Modeling Deliberation Over Combinatorially-Structured Domains: Similarity, Attraction and Compromise Effects

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Abstract

We consider a scenario where a user has to make a set of correlated decisions and we propose a computational model of the deliberation process. We assume the user compactly expresses her preferences as soft constraints. We design a sequential procedure which decomposes the complex decision task by using Decision Field Theory (DFT) to model deliberation on each variable. We compare our sequential approach to one in which a single deliberation is made over the set of complex objects. Our results show that the sequential approach outperforms the non-sequential one in terms of execution time and returns choices of similar quality. Finally, we consider three important effects which have been observed in human decision making: the similarity, attraction and compromise effect. These are modeled effectively by DFT in the case of non-structured alternatives. We show that our approach captures these phenomena for complex decisions which are decomposed in a combinatorial structure.

Introduction and Motivation

Preferences play a crucial role in decision making. As such they have been studied in multiple disciplines. Preference reasoning has grown into an important topic in Artificial Intelligence (Rossi, Venable, and Walsh 2011) where Preference models are currently used in many applications, such as scheduling (Bartak and Venable 2011) and recommendation engines (Jannach et al. 2010). On the other hand, in psychology, Decision Field Theory (DFT) (Busemeyer and Townsend 1993) formalizes the evolution of preferences during the process of deliberation. While DFT has mainly tackled the problem of making a single decision, both in real life and in artificial intelligence applications, decision are more complex and it can be helpful to organize them in a combinatorial structure over which decisions can be applied sequentially. To this end, we propose here an approach which brings together compact preference models and DFT.

There are several approaches to modeling preferences compactly (Rossi, Venable, and Walsh 2011), such as, for example, soft constraints (Meseguer, Rossi, and Schiex 2005). This is the formalism we use in this paper: variables are assigned values from their domains, and there are constraints, involving subsets of variables and associating to simultaneous assignments of the constrained variables a preference value. We use soft constraints to support a deliberation process performed through DFT. In particular, preferences over the set of alternatives according to a criteria are decomposed into a soft constraint satisfaction problem. We compare the results obtained applying the deliberation process sequentially to those obtained by a single deliberation step over the entire combinatorial domain. As expected the sequential approach outperforms the one-shot procedure in terms of time. Moreover, it focuses on a relatively small set of alternatives compared to the size of the choice space and the deliberated assignments are, on average, of high quality with respect to the preferences in the soft constraint problems representing the criteria.

Our work has two objectives: the first one is to provide a computational model which can help understand human decision making over complex domains; the second one is to investigate DFT as means of incorporating a form of uncertainty into the soft constraint formalism. To the first end, we note that our sequential approach appears to be cognitively more plausible as it is far more likely for humans to break complex decisions into interconnected sub-problems, rather than try to deliberate directly on a large set of complex objects. We further support this claim by showing that this sequential procedure is capable of capturing three well known behavioral effects which have been observed in human decision making, namely, the similarity, attraction and compromise effect. In terms, of using DFT as a paradigm to extend soft constraints, we note that the DFT simulation of deliberation, which is modeled as an oscillation between evaluation criteria for different scenarios, can be used to model uncertainty about which are the true preferences. Furthermore, DFT allows to specify a similarity measure between options and to express how a high preference on an option can influence the preference of similar or dissimilar options. Both aspects extend the expressive power of soft constraints where preferences are assumed to be known (or static), and those of different options influence each other only through shared variable assignments.

This paper is organized as follows. The first section discusses related work and is followed by a background section on soft constraints and DFT. The third section presents the description of sequential decision making over soft-constraint networks exemplified on a detailed running example. The fourth section, instead, describes a non-sequential approach to decision making using the same running example. The last section of the paper, before concluding, presents and discusses experimental result.

Related Work

While a large literature has been dedicated to studying the human deliberation process in settings with few, unstructured alternatives, understanding how humans make decisions over complex or combinatorial structures is for the most part an unexplored topic. In a recent paper, (Gershman, Malmaud, and Tenenbaum 2017) the authors developed a theory of decision making on combinatorial domains. They adopt compositionally structured utility functions as a means to represent preferences and they use probabilistic reasoning to predict preferences over new unseen alternatives. While the combinatorial structure of the alternatives is shared, our approach is very different from theirs in methods and objectives. The preference models are different as they use utilities and we use a fuzzy constraint-based approach. Furthermore, our goal is to model the variability which is observed in human decision making when preferences come from different criteria as opposed to predicting preferences over unseen options from known ones.

