Monitoring Daily Hedge Fund Performance When Only Monthly Data is Available

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Abstract

This paper introduces a new approach to monitoring the daily risk of investing in hedge funds. Specifically, we use low-frequency (monthly) models to forecast high-frequency (daily) hedge fund returns. This approach addresses the common problem that confronts investors who wish to monitor their hedge funds on a daily basis—disclosure of returns by funds occurs only at a monthly frequency, usually with a time lag. We use monthly returns on investable assets or factors to fit monthly hedge fund returns, then forecast daily returns of hedge funds during the following month using the publicly observed daily returns on the explanatory assets. We show that our approach can be used to forecast daily returns of long/short hedge funds. In addition, for diversified portfolios such as hedge fund indices and funds-of-hedge-funds, it forecasts daily returns very accurately. We illustrate how this approach can be used to (1) hedge daily hedge fund risk and (2) estimate and control value-at-risk.

Investors have long understood the value of frequent disclosure of information from their portfolio managers. For example, arguments for increased disclosure by consumer groups representing individual investors helped to bring about new regulations requiring that U.S. mutual funds disclose their complete portfolio lists at the end of each fiscal quarter, rather than twice per year, starting in May 2004. However, portfolio managers usually resist high frequency disclosure, due to the potential for harmful effects, which include front-running of fund trades by outsiders as well as free-riding by investors on the costly expenditures made to research securities—by replicating fund holdings, instead of investing in the mutual fund itself (Wermers, 2001).

Hedge fund management companies have successfully argued that the costs of frequent portfolio or return disclosure far outweigh the benefits. As a result, unlike mutual funds, which provide daily per-share net asset values (NAVs) widely to the media within two hours of the close of the NYSE, hedge funds typically report NAVs monthly, sometimes with a significant delay.[1] Further, hedge fund database vendors (e.g., TASS, HFR) provide only monthly performance, also with a delay. In addition, hedge funds routinely implement barriers to inflows and outflows, such as subscription or redemption periods, which serve to further slow any response of investors to such infrequent information disclosure (Ding, Getmansky, Liang, and Wermers, 2008). With such barriers in an infrequent disclosure environment, investors are limited in their ability to respond to dramatic market shifts, such as the recent market turmoil of 2008-2009.

However, hedge fund investors may face an even greater need to understand their risk-exposures than mutual fund investors. Hedge funds, while often less volatile to general market conditions, can experience greater volatility during market crises due to their exposures to common strategies (and the risks from correlated flows that such common strategies create—see, for example, Asness, 2007). Also, each hedge fund investor is exposed to negative externalities imposed by the actions of all other investors, such as the costs imposed by investors who are first to exit a fund during a liquidity crisis (see Chen, Goldstein, and Jiang, 2009, for evidence in illiquid mutual funds). Thus, timely information can be of significant value to hedge fund investors who would want to understand daily hedge fund gains/losses, to plan redemptions or contributions, or even to consider potential hedging strategies. For example, while many investors expected poor performance from their hedge fund investments during October 2008, they had little ability to evaluate the magnitude of losses before receiving a performance report after the end of the month, making prompt risk management impossible.

Accordingly, this paper develops a new approach for hedge fund investors to infer timely high-frequency (i.e., daily) results from low-frequency (i.e., monthly) returns data. This new method relies on relatively simple computation methods that use only observed monthly hedge fund returns. Our approach develops a replication strategy for hedge fund

[1] Our discussions with fund-of-fund managers indicate that individual hedge funds offer an estimate of the end-of-month NAV by the 5th business day of the following month, and that the official NAV is usually issued before the next month ends (usually during the 2nd and 3rd week of that month). This includes a capital balance statement from an administrator.

returns that uses only investable assets for which daily returns information is widely available with little time lag. As such, our approach is related to a long line of literature that attempts to replicate *monthly* hedge fund returns with primitive securities, factor exposures, or distribution-based replication procedures (see Gupta, Szado, and Spurgin, 2008, for a survey of these approaches). For example, Hasanhodzica and Lo (2007) perform in-sample and out-of-sample replication of monthly returns of individual hedge funds and compare distribution properties of original funds and their "clones."

However, our goal is different from prior replication techniques. Rather than creating a portfolio that replicates the long-term returns of hedge funds, we wish to create a very simple approach to allow investors to model the risk of their hedge fund investments on a daily basis, using commonly observed investable assets with liquid, daily market prices. Such daily monitoring is especially important to investors during periods of high market volatility, such as during the Fall 2008 crisis. In addition, our intent is to arm practitioners with a useful methodology that allows them to measure the impact of model (static vs dynamic) risk, data and factor specification risk on their results. The significant dispersion of the performance of actual replication investment products noted in Gupta, Szado, and Spurgin (2008) makes such a framework especially important.

Specifically, we project the daily performance of hedge funds by creating synthetic replication portfolios based on a monthly factor model that uses common investable indices. We implement several models, including "returns-based style analysis" (RBSA), introduced by Sharpe, where exposure to the indices are made dynamic by updating the estimation window at the end of each month. In addition, we implement an improved model that uses dynamic style analysis (DSA), which captures dynamic index exposures without a moving window. The DSA model is especially important in modeling high-frequency returns using a low-frequency model, since it quickly detects shifts in factor loadings over time.

Once a high quality in-sample replication is achieved, we then use daily returns of replicating assets to create a daily proxy of the hedge fund. Naturally, our methodology builds on and contributes to a large and growing literature on the exposure of hedge funds to market indices. A number of studies, including Fung and Hsieh (1997, 1999, 2001); Brown, Goetzmann and Ibbotson (1999); Ackermann, McEnally and Ravenscraft (1999); Liang (1999); Agarwal and Naik (2000, 2004); and Markov, Mottl, Muchnik and Krasotkina (2006) have examined the relationship between hedge fund returns and broad market returns.

To backtest the effectiveness of the proposed methodology, we calculate the tracking errors between actual (ex-post) daily hedge fund returns and the corresponding (ex-ante) daily replication portfolios (using common investable indices) created using lagged monthly returns. We find that our approach closely tracks actual daily hedge fund index returns with low tracking error; this small tracking error demonstrates that our replication portfolio can successfully mimic the daily variation of actual hedge fund index returns using only monthly data as inputs. For instance, the daily tracking error for the HFRX EH index is as small as 24 bps in 2007 and 33 bps in 2008. Further, fitting the monthly model with one-month lagged returns (e.g., forecasting September with the July

31 model) only slightly increases daily tracking errors to 28 and 41 bps during 2007 and 2008, respectively. Thus, a one-month delay in receiving returns on the target hedge fund or fund-of-funds does not substantially reduce the power of our model. Further, the correlation between our daily projection returns and actual index returns are close to 90% both with and without a one-month lag.

We also apply our technique to a sample of 17 individual mutual funds with hedge fund-like strategies. We choose these funds since we can track their actual performance with daily returns, thus, we can backtest the success of our replication approach that fits monthly models to replicate daily returns. Our replication technique generates median daily tracking errors of 42 bps in 2007 and 79 bps in 2008 using generic style factors, but this shrinks to 29 bps and 72 bps with style-specific factors derived from knowledge of the specific type of strategy followed by each fund. Even more dramatic is that we use generic style factors to replicate the daily returns of an equal-weighted portfolio of the 17 funds, and obtain daily tracking errors of 15 bps and 27 bps during 2007 and 2008, respectively. This result indicates that fund-of-funds managers can closely track their daily performance with only general knowledge of the styles that might be employed by their individual hedge fund managers.

There are several potential applications of our methodology. First, these replication hedge fund portfolios could serve as an early warning system, thereby enabling investors to make prompt investment and risk management decisions at any time, instead of waiting until the end of the month.[2] As an example, we illustrate this application where we hedge unwanted risks of a hedge fund. Second, this methodology could be used to improve the VaR measure of hedge funds. Jorion (2008) argues that estimating VaR with the realized monthly return distribution fails to capture the dynamic risk of hedge funds. And, Goetzmann, Ingersoll, and Ivkovic (2000) show that monthly models fit to funds that use daily dynamic strategies can be problematic. We demonstrate that the daily ex ante VaR measures (created using the projection technique) successfully capture the dynamics of hedge fund risks that are not reflected in monthly risk measures. Finally, our projection methodology is generic enough to apply to any investment portfolio that reports only monthly or lower frequency performance data. In addition to hedge funds and hedge fund of funds, this group also includes institutional and separately managed accounts, which is left for future research.

