Endogenous Comparative Advantage*

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Abstract

We develop a model of trade between identical countries. Workers endogenously acquire skills that are imperfectly observed by firms, who therefore use aggregate country investment as the prior when evaluating workers. This creates an informational externality interacting with general equilibrium effects on each country's skill premium. Asymmetric equilibria with comparative advantages exist even when there is a unique equilibrium under autarky. Symmetric, no-trade equilibria may be unstable under free trade. Welfare effects are ambiguous: trade may be Pareto improving even if it leads to an equilibrium with rich and poor countries, with no special advantage to country size.

Keywords: Trade, Specialization, Human Capital, Reputation.

JEL Classification Number: D62, D82, F11, O12

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I Introduction

In this paper we develop a stylized model of international trade in which a country can establish a reputation for having a high quality labor force, providing new insights to the understanding of the causes of trade, specialization, and inequality across countries.

A reputation for high or low quality of the labor force may arise when employers do not perfectly observe workers' competencies and skills. Workers acquire human capital not only through education and work experience, but also with personal effort and investments that are not as easily observable. We focus on this informational asymmetry, showing that it may generate self-fulfilling reputational differences across countries.

Research has shown that informational asymmetries of this kind are empirically relevant. **?** and **?** first showed that employers' learning is significant, supporting the assumption that employers initially observe workers' skills with noise.[1] Recent literature confirms these results[2] suggesting a significant scope for the mechanism proposed in this paper to play a role in determining workers' wage distribution, incentives to acquire skills, and sorting across industries.

Based on this evidence, one cannot dismiss the possibility that labor market informational asymmetries may play a role in explaining, at least in part, trade and specialization across countries. In this paper, we demonstrate that they are sufficient to generate self-fulfilling cross-country differences in reputation that imply human capital differences, trade, and specialization between otherwise identical countries. There is arguably an incomplete

[1]In most of the literature, the identification of the main effect exploits panel data where workers are observed over time. If employers imperfectly observe workers' skills, but learn over time through the observation of productivity signals, then as tenure increases wages should become more correlated with measures of productivity available to the researcher (typically, workers' scores in aptitude tests).

[2]In particular, **?** measured the "speed" of employer learning finding that, according to the best estimates, it takes three years for an employer to reduce her expectation error to 50 percent of its initial value, and 26 years to reduce it to less than 10 percent of its initial value. Note that median employee tenure is currently just above 4 years, (January 2016, see the U.S. Bureau of Labor Statistics news release "Employee Tenure", https://www.bls.gov/news.release/tenure.toc.htm, last accessed: February 9, 2018). See also **?**, **?**, and **?** using U.S. data, and **?** with Danish data. **?** use Brazilian data to show that employer variation in workers' perceived race significantly affects wages.

understanding of the patterns of trade and specialization observed in the real world, which suggests that exploring alternative models could provide new insights.[3]

In our model, technology has constant returns to scale, a *country* is defined as a labor market, and international trade is frictionless. Countries are symmetric in every respect, therefore the model always admits symmetric equilibria that replicate autarky, without gainful trade. The only aspect of the model that is non-standard is workers' skill acquisition. We investigate the conditions that generate equilibria with asymmetric country reputations for skill investments and show the properties of such equilibria.

Workers can acquire skills at a cost that varies across workers. There are two sectors demanding labor, a "high tech" sector and a "low tech" sector, and skills increase productivity only in the high tech sector. Incentives to acquire skills are affected by an informational asymmetry: workers' skills are only observed by employers with noise, through a signal of productivity that may be thought of as aggregating information provided by the worker's curriculum, interviews, and observation in the workplace. A worker without skills, which we henceforth call an *unqualified* worker, may send a good signal, but this is less likely than a *qualified* worker (a worker with skills) sending a good signal.

Before observing the noisy signal, the prior probability of investment is determined in each country by aggregate investment rates summarized by the proportion of qualified workers. The probability of investment of each worker is then computed using her signal, but is also affected by the prior. Hence, the actual proportion of qualified workers, together with endogenous relative prices, *determines incentives to invest.* There is no point in investing in skills if there are very few qualified workers in the country because firms interpret a good signal as most likely being noise and the good signal raises the wage very little. Symmetrically, if almost all workers invest, firms ignore bad signals as "bad luck,"and again there is no point in investing since all workers get high wages regardless of the signal. Incentives to invest are at the highest at some intermediate level of aggregate investment because this is

[3] A full empirical investigation of the model implications, which would require accounting for (and separately identifying) other relevant factors, is outside the scope of this paper.

when firms pay most attention to the noisy signals.

Hence, starting from a relatively low level of investments, the value to acquire human capital *increases* if the proportion of skilled workers in the economy *increases* as the signal to noise ratio decreases. Working against this there are relative price effects that make the high-tech good less valuable when its supply increases, but these effects are smaller when countries trade than in autarky. Additionally, when skills increase in one country, the incentives to acquire skills in the other country are unambiguously reduced because of the price effects. Hence, an asymmetric allocation of human capital and goods production may arise even if countries are fundamentally identical. As far as we know, this is an explanation of trade and specialization that is novel in the international trade literature. What is crucial for this result is that the reputation for having a qualified labor force is like a public good, operating within a country regardless of its size.

While our mechanism is novel, there are some similarities with models of agglomeration. Scale economies and network effects can also create asymmetries between countries. However, these models usually assume some exogenous differences that are being accentuated in equilibrium. Moreover, in these models it is typically an advantage to have a large domestic market, whereas in our model there is no systematic effect favoring large countries. It is not the *number*, but the *proportion* of qualified workers that is critical in generating the reputational externality because employers, when assessing workers, use the proportion of qualified worker as their prior for human capital investment. A worker is more likely to be qualified the higher her country's proportion of qualified workers.

We highlight this irrelevance of country size by showing that large economies have no systematic advantage. In many parameterizations where country sizes are allowed to differ, there is an equilibrium where the large country is the richer as well as an equilibrium in which the small country is the richer. Which of these equilibria leads to more inequality or higher welfare is also a matter of parameter choices.

Asymmetric equilibria arise under free trade, but as already noted, there is always at least one symmetric equilibrium with no gainful trade that replicates the autarky allocation. However, several properties of our model suggest that coordinating on an asymmetric equilibrium may be plausible.[4] First, this is *not* a model in which some countries are trapped in a coordination failure and others are not. Incentives in one country depend on investments in the other and relative price effects are crucial. Asymmetric equilibria may therefore occur even if the autarky equilibrium is unique. Moreover, the stability conditions under autarky differ from the stability conditions under free trade. Opening up international trade may destabilize the unique and stable autarky equilibrium, so cross country income differences may be an inevitable aspect of free trade even if there are no exogenous differences that "explain" which country becomes richer.