Sequencing preference reasoning over combinatorial structures is not new (Chevaleyre et al. 2008). The most related work to ours in this context is (Pozza et al. 2011) where a sequential voting method over fuzzy constraint problems is presented. In that setting each voter expresses her preferences as a SCSP and votes are aggregated variable by variable using a voting rule. Our approach bears some similarities, in that, also in that case, constraint propagation is used as a preprocessing step before voting on each variable. However, while one could view attributes as voters, the way preferences is aggregated by DFT is very different from the application of a voting rule. In (Rajkumar et al. 2014) combinatorial structures are the candidate options of a online combinatorial decision problem where costs are associated with a specific choice. The authors present an algorithm to minimize the total regret over a sequence of trials. While costs associated to choices can be seen as the flip side of preferences, the setting is very different from ours as their is no cognitive modeling component to their approach. Extension of DFT to more complex domains have been proposed in the literature, for example addressing dynamic environments (Lee, Son, and Jin 2008). In (Gao and Lee 2006) an extension is presented to model trust and reliance on automation. Similarly to what we consider here, the authors study an iterated decision process where previous decisions may influence later ones. However, while we represent the correlation between decisions via soft constraints they link sequential decision processes by a dynamic updating of beliefs.

Background

In this section, we provide basic background on soft constraints and DFT.

Soft Constraints In our approach, preferences of decision makers are represented as a set of constraints. A soft constraint (Meseguer, Rossi, and Schiex 2005) requires a set of variables and associates each instantiation of its variables to a value from a partially ordered set. More precisely, the underlying structure is a c-semiring which consist of the following, $\langle A, +, \times, 0, 1\rangle$, where A is the set of preference values, $+$ induces an ordering over A (where $a \leq b$ iff $a+b=b$), $\times$ is used to combine preference values, and 0 and 1 are respectively the worst and best element. A Soft Constraint Satisfaction Problem (SCSP) is a tuple $\langle V, D, C, A\rangle$ where V is a set of variables, D is the domain of the variables and C is a set of soft constraints (each one involving a subset of V) associating values from A. An instance of the SCSP framework is obtained by choosing a specific preference structure. Choosing $S_{FCSP} = \langle [0,1]$, max, min, 0, $1\rangle$, where FCSP stands for Fuzzy constraint satisfaction problem (Schiex 1992; Meseguer, Rossi, and Schiex 2005), means that preference values are in $[0,1]$, we want to maximize the minimum preference value, the worst preference value is 0 and the best preference value is 1. This is the setting , that we use this paper. Figure 1 shows the constraint graph of an FCSP where $V = \{X, Y, Z\}$, $D = \{a, b\}$ and $C = \{c_X, c_Y, c_Z, c_{XY}, c_{YZ}\}$. Each node models a variable and each arc models a binary constraint, while unary constraints define variables' domains. For example, c_Y is defined by the preference function f_Y that associates preference value 0.4 to $Y = a$ and 0.7 to $Y = b$. Default constraints such as c_X and c_Z, where all variable assignments get value 1, will often be omitted in the following examples.

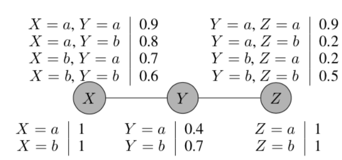


Figure 1: Example of an FCSP.

Given an SCSP, a complete assignment to all of its variables is associated with a preference value obtained combining the preferences associated to its projection on each of the constraints. In our example the preferences of $(X = a, Y = b, Z = b)$ is $0.5 = min(1, 0.9, 0.7, 0.5, 1)$. These global preferences induce, in general, a partial order with ties over the set of complete assignments, which is total for FCSPs.

Often, solving an SCSP is interpreted as finding an optimal solution, that is, a complete assignment with an undominated preference value (e.g. $(X = a, Y = b, Z = b)$ in this example). Unless certain restrictions are imposed, such as a tree-shaped constraint graph, finding an optimal solution is an NP-hard problem.

Constraint propagation may improve the search for an optimal solution. In particular, given a variable ordering o, a FCSP is directional arc-consistent (DAC) if, for any two variables X and Y linked by a constraint, such that X precedes Y in the ordering o, we have that, for each a in the domain of X, $f_X(a) =$

$max_{b\in D(Y)}(min(f_X(a), f_{XY}(a,b), f_Y(b)))$, where f_X, f_Y, and f_{XY} are the preference functions of c_X, c_Y and c_{XY}. When the constrained graph is tree shaped and the variable ordering is compatible with the father-child relation of the tree, DAC is enough to find the preference level of an optimal solution (Meseguer, Rossi, and Schiex 2005). Such an optimal preference level is the best preference level in the domain of the root variable after DAC and an optimal solution, can be found by a backtrack-free search which instantiates variables in the same order used for DAC.

In our running example, if we choose the variable ordering $\langle X, Y, Z\rangle$, achieving DAC means first enforcing the property over the constraint on Y and Z and then over the constraint on X and Y. The first phase modifies the preference value of $Y = b$ to $max(min(f_Y(b), f_{Y,Z}(b,a), f_Z(a)), min(f_Y(b), f_{Y,Z}(b,b), f_Z(b))) = max(min(0.7, 0.2, 1), min(0.7, 0.5, 1)) = max(0.2, 0.5) = 0.5$. Similarly, the second phase sets the preference values of both $X = a$ and $X = b$ to 0.5.

We also note that, by achieving DAC w.r.t. ordering o, we obtain a total order with ties over the values of the first variable in o, where each value is associated to the preference of the best solution having such a variable instantiated to such a value.