The remainder of this paper is organized as follows. Section I introduces our methodology for daily hedge fund return projection using past monthly returns data. Actual daily returns of HFRX investable hedge fund indices as well as hedge-fund-like mutual funds are then used to test the methodology effectiveness. In addition, we compare daily projection backtesting results using various estimation models and market indices. Empirical applications of our methodology are the focus of Section II. Section III summarizes our findings.

[2] Usually investors must notify hedge funds for redemption 45-65 days before quarter end. However, this process could start earlier with such a warning system. Also, investors can apply other methods to hedge their hedge fund exposures if they foresee a potential loss in the short-term.

I. Data and Methodology

Our proposed projection methodology replicates the monthly returns of hedge fund portfolios with investable assets or factors that have daily pricing available to the public with little delay. Several past papers attempt to model hedge fund returns with multifactor models. For instance, Amenc, El Bied, and Martellini (2003) find that multifactor models, augmented with macroeconomic variables, successfully forecast out-of-sample hedge fund index returns.[3]

In this paper, we apply one of the most effective methods, the Returns-Based Style Analysis (RBSA) technique, to forecast high-frequency (daily) hedge fund index returns using low-frequency (monthly) data to estimate our model. RBSA was first proposed by William Sharpe in the late 1980's to identify a credible portfolio of systematic market factors that explain or best mimic a given mutual fund's performance variability. Since then, it has been widely applied to hedge fund analysis (Fung & Hsieh 1997, Agarwal & Naik, 2002). Sharpe's model, applied to a hedge fund index, is the following:

$$\begin{cases} R_{HFI,t} = \beta_1 F_{1t} + \beta_2 F_{2t} + \ldots + \beta_n F_{Nt} + \varepsilon_t \\ s.t. \sum_{i=1}^{N} \beta_i = 1 \end{cases} \tag{1}$$

where $R_{HFI,t}$ is a hedge fund index return during the month ending at time t, and F_{it} is the return on factor i during this month. If all N factors represent investable market indices, there is a constraint that the summation of all asset loadings (betas) equals one (the "budget constraint")—this is a simplified generic model that doesn't include any margin requirements. As hedge fund indices employ leverage and shorting, we do not impose any non-negativity constraints, in contrast to Sharpe (1992), where analysis was performed on long-only mutual funds and institutional portfolios.

The absence of intercept ("alpha") term in equation (1) deserves separate attention. While the model intercept or "alpha" is frequently used in modeling mutual fund and hedge fund returns, its use in a model with an application to short-term projections, replication and hedging presents certain risks since alpha is exogenous to the model and cannot be

[3] See, also, Gupta, Szado, and Spurgin, 2008, and Fung and Hsieh, 2007, for a survey of replication approaches. Asness (2004) argues that hedge fund strategies have a tendency to move over time from alpha to systematic beta trading strategies; by exploiting arbitrage opportunities, the market becomes more efficient and alpha returns quickly disappear. In a similar paper, Jaeger and Wagner (2005) state that the main component of hedge fund returns corresponds to risk premiums rather than market inefficiencies. Therefore, we would expect replication strategies that use common factors to capture the vast majority of hedge fund return variation (especially for hedge fund indexes, which likely have much smaller alphas than some subgroups of hedge funds).

replicated or hedged. Therefore, we will assume zero intercept for all regression models in this paper.[4]

To illustrate our methodology, we use the HFRX Equity Hedge (HFRX EH) index as an example. HFRX Indices are a series of benchmarks of hedge fund industry performance which are engineered to achieve representative performance of a larger universe of hedge fund strategies—in essence, these indices represent investable fund-of-funds strategies of live funds, net of all fees and expenses. One important reason that we analyze HFRX is that the daily returns are available with little delay, thus providing an ideal test for our methodology in a realistic setting. That is, we can test the effectiveness of our daily performance projection by comparing our forecasted returns to actual daily index returns.

The methodology consists of two steps. First, we fit a model to replicate the monthly returns of a given hedge fund index during an estimation period. These monthly index returns are fitted using common factors based on implementable strategies, such as a factor based on a portfolio of small-capitalization growth stocks. The factors that we use include the Russell Equity Style Indices for U.S. domestic equity stocks (Russell 1000 Value, Russell 1000 Growth, Russell 2000 Value and Russell 2000 Growth), MSCI EAFE to capture international equity exposure, and Merrill Lynch 3 month T-bill quotes as our cash proxy.

Second, we use the fitted model to project the daily returns of the hedge fund indices, using daily returns on the underlying assets or factors that were used in the estimation step above as explanatory forecasting variables. For example, to project daily returns of the HFRX EH index for the month of September 2008, we perform a RBSA on the index monthly returns for the window from September 2005 through August 2008. To capture the time-varying exposures of the index to the market factors, we use two algorithms—a 36-month static trailing Sharpe (1992) RBSA style analysis, and a dynamic filtering technique, Dynamic Style Analysis (DSA) (Markov, Mottl, Muchnik, Krasotkina, 2006), which is explained in detail in the Appendix.

The estimated factor loadings for the period ending August 31, 2008 are reported in Table I, for both the 36-month Sharpe approach and the DSA approach, which uses an expanding window for estimation. The high R-squared and predicted R-squared indicate successful in-sample and out-of-sample replication, respectively.[5] Note, also, that the DSA approach to estimating factor exposures substantially improves out-of-sample forecasting.

[4] A similar rationale is used by Hasanhodzicy and Lo (2007). At the same time, the authors have incorrectly assumed that RBSA model (Sharpe, 1992) has zero intercept. Even though the intercept term is not included in the model (1), Sharpe used error variance minimization as the estimation technique; in an unconstrained (no non-negative constraints) model (1), this is equivalent to using a regression model with an intercept.

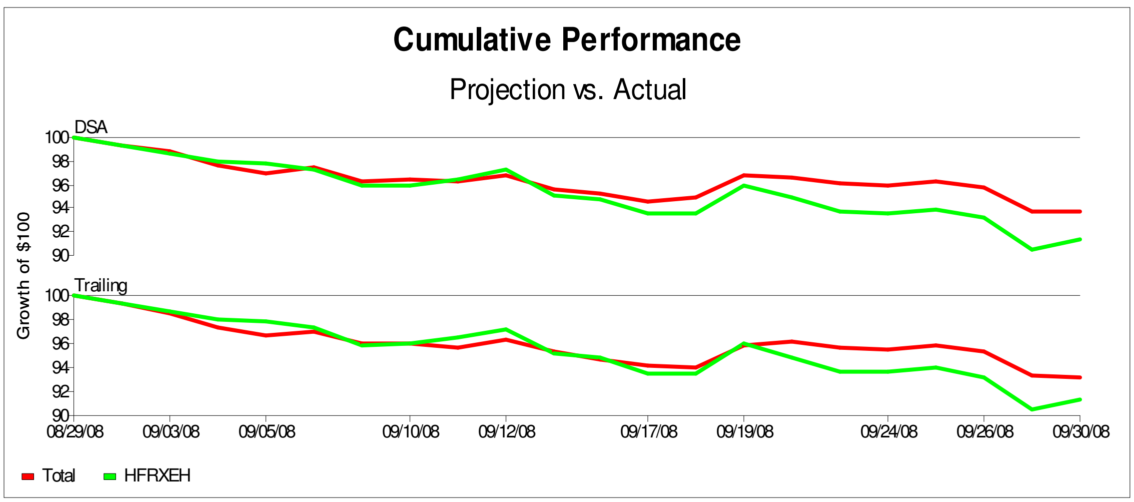
[5] The predicted R-squared is a cross-validation statistic for out-of-sample prediction computed using a method similar to the PRESS statistic (Allen, 1971). See the Appendix for further details.