In any asymmetric equilibrium, a country with more human capital is richer and better off than the other country. However, this does not necessarily imply that the poor country is worse off under trade than autarky. Welfare in the poor country can go either way, but we emphasize the less intuitive possibility by showing an example where an asymmetric equilibrium Pareto dominates the autarky equilibrium. The intuition is that an increase in the skill level abroad may drive down the relative price of the high-tech good so much that exchanging the low-tech good for the high-tech good generates higher welfare in the poor country compared to domestic production.

Our results are robust to introducing exogenous productivity differences. If one country has a "fundamental" comparative advantage in the high-tech industry, it may still specialize as a low-tech industry as a result of the mechanism in our model, provided that the exogenous differences are not too large.

The next section discusses the contribution of this paper relative to existing literature. Section **??** introduces the model, defines the equilibrium, and shows that it can be characterized as a planner's problem, simplifying the analysis that follows. Section **??** characterizes the equilibria under autarky. The main result, the existence of equilibria with trade and special-

[4] **?** argues that multiplicity by itself does not offer a compelling reason for observed asymmetries.

ization, is presented in Section ??. Section ?? discusses the stability and welfare properties of equilibria with trade, and the irrelevance of size. Section ?? concludes discussing the robustness of the results to extending the model to multiple countries, to including physical capital, and migration.

II Related literature

Our main contribution to the literature is to suggest a novel source of trade and comparative advantage between *identical* countries. There are several papers in the literature that include some of the crucial elements of our model, imperfectly observed human capital accumulation, but in those models either some exogenous differences are posited, or equilibrium multiplicity in a baseline autarky model is the driving source of specialization.

Our model relates to a literature on trade and endogenous skill formation initiated by ?, who develop a general equilibrium model where the driver of trade is endogenous human capital acquisition as in our model. Countries specialize because of exogenous differences in the availability of inputs needed to acquire human capital, generating what we refer to as price effects. In our setup instead, countries are identical also in the cost of acquiring human capital.[5]

Among the papers presenting models with asymmetric information, ? and ? have elements that are similar to our setup: a Hecksher-Ohlin model with imperfectly observable skills. They focus on comparative statics with respect to changes in the skill distribution. For their purposes it is sufficient to consider how trade is affected by exogenous differences in the talent distribution across countries, ignoring the incentives to acquire skills that are central in our model.[6]

[5]The focus of this literature is mainly in showing how even if factor price equalization holds (for the marginal worker), trade induces different incentives to acquire human capital across countries. For recent extensions see also ?, ?, ?, ?, ?, and ?. In some cases, the exogenous country differences are assumed by analyzing the effects of trade on a small open economy that takes the world price as given, as in ?, ?, ?, or ?.

[6]Several papers study the effects of informational asymmetries on trade, without focusing on how trade arises in equilibrium. ?, studies the effect of institutional quality on reducing workers' moral hazard. ?,

?, like us, seeks to formulate a more fundamental theory of comparative advantage. The technology is also based on the idea that human capital is more important for some firms than for others. The main difference is that the model ultimately derives country differences from exogenous differences in institutional quality and human capital.

? derives trade in a model where products may acquire, in equilibrium, different reputation for quality. Self-fulfilling reputation determines the average quality of a country exports, and comparative advantages arise endogenously because countries coordinate on selecting different equilibria.[7] Similarly, in **?** comparative advantages emerge endogenously as a Nash equilibrium of a game in which countries choose policies that affect sector-specific productivities or relative factor endowments. In these papers equilibrium multiplicity is needed to generate the comparative advantage. In our model instead, trade may arise even when there is a unique autarky equilibrium.

While our underlying assumptions are very different, our model shares many features with trade models with increasing returns (**?**, **?**), their versions usually referred to as "agglomeration models" (**?**, **?**), and the "symmetry-breaking" literature (see **??**). Agglomeration models can sustain a concentration of (high-income) manufacturing because production costs decrease with the size of the industry. Manufactured goods are inputs in the production of other goods, implying that being close to other producers saves on transportation costs. This creates incentives to concentrate production. When production costs are neither too small nor too large, there are equilibria where manufacturing is concentrated in one country that becomes richer.

While our model is considerably less complicated and closer to the neoclassical bench-

study how trade affects one country's human capital accumulations when education has a signaling role. **?** analyzes trade agreements under imperfect public monitoring, **?** consider effects of asymmetric information when exporters are credit constrained, and **?** show that weak institutions may result in welfare-reducing trade in an adverse selection model. **?** use an informational asymmetry to model the role of foreign direct investments, **?** derive a role for minority groups in international trade using an informational friction, and **?** considers the impact of asymmetric information in bargaining about trade agreements. **?** considers a model that is significantly richer than ours in many ways, but the informational asymmetry is modeled in reduced form.

[7] Other models based on trust and endogenous quality reputation are **?**, **?**, **?**, and **?**, who assume countries are initially asymmetric as they differ in the endowment of human capital.

mark than models with increasing returns, there is a close similarity in how a pecuniary externality interacts with local market conditions. There are also crucial differences: our model resorts to imperfect information rather than global increasing returns. Agglomeration models predict a positive relation between size and development whereas our model has no such implications, as illustrated in Section **??**. This is because what matters in determining a country's reputation is the proportion, not the number of skilled workers.

We borrow some of the modeling assumptions from the statistical discrimination literature. In **?** racial differences arise in a statistical discrimination model because groups specialize in the level of acquired skill. Here, countries take the role of racial groups, but embedding the reputational effects in a model in which spillover are carried by equilibrium price effects creates some additional complications. To make the analysis more transparent we have therefore simplified the information technology (the noisy signal has support on two realizations), the production technology (it is linear), so complementarities arise here because of convexity in preferences only. All these simplifications can be relaxed at the cost of some additional complexity of the analysis.

III The Model

Two countries, labeled by $j = h, f$, are populated by a continuum of agents of mass λ^h and $\lambda^f = 1 - \lambda^h$, respectively. Agents are price takers. We build on a simple $2 \times 2 \times 2$ trade model but with factors of production being workers with and without human capital. The model is closed by a stylized human capital acquisition and an informational technology borrowed from the statistical discrimination literature.[8] Workers cannot migrate.

Agents can invest in human capital. Investment is binary, the investment cost c is private information, drawn from a cumulative density G independent of which country the agent lives in, defined on the interval $[\underline{c}, \overline{c}]$. We call workers who invest in human capital *qualified* and the others *unqualified*. Agents have the same preferences. The utility of an agent consuming

[8]See **?**

the bundle (x_1, x_2) is $u(x_1, x_2) - c$ if the agent invests and $u(x_1, x_2)$ otherwise, where u is a homothetic and strictly quasi-concave.

After the investments, nature assigns each worker a signal $\theta \in \{g, b\}$ observed by employers. For simplicity we assume that $\Pr[g|\text{worker qualified}] = \Pr[b|\text{worker unqualified}] = \eta > \frac{1}{2}$ (that is, g is good news). Our preferred interpretation is that the unobservable investment is a costly effort decision and the signal is an imperfect measure of the costly effort, aggregating information from letter of recommendation, grades, tests, etc....