Multialternative Decision Field Theory Decision Field Theory attempts to formalize the deliberation process by assuming that a decision maker's preference for each option evolves during deliberation and by integrating a stream of comparisons of evaluations among options on attributes over time (Busemeyer and Townsend 1993). DFT has been extended to multialternative preferential choice, where settings with more than two options are considered. In DFT a valence value $v_i(t)$ is associated with a choice i at time t, and represents the advantage or disadvantage of option i when compared with other options with respect to some attribute(Roe, Busemeyer, and Townsend 2001). For example, options could be different car models with attributes being fuel efficiency and comfort.

More formally, MDFT, in its basic formulation (Roe, Busemeyer, and Townsend 2001), is composed of:

Personal Evaluation: We assume a set of options $\{o_1, \ldots, o_n\}$ and a set of attributes $\{A_1, \ldots, A_J\}$. The subjective value of option o_i on attribute A_j is denoted by m_{ij} and stored in matrix $\mathbf{M}$ for all options and attributes. Let us assume, for example, that a person is considering to buy a car and that the car options are *SUV (S)*, *Truck (T)* and *Coupe (C)* and that the attributes considered are *Fuel Efficiency* and *Comfort*. Matrix $\mathbf{M}$ containing the persons's initial preferences for the three options according to the two attributes could be defined as follows:

$$\mathbf{M} = \begin{bmatrix} 2 & 8 \\ 1 & 7 \\ 7 & 3 \end{bmatrix}$$

In this matrix the rows correspond to the options in order (S, T, C) and the columns to the attributes $Fuel\ Efficiency$ and $Comfort$. For example, we can see that $Coupe$ has a high preference in terms of fuel efficiency but low in terms of comfort.

Attention Weights: Attention weights are used to express how much attention is allocated to each attribute at each particular time t during the deliberation process. We denote them by a one column vector $\mathbf{W}(t)$ where $W_j(t)$ is a value denoting the attention to attribute j at time t. We adopt the common simplifying assumption that, at each point, the decision maker attends to only one attribute. Thus, $W_j(t) \in \{0, 1\}, \forall t, j$. In our example, where we have two attributes, at any point in time t, we will have $\mathbf{W}(t) = [1, 0]$, or $\mathbf{W}(t) = [1, 0]$, representing that the buyer is attending to, respectively, $Fuel\ Efficiency$ or $Comfort$. In general, the attention weights change across time according to a stationary stochastic process with probability distribution $\mathbf{w}$, where w_j is the probability of attending to attribute A_j. In our example, defining $w_1 = 0.55$ and $w_2 = 0.45$ would mean that at each point in time, the person will be attending $Fuel\ Efficiency$ with probability 0.55 and $Comfort$ with probability 0.45. In other words, $Fuel\ Efficiency$ matters slightly more to this particular person than $Comfort$.

Contrast Matrix: Contrast matrix $\mathbf{C}$ is used to compute the advantage (or disadvantage) of an option with respect to the other options. For example, $\mathbf{C}$ can be defined by contrasting the initial evaluation of one alternative against the average of the evaluations of the others. In this case, for three options, we have:

$$\mathbf{C} = \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}$$

At any moment in time, each alternative in the choice set is associated with a **valence** value. The valence for option o_i at time t, denoted $v_i(t)$, represents its momentary advantage (or disadvantage) when compared with other options on some attribute under consideration. The valence vector for n options $o_1, \ldots, o_n$ at time t, denoted by column vector $\mathbf{V}(t) = [v_1(t), \ldots, v_n(t)]^T$, is formed by:

$$\mathbf{V}(t) = \mathbf{C} \times \mathbf{M} \times \mathbf{W}(t) \qquad (1)$$

In MDFT preferences for each option are accumulated across the iterations of the deliberation process until a decision is made. This is done by using the **Feedback matrix**, which defines how the accumulated preferences affect the preferences computed at the next iteration. This interaction depends on how similar the options are in terms of their initial evaluation contained in matrix $\mathbf{M}$. Intuitively, the new preference of an option is affected positively and strongly by the preference it had accumulated so far, it is strongly inhibited by the preference of other options which are similar and this lateral inhibition decreases as the dissimilarity between options increases.

Clearly, the concept of similarity plays a crucial role. A common way to define it is to project the initial evaluations contained in $\mathbf{M}$ on the directions of indifference and dominance respectively (Busemeyer and J.M. Hotaling 2010). Formally, this is done by defining distance matrix $\mathbf{D} = [D_{ij}]$ where $D_{ij} = (\mathbf{M}_i - \mathbf{M}_j) \times \mathbf{H}_b \times (\mathbf{M}_i - \mathbf{M}_j)^T$ and $\mathbf{H}_b$ is defined as

$$\mathbf{H}_b = \frac{1}{2} \times \begin{bmatrix} b+1 & b-1 \\ b-1 & b+1 \end{bmatrix} \tag{2}$$

where constant b determines the ratio of emphasis on the dominance direction with respect to the indifference direction. Intuitively, the difference between two options where one is dominating will have a stronger component along the line of dominance while the difference between competitive options will have a larger component along the line of indifference