Table I. Estimated Asset Loadings for HFRX EH (August 31, 2008)

Models/Market Factors	3M T-bills	Small Growth	Small Value	Large Growth	Large Value	EAFE	R^2	PR^2
Trailing RBSA	61.34	39.93	-33.46	4.16	6.18	21.83	86.81	56.75
DSA	60.95	37.43	-24.89	2.09	5.57	18.86	86.51	70.19

To project the September daily returns, we construct a replication portfolio using the weights of the fitted model from August (Table I) with the publicly available daily market factor returns during September. Since a hedge fund index, as an investment portfolio, should not have significant turnover, our buy-and-hold replication portfolio should successfully forecast the performance of the index. We plot our replication portfolio returns against actual index returns for September and show them in Figure 1 below.

Figure 1. Projection vs. Actual Returns during September 2008



Created with mpi Stylus

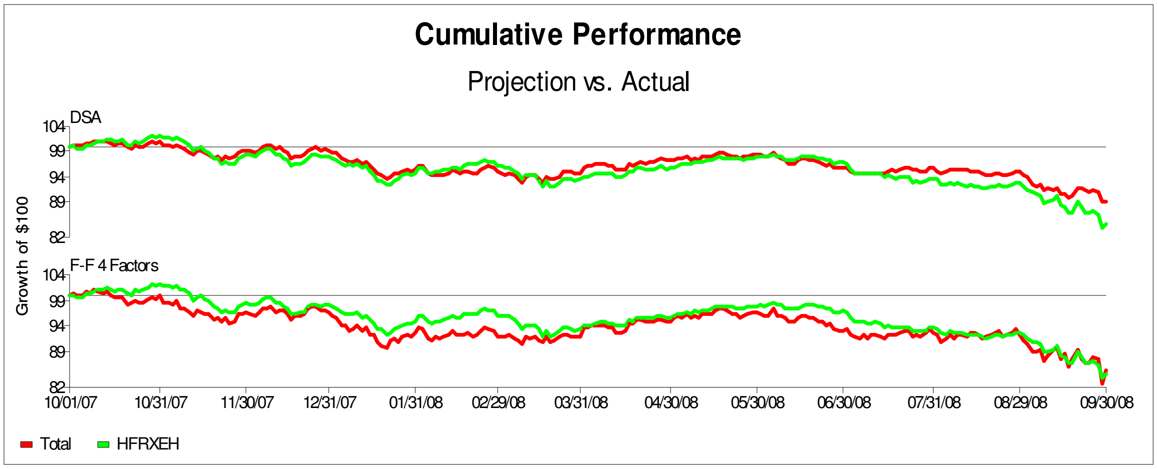
The cumulative performance in Figure 1 illustrates that our simple buy-and-hold replication portfolio (labeled "total") tracks the daily movement of the actual HFRX EH index quite well during most of the forecast month. The projected returns move closely with actual returns until late into September 2008 (when volatility was extremely high following the Lehman bankruptcy filing), showing the effectiveness of our projection methodology.

To show the long-term performance of our replication approach, we repeat the above procedure for each of the 12 months during the date range October 1, 2007 to September 30, 2008. At the end of each month, we adjust the projection portfolio weights by using the trailing 36-month return window, then use daily returns in the fitted model to forecast the HFRX EH daily returns during the following month.[6] For comparison purposes, the popular Fama-French 4 factors (used by Carhart, 1997) in an OLS regression are also used for monthly asset loading estimation and performance projection.

[6] For DSA analysis we start with the initial 36 months and then expand the estimation window monthly rather than using trailing 36 months as in Sharpe's RBSA. In addition, we re-calibrate the parameter λ.

Figure 2 shows the performance of the index and its projection (Total) from DSA with Russell style indices and EAFE as market factors, as well as 36 month trailing window OLS regression with Fama-French-Carhart 4 factors.

Figure 2. One Year Daily Projection vs. Actual Returns



Created with mpi Stylus

Because hedge fund monthly returns are often reported with a delay, we performed additional projections using factor loadings computed with a one month lag to allow a hypothetical investor time to implement the tracking portfolio. For instance, September 2008 daily returns are forecasted with a model fitted to monthly returns ending at July 31, 2008.

To quantify the quality of our daily projection, we also compute the tracking errors between our out-of-sample projection returns and the actual daily index returns. The tracking error is calculated as the standard deviation of actual returns relative to the projection returns. Since we consider one important missing factor in Fama-French Carhart's 4 factor model—international equity—we also calculate projections by adding MSCI EAFE to the 4 factor model to make our comparison fair. These tracking errors, computed over the 24-month interval January 1, 2007 – December 31, 2008, are reported in Table II, together with correlations between actual and projected returns. Two pairs of columns for each model represent two sets of projections: without a lag (left column), and with a one month lag (right column).

As shown in Table II, the quality of projection, judging by the tracking errors, varies with the models used for projection and the time period selected. Overall, the DSA model with 6 factors provides better projections across different time periods. Projection results are improved with the Carhart model when we add MSCI EAFE as an additional factor. As expected, projections without the lag are more accurate than the ones with one month lag (except for 36 month OLS); however, the 36-month RBSA and the DSA method exhibit tracking errors that are not particularly sensitive to a one-month lag. In addition, as hedge funds typically report their return estimates within the first week after the end of a month, the real tracking error will be closer to the zero lag estimate.[7]

Table II. Two Year Daily Projection Tracking Errors
(HFRX EH, Jan-Dec 2007 and Jan-Dec 2008)

This table shows the daily tracking errors and correlations between projection returns and actual returns of the HFRX Equity Hedge Index. Monthly asset loadings were estimated with 36 month trailing window RBSA analysis and expanding window dynamic style analysis (DSA) using Russell 1000 and 2000 Value/Growth Indices, MSCI EAFE Index and 3M T-bill as market factors. For comparison purpose, Fama-French-Carhart 4 factors and Fama-French-Carhart 4 factors plus MSCI EAFE were used for monthly asset loadings estimation with 36 month trailing window. Daily projection returns for each month were calculated using previous monthly estimated asset loadings times current month's daily market factor returns. The daily tracking error is calculated as the standard deviation of actual returns relative to projection returns. Correlation is between actual returns and projection returns. The "No lag" calculations assume that a monthly fund returns is available on the last day of the month and can be used to project the next month daily returns. The "Lag" calculations use monthly returns with a one-month lag to fit the model in projecting daily returns (e.g., monthly returns available on July 31 are used to forecast September daily returns).

	36 Month RBSA 6-index		DSA 6-index		Fama-French-Carhart		Fama-French-Carhart & EAFE	
	No lag	Lag	No lag	Lag	No lag	Lag	No lag	Lag
2007 Daily TE (bps)	28	27	24	28	41	56	40	45
2007 Daily Corr.	0.85	0.83	0.90	0.85	0.84	0.80	0.88	0.80
2008 Daily TE (bps)	45	47	33	41	103	127	61	76
2008 Daily Corr.	0.84	0.83	0.87	0.88	0.83	0.70	0.83	0.84

The quality of tracking, judging by the correlations, has not changed from 2007 to 2008. However, tracking errors have increased, due to increased overall market volatility. Because HFRX EH is a diversified portfolio of hedge funds, a similar projection error could be expected when replicating a diversified portfolio of hedge funds, e.g., a hedge fund-of-funds, which may contain dozens of individual hedge funds.

II. Applications

A. Monitoring Daily Hedge Fund Performance

As shown in the previous section, we construct a replication portfolio (with prior-month estimated asset loadings and current-month daily market factor returns) that closely tracks the daily performance of a hedge fund index. This creates a simple system that allows investors to monitor the movements of their hedge fund investments on a day-to-day basis, using publicly available returns data on common indices.

[7] If partial (e.g., mid-month) return estimates could be obtained from a fund, they can be appended to monthly return stream and together with partial factor returns could be used to obtain factor exposure estimates intra-month.

To test how closely our projection system can monitor returns in individual funds, we apply this methodology to a sample of long/short equity mutual funds, which employ similar long/short equity strategies to those of hedge funds. These mutual funds have daily returns available through Lipper/Reuters, allowing us to measure the success of our daily replication strategy. Out of 50 active mutual funds (each with a unique share class) classified by Lipper as long/short equity, we identified 17 funds with an inception date prior to January 1, 2006. This requirement was necessary to provide a sufficient return history for our analysis.[8] In Table III.A, we report daily tracking errors between our projection returns and the actual daily returns of these 17 long/short equity mutual funds. Table III.B presents results obtained by using factors specific to each fund's investment strategy.