The two consumption goods are produced solely from qualified and unqualified labor, denoted q and n respectively, according to

$$y_1(q, n) = q; \quad y_2(q, n) = q + n. \tag{1}$$

All workers are thus perfect substitutes in industry 2, whereas only qualified workers contribute to the production of good 1.[9]

Next, after defining equilibrium, we show that given human capital investment the equilibrium in the goods and labor markets can be characterized as the solution to a planners' problem, simplifying the derivations that follow. The section concludes with a graphical representation of the production possibilities set.

Equilibrium

Our notion of equilibrium is analogous to a competitive equilibrium in a perfect information environment, but the informational asymmetry makes the treatment of the "labor supply" somewhat non-standard: skilled labor is endogenously determined by incentives that depend on prices derived from the goods markets.

Consider an agent with realized wage w deciding how to allocate her earnings between

[9] This extreme technology is for simplicity only. In previous versions we considered a more general technology with one good being more intensive in skilled labor than the other. This generalization creates no additional qualitative insights. Qualitatively, we need two sectors with different factor intensities, just like in the Heckscher-Ohlin model with fixed factor endowments.

the two goods given prices $p=(p_1,p_2)$. The (ex-post) maximized utility of the worker is

$$v(w,p)=\max_{x_1,x_2} u\left(x_1,x_2\right), \text{ subject to } p_1x_1+p_2x_2\leq w. \tag{2}$$

By strict quasi-concavity of $u\left(x_1,x_2\right)$, the optimization problem in (**??**) has a unique solution. We denote the demand functions by $x_1(w,p), x_2(w,p)$.

Employers cannot observe if a worker is qualified, so a labor demand is a map $l:\{g,b\}\rightarrow R_+$. Denote with π any fraction of qualified workers in a country, which can be thought of as the *prior* probability that a worker is qualified, before observing the signal. Employers then use Bayes' rule to form the posterior, conditional on her signal:

$$\mu\left(g,\pi\right)\equiv\frac{\eta\pi}{\eta\pi+\left(1-\eta\right)\left(1-\pi\right)}\qquad\mu\left(b,\pi\right)\equiv\frac{\left(1-\eta\right)\pi}{\left(1-\eta\right)\pi+\eta\left(1-\pi\right)}. \tag{3}$$

Associated with any fraction of qualified workers, π, and a given labor demand l, the corresponding quantities of qualified and unqualified workers are:

$$\begin{aligned} q &= l\left(g\right)\mu\left(g,\pi\right)+l\left(b\right)\mu\left(b,\pi\right) && (4)\\ n &= l\left(g\right)\left(1-\mu\left(g,\pi\right)\right)+l\left(b\right)\left(1-\mu\left(b,\pi\right)\right), \end{aligned}$$

We assume that a strong law of large numbers applies and treat q and n in (**??**) as both expected and realized inputs of labor.

Without loss of generality there is a representative firm in each sector and each country, which takes a *wage schedule* $w^j:\{g,b\}\rightarrow R_+$ and output prices p_i as given.[10] Using (**??**) and (**??**), the profit maximization problem for firms in either sector is:

$$\text{Sector 1:}\qquad \max_{l} p_1\left(l\left(g\right)\mu\left(g,\pi^j\right)+l\left(b\right)\mu\left(b,\pi^j\right)\right)-w_g^jl\left(g\right)-w_b^jl\left(b\right) \tag{5}$$

$$\text{Sector 2:}\qquad \max_{l} p_2\left(l\left(g\right)+l\left(b\right)\right)-w_g^jl\left(g\right)-w_b^jl\left(b\right) \tag{6}$$

Agents have rational expectations about the wages and prices, but face uncertainty about the realization of the signal. Denoting $v\left(w,p\right)$ the indirect utility function defined in (**??**),

[10] The caveat is that the informational asymmetry would disappear if (qualified) workers could start their own firms. We rule this and other contractual solutions to the informational asymmetry out by assumption. One way to justify this is to assume that there is a minimum efficient scale for production and that only aggregate output, and not the performance of individual workers, can be observed.

the expected utility for an agent in country j with investment cost c is

$$\eta v(w_g^j, p) + (1-\eta)\, v(w_b^j, p) - c \qquad \text{if the worker invests in human capital} \tag{7}$$

$$(1-\eta) v(w_g^j, p) + \eta v(w_b^j, p) \qquad \text{if not} \tag{8}$$

The worker is better off investing if and only if (**??**) exceeds (**??**), or if the cost of investment is less than the gross incentives. The implied proportion of investors in country j is thus

$$\pi^j = G\left((2\eta - 1)\left(v(w_g^j, p) - v(w_b^j, p)\right)\right). \tag{9}$$

To sum up: optimal consumption plans are defined in (**??**), (**??**) and (**??**) describe the profit maximization problems for each sector, and (**??**) summarizes the individually optimal human capital investments. What remains to describe are the market clearing conditions. Factor market clearing requires that the aggregate demand for workers with each signal equals the mass of agents who draw the signal. That is, let $l_i^j = \left(l_i^j(g), l_i^j(b)\right)$ be a labor demand scheme in industry i and country j. The labor market clearing conditions are:

$$\begin{aligned} l_1^j(g) + l_2^j(g) &= \eta\pi^j + (1-\eta)(1-\pi^j) \qquad (10) \\ l_1^j(b) + l_2^j(b) &= (1-\eta)\,\pi^j + \eta(1-\pi^j). \end{aligned}$$

Finally, for the product market equilibrium conditions let x_i^j be the output in industry i and country j. That is $x_1^j = l_1^j(g)\,\mu(g, \pi^j) + l_1^j(b)\,\mu(b, \pi^j)$ and $x_2^j = l_2^j(g) + l_2^j(b)$, which allows us to write the product market clearing conditions for the world market as

$$\sum_{j=h,f} \lambda^j (x_i^j - \underbrace{\left[\eta\pi^j + (1-\eta)(1-\pi^j)\right]}_{\#\text{agents with wage } w_g^j} x_i(w_g^j, p) - \underbrace{\left[(1-\eta)\,\pi^j + \eta(1-\pi^j)\right]}_{\#\text{agents with wage } w_b^j} x_i(w_b^j, p)) = 0 \tag{11}$$

Definition 1 *A competitive equilibrium consists of output prices p^*, wages w^{j*}, labor demands l_i^{j*}, outputs x_i^{j*}, and fractions of qualified workers π^{j*} for each country $j = h, f$ and industry $i = 1, 2$, satisfying:*

*(a) l_1^{j*} solves (**??**) and l_2^{j*} solves (**??**) given $p_i = p_i^*$ and x_1^{j*} and x_2^{j*} are the associated profit maximizing outputs in $j = h, f$*

*(b) the product market clearing conditions in (**??**) are satisfied.*

*(c) the factor market clearing conditions in (**??**) are satisfied.*

(d) π^{j*} *satisfies (**??**) given* $p = p^*$ *and wages* $w^j = w^{j*}$*for* $j = h, f$

We refer to a situation where all equilibrium conditions except the optimal investment condition (d) are fulfilled as a *continuation equilibrium.*[11]

A Planning Characterization of Continuation Equilibria

For an informationally unconstrained planner, a continuation equilibrium is inefficient: qualified and unqualified workers with the same signal are treated symmetrically, resulting in job misallocations. However, if the informational asymmetry is viewed as a property of the environment, then the equilibrium is (constrained) efficient *conditional on the investment behavior.* This allows us to describe aggregate equilibrium allocations as solutions to the planning problem:

$$\max_{(x_1,x_2)\in X^W\left(\pi^h,\pi^f\right)} u\left(x_1,x_2\right), \tag{12}$$

where $X^W\left(\pi^h,\pi^f\right)$ is the world production possibilities set.