More emphasis should be given to differences in the dominance direction than in the indifference direction in order to simulate the common human behavior where dominated options are rapidly discarded during a decision process. This is achieved whenever $b > 1$ At this point matrix $\mathbf{S}$ can be defined by mapping the distance via Gaussian function:

$$\mathbf{S} = \delta - \phi_2 \times \exp\left(-\phi_1 \times \mathbf{D}^2\right) \tag{3}$$

where δ, ϕ_1, ϕ_2 are the identity matrix, the decay parameter, and the sensitivity parameter respectively. The identity matrix δ is used to model positive self feedback. We can also see that when $i \neq j$, and, thus, $\delta_{ij} = 0$, we a have lateral inhibition $-\phi_2 \times \exp\left(-\phi_1 \times \mathbf{D}^2\right)$ which depends on the distance $\mathbf{D}$ between options. In our example, if we set $b = 10$, $\phi_1 = 0.01$ and $\phi_2 = 0.1$ we get the following $\mathbf{S}$ matrix:

$$\mathbf{S} = \begin{bmatrix} +0.9000 & -0.0000 & -0.0405 \\ -0.0000 & +0.9000 & -0.0047 \\ -0.0405 & -0.0047 & +0.9000 \end{bmatrix}$$

At any moment in time, the preference of each alternative is calculated by

$$\mathbf{P}(t+1) = \mathbf{S} \times \mathbf{P}(t) + \mathbf{V}(t+1) \tag{4}$$

where $\mathbf{S} \times \mathbf{P}(t)$ is the contribution of the past preferences and $\mathbf{V}(t+1)$ it the valence computed at that iteration. Usually the initial state $\mathbf{P}(0)$ is defined as $\mathbf{0}$, unless defined otherwise due, for example, to prior knowledge on past experiences.

Given an MDFT model one can simulate the process of deliberating among the options by accumulating the preferences for a number of iterations. The process can be stopped either by setting a threshold on the preference value and selecting whichever option reaches it first or, by fixing the number of iterations and then selecting the option with highest preference at that point.

Because of the uncertainty on the attention weights distribution, different runs of the same MDFT model may return different choices. This allows MDFT to effectively replicate behaviors observed in humans (Busemeyer and Townsend 1993).

Effects in Human Decision Making Certain behaviors have been identified as characterizing human decision making and have been modeled effectively by DFT (Roe, Busemeyer, and Townsend 2001). Consider, for example, the purchase of a new car among three option (A,B or C) , and according to two attributes: performance and fuel efficiency. First, assume that A has better performance but poorer efficiency than B, and B has worse performance and better efficiency than A. It has been shown that introducing a third option C, similar to A will decrease A's probability of being chosen. This is know as the similarity effect. It is formally define as the following reversal of choice probabilities $Pr[A|A,B] > Pr[B|A,B]$ but $Pr[A|A,B,C] < Pr[B|A,B,C]$.

Now, let A and B be as the above and assume a new option C, similar to A but dominated by A in all attributes, is introduced. It has been shown that this will increase the probability that option A is chosen. This effect is known as the attraction effect, and can be formally described in terms of choice probabilities as: $Pr[A|A,B] < Pr[A|A,B,C]$.

Finally, a third effect called the compromise effect refers to the fact that when three options are available, the compromise option is chosen more frequently than either of the extremes. Suppose there are three equally attractive cars, A, B, and C, as indicated by their pairwise preferences, but suppose two of the cars, say A and B, are extremely different, and the third one, C, is a compromise that lies in between these two extremes. The compromise effect refers to the empirical finding that $Pr[A|A,B] = Pr[A|A,C] = Pr[B|B,C]$, but $Pr[C|A,B,C] > Pr[A|A,B,C]$ and $Pr[C|A,B,C] > Pr[B|A,B,C]$.

Sequential decision making over soft constraint networks

The deliberation tasks we consider involve choosing an option which can be decomposed into a set of variables, $X = \{X_1, \ldots, X_n\}$, where each variable X_i can take different values from its domain $D(X_i) = \{\sigma_1, ..., \sigma_m\}$. To represent the agent's personal evaluation we use a FCSP defined over the variables in X. We use one FCSP for each attribute. In the car example mentioned above, variables would describe different car models, and could be, for example, brand, engine type and number of seats. We consider FCSPs where the constraint-graph is tree-shaped. This allows us to topologically sort the variables in an ordering which we denote by $O = X_1 > X_2 > ..... > X_n$. The first approach to deliberation we consider is to sequentially finds a value for each variable X_i via a DFT run following order O. For each variable X_i in order O:

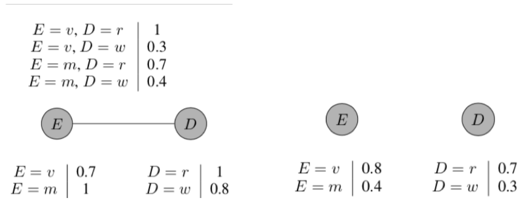
(a) We extract the subjective preference of the user on the values in the domain of X_i. To do this, we enforce DAC on the FCSP, in reverse order w.r.t. O.