1) Results using generic factors

The average/median daily tracking error in Table III.A is 49/42 bps for 2007, and 98/79 bps for 2008. These figures represent a significant increase, as compared with our Table II replication of a diversified index. We do, however, observe a similar trend—tracking errors during 2008 increase two-fold, as compared to 2007, which is likely due to a significant increase in the trading activity of funds during 2008. In addition, the higher market volatility during 2008 exacerbates any replication errors. Note that median tracking errors are smaller, due to the poor replication of a few funds that are outliers: these funds have significant tracking errors and low correlations with the projection portfolios.

2) The impact of portfolio turnover

In order to further understand the impact of portfolio turnover on the quality of return projections, we provide annual turnover figures for each fund, where available, for both years 2007 and 2008. These represent trailing 12 -month turnover figures, as of the latest available date, prior to year -end.[9] The data was obtained using various fund filings and data vendors, such as Morningstar. Although investors rarely have access to hedge fund turnover figures, these data provide some useful insights.

Based on the 2007 projection results, two outliers in both the tracking error (TE) and correlation stand out: funds #5 and #17. These funds have the lowest daily correlation to the projection portfolio, and the highest daily TE in the sample. A close analysis of the portfolio of these funds reveals strategies with an unusually high annual turnover: 529% for fund #5 and 2,110% for fund #17.[10]

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[8] Fund #1 in our sample was closed on Dec 23, 2008, nevertheless, it was included in the sample, and we use partial Dec 2008 data for this fund.

[9] In some cases, the turnover may not represent the exact year-end, but, rather, the third-quarter report.

[10] In its marketing material fund #17 describes its strategy as a "short-term style rotation" and "the Fund's holding period averages three to four weeks." For a short-term style rotation strategy using daily estimations is imperative.

Clearly, such a high level of trading activity presents a challenge to using monthly estimations for projections of daily returns, unless funds are diversified and maintain stable overall factor or sector exposures. For example, fund #2 has the second highest turnover in the sample—1,697% for 2007 and 1,248% for 2008—yet some of the highest correlations (.96 and .97) for its daily projections, and a below-average TE of 30 bps for 2007 and 57 bps for 2008.

3) Attributing projection quality

In order to understand what portion of the projection error could be attributed to using monthly data instead of daily for estimation of the tracking portfolio, we create an "ideal" daily benchmark. Such a benchmark represents a tracking portfolio with index exposures estimated using daily returns rather than monthly, assuming that both fund and index returns are available. By definition, such a portfolio will track a hedge fund more closely than one created using monthly data. To construct such a benchmark, first, a 40-day (approximately 2-month) trailing window daily RBSA analysis is used to calculate daily estimated factor loadings. Then, the following-day return is forecasted using the fitted model and the following-day factor return.

We present the tracking error between the out-of-sample replication portfolio (the synthetic portfolio using daily estimated asset loadings times daily out-of-sample index returns) and the actual fund's daily returns in Table III.A as the "Benchmark TE." We observe that the tracking errors of such a daily benchmark are, on average, half the size of our tracking errors, which are based on monthly out-of-sample projections. Surprisingly, we lose only about one half of the precision by using monthly data in estimation vs. daily for individual hedge funds.[11]

4) Strategy-specific factors

For Table III.A, we use the same generic set of six factors to replicate each of the funds in the set: Russell 4 Style (1000 Value/Growth and 2000 Value/Growth), MSCI EAFE, and 3M T-bill. In this section, we explore whether additional information about an individual fund's strategy and the use of more specific factors, such as industry sectors, results in a better projection.

Based on a detailed study of each fund's investment strategy, available from Lipper/Reuters, Morningstar, the fund prospectus, and the fund website, we assigned a more specific set of indices to each fund.[12] The entire list of factors considered include: 3M T-bill, Russell 6 Style Indices (Top 200 Value/Growth, Midcap Value/Growth, 2000 Value/Growth), MSCI EAFE, MSCI Pacific, MSCI Europe, MSCI Emerging Markets,

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[11] Note that the best improvement in projection quality using daily vs. monthly estimation (Fund TE/Benchmark TE = 1.20/0.44 = 2.7) during 2007 was attained for fund #17, which has the highest turnover. A similar improvement was obtained in 2008 for this fund.

[12] For instance, fund #15 states in its prospectus that it may invest "at least" 40% in non-US equities. Moreover, it is classified by Morningstar as a "World Stock" fund. Adding detailed foreign equity indices allows us to decrease the TE from 42 bps to 24 bps during 2007 (Table III.A vs. III.B)

and S&P 500/400 Economic Sectors. We then used stepwise regression and PRESS criterion (Allen, 1971) in our monthly analysis to find a small subset of indices before we made our daily projections.

We present results of the above analysis in Table III.B. Assigning specific factors results, on average, in about 30% improvement of 2007 results (35 vs. 49 bps average) and only about 10% improvement of 2008 results (86 vs. 98 bps), relative to generic factors discussed in Section 1 above.

5) Weekly aggregation of results

We also performed a weekly aggregation of tracking results to demonstrate that a significant portion of the daily tracking error could have come from other sources unrelated to the model estimation and prediction quality, such as pricing fluctuations, accounting errors, fees, etc. For an IID process, one would expect weekly volatility to be $\sqrt{5} = 2.24$ times the daily volatility. Therefore, if a significant portion of the tracking error is indeed related to noise in fund NAV calculation, it is likely that the noise will cancel out if the daily data is aggregated into weekly, and the resulting weekly TE will be smaller than 2.24 times the daily TE.

In Table III.A and B, the weekly columns represent tracking errors and correlations between weekly return series. We didn't redo any monthly exposure computations, but simply compounded weekly each daily return series used in the daily portion of the table to obtain 52 weekly returns for years 2007 and 2008. We observe that weekly aggregation indeed improved the projection. The median ratios of weekly TE to the daily ones for Table III.A are 1.58 (2007) and 1.84 (2008). The ratios for Table III.B are 1.30 (2007) and 1.53 (2008). These figures indicate that up to one-half of the daily observed projection discrepancies could cancel out in the long run. Therefore, investors should expect much smaller cumulative under/over performance (weekly, monthly, annually) than the daily TE numbers suggest.

6) Portfolio aggregation

We also performed an analysis of a portfolio of all 17 long/short funds. For the Table III.A, we created an equal-weighted monthly rebalanced return series of the 17 funds and performed all the steps in analyzing it as we did for every fund in the table. The results are presented in the row denoted "*EW Port*." One would expect the tracking error (if it's entirely diversifiable non-systematic) to decrease by the factor of $\sqrt{17} = 4.12$ due to diversification. We observe a very similar decline in TE of the analyzed portfolio, where two-thirds of the TE is diversified away: daily TE of 16 bps vs. average 49 bps (2007) and 31 bps vs. average 98 bps (2008). Despite the fact that many of the funds in the portfolio have very significant annual turnover of 1,000-2,000% and we weren't able to successfully project several funds, the portfolio incorporating *all* such funds had daily projection TE between 16 and 31 bps. This should be very encouraging for investors in diversified portfolios, such as funds-of-funds.

We performed a different procedure for Table III.B. Because the total number of specific factors is very significant, and bigger than the estimation data window, we used results of each fund analysis and created an equal-weighted combination of all 17 funds' exposures to all factors to perform daily return projections. Such approach is possible only if one knows the structure of the analyzed portfolio, unlike the approach we used for the Table III.A, where we used the composite return on the 17-fund portfolio, disregarding the structure. The results obtained combining individual fund exposures are very similar to the composite return analysis: 15 and 27 bps daily TE for years 2007 and 2008 respectively.

To summarize the results of this section, strategy-specific factors should be used, where possible, to replicate individual hedge fund daily returns, while generic factors can be used to replicate fund-of-funds daily returns.

Table III.A. Daily Projection Results with Six Generic Factors

This table shows the daily and weekly tracking errors between projection returns and actual returns of Lipper Long/Short mutual funds. Monthly factor loadings were estimated with expanding window dynamic style analysis (DSA) using Russell four, MSCI EAFE, and 3M T-bill as market factors for each of the 17 funds. Daily projection returns for each month were calculated using previous monthly estimated factor loadings times the current month's daily market factor returns. The daily tracking error is calculated as the standard deviation of actual returns relative to projection returns. Correlation is also between actual returns and projection returns. For benchmark tracking error, a 40-day trailing window daily analysis is used to calculate daily estimated factor loadings. Daily projected returns are aggregated to weekly frequency to calculate weekly statistics. EW Port. represents an analysis of the equally-weighted portfolio consisting of all listed 17 funds.

