and c) computing the potential directly and then taking the gradient. In method c), show that the mathematics breaks down and give an explanation for this. Can you find a way around the problem?
5. Griffiths, Chapter 2, Problem 2.6.
6. Griffiths, Chapter 2, Problem 2.15.
7. Griffiths, Chapter 2, Problem 2. 21
8. Griffiths, Chapter 2, Problem 2.34, parts a) and b only; and Problem 2.35.
9. At an early stage in the development of the atomic theory, J. J. Thompson proposed an atom consisting of a positive charge Ze spread uniformly throughout a sphere of radius R where Z is the number of electrons each with charge -e and e is the fundamental unit of charge

($1.6 \times 10^{-19}$ coulombs). The point-like electrons are embedded in the uniform positive charge like raisins in raisin pudding, hence the name "raisin pudding" model of the atom.

a.) Find the force acting on one electron as a function of its distance r from the center of the sphere. Assume that the charge of the remaining electrons is smeared out uniformly throughout the sphere.

b) What type of motion does the electron execute in the radial direction?

c) What is the frequency of this motion for a typical atomic radius of 1 angstrom (0.1 nm)?

d) What is the frequency of an electron that is in a circular orbit about the center? Interesting, no?

e) How does this frequency compare with that of the fundamental Lyman $\alpha$ transition in hydrogen? The wavelength of a Lyman $\alpha$ photon is 121.6 nm.

10. Find the charge distribution (i.e., the charge density) that produces the Yukawa potential (a model for the nuclear force), i.e.,

$$V(r) = \frac{A}{r} e^{-r/a}$$

Hint: In addition to a continuous charge density, you must place a point charge at the origin. Why? Be sure to give the value of this point charge, which you should be able to identify from the form of the electric field. If not, try Gauss's law.