(b) Then, DFT is applied to X_i, returning a deliberated assignment for variable X_i, say σ_i. We write this as: $DFT(X_i) = \sigma_i$.

(c) Finally, DAC is applied,(following order O), to propagate the effect of the assignment.

After n steps, we obtain a final combinatorial decision, that is, we will have selected one value for each variable. In step (a), the preferences, as updated by DAC, take into account the information induced on that assignment by the rest of the network.

We will now describe the procedure in more detail using an example. Assume we have a user who has to decide on what to have for dinner. Her preferences in terms of attributes taste and health on the available options are expressed by the two FCSPs depicted in Figure 2.



(a) Taste FCSP (b) Health FCSP

Figure 2: Taste and Health preferences expressed as Fuzzy CSP.

Both FCSPs are defined on same set of variables, that is, Entree, denoted by E, with domain $\{m, v\}$ for meat and vegetables respectively, and Drink, denoted with D, with values $\{w, r\}$ for white wine and red wine. From Figure 2 we see, for example, that the user, prefers the taste of meat to the taste of vegetables and prefers red wine with it. On the other hand, the user is also aware that vegetables are healthier than meat and that red wine is healthier than white wine.

In step (a) of our procedure, DAC is enforced on both graphs. This, for example, modifies the taste preference value of $E = v$ to the maximum preference of any complete assignment involving it, that is, to $max(min(0.7, 1, 1), min(0.7, 0.3, 0.8)) = 0.7$. Similarly, it modifies the preference value of $E = m$ to 0.7. In what follows we will write X_a and $X = a$ Interchangeably

The Taste FCSP is already DAC since there is no constraint between the two variables. In step (b), DFT is applied to variable E, using the four preference values from the two FCSPs as input to evaluation matrix M_E. We will come back to this step later. For now, let us assume, that DFT(E) returns $E = v$. Then, in step (c), all of the preferences of tuples where $E = m$ are set to 0 in both FCSPs and the effect is propagated to all the network through DAC following O (see Figure 4).

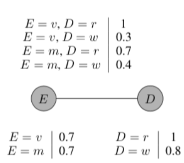
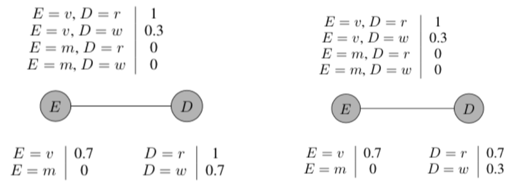


Figure 3: Taste preferences expressed by a FCSP after achieving DAC.

This step modifies the preference value of $D = r$ to $max(min(1, 1, 0.7), min(1, 0, 0)) = 0.7$ and then modifies the preference value of $D = w$ to $max(min(0.7, 0.3, 0.7), min(0.7, 0, 0)) = 0.3$ (see Figure 4). Finally, DFT is applied to D with inputs 0.7 and 0.3 and the final result is reached. For example, if DFT($D = r$) then, overall, we would have chosen to have vegetables and red wine.

Let us now focus on the DFT process performed in step (b) for variable E. Intuitively, this means that we will simulate the deliberation process carried out by the user when choosing between vegetable and meat. This is done by assuming she alternates between considering the options in terms of taste and health and converges to a final preference after a number of iterations. More specifically, in our example we are considering two alternatives, i.e. m and v, and two attributes, i.e. taste and health. At any moment in time each alternative is associated with a valence value. For example, the choice among entrees produces a two dimensional valence vector $V(t) = [v_{E_m}(t), v_{E_v}(t)]$, where for example $v_{Em}(t)$ is the valence of $E = m$ at time t. As mentioned before, this valence vector is determined by three different components: $V(t) = CMW(t)$. We recall that matrix, M, contains the personal evaluation of each option with respect to its attribute, $W(t)$, is the vector containing the weights associated to each attribute at a given point in time t and matrix C contains parameters describing how to aggregate the evaluation of an option with the evaluation of the other options.



(a) After applying DFT to S, and assuming $S = v$ is deliberated.

(b) After the second round of DAC,

Figure 4: FCSPs exemplifying deliberation steps.

In this example, matrix C is the same for all variables and is represented by the following values:

$$C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

In other words, the advantage/disadvantage of an option w.r.t. the other is simply the difference between their preference values. The attention weights, $[W_T(t), W_H(t)]$ where T and H stand for Tase and Health, respectively, are assumed to fluctuate between $[1, 0]$ and $[0, 1]$ over time steps according to a simple Bernoulli process where we assume that the probability of attending taste (i.e. of picking $[1, 0]$) is 0.55, and health (i.e. $[0, 1]$) is 0.45.