Year 2007

	Daily			Weekly			
			Benchmark			Benchmarl	Ann.
Fund	**TE,%**	**Corr.**	**TE,%**	**TE,%**	**Corr.**	**TE,%**	**Turnover**
1	0.67	0.66	0.43	1.20	0.55	0.83	262
2	0.30	0.96	0.17	0.63	0.99	0.45	1697
3	0.42	0.71	0.20	0.48	0.83	0.57	172
4	0.55	0.90	0.34	0.98	0.96	0.67	33
5	0.79	0.25	0.43	1.04	0.49	0.78	529
6	0.29	0.76	0.20	0.57	0.68	0.55	89
7	0.47	0.83	0.32	0.93	0.84	0.63	191
8	0.25	0.92	0.19	0.55	0.96	0.56	40
9	0.23	0.78	0.16	0.36	0.74	0.49	609
10	0.31	0.84	0.24	0.42	0.78	0.58	94
11	0.39	0.88	0.27	0.61	0.78	0.56	
12	0.43	0.61	0.25	0.56	0.82	0.60	93
13	0.59	0.71	0.27	0.90	0.69	0.59	72
14	0.40	0.92	0.16	0.61	0.86	0.51	105
15	0.42	0.63	0.24	0.73	0.68	0.55	59
16	0.55	0.81	0.37	1.19	0.91	0.76	178
17	1.20	-0.27	0.44	1.42	0.31	0.74	2110
Mean	**0.49**	**0.70**	**0.28**	**0.78**	**0.76**	**0.61**	**396**
Median	**0.42**	**0.78**	**0.25**	**0.63**	**0.78**	**0.58**	**139**
EW Port.	*0.16*	*0.95*	*0.11*	*0.30*	*0.96*	*0.32*	

Year 2008

	Daily			Weekly			
			Benchmark			Benchmarl	Annual
Fund	**TE,%**	**Corr.**	**TE,%**	**TE,%**	**Corr.**	**TE,%**	**Turnover**
1	0.67	0.66	0.55	1.72	0.84	1.10	242
2	0.57	0.97	0.31	0.72	0.90	0.68	1248
3	1.10	0.67	0.53	1.55	0.80	0.90	71
4	1.44	0.94	0.49	1.79	0.98	0.82	38
5	1.04	0.14	0.36	2.39	0.41	0.73	712
6	0.37	0.73	0.19	1.04	0.99	0.65	
7	0.77	0.87	0.44	1.41	0.75	0.75	
8	0.58	0.92	0.32	0.89	0.98	0.73	44
9	0.57	0.54	0.23	0.79	0.69	0.62	574
10	0.85	0.89	0.40	1.08	0.95	0.85	86
11	0.72	0.92	0.40	1.41	0.89	0.72	
12	0.88	0.91	0.43	2.81	0.76	0.82	124
13	0.79	0.77	0.29	0.97	0.94	0.60	138
14	0.71	0.94	0.29	1.48	0.97	0.66	175
15	1.06	0.86	0.35	1.15	0.95	0.85	59
16	1.35	0.82	0.87	2.79	0.76	1.26	220
17	3.13	0.16	1.15	6.16	-0.09	1.72	2121
Mean	**0.98**	**0.75**	**0.45**	**1.77**	**0.79**	**0.85**	**418**
Median	**0.79**	**0.86**	**0.40**	**1.41**	**0.89**	**0.75**	**157**
EW Port.	*0.31*	*0.98*	*0.19*	*0.65*	*0.98*	*0.44*	

Table III. B Daily Projection Results with Strategy Specific Factors

Unlike in the table III.A, factors are selected based on a detailed study of each fund's investment strategy available from Lipper/Reuters, fund prospectus, and fund website. The entire list of factors considered include: 3M T-bill, Russell 6 Style Indices (Top 200 Value/Growth, Midcap Value/Growth, Smallcap 2000 Value/Growth), MSCI EAFE, MSCI Europe, MSCI Emerging Markets, S&P 500/400 Economic Sectors. Unlike the analysis using 6 generic factors, EW Port projections for this table are obtained using equal-weighted factor weights resulting from 17 individual fund analysis. The rest of statistics calculation is the same as those in the Table III. A.

Year 2007

	Daily			Weekly			
			Benchmark			Benchmarl	Annual
Fund	**TE,%**	**Corr.**	**TE,%**	**TE,%**	**Corr.**	**TE,%**	**Turnover**
1	0.49	0.71	0.38	1.05	0.56	0.70	262
2	0.24	0.92	0.09	0.38	0.93	0.34	1697
3	0.28	0.68	0.14	0.32	0.76	0.40	172
4	0.41	0.94	0.31	0.81	0.91	0.57	33
5	0.70	0.08	0.37	0.54	0.56	0.65	529
6	0.11	0.76	0.12	0.15	0.75	0.40	89
7	0.31	0.79	0.26	0.31	0.87	0.51	191
8	0.05	0.85	0.16	0.12	0.90	0.40	40
9	0.17	0.55	0.08	0.05	0.75	0.35	609
10	0.23	0.87	0.18	0.18	0.87	0.45	94
11	0.29	0.99	0.24	0.29	0.81	0.46	
12	0.27	0.92	0.22	0.49	0.64	0.43	93
13	0.40	0.76	0.21	0.75	0.66	0.49	72
14	0.34	0.92	0.11	0.36	0.92	0.39	105
15	0.24	0.89	0.17	0.31	0.69	0.43	59
16	0.36	0.77	0.30	0.90	0.85	0.59	178
17	1.03	0.22	0.36	1.20	0.11	0.62	2110
mean	**0.35**	**0.74**	**0.22**	**0.48**	**0.74**	**0.48**	**396**
median	**0.29**	**0.79**	**0.21**	**0.36**	**0.76**	**0.45**	**139**
EW Port.	*0.15*	*0.95*	*0.09*	*0.24*	*0.95*	*0.27*	