Proposition 1 *Suppose that* $u\left(x_1,x_1\right)$ *is homothetic. Then:*

*1. In any continuation equilibrium, aggregate world consumption i is a solution to (**??**)*

2. Suppose that $\left(x_1^*,x_2^*\right)$ *solves (**??**),* $\left(p_1^*,p_2^*\right)$ *is a normal to a hyperplane that separates the set of bundles such that* $u\left(x_1,x_2\right)\geq u\left(x_1^*,x_2^*\right)$ *and* $X^W\left(\pi^h,\pi^f\right)$*, and that* $w_g^{j*}=\max\left\{p_1^*\mu\left(g,\pi^j\right),p_2^*\right\}$ *and* $w_b^{j*}=\max\left\{p_1^*\mu\left(b,\pi^j\right),p_2^*\right\}$ *in each country j. Then these prices, wages and aggregate consumption are part of a continuation equilibrium.*[12]

The proof is in Appendix **??**. The proposition shows that, for *fixed* investments, versions of the welfare theorems hold: the equilibrium is characterized by a planning problem where the informational asymmetry is built into the feasible set. It immediately follows that, given

[11] This term is mainly due to lack of a better alternative. Due to the workers being non-atomic it does not make a difference whether investments are made before or simultaneously with the wage posting.

[12] The allocation of workers in each country is somewhat complicated to describe in general, but is implicitly pinned down as the (almost always) unique allocation that can produce the equilibrium bundle.

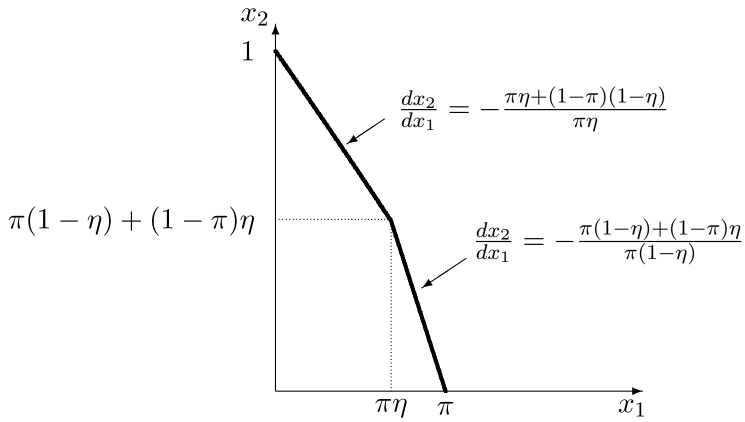

Figure 1: Per capita production possibilities in a country

any $\left(\pi^h, \pi^f\right)$ there is a unique continuation equilibrium up to a re-normalization of the prices. This allows us to appeal to simple graphs in the analysis.

The Production Possibilities Set

A useful way to represent technology is in terms of the *production possibilities set.* The set of feasible production plans in a country depends on the fraction of workers that invest in human capital, π. Figure **??** illustrates the (per capita) production possibilities set in a country, which we denote with $X(\pi)$.

To understand the figure, first observe that $(x_1, x_2) = (0, 1)$ if all workers are producing good 2, and that $(x_1, x_2) = (\pi, 0)$ if all workers are producing good 1, because only a fraction π of the workers are productive in Sector 1. There are $\pi\eta + (1-\pi)(1-\eta)$ workers with signal g and $\pi(1-\eta) + (1-\pi)\eta$ workers with signal b. If all signal g workers are in Sector 1 ($\pi\eta$ of these workers are productive) and all signal-b workers are in Sector 2, then the outputs are given by the point at the kink in the graph. The frontier to the right of the kink is steeper because in that region all g workers are employed in Sector 1, therefore to increase production firms must employ more b workers, who are less likely to be qualified. To the left of the kink instead, only g workers are employed in Sector 1.

The *world production possibilities* set is given by $X^W\left(\pi^h, \pi^f\right) = \lambda^h X\left(\pi^h\right) + \lambda^f X\left(\pi^f\right)$ and is convex by convexity of $X(\pi)$. The next proposition immediately follows, since the production possibilities set becomes (weakly) flatter as investment increases:

Proposition 2 *Suppose that $u(x_1, x_1)$ is homothetic. Then in any continuation equilibrium the relative price of the high-tech good is (weakly) decreasing in the countries' investment π^h and π^f.*

IV A Parametric Specification

While the results presented below are more general, for simplicity of exposition in the remainder of the paper we will restrict attention to Cobb-Douglas preferences, $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$, which imply demand functions:

$$x_1(p, w) = \frac{\alpha w}{p_1} \qquad x_2(p, w) = \frac{(1-\alpha) w}{p_2}. \tag{13}$$

The continuation utility for a worker that earns wage w is therefore:

$$v(w, p) = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha} p_2^{1-\alpha}} w. \tag{14}$$

We normalize setting $p_2 = 1$ and, abusing notation, write $p(\boldsymbol{\pi}), w_g^j(\boldsymbol{\pi})$ and $w_b^j(\boldsymbol{\pi})$ for the unique continuation equilibrium prices and wages in good 2 units, where $\boldsymbol{\pi} = \left(\pi^h, \pi^f\right)$.

A qualified worker earns $w_g^j(\boldsymbol{\pi})$ with probability η and $w_b^j(\boldsymbol{\pi})$ with probability $1-\eta$. Symmetrically, an unqualified worker earns $w_g^j(\boldsymbol{\pi})$ with probability $1-\eta$ and $w_b^j(\boldsymbol{\pi})$ with probability η. Computing the expectation of $v(w, p)$ in (**??**) *conditional on investment* and subtracting from this the expectation of $v(w, p)$ *conditional on not investing* we get the *gross benefits of investment* for an agent in country j, denoted $B^j(\boldsymbol{\pi})$:

$$\begin{aligned} B^j(\boldsymbol{\pi}) &= E\{v(w, p) \,|\text{qualified}\} - E\{v(w, p) \,|\text{unqualified}\} \\ &= \alpha^{\alpha}(1-\alpha)^{1-\alpha}(2\eta - 1)\frac{(w_g^j(\boldsymbol{\pi}) - w_b^j(\boldsymbol{\pi}))}{(p(\boldsymbol{\pi}))^{\alpha}}. \end{aligned} \tag{15}$$

Using condition (d) in Definition **??** we see that any $\boldsymbol{\pi}$ such that $\pi^j = G(B^j(\boldsymbol{\pi}))$ for $j = h, f$ gives an equilibrium fraction of investors in each country. All that remains to calculate full equilibria is to derive expressions for the continuation equilibrium prices.