Each variable has a different M matrix denoted M_E and M_D for entree and drink, respectively. The entries for M_E are obtained from the preferences over the domain values of E after DAC:

$$M_E = \begin{array}{cc} \begin{array}{cc} \mathrm{T} & \mathrm{H} \end{array} & \\ \begin{bmatrix} 0.7 & 0.8 \\ 0.7 & 0.4 \end{bmatrix} & \begin{array}{c} E_v \\ E_m \end{array} \end{array}$$

We write, for example m_{E_mT} to denote the element of matrix M_E corresponding to the assignment $E = m$ and attribute Taste and similarly for the other elements. If we assume that at time t we have $W(t) = [0, 1]$, then $V(t) =$

$[v_{E_m}(t), v_{E_v}(t)]$, where: $v_{E_m}(t) = W_T(t) \cdot m_{E_mT} + W_H(t) \cdot m_{E_mH} - W_T(t) \cdot m_{E_vT} - W_H(t) \cdot m_{E_vH} = 0 \cdot 0.7 + 1 \cdot 0.4 - 0 \cdot 0.7 - 1 \cdot 0.8 = -0.4$ and $v_{E_v}(t) = W_T(t) \cdot m_{E_vT} + W_H(t) \cdot m_{E_vH} - W_T(t) \cdot m_{E_mT} - W_H(t) \cdot m_{E_mH} = 0 \cdot 0.7 + 1 \cdot 0.8 - 0 \cdot 0.7 - 1 \cdot 0.4 = 0.4$.

A choice among both entrees produces a two-dimensional preference state $P(t) = [P_{E_m}(t), P_{E_v}(t)]$, and a new state of preference $P(t+1) = SP(t) + V(t+1)$, where S is the feedback matrix.

To calculate the S matrix we use the distance function defined in (Busemeyer and J.M. Hotaling 2010). The details of this computation are beyond the scope of this paper. For here, it suffice to say that the S matrix is obtained from the differences in preferences according to attributes in a way such that similar options influence each other in a positive way and dissimilar options have a small inhibitory connection.

$$S = \begin{array}{cc} \begin{array}{cc} E_v & E_m \end{array} & \\ \begin{bmatrix} 0.94 & \text{-}1.4 \times 10^{-35} \\ \text{-}1.4 \times 10^{-35} & 0.94 \end{bmatrix} & \begin{array}{c} E_v \\ E_m \end{array} \end{array}$$

As a criterion to stop deliberation, we set a bound on the number of iterations for each variable.

Non-Sequential approach

Another possibility for making an overall decision is to run the deliberation process only once over the set of candidate options consisting of all complete assignments. Indeed this approach is cognitively less plausible. DFT is intended to model human decision making under the assumption that the number of possible options is manageable. Nevertheless, we consider this alternative method as a means of comparison with the sequential approach.

We assume a single decision to be made over a combinatorial structure. In other words, each complete assignment to all variables in X is treated as an option, and its preference according to the FCSPs is its evaluation with respect to the associated attribute. For example, $(E = m, D = r)$ has preference 0.7 accordingly to taste and 0.4 w.r.t health. We evaluate all of the complete assignments via the FCSPs and we use such preferences to populate matrix M. The non-sequential procedure consists of two steps:

(a) We compute the subjective preferences for all complete assignments for each attribute. This is done, as described in the background section, by projecting each assignment onto the constraints and then taking the minimum preference.

(b) DFT is applied to the set of all complete assignments returning a final deliberation $(X_1 = \sigma_1, X_1 = \sigma_2, \ldots, X_n = \sigma_n)$.

We now exemplify the procedure using the same example with two binary variables and two attributes introduced in the previous section (see Figure 2). The preferences of all complete assignments according to both FCSPs, computed in step (a) are coded into matrix M:

$$M = \begin{array}{cc} \begin{array}{cc} \text{T} & \text{H} \end{array} & \\ \begin{bmatrix} 0.7 & 0.7 \\ 0.3 & 0.3 \\ 0.7 & 0.4 \\ 0.4 & 0.3 \end{bmatrix} & \begin{array}{c} E_v, D_r \\ E_v, D_w \\ E_m, D_r \\ E_m, D_w \end{array} \end{array}$$

Then, similarly to what was done for each variable in the case of the sequential procedure, DFT is applied to the set of all complete assignments (4 in this small example). The values for the C and S matrices are defined using the same rationale as in case involving single variables:

$$C = \begin{bmatrix} 1 & \text{-}1/3 & -1/3 & -1/3 \\ -1/3 & 1 & -1/3 & -1/3 \\ -1/3 & \text{-}1/3 & 1 & -1/3 \\ -1/3 & \text{-}1/3 & -1/3 & 1 \end{bmatrix}$$

$$\mathbf{S} = \begin{array}{cc} \begin{array}{cccc} E_vD_r & E_vD_w & E_mD_r & E_mD_w \end{array} & \\ \begin{bmatrix} 0.94 & 0 & -1.37 \times 10^{-12} & -1.07 \times 10^{-263} \\ 0 & 0.94 & -8.83 \times 10^{-75} & -0.044 \\ -1.37 \times 10^{-12} & -8.83 \times 10^{-75} & 0.94 & -3.77 \times 10^{-31} \\ -1.07 \times 10^{-263} & \text{-}0.044 & -3.77 \times 10^{-31} & 0.94 \end{bmatrix} & \begin{array}{c} E_vD_r \\ E_vD_w \\ E_mD_r \\ E_mD_w \end{array} \end{array}$$

The advantage of an option is its valence minus the average of the other 3 (thus we have $-1/3$ in the C matrix). As far as the attention weights on attributes, they are defined exactly as in the previous section, that is, as fluctuating between $[1, 0]$ and $[0, 1]$ over time steps with probability of attending taste of 0.55 and health of 0.45. The stopping criterion is a limit on the number of iterations. For example, in our experiments, which we describe below, we set this number to be a multiple of the number of variables.