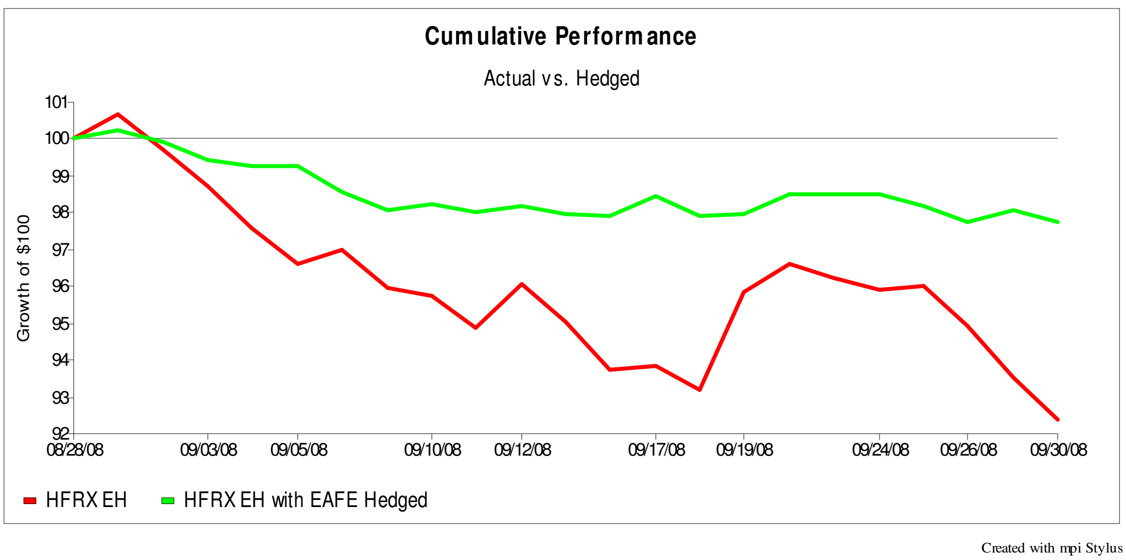
Year 2008

	Daily			Weekly			
			Benchmark			Benchmarl	Annual
Fund	**TE,%**	**Corr.**	**TE,%**	**TE,%**	**Corr.**	**TE,%**	**Turnover**
1	0.50	0.65	0.50	1.66	0.77	0.98	242
2	0.49	0.99	0.23	0.21	0.98	0.52	1248
3	1.02	0.65	0.46	1.07	0.85	0.74	71
4	1.34	0.89	0.41	1.26	0.95	0.66	38
5	0.88	0.20	0.31	2.07	0.36	0.56	712
6	0.25	0.72	0.13	0.89	0.89	0.51	
7	0.63	0.79	0.41	0.90	0.83	0.61	
8	0.46	0.99	0.27	0.70	0.92	0.56	44
9	0.37	0.52	0.17	0.30	0.69	0.46	574
10	0.72	0.90	0.37	0.78	0.94	0.74	86
11	0.60	0.85	0.35	1.26	0.87	0.61	
12	0.79	0.84	0.37	2.40	0.84	0.68	124
13	0.72	0.79	0.23	0.52	0.86	0.44	138
14	0.61	0.96	0.23	1.17	0.95	0.48	175
15	0.93	0.79	0.29	0.99	0.94	0.68	59
16	1.24	0.86	0.79	2.56	0.80	1.09	220
17	3.06	0.14	1.09	6.05	-0.06	1.54	2121
mean	**0.86**	**0.74**	**0.39**	**1.46**	**0.79**	**0.70**	**418**
median	**0.72**	**0.79**	**0.35**	**1.07**	**0.86**	**0.61**	**157**
EW Port.	*0.27*	*0.96*	*0.19*	*0.49*	*0.94*	*0.43*	

B. Hedging Market Exposures

In addition to passively tracking hedge fund performance, investors can proactively hedge unwanted risk (if they expect their investment is deteriorating due to specific market exposures). As an example, Table I showed that HFRX Equity Hedge index had about 20% international equity exposure, represented by EAFE, as estimated using the 36-month window ending at August 31, 2008. If investors foresee further decline in this sector, they could actively hedge this sector by shorting 20% MSCI EAFE by taking, for example, a position in an ETF such as EFZ (NYSE listed ProShares Short MSCI EAFE ETF). Figure 3 presents two projection returns for HFRI EH: one with EAFE exposure hedged (green) and one without (red). The portfolio with EAFE hedged using EFZ significantly outperformed the actual HFRX EH index. Moreover, the hedged portfolio also has a much smaller volatility; the annualized standard deviation is 5.48% for the EAFE hedged portfolio, compared to 18.24% for HFRX EH index during September 2008.

Figure 3.HFRX EH Actual Returns vs. EAFE Hedged Returns



Unlike in our hypothetical hedging example above, in some cases, using indices involved in the analysis as hedging instruments could be either impossible or impractical. For instance, small-cap equity indices could be very expensive to short. Nevertheless, a hypothetical analysis using generic indices provides an important benchmark for further steps, when such indices are replaced with actual hedging instruments. This way we're able to attribute the loss of hedging precision to selection of more efficient (albeit less precise) hedging instruments.

C. Estimating Daily VaR

Hedge funds usually have a short history, provide only low frequency return data (typically monthly) and employ dynamic investment strategies. This presents challenges for value-at-risk as an effective risk measure. Jorion (2008) argues that, for hedge funds, using monthly returns to compute ex-post risk measures such as standard deviation and/or VaR is insufficient to capture dynamic portfolio risks. For example, Goetzmann, Ingersoll, and Ivkovic (2000) show that monthly models fit to funds using daily dynamic strategies can be problematic. Further, Lo (2001) calculates the required sample size for accurate VaR estimation, and concludes that, to achieve a 95% confidence level, one needs more than 475 data points. Most hedge funds don't meet this data requirement.

In this section, we apply our replication methodology to generate daily data to calculate daily VaRs. This provides enough return data points for us either to fit in a parametric distribution function or use empirical quantiles for VaR estimation.[13] As illustrated below, daily VaR also reveals a much more dynamic picture for hedge fund risk through time, as opposed to monthy VaR.

We demonstrate the method, again using HFRX EH as an example. In order to estimate the daily VaR value for day T_m in month m, we construct a daily hypothetical portfolio by multiplying the previous month-end estimated factor loadings by the time series of historical daily factor returns, that is,

$$R_t^d = \sum_{i=1}^{N} \widehat{\beta}_{i,m-1} F_{i,t}^d, \; t = T_m - W + 1, ..., T_m \tag{2}$$

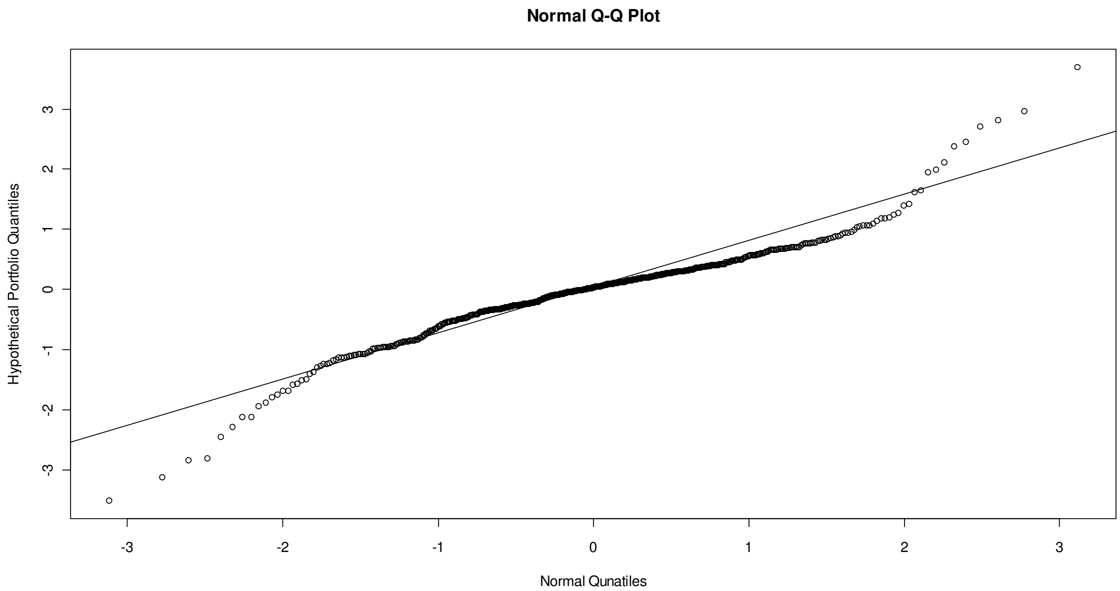
where F_t^d represent daily historical factor returns; $\widehat{\beta}_{i,m-1}$ are the factor loadings estimated using style analysis approach (1) applying it to available monthly return data through month $(m-1)$. As a result, R_t^d does not represent an actual portfolio but rather reconstructs the history of a hypothetical portfolio having constant factor loadings. The VaR estimate as of the end of day T_m is then calculated using W daily hypothetical returns $\left\{R_t^d\right\}$ within the VaR estimation window ending on T_m.

One advantage of this hypothetical portfolio is that it could account for fat tails to the extent that they are present in the daily historical data. To illustrate, we construct the hypothetical portfolio to compute VaR on September 15, 2008 for the HFRX EH index. We use the same estimated factor loadings in August as those in Table I, and the hypothetical portfolio returns are constructed using a two-year (503 days) VaR estimation window from September 18, 2006 to September 15, 2008. Figure 4 depicts the Q-Q plot of these returns against a standard normal distribution. Accordingly, the plot substantially deviates from linearity at both ends and indicates the existence of fat tails for hypothetical portfolio return distribution. The Jarque-Bera test also rejects the normality hypothesis with a p-value close to zero. As a result, the 95% daily VaR for September 15, 2008 is

[13] A large sample size is required to obtain meaningful quantiles. For instance, a 95% daily VaR estimated over a window of 100 days only produces 5 observations in the tail on average. According to Jorion (2007) Chapter 10, in practice, most banks use periods between 250 and 750 days for daily VaR calculation.

0.91% using an empirical quantile method, and 0.7% by assuming a normal distribution. Therefore, empirical and parametric methods produce very different VaR estimates.

