Count-min sketch: range queries

Show and analyse the application of count-min sketch to range queries (i, j) for computing $\sum_{k=i}^{j} F[k]$. Hint: reduce the latter query to the estimate of just $t \leq 2\ log\ n$ counters $c_1, c_2, ..., c_t$. Note that in order to obtain a probability at most δ of error (i.e. that $\sum_{l=1}^{t} c_l > \sum_{k=i}^{j} F[k] + 2\epsilon\ log\ n||F||$), it does not suffices to say that it is at most δ the probability of error of each counter c_l: while each counter is still the actual wanted value plus the residual as before, it is better to consider the sum V of these t wanted values and the sum X of these residuals, and apply Markov's inequality to V and X rather than on the individual counters.

SOLUTION

The proposed solution is based on a range tree, where we store a counter for each level of the tree.
At each level we keep a count-min sketch counter with parameter ϵ and δ, fixed. Then every time an items k arrives from the streaming we update the counter at each level i of the tree with the following rule: $\frac{k}{2^{h-i}}$ where h is the hight of the tree (i.e. $h = log_2$). Notice that at the leaf level we store the number as is it, and at each level we store the relative range.