

Randomized min-cut algorithm

Consider the randomized min-cut algorithm discussed in class. We have seen that its probability of success is at least $\frac{1}{\binom{n}{2}}$, where n is the number of its vertices.

- Describe how to implement the algorithm when the graph is represented by adjacency lists, and analyze its running time. In particular, a contraction step can be done in $O(n)$ time.
- A weighted graph has a weight $w(e)$ on each edge e , which is a positive real number. The min-cut in this case is meant to be min-weighted cut, where the sum of the weights in the cut edges is minimum. Describe how to extend the $\frac{1}{\binom{n}{2}}$ [hint: define the weighted degree of a node]
- Show that running the algorithm multiple times independently at random, and taking the minimum among the min-cuts thus produced, the probability of success can be made at least $1 - 1/n^c$ for a constant $c > 0$ (hence, with high probability).

SOLUTION