Figure 4. Q-Q Plot of Hypothetical Portfolio Returns from August 16, 2006 to August 15, 2008



To test whether our daily VaR provides a reasonable risk measure of the HFRX EH index, we performed an out-of-sample test by computing daily VaR values for each day in 2008 and compared them with the next day actual index returns. More specifically, for each month $m = 1,...,12$ in year 2008, we used 24 monthly returns preceding month m to estimate the effective asset loadings $\{\hat{\beta}_{i,m-1}\}$ for the month m daily VaR calculation. We then computed the daily VaR according to (2), using computed factor loadings and daily factor returns for each day in our VaR estimation window W. According to the VaR definition, the percentage of exceptions when the observed actual daily loss is greater than the estimated VaR value should be close to the theoretical probability value for which VaR was computed (e.g. 5% of exceptions for 95% VaR). The test statistics that is commonly used to measure the quality of VaR backtesting is Kupiec's (1995) *Proportion Of Failures* or *POF*, which has the following form,

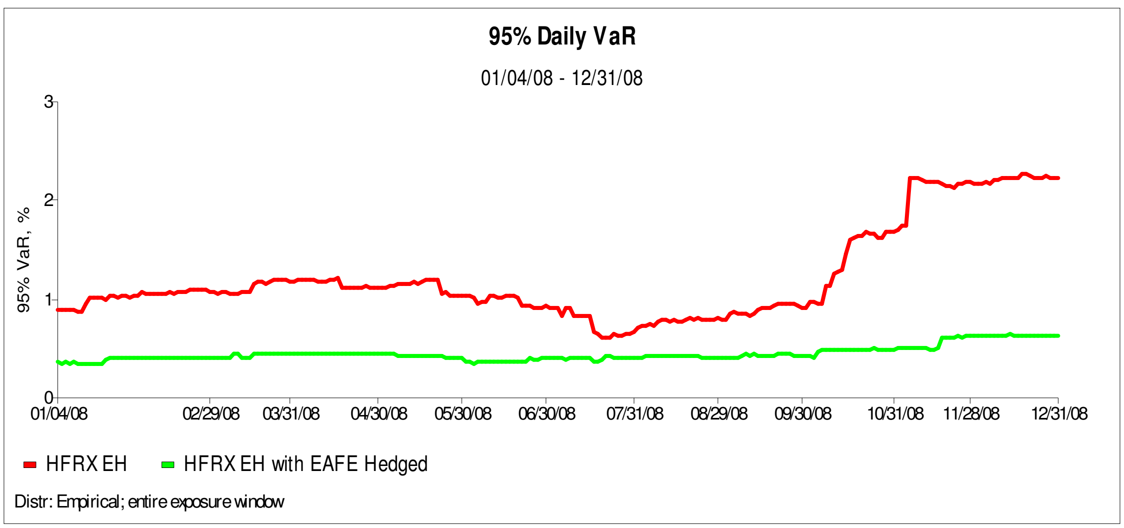
$$POF = 2\ln\left(\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-x}\left(\frac{\hat{\alpha}}{\alpha}\right)^{x}\right) \tag{3}$$

where T is VaR estimation window, x is the number of times actual loss exceeds VaR value (exception). α is the significance level for VaR (0.05 for 95% VaR in our test) and $\hat{\alpha}$ is the observed frequency of exceptions, which equals $\frac{x}{T}$. The null hypothesis is H0: $\hat{\alpha} = \alpha$. Accordingly, POF is asymptotically distributed as chi-squared with one degree of freedom. With 253 daily 95% VaR values estimated for year 2008 using empirical quantiles, we observed fourteen daily index losses exceeding their daily VaR estimates, so that the observed frequency $\hat{\alpha} = 0.055$. The corresponding POF statistic is equal to 0.15 and the null hypothesis of the 95% VaR estimate being accurate is accepted with p-value=0.69.

As stated in Jorion (2008), daily VaRs would also reveal some volatility that could not be captured in monthly VaRs. In Figure 5, we plot daily VaR values of the HFRX EH index during 2008. The daily risk profile is nothing but volatile, and it shows the VaR value spiking up significantly since October. Such daily level risk information is important but hard to come across with monthly VaR estimation. If only monthly data were used to compute VaR, an increase in VaR may only be noticed in several months. However, an investor monitoring risk using our daily VaR would discover this much more quickly.

Similar to application B in this section, the investor can hedge the unwanted risk exposure if they foresee the further deterioration to this specific exposure. In Figure 5, we also plot the daily 95% VaR of the HFRX EH index, but with EAFE position hedged**.** As depicted, this simple hedging strategy places daily VaRs within reasonable limits.

Figure 5. Daily VaR



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D. Monitoring Daily Performance of Other Hedge Fund Strategies

To test our projection system on other hedge fund strategies besides L/S equity, we apply the same analysis as in section A to all HFRX indices and report their results in Table IV below. Similar to Section A part (4), we match each index with its own strategy-based factors for monthly replication. Besides the factors listed in part (4), we also include Barclays Aggregate Bond and Barclay High Yield Corporate Bond Indices, Merrill Lynch Convertible Bond, Goldman Sachs Commodity as additional factors.

Overall, the projection qualities, judging by tracking error and correlation, are better for those hedge fund strategies that are known to be easily replicated, such as hedge fund composite indices, equity hedge, and event driven indices. The average tracking error for those indices is around 22 bps in year 2007 and 2008, and the number doubles in 2008 as the market volatility increases. The tracking errors are high and correlations are low for relative value, convertible arbitrage and equity market neutral strategies, especially during 2007 and 2008. However, the numbers for convertible arbitrage strategy are apparent outliers, as the strategy itself suffered a breakdown in 2008 due to the market turmoil and some rule changes by the SEC.[14] The huge distortion in performance makes consistent projection of this strategy almost impossible.

[14] See "Convertible-Bond Arbitrage Loses Its Shirt," *Wall Street Journal*, Nov. 11, 2008.

Table IV Daily Projections for All HFRX Hedge Fund Indices

This table shows the daily and weekly tracking errors between projection returns and actual returns of all HFRX hedge fund indices with more than three years data available. Numbers in parenthesis after the indices indicate the number of funds composed of the indices, composite means using all hedge funds for the index construction, but with different weighting scheme. For details, please refer to HFRX index methodology. Monthly asset loadings were estimated with expanding window dynamic style analysis (DSA) using strategy-based factors for each index. All other calculations are the same as in table III.

Year 2006

	Daily			Weekly		
HFRX indices(# of funds)	**TE,%**	**Corr.**	**Benchmark TE,%**	**TE,%**	**Corr.**	**Benchmark TE,%**
Abs. Return (composite)	0.17	0.32	0.08	0.3	0.51	0.57
Conv. Arb. (3)	0.22	0.07	0.16	0.44	0.10	0.28
Equity Hedge (17)	0.18	0.93	0.10	0.25	0.95	0.21
Equity Market Neutral (4)	0.19	0.46	0.16	0.33	0.77	0.22
Event Driven (13)	0.22	0.84	0.11	0.51	0.87	0.21
EW Strategy Index (composite)	0.12	0.85	0.07	0.27	0.89	0.15
Global Hedge Fund (composite)	0.19	0.90	0.07	0.40	0.94	0.17
Macro Index (8)	0.38	0.60	0.20	0.87	0.69	0.53
Market Directional (composite)	0.32	0.89	0.12	0.69	0.90	0.58
Relative Value (10)	0.29	0.23	0.09	0.54	0.53	0.19
Mean	**0.23**	**0.61**	**0.12**	**0.46**	**0.72**	**0.31**
Median	**0.21**	**0.72**	**0.11**	**0.42**	**0.82**	**0.22**