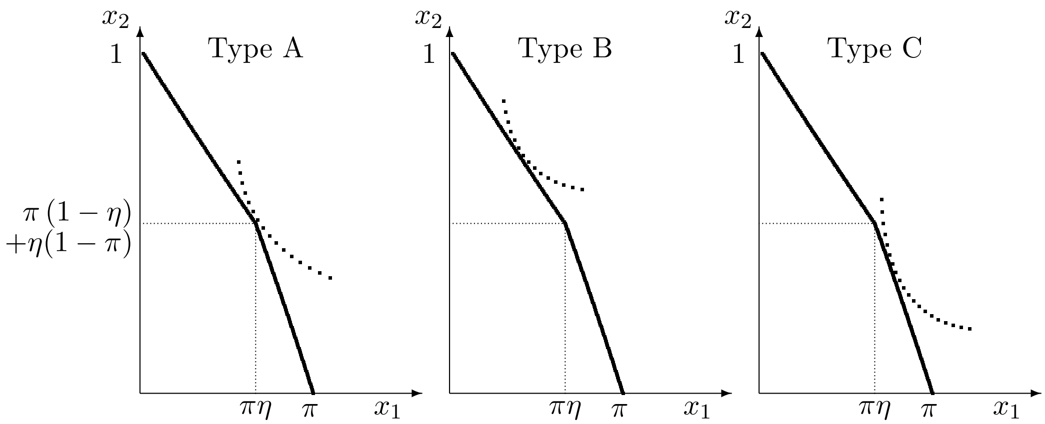

Figure 2: Three types of continuation equilibria

Continuation Equilibria in Autarky

As a benchmark, we first consider a closed economy. Suppressing the country index, we write π for the proportion of qualified workers. There are three possible types of continuation equilibria, illustrated in Figure **??**.[13]

Type A equilibria (allocation of workers "according to signals"). Graphically, this type occurs when the tangency is at the kink of the feasible set. All workers with signal b (g) are working in the low (high) tech sector. Outputs are $x_1 = \eta\pi$ and $x_2 = (1-\eta)\pi + \eta(1-\pi)$, so the demands in (**??**) pin down the price of the high-tech good as

$$p(\pi) = \frac{\alpha}{1-\alpha} \frac{(1-\eta)\pi + \eta(1-\pi)}{\eta\pi}. \tag{16}$$

Candidate equilibrium wages are obtained by the zero profits condition. Since $p_2 = 1$, this immediately gives $w_b(\pi) = 1$. The high-tech firm sells $\eta\pi$ units at price $p(\pi)$ and hires $\eta\pi + (1-\eta)(1-\pi)$ workers with signal g. Zero profits in Sector 1 therefore implies that the wage in that sector, $w_g(\pi)$, equals the price of good 1 times the expected probability that a

[13]This is a somewhat unfortunate aspect of having only 2 signals. With a continuum of signals we would get a strictly convex production possibilities set and the tangency condition would determine a unique threshold signal. However, as it is much simpler to compute explicit examples with two signals we decided to stick with the more inelegant case. Calculations are straightforward but may be tedious. We provided more detailed steps in the online appendix

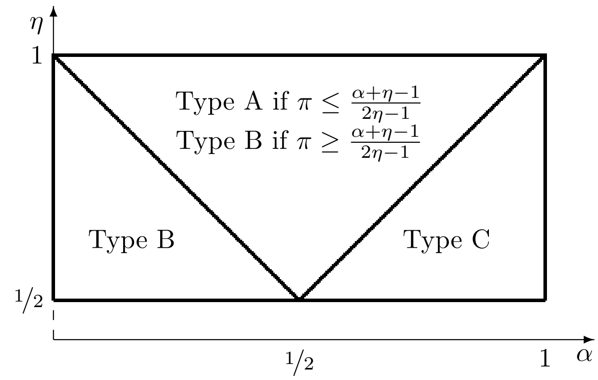

Figure 3: Types of autarky equilibria in the (α, η) space

worker with signal g is productive in that sector $\mu(g, \pi)$:

$$w_g(\pi) = p(\pi)\,\mu(g, \pi) = p(\pi)\,\frac{\pi\eta}{\pi\eta + (1-\eta)(1-\pi)}, \tag{17}$$

which has the alternative interpretation that wage equals the expected value of output. Finally, we have to check that a high-tech firm has no incentive to hire signal b workers, and that a low-tech firm has no incentive to hire signal g workers. These conditions determine the region where a Type A equilibrium exists (see Figure **??**).

Type B equilibria (mixing of good signals). In Figure **??**, this corresponds to a tangency to the left of the kink. Some workers with signal g work in Sector 2. These workers earn the same wage as g-signal workers in Sector 1, and, since all workers in the low-tech sector are paid their marginal product, 1, it follows that $w_g(\pi) = w_b(\pi) = 1$. This provides workers zero incentives to invest. Because this makes the case less interesting for the full equilibrium, we refer the reader to the online appendix for details.

Type C equilibria (mixing of bad signals). This type occurs when the demand for good 1 is strong (i.e. when the Cobb-Douglas share α is high). In Figure **??**, this corresponds to a tangency to the right of the kink. In this case a fraction β of workers with signal b (defined below) works in Sector 1. Workers with signal b are paid 1 if employed in the low-tech sector. This must equal to the wage when employed in the high tech sector, which is equal to their expected productivity (price times their probability of being productive). Therefore, $w_b(\pi) = p(\pi)\cdot\mu(b, \pi) = 1$ pins down the price of good 1 as the inverse of the probability

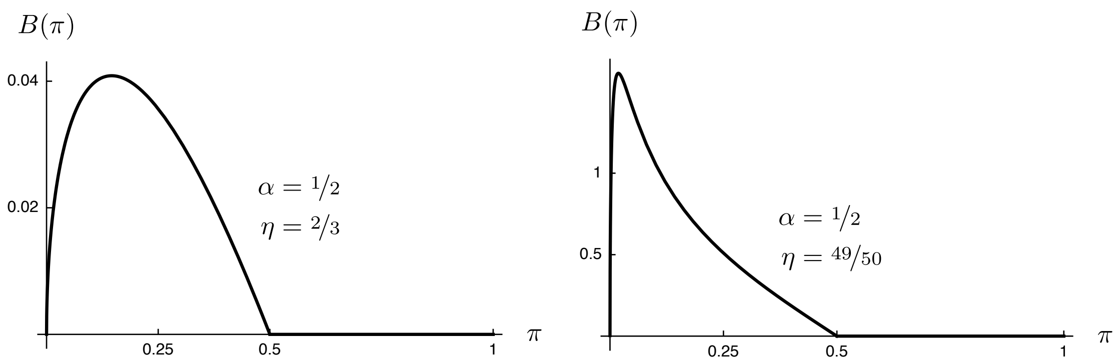

Figure 4: Gross incentives to invest under autarky

that a b-signal worker is productive in the high-tech sector,

$$p(\pi) = \frac{1}{\mu(b,\pi)} = \frac{\pi(1-\eta) + (1-\pi)\eta}{\pi(1-\eta)} \tag{18}$$