Experimental results

We have implemented both the sequential and non-sequential decision making approach and we have tested them on randomly generated problems. We consider a setting with two attributes. Thus, each generated instance comprises of a pair of tree-shaped fuzzy problems, one for each attribute, defined over the same set of binary variables. We consider a number of variables ranging between 2 and 8 with increments of 2 and a constraint tightness of 20%, meaning that, in each constraint, 20% of the tuples are associated with preference 0. For both of the approaches, the values for the DFT matrices are set as described in our running example accordingly to the number of variables. In the sequential case, deliberation is stopped after 20 iterations on each variable and in the non-sequential case after $20 \cdot n$ iterations where n is the number of variables.

Runing Time In Figure 5 we show the comparison of the two approaches in terms of running time when varying the number of variables. The plotted results are an average over 10000 runs for each number of variables. For both approaches, we take into account also the time necessary to

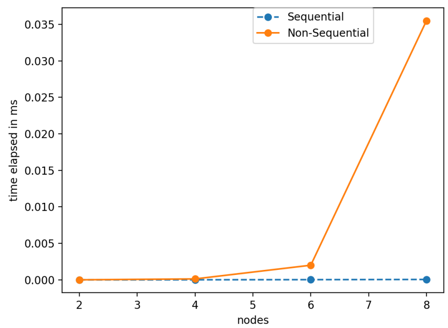


Figure 5: Average execution time for the sequential and non sequential approach when varying the number of variables.

extract the values for the M matrices from the FCSPs. Not surprisingly, the sequential procedure is faster and the gap between the two running times becomes exponentially larger as the number of variables grows. This means that the sequential approach is more efficient model of decision making on combinatorial structures. The rest of the experiments address the quality of the model.

Output Quality By this is not the case, as, by decomposing the decision-making into a set of local deliberation steps, we incur in a situation similar to the discursive dilemma in judgment aggregation (Kornhauser and Sager 1986) and a similar effect observed in sequential voting (Lang and Xia 2009; Pozza et al. 2011).

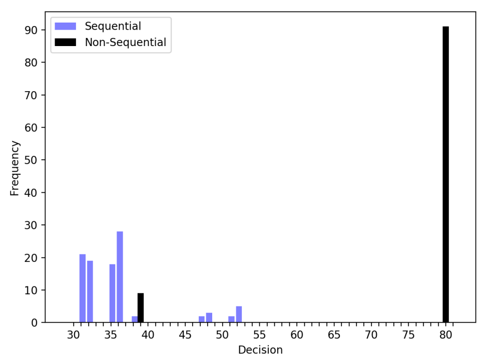


Figure 6: Frequency with which options are returned as deliberated in one instance of eight variables.

In fact, figure 6 shows the frequency with which solutions are returned in a problem with 8 nodes after running both approaches 100 times. As we can see, out of the 2^8 possible choices, only 9 are returned by the sequential approach and 2 are returned by the non-sequential one. We performed this same experiment on 100 different instances of 8 nodes and we observed a similar trend. The average size of the set containing solutions returned at least once by the sequential approach is 3.57, over a total of 256, while the size of the non-sequential approach is 1.61. Indeed most of the time the two sets are disjoint, and the average size of their intersection is 0.07. It is not surprising that the sequential approach has slightly more variability in its outputs. In fact, the uncertainty modeled by the probability distribution over the weights of the attributes affects the decision at each variable. Instead, in the case of the non-sequential approach it only contributes once and at the global level. Nonetheless, both methods are capable of focusing only a few alternatives and manage to efficiently weed out unattractive candidates.

This is further corroborated by an analysis we have performed on the relationship between the options deliberated by the DFT-based decision making procedures and the optimal solutions of the FCSPs (that is, the most preferred complete assignments according to each attribute). The results, obtained from the same set of 100 instances are shown in Figure 7.

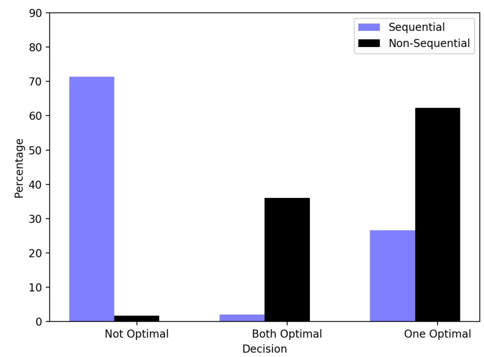


Figure 7: Percentage of times the decision making procedures return options that are not optimal or optimal in one or optimal in both FCSPs.

