



Table of Fourier transforms

$s(t)$	$S(f)$
$\Pi_T(t)$	$\frac{\sin(\pi f T)}{\pi f} = T \text{Sinc}(fT)$
$\Lambda_T(t)$	$\frac{1}{T} \left[\frac{\sin(\pi f T)}{\pi f} \right]^2 = T \text{Sinc}^2(fT)$
$e^{-t^2/2\sigma^2}$	$\sqrt{2\pi}\sigma^2 e^{-2\pi^2 f^2 \sigma^2}$
$e^{-\alpha t} u(t) \quad \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
$te^{-\alpha t} u(t) \quad \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
$\frac{t^n}{n!} e^{-\alpha t} u(t) \quad \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^{n+1}}$
$e^{-\alpha t } \quad \alpha > 0$	$\frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$
$\rho_{T,\alpha}(t)$	$T \text{Sinc}(fT) \frac{\cos(\pi\alpha fT)}{1 - (2\alpha fT)^2}$
$u(t)$	$\frac{\delta(f)}{2} + \frac{1}{2j\pi f}$
$tx(t)$	$\frac{j}{2\pi} \frac{d}{df} X(f)$
$\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$	$f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0) \quad , \quad f_0 = 1/T_0$

Table of z-transforms

$x[n]$	$X(z)$
$a^n u[n]$	$\frac{z}{z-a}$, ROC: $ z > a $
$-a^n u[-n-1]$	$\frac{z}{z-a}$, ROC: $ z < a $
$na^n u[n]$	$\frac{az}{(z-a)^2}$, ROC: $ z > a $
$n^2 a^n u[n]$	$\frac{az(z+a)}{(z-a)^3}$, ROC: $ z > a $
$r_N[n]$	$\frac{1-z^{-N}}{1-z^{-1}} = 1 + z^{-1} + \dots + z^{-(N-2)} + z^{-(N-1)}$
$nx[n]$	$-z \frac{d}{dz} X(z)$
$x[0]$	$\lim_{z \rightarrow \infty} X(z)$
$\lim_{n \rightarrow \infty} x[n]$	$\lim_{z \rightarrow 1} [X(z) - z^{-1} X(z)]$

$$u[n] = 1 \text{ for } n \geq 0, u[n] = 0 \text{ for } n < 0$$

$$r_N[n] = 1 \text{ for } n = 0, \dots, N-1, \quad r_N[n] = 0 \text{ *elsewhere*}$$

$$\text{sign}[n] = 0 \text{ for } n = 0, \text{ sign}[n] = 1 \text{ for } n > 0, \text{ sign}[n] = -1 \text{ for } n < 0$$

Basic trigonometric formulas

$$\cos\left(\alpha - \frac{\pi}{2}\right) = \sin(\alpha) \quad \sin\left(\alpha + \frac{\pi}{2}\right) = \cos(\alpha)$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \quad \cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos^2(\alpha) = \frac{1}{2} + \frac{1}{2}\cos(2\alpha) \quad \sin^2(\alpha) = \frac{1}{2} - \frac{1}{2}\cos(2\alpha)$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha + \beta) + \frac{1}{2}\cos(\alpha - \beta)$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha + \beta) + \frac{1}{2}\sin(\alpha - \beta)$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha - \beta) - \frac{1}{2}\cos(\alpha + \beta)$$

Series and integrals:

$$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

Some change of integration variable rules:

$$\int_{t_0}^{t_1} s(\alpha \cdot [t - t_d]) dt = \begin{cases} \frac{1}{|\alpha|} \int_{\alpha[t_0 - t_d]}^{\alpha[t_1 - t_d]} s(t) dt & \alpha > 0 \\ \frac{1}{|\alpha|} \int_{\alpha[t_1 - t_d]}^{\alpha[t_0 - t_d]} s(t) dt & \alpha < 0 \end{cases}$$

$$\int_{t_0}^{t_1} s(\alpha \cdot [t - t_d]) g(t) dt = \begin{cases} \frac{1}{|\alpha|} \int_{\alpha[t_0 - t_d]}^{\alpha[t_1 - t_d]} s(t) g\left(\frac{\tau}{\alpha} + t_d\right) dt & \alpha > 0 \\ \frac{1}{|\alpha|} \int_{\alpha[t_1 - t_d]}^{\alpha[t_0 - t_d]} s(t) g\left(\frac{\tau}{\alpha} + t_d\right) dt & \alpha < 0 \end{cases}$$

Train of Dirac's Deltas generation formula:

$$\frac{1}{a} \sum_{n=-\infty}^{\infty} e^{\pm j \frac{2\pi}{a} nx} = \sum_{k=-\infty}^{\infty} \delta(x - n \cdot a) \quad x \in \mathbb{R}$$

The delta-generating integral:

$$\int_{-\infty}^{\infty} e^{\pm j 2\pi f t} df = \delta(t) = \delta(-t)$$

The Fourier basis for L_1^2

$$\Phi = \{\hat{\phi}_n(t)\}_{n=-\infty}^{+\infty} = \left\{ \frac{1}{\sqrt{T}} e^{j \frac{2\pi \cdot t}{T} n} \right\}_{n=-\infty}^{+\infty}, \quad t \in [t_0, t_1], \quad T \triangleq t_1 - t_0$$

The autocorrelation and cross-correlation functions for finite-energy signals

$$R_x(\tau) \triangleq (x(t), x(t-\tau)) = \int_{-\infty}^{+\infty} x(t)x^*(t-\tau)dt$$

$$R_{xy}(\tau) \triangleq (x(t), y(t-\tau)) = \int_{-\infty}^{+\infty} x(t)y^*(t-\tau)dt$$

The Expectation operator

$$E_{\xi}\{g(\xi)\} \triangleq \int_{-\infty}^{+\infty} g(x)f_{\xi}(x)dx$$

where $g(\cdot)$ is any function, ξ is a random variable and $f_{\xi}(x)$ is the pdf of ξ

The fundamental formula of joint pdfs

$$f_{\xi,\eta}(x,y) = f_{\xi|\eta}(x|y)f_{\eta}(y) = f_{\eta|\xi}(y|x)f_{\xi}(x)$$

The joint Expectation operator

$$E_{\xi\eta}\{g(\xi,\eta)\} \triangleq \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x,y)f_{\xi\eta}(x,y)dxdy$$

Gaussian pdf and its integral:

$$f_{\xi}(x) = \frac{1}{\sqrt{2\pi\sigma_{\xi}^2}} e^{-\frac{(x-\mu_{\xi})^2}{2\sigma_{\xi}^2}}, \quad \int f_{\xi}(x)dx = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{(x-\mu_{\xi})}{\sqrt{2\sigma_{\xi}^2}}\right) + C$$

The Joint Gaussian pdf

$$f_{\xi_1\xi_2\ldots\xi_N}(x_1,x_2,\ldots,x_N) = f_{\xi}(\mathbf{x}) = \frac{1}{(2\pi)^{N/2} \sqrt{\det\{\mathbf{\Sigma}\}}} \cdot \exp\left\{-\frac{1}{2}(\mathbf{x}-\mathbf{\mu})^T \cdot \mathbf{\Sigma}^{-1} \cdot (\mathbf{x}-\mathbf{\mu})\right\}$$

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \quad \mathbf{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix} \quad \mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & & \\ \vdots & & \ddots & \\ \sigma_{N1} & & & \sigma_N^2 \end{bmatrix}$$