The price must also satisfy a relationship imposed by demand shares (**??**):

$$p\left(\pi\right) = \frac{\alpha}{1-\alpha} \frac{\overbrace{(1-\beta)\left(\left(1-\eta\right)\pi + \eta\left(1-\pi\right)\right)}^{x_2 \text{ produced by } b\text{-workers}}}{\underbrace{\eta\pi}_{x_1\text{produced by } g \text{ workers}} + \underbrace{\beta(1-\eta)\pi}_{x_1\text{produced by } b \text{ workers}}} \tag{19}$$

Equating the right-hand sides of (**??**) and (**??**) determines the fraction of b-signal workers employed in Sector 1, β. The solution reveals that a positive β exists if and only if $\alpha > \eta$, as illustrated in Figure **??**. We refer the reader again to the online appendix for details.

Equilibrium investments in Autarky

To obtain a closed form expression for the incentives to invest as a function of π substitute the wages and prices derived above into (**??**). If $\alpha \leq \eta$, this function may be written as:[14]

$$B\left(\pi\right) = \max\left\{(2\eta-1)\left(\frac{\pi\eta}{\pi(1-\eta)+\left(1-\pi\right)\eta}\right)^{\alpha}\left(\frac{\alpha-(\pi\eta+(1-\pi)(1-\eta))}{\pi\eta+(1-\pi)(1-\eta)}\right), 0\right\}. \tag{20}$$

Figure **??** plots $B\left(\pi\right)$ for two sets of parameter values. All values where $B(\pi) > 0$ in the figure correspond to type-A continuation equilibria, where g workers produce good 1 and b workers produce good 2. $B\left(\pi\right)$ is single-peaked, but not necessarily concave (example in the right panel). Under different specifications of information and output technology the single-peakedness may break down, but what remains true is that the function is equal to

[14]See the online appendix for a detailed derivation.

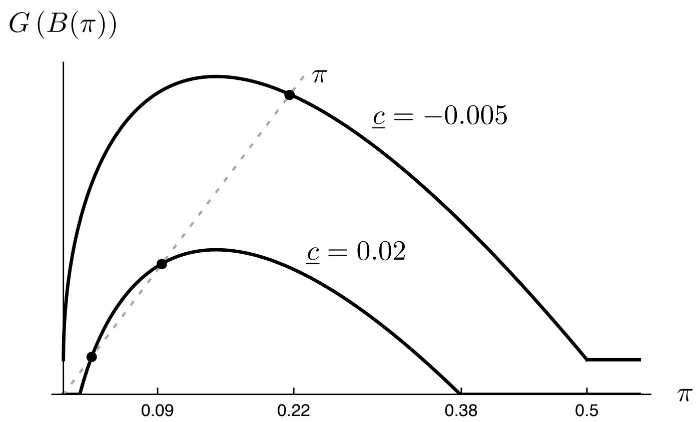

Figure 5: Equilibrium fixed point maps for two values of $\underline{c}$, with $\eta = 2/3, \alpha = 1/2$

zero at the extremes, and therefore must be initially increasing, and eventually decreasing. The reason is that if $\pi = 0$ or $\pi = 1$ workers are all equally productive in the production of both goods regardless of their signal (in particular, they are all unproductive in Sector 1 when $\pi = 0$), therefore their wage does not depend on the signal. But if better signals are not rewarded with higher wages, incentives to invest are zero. Only when $0 < \pi < 1$ the signal carries information; workers that receive a good signal are paid higher wages, generating positive incentives to invest.

Any π such that $\pi = G(B(\pi))$ is an equilibrium fraction of investors. Since $G(B(\pi))$ is continuous and takes values on $[0, 1]$, existence follows trivially. The fixed point condition is illustrated in Figure **??**, computed with $\eta = 2/3$, $\alpha = 1/2$ and G uniform over $[\underline{c}, \overline{c}]$, with $\overline{c} - \underline{c} = 0.2$. Changes in $\underline{c}$ correspond to shifts in the cost distribution. If $\underline{c} < 0$ (i.e. when some workers prefer to invest even without incentives) the equilibrium is unique. For $\underline{c} = 0$, there is a trivial equilibrium with no investment and an equilibrium with $\pi > 0$. As $\underline{c}$ gets slightly larger there are three equilibria (one with $\pi = 0$), whereas if $\underline{c}$ is sufficiently large (not shown in the figure), as the curve shifts to the right only the trivial equilibrium with no investment remains.

In many examples that follow we assume that a unique equilibrium with $\pi > 0$ exists under autarky.[15] This is to highlight that country specialization does not rely on countries

[15]Sufficient conditions are that $G \circ B$ is concave and $\underline{c} < 0$. The first is a technical assumption needed

coordinating on different equilibria of the autarkic model (with multiplicity under autarky, further possibilities for specialization with trade arise). This assumption also eliminates "nuisance equilibria" with zero investments and makes welfare analysis sharper, not having to rely on comparisons between sets of equilibria.

V Equilibria in the Trade Regime

In this section we assume that h and f trade on a frictionless world market. We will first prove by construction the main result of the paper: the existence of a asymmetric equilibria with trade and specialization. Next, we will provide some evidence of the generality of the result. While the replication of the autarky equilibrium in both countries remains an equilibrium of the two-country model (with no trade), we will show in the next section that this equilibrium may be unstable and conclude the analysis illustrating some welfare properties of the equilibria with trade.

Illustration of the existence of asymmetric equilibria by construction

The simplest asymmetric equilibrium we can construct occurs when the poor country, which we label as country h, is fully specialized in the low-tech sector. In such an equilibrium, the wage gap in h is zero, so the fraction of qualified workers is pinned down as $\pi^h = G(0)$. The proportion of qualified workers in f solves a single variable fixed point equation similarly to the autarky case, but with some extra production of x_2 performed in country h. Once π^f is obtained from this condition, it only remains to check that firms in h have no incentives to hire workers with signal g to produce the high-tech good.

