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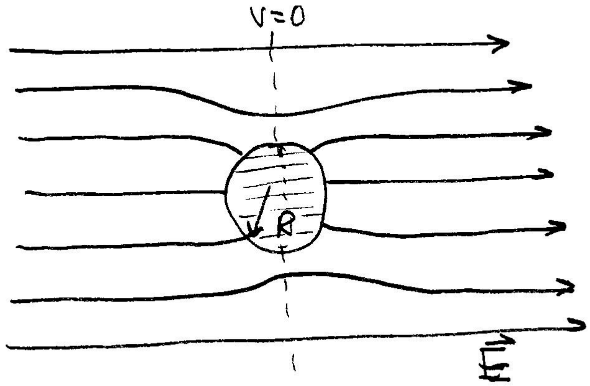

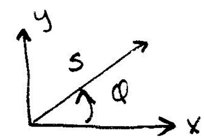

Far from the cylinder clearly

$$V = -E_0 x = -E_0 s \cos\varphi$$

B.C. i) $V \xrightarrow[s\to\infty]{} -E_0 s \cos\varphi$

Since we've pick the above form then clearly (by symmetry $V(0) = 0$,

Thus our boundary conditions are:

i) $r \to \infty \quad V = -E_0 s \cos\varphi$

ii) $r = 0 \quad V = 0$

iii) $r = R \quad V_{inside} = V_{outside}$

iv) $r = R \quad D_{\perp in} = D_{\perp out} \Rightarrow \varepsilon \frac{\partial V_{inside}}{\partial s} = \varepsilon_0 \frac{\partial V_{outside}}{\partial s} \Rightarrow \varepsilon_r \frac{\partial V_{in}}{\partial s} = \frac{\partial V_{out}}{\partial s}$

The general solution is (see problem #4 of assignment #3)

$$V(s,\varphi) = \sum_{n=1}^{\infty} \left(A_n r^n + B_n/r^n\right)\left(C_n \sin n\varphi + D_n \cos n\varphi\right) + A_0 \ln r + B_0$$

with different coefficients for "inside" and "outside" solutions. Why?

Let's use $A_n, B_n, \cdots$ for the "outside" solution and $a_n, b_n, \cdots$ for the inside.

Then B.C. i) $\Rightarrow$ $A_0 = B_0 = A_n C_n = A_n D_{n\neq 1} = 0$ and $A_1 D_1 = -E_0$

Thus $V(s,\varphi)_{\text{outside}} = -E_0 s \cos\varphi + \sum_{n=1}^{\infty} (C_n \sin n\varphi + D_n \cos n\varphi)/r^n$

where I've absorbed B_n into the C_n and D_n.

Now B.C. ii) $\Rightarrow$ $a_0 = b_0 = b_n = 0$ and the general solution inside is

$$V(s,\varphi)_{\text{inside}} = \sum_{n=1}^{\infty} (c_n \sin n\varphi + d_n \cos n\varphi) r^n \qquad \text{absorbing } a_n \text{ into } c_n \,\&\, d_n$$

B.C. iii) $\Rightarrow$

$$\sum_{n=1}^{\infty} (c_n \sin n\varphi + d_n \cos n\varphi) R^n = -E_0 R \cos\varphi + \sum_{n=1}^{\infty} (C_n \sin n\varphi + D_n \cos n\varphi)/R^n$$

matching coefficients: for $n=1$ $\quad c_1 R = C_1/R$ and $d_1 R = -E_0 R + D_1/R$

for $n \neq 1$ $\quad c_n R^n = C_n/R^n$ and $d_n R^n = D_n/R^n$

B.C. iv) $\Rightarrow$

$$\varepsilon_r \sum_{n=1}^{\infty} n (c_n \sin n\varphi + d_n \cos n\varphi) R^{n-1} = -E_0 \cos\varphi - \sum_{n=1}^{\infty} n (C_n \sin n\varphi + D_n \cos n\varphi)/R^{n+1}$$

Again matching coefficients: for $n \neq 1$ $\quad \varepsilon_r n c_n R^{n-1} = -n C_n/R^{n+1}$; $\varepsilon_r d_n n R^{n-1} = -n D_n/R^{n+1}$

and for $n=1$ $\quad \varepsilon_r c_1 = -C_1/R^2$; $\varepsilon_r d_1 = -E_0 - D_1/R^2$

From these last two conditions clearly $c_n = C_n = 0 = d_{n\neq 1} = D_{N\neq 1}$

Also $\quad d_1 = -E_0 + D_1/R^2$ and $\varepsilon_r d_1 = -E_0 - D_1/R^2 \Rightarrow d_1 = -2E_0/(1+\varepsilon_r)$

and $D_1 = R^2\left(E_0 - 2E_0/1+\varepsilon_r\right) = E_0 R^2 \dfrac{\varepsilon_r - 1}{\varepsilon_r + 1}$

Thus

$$\boxed{V(r<R) = -\frac{2E_0 s}{\varepsilon_r + 1} \cos\varphi}$$

and

$$\boxed{V(r>R) = -E_0 \cos\varphi \left[s - \frac{R^2(\varepsilon_r - 1)}{(\varepsilon_r + 1) s}\right]}$$