Year 2007

	Daily			Weekly		
HFRX indices(# of funds)	**TE,%**	**Corr.**	**Benchmark TE,%**	**TE,%**	**Corr.**	**Benchmark TE,%**
Abs. Return (composite)	0.43	0.40	0.11	0.93	0.24	1.61
Conv. Arb. (3)	0.59	-0.01	0.20	0.92	0.46	0.33
Equity Hedge (17)	0.24	0.90	0.12	0.68	0.84	0.28
Equity Market Neutral (4)	0.42	0.22	0.30	0.94	0.36	0.60
Event Driven (13)	0.35	0.81	0.17	0.64	0.88	0.28
EW Strategy Index (composite)	0.22	0.79	0.11	0.49	0.81	0.24
Global Hedge Fund (composite)	0.28	0.88	0.11	0.57	0.90	0.27
Macro Index (8)	0.52	0.72	0.37	1.22	0.84	0.88
Market Directional (composite)	0.32	0.88	0.15	0.72	0.88	0.81
Relative Value (10)	0.38	0.47	0.18	0.75	0.56	0.38
Mean	**0.38**	**0.61**	**0.18**	**0.79**	**0.68**	**0.57**
Median	**0.37**	**0.76**	**0.16**	**0.74**	**0.83**	**0.36**

Year 2008

	Daily			Weekly		
HFRX indices(# of funds)	**TE,%**	**Corr.**	**Benchmark TE,%**	**TE,%**	**Corr.**	**Benchmark TE,%**
Abs. Return (composite)	0.68	0.02	0.89	0.95	0.21	0.46
Conv. Arb. (3)	4.43	-0.06	0.64	7.12	0.50	1.12
Equity Hedge (17)	0.33	0.87	0.20	0.89	0.87	0.48
Equity Market Neutral (4)	0.39	0.30	0.32	0.83	0.50	0.64
Event Driven (13)	1.06	0.68	0.30	2.05	0.75	0.39
EW Strategy Index (composite)	0.67	0.61	0.17	1.39	0.68	0.37
Global Hedge Fund (composite)	0.72	0.77	0.18	1.65	0.69	0.52
Macro Index (8)	1.09	0.39	0.32	1.83	0.61	0.71
Market Directional (composite)	0.83	0.81	0.23	1.58	0.84	0.85
Relative Value (10)	1.02	0.42	0.43	1.82	0.65	1.04
Mean	**1.12**	**0.48**	**0.37**	**2.01**	**0.63**	**0.66**
Median	**0.78**	**0.52**	**0.31**	**1.62**	**0.67**	**0.58**

III. Conclusion

This paper presents a methodology for using low-frequency (monthly) hedge fund returns to model high-frequency (daily) out-of-sample returns. We show that our technique successfully tracks the actual out-of-sample daily returns of a diversified portfolio of hedge funds, such as the HFRX EH index, as well as the out-of-sample daily returns of several individual long/short mutual funds. We compare several regression estimation methods, and find that dynamic filtering techniques provide an improvement over static regressions.

Our simple and easy-to-implement methodology has important and valuable applications. It allows hedge fund investors and analysts to monitor daily hedge fund "proxy" returns and to make proactive investment decisions (e.g., allocating inflows and outflows) intra-month, rather than after they receive month-end performance results from the fund. For hedge fund investors faced with redemption restrictions (e.g., lockups and gate clauses), the proposed methodology provides a means to implement risk controls and to effectively hedge unwanted risks. Portfolio managers and investors can apply this approach to improve their existing risk management measures, such as value-at-risk.

Obviously, this is a clear approximation of daily performance and the model does not attempt nor claim to understand the trades, leverage or positions that a hedge fund could take on a daily basis (which can greatly alter the risk exposure of the fund). However, given the lack of actual daily hedge fund returns or holdings, our proposed approach attempts to provide investors with some insight into how their hedge funds might be performing each day.

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Appendix. Dynamic Style Analysis (DSA) Methodology Overview

In this appendix we describe the basic elements of DSA. In Sharpe's RBSA the return on a portfolio $r_t^{(p)}$, $t = 1,2,3,...,T$ is approximated by the return on a linear combination of indices $\mathbf{r}_t = \left(r_t^{(1)},...,r_t^{(N)}\right)$ with the factor loadings $\boldsymbol{\beta} = \left(\beta^{(1)},...,\beta^{(N)}\right)$ and the intercept α such that

$$r_t^{(p)} \cong \alpha + \sum_{i=1}^{N} \beta^{(i)} r_t^{(i)} = \alpha + \boldsymbol{\beta}' \mathbf{r}_t$$

for $t = 1,2,3,...,T$. The parameters α and $\boldsymbol{\beta}$ are determined by solving the following *least-squares* problem[15]

$$\begin{cases} (\alpha, \boldsymbol{\beta}) = \arg\min\limits_{\alpha,\boldsymbol{\beta}} \sum\limits_{t=1}^{T} \left(r_t^{(p)} - \alpha - \boldsymbol{\beta}' \mathbf{r}_t \right)^2 \\ \text{s.t.} \;\; \boldsymbol{\beta}' 1 = 1 \end{cases}$$

We note that in Sharpe's RBSA factor loadings $\boldsymbol{\beta}$ are assumed constant within the estimation window T .

In contrast, in DSA method factor loadings evolve slowly over time, satisfying the relationship:

$$\boldsymbol{\beta}_{t+1} \cong \mathbf{V}_t \boldsymbol{\beta}_t$$

where the transition matrices, $\mathbf{V}_t$, determine the dynamics (Hidden Markov Model) of the factor loadings. In DSA, the parameters α and $\boldsymbol{\beta}_1,...,\boldsymbol{\beta}_T$ are determined through time-varying regressions referred to as Flexible Least Squares (Kalaba, Tesfatsion, 1989)[16]:

$$\begin{cases} \left(\alpha, \boldsymbol{\beta}_1,...,\boldsymbol{\beta}_T\right) = \arg\min\limits_{\alpha,\boldsymbol{\beta}_1,...,\boldsymbol{\beta}_T} \sum\limits_{t=1}^{T} \left(r_t^{(p)} - \alpha - \boldsymbol{\beta}_t' \mathbf{r}_t \right)^2 \\ + \lambda \sum\limits_{t=1}^{T-1} \left(\boldsymbol{\beta}_{t+1} - \mathbf{V}_t \boldsymbol{\beta}_t \right)' \mathbf{U}_t \left(\boldsymbol{\beta}_{t+1} - \mathbf{V}_t \boldsymbol{\beta}_t \right) \\ \text{s.t.} \;\; \boldsymbol{\beta}_t' 1 = 1 \end{cases}$$

We observe that the objective function consists of two terms, each term penalizing a different component of the model specification error. The first is the sum of the squared residuals, measuring the goodness of fit of the regression. The second term provides penalty for non-smoothness of the dynamic factor loadings. The matrices $\mathbf{U}_t$ are weighting matrices. The positive parameter λ measures the relative importance between the goodness of fit and the smoothness of the regression coefficients.

[15] RBSA non-negativity constraints on $\boldsymbol{\beta}$ are typically relaxed or removed when analyzing hedge funds and the budget constraint is not utilized when regressors represent non-investable factors.

[16] Here we assume static intercept for simplicity. The intercept can also be assumed time-varying within the same model.

We use leave-one-out cross-validation to find an optimal λ. For this purpose, the optimal dynamic model is constructed for the data after one observation has been removed from the sample, and the prediction error is calculated on the removed observation. We repeat this procedure for each observation in the sample, and the sum of squared errors is computed. A linear complexity algorithm to perform such cross-validation is presented in (Markov, Muchnik, Mottl, Krasotkina, 2006). Assuming we omit the *s*-th observation, we denote the optimal DSA solution as follows:

$$\left(\alpha(s,\lambda),\mathbf{\beta}_1(s,\lambda),\mathbf{\beta}_2(s,\lambda),\ldots,\mathbf{\beta}_T(s,\lambda)\right)$$

The residual error of the regression is calculated for each omitted observation. The so-called cross-validation estimate of the noise variance is found as the average over all the local squared prediction errors

$$D(\lambda)=\frac{1}{T}\sum_{s=1}^{T}\left(r_s^{(p)}-\alpha(s,\lambda)-\mathbf{\beta}'_s(s,\lambda)\mathbf{r}_s\right)^2$$

We define the Predicted R^2 by

$$PR^2(\lambda)=1-\frac{D(\lambda)}{D\left(r^{(p)}\right)}=1-\frac{\frac{1}{T}\sum_{s=1}^{T}\left(r_s^{(p)}-\alpha(s,\lambda)-\mathbf{\beta}'_s(s,\lambda)\mathbf{r}_s\right)^2}{\frac{1}{T}\sum_{s=1}^{T}\left(r_s^{(p)}\right)^2}$$

and choose the parameter λ such that the Predicted R^2 is maximized.