To formalize the argument, assume $G = U[0, 0.2]$. Assuming all workers in country h specialize in the production of x_2, this induces zero incentives to invest, implying $\pi^h = G(0) = 0$ and no incentives to place any worker in Sector 1 in country h. There is always a

to ensure that $G()$ does not intersect the 45 degree line from below. The second assumption posits that an arbitrarily small fraction of workers like to make the investment even if there are no monetary gains. We do not believe this to be unrealistic.

trivial equilibrium with $\pi^f = 0$, zero production of good 1, and zero utility for all, but we look for non-trivial equilibria with positive incentives to invest in f. If these equilibria exist, the equilibrium in country f is of type A or C (a fraction $0 < \beta \leq 1$ of bad signal workers producing good 1).[16] The relative price of good 1 is pinned down by conditions similar to the autarky case, but modified to take into account the production of good 2 occurring in country h. The equivalent of (**??**) is:[17]

$$p\left(\pi^f\right) = \frac{\alpha}{1-\alpha} \frac{\overbrace{\lambda^f\left(\left(1-\beta\right)\left(1-\eta\right)\pi^f + \eta\left(1-\pi^f\right)\right)}^{x_2 \text{ produced in f}} + \overbrace{\lambda^h}^{x_2 \text{ in } h}}{\underbrace{\lambda^f\left(\eta\pi^f + \beta(1-\eta)\pi^f\right)}_{x_1 \text{ produced in } f}} \tag{21}$$

Where $\beta = 0$ if the equilibrium is of type A (no workers with signal b produce good 1) and $0 < \beta < 1$ if the equilibrium is of type C (some b-signal workers produce good 1). This equation defines the relative price of good 1 in a type-A equilibrium, and β in a type-C equilibrium (because b workers are employed in both sectors, the price is determined by equalizing their marginal productivity in the two sectors: $p(\pi^f)\mu\left(b, \pi^f\right) = 1$).

To derive incentives to invest, we make two additional assumptions that do not hinder the generality of the result, as we discuss below, but simplify the derivations: we set equal Cobb-Douglas shares $\alpha = 1/2$, information technology parameter $\eta = 2/3$, and equal country sizes: $\lambda^h = \lambda^f = 1/2$. Simple algebraic simplifications, which we relegate to the web appendix, show that the continuation equilibrium in country f is of type C. Workers with signal b in f are employed in both sectors, therefore the price is pinned down by equating the marginal product of these workers in the two sectors $1 = p(\pi^f)\mu(b, \pi^f)$, which, using (**??**), and $\eta = 2/3$ implies $p(\pi^f) = \left(2 - \pi^f\right)/\pi^f$. Wages are:

$$w_b^f = 1, \qquad w_g^f = p(\pi^f)\mu(g, \pi^f) = \frac{2-\pi^f}{\pi^f}\frac{2\pi^f}{1+\pi^f}.$$

Solving (**??**) for β, the fraction of b-signal workers in country f employed in Sector 1 is

[16] In equilibria of type B (mixing of good signals) some good signal workers produce good 2 and therefore receive wage 1, which is the same as the wage of bad signal workers. This provides no incentives to invest leading to the uninteresting equilibrium $(\pi^h, \pi^f) = (0, 0)$.

[17] Since we are looking for equilibria where $\pi^h = 0$ we can drop the dependency of the price on π^h.

$\beta = (1+\pi^f)/(4-2\pi^f)$. We are now in a position to derive incentives to invest in country f. We substitute our derivations into (**??**) to obtain,

$$B^f(\pi^f) = \frac{1}{6}\left(\sqrt{p(\pi^f)}\mu\left(g,\pi^f\right) - \frac{1}{\sqrt{p(\pi^f)}}\right) = \left(\frac{4-2\pi^f}{1+\pi^f} - 1\right)\sqrt{\frac{\pi^f}{2-\pi^f}}, \tag{22}$$

with $\mu(g,\pi) = 2\pi/(1+\pi)$ from (**??**). Note that (**??**) is equal to zero for $\pi^f = 0$ or 1. The equilibrium in country f is defined by the fixed-point equation $\pi^f = G\left(B^f\left(\pi^f\right)\right)$ with one interior solution at $\pi^f = 0.49$ with $p = 3.095$. As will be shown next, this type of trade equilibrium is robust to perturbations of the parametric assumptions we made.

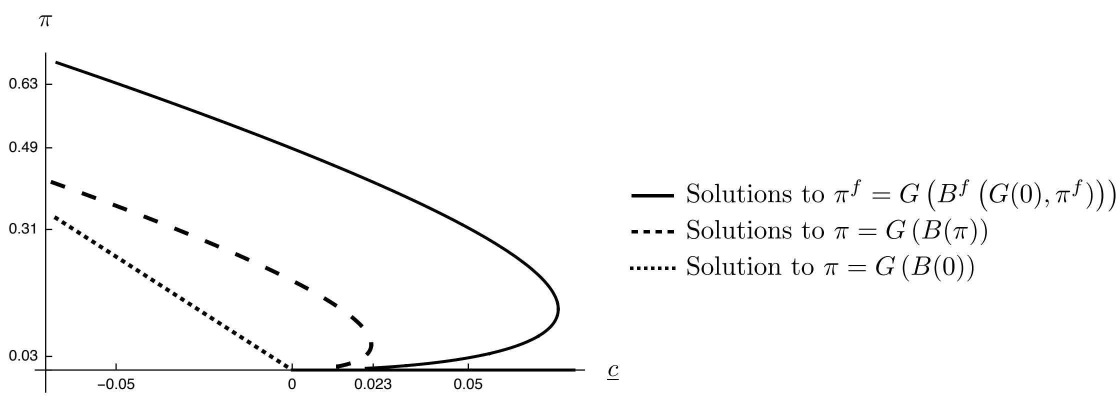

Figure 6: Equilibrium investments under trade with $\eta = 2/3, \alpha = 1/2$ for different values of $\underline{c}$.

Robustness of the equilibria with trade

The cost distribution. We explore first how shifts in the cost distribution affect the existence of asymmetric equilibria of the type we computed above (with full specialization of h-country workers). Assume a uniform G over $[\underline{c}, \underline{c}+0.2]$, and treat the lower bound of the distribution $\underline{c}$ as a variable, holding the other parameters fixed.

Figure **??** illustrates the results. The solid line represents equilibrium investments in country f if there were no incentives to invest in country h. The dotted line is the fraction that is willing to invest without incentives, and the line in between represents equilibrium investments in autarky. It cannot be seen in the figure, but it can be shown that $\pi^h = G\left(B\left(0\right)\right)$ is a best response given that the country f invests in accordance with the solid line, so country h investing in accordance with the dotted line and f in accordance with the solid line is an equilibrium under trade.

Both curves bend backwards, so there is a range with multiple equilibria in the autarky model (see dashed line where $\underline{c} > 0$). If $\underline{c} > 0$ zero incentives in county h implies $\pi^h = 0$. As can be seen from the solid line bending backwards in this region, there are three best responses in country f to $\pi^h = 0$: one is the trivial equilbrium $\pi^f = 0$ whereas two have positive investment. There is also a range to the right of approximately $\underline{c} = 0.023$ where there are two non-trivial asymmetric trade equilibria, despite the unique autarky equilibrium being a trivial zero investment equilibrium (the dashed line can't be seen but it corresponds to the horizontal axis in this range). For example if $\underline{c} = 0.05$, $\pi^f = \{0, 0.03, 0.31\}$ are all best responses to $\pi^h = 0$.