In most cases the non sequential deliberation process returns an option which is optimal in at least one of the FCSPs. This suggests that the subjective preferences given in input by matrix M have the effect of focusing the deliberation process only on solutions which are highly ranked by at least one attribute. The results of the sequential approach, instead, reflect the decomposition of the decision process. In fact, running the deliberation process variable by variable implies applying the effect of both FCSPs variable by variable. The selected assignments to previous variables affect future deliberations on those which are connected to them via the constraints. We also computed the average difference in preference between the options returned by the two procedures and the the optimal preference of the FCSPs. In the table below we show these results.

	Min Distance	Max Distance
Sequential	0.06	0.17
Non-sequential	0.01	0.04

The averages for the non-sequential approach are aligned with the fact that it often returns an option which is optimal in at least one FCSP. However, the distance from optimal of the options deliberated by the sequential approach is also very small. Thus, it appears that the preference propagation obtained via DAC is sufficient to guarantee the selection of a solution which is of high quality for both attributes. Moreover, the sequential approach can be seen as better compromising between possible conflicting attributes.

Behavioral Effects We have further investigated the sequential approach, due to its higher cognitive plausibility, in terms of modeling well known behavioral effects. We focus on the similarity, attraction and compromise effects, which DFT has been shown to model effectively in the case of the case of single choices.

In this set of experiments we consider problems with two attributes and three variables. Each generated instance comprises of a pair of tree-shaped FCSPs over the same set of variables. We consider variable domain sizes between 2 and 8 with increments of 2 and a number of deliberation iterations ranges between 20 and 50. We replicate the scenarios corresponding to the effects in our combinatorial setting. In particular, we consider both settings in which the FCSPs are constrained to have only up to 3 solutions as well as settings where the effects are observed on a subset of solutions among many others. In both cases we define the preference in the constraints so to observe the targeted behavior.

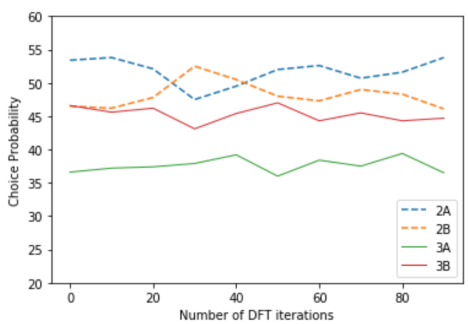


Figure 8: Similarity effect: probability of choice over 450 runs as a function of the number of deliberation iterations. 2A/2B probability of choosing option A/B without C. 3A/3B probability of choosing A/B when option C is available.

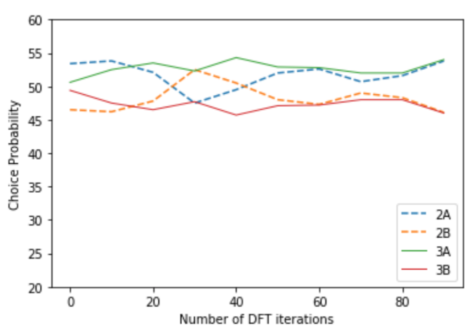


Figure 9: Attraction effect: 2A, 2B, 3A and 3B as in the caption of figure 8

In all of the experiments we define two options A and B, each corresponding to a complete variable assignment, with

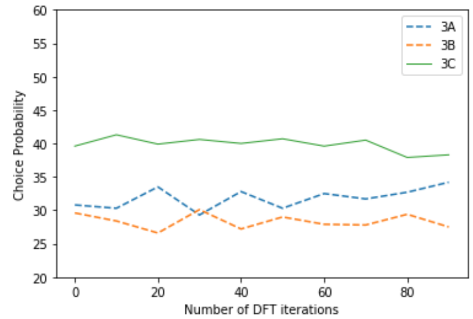


Figure 10: Compromise effect: 3A, 3B and 3C as in the caption of figure 8

asymmetric preferences in the two FCSPs (i.e., A is hugely preferred in one and disliked in the other FCSP and viceversa for B). Then a third option C was introduced with a preference defined by the constraints according to the effect that we want to observe. In the similarity case C's preferences are similar to A's in both FCSPs, in the attraction effect C's preferences are slightly below A's and in the compromise effect they are in between A's and B's preferences in both FCSPs. As it can be seen, our model reproduces all three effects for the case of decision making over complex structures. In Figure 8, introducing C decreases the probability for A being chosen. In Figure 9, C's introduction increases the probability for A being chosen when we consider 15 or more iterations and in Figure 10, C is the favored compromising option.

Future Work

On our agenda, for the near future, is to test our model on behavioral data of human decision making over complex domains and on data sets from the preference library PrefLib (Mattei and Walsh 2013). We also intend to investigate the combination of other compact preference models, such as probabilistic CP-nets(Boutilier et al. 2004), with DFT.

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