Multiple autarky equilibria are not necessary for trade to occur. For $\underline{c}$ approximately between -0.07 and 0 there is an asymmetric equilibrium with trade, and a unique autarky equilibrium. For example, when $\underline{c} = -0.05$, 25 percent of workers from country h invest even when there are no incentives to do so. Assuming that this is the case, and placing all workers of country h in Sector 2, in country f most workers specialize in Sector 1, generating incentives so that $\pi^f = 0.63$ is the optimal response, with a relative price of good 1 equal to 2.16. It remains to be checked that it is optimal to employ country h workers with good signals in Sector 2. With $\pi^h = .25$, the expected probability of being qualified for a good worker is $\mu(g, 0.25) = 0.4$, which multiplied by the price 2.16 gives an expected productivity of 0.865, less than the unit productivity in Sector 2. In general, one can verify that this condition, $p(\pi^f)\eta(g, \pi^h) \leq 1$, is satisfied if $\frac{4\pi^h}{1+3\pi^h} \leq \pi^f$, which holds as long as $\pi^h = G(0)$ is small enough. Indeed for lower values of the lower bound of the cost distribution not displayed in the figure, as the number of qualified workers in country h increases, it becomes impossible to sustain this type of asymmetric equilibrium.

Country size and preference parameter. Existence of this type of trade equilibria also does not hinge on our choice of the values of relative country size λ^h and of the Cobb-

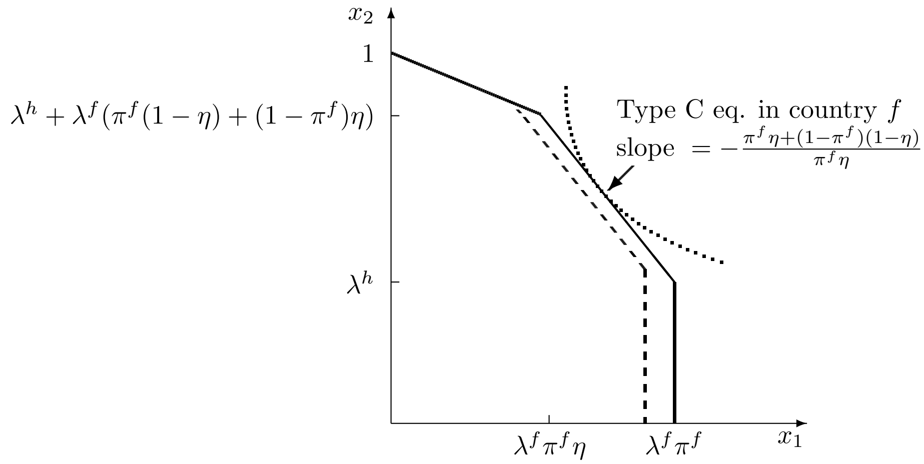

Figure 7: World production possibilities frontier when all in h produce good 2

Douglas preference parameter α. Figure **??** shows the production possibilities frontier when all workers in country h produce good 2. An increase in λ^h shifts the production possibilities frontier from the solid to the dashed line, but does not change the slope of the frontier in correspondence to a type-C equilibrium, because the relative productivity of workers in country f in the two sectors, determined by the information technology, does not change. Similarly, a change in α changes the slope of the indifference curves. Therefore, perturbations of α and λ^h (small enough so that the equilibrium remains of type C in country f) change the point of tangency but not the equilibrium price, which is defined by the slope of the production possibilities set.[18] Expected productivities, determined by the price and the information technology, do not change, therefore wages do not change. Incentives and equilibrium investment remain the same in both countries.

Extreme specialization in country h. Asymmetric equilibria also do not depend on the extreme specialization in country h we assumed to construct the equilibrium so far. The analysis gets more complicated because when positive incentives to invest exist in both countries, solving for equilibrium implies computing the solution to a system of two equations.

[18]Prices are constant because of the simplifying assumption that information technology has only two signals available. With a more general information structure the production possibilities set would be strictly convex, and small perturbations of λ^h or α would have a small effect on equilibrium prices. To make the case that a nearby trade equilibrium still exists we would have to rely on continuity arguments.

For an intuition, recall from Proposition **??** that the equilibrium price is decreasing in π^f (strictly, in some regions). From (**??**), incentives are increasing in price because price increases wages of g-signal workers more than wages of b-signal workers.[19] Hence, an increase in investments abroad decreases prices and incentives at home. Symmetrically, an increase in investments at home reduces incentives abroad. This is a negative cross-country externality in human capital acquisition. These effects create equilibria where countries specialize: rich countries export the high-tech good and poor countries export the low-tech good, even with a unique autarky equilibrium.

Formally, consider the region of the parameter space where equiilbria are of type C or A in both countries so that $w_g^j = p(\pi)\mu(g, \pi^j)$ and $w_b^j = 1$.[20] Differentiate (**??**) to obtain, using p as shorthand for $p(\pi^h, \pi^f)$ and introducing notation $\Psi = (2\eta - 1)\alpha^\alpha(1-\alpha)^{1-\alpha}$:

$$\frac{\partial B^f\left(\pi^h, \pi^f\right)}{\partial \pi^f} = \underbrace{\Psi p^{1-\alpha}\frac{d\mu(g,\pi^f)}{d\pi^f}}_{\text{“information effect”}} + \underbrace{\Psi p^{-\alpha}\left((1-\alpha)\mu(g,\pi^f) + \frac{\alpha}{p}\right)\frac{\partial p}{\partial \pi^f}}_{\text{“price effect”}} \tag{23}$$

$$\frac{\partial B^h\left(\pi^h, \pi^f\right)}{\partial \pi^f} = \underbrace{\Psi p^{-\alpha}\left((1-\alpha)\mu(g,\pi^f) + \frac{\alpha}{p}\right)\frac{\partial p}{\partial \pi^f}}_{\text{“price effect”}} \tag{24}$$

The price effect labeled in the equations is, as discussed, negative, and occurs in both countries whereas the information effect bites only in the country where investment changes. The information effect is positive because as the proportion of investors increases, the probability that an individual with good signal is productive increases as well, but its size depends on the size of π^f. Hence, starting from a non-trivial autarky equilibrium in which $\pi^A = \pi^f = \pi^h$, an increase in π^f either decreases function B^h and increases B^f, or it shifts B^h downwards more than it shifts B^f. A decrease in π^h has the symmetrically opposite effect. These derivations illustrate why the informational externality pushes countries to specialize. One can then find values $\pi^h < \pi^f$ such that $B^h(\pi^h, \pi^f) < B^f(\pi^h, \pi^f)$. Whether these values satisfy the

[19] Either b workers are employed only in Sector 2, in which case their wage is fixed at 1, or some are employed in Sector 1, in which case their wage is $p(\pi)\mu(b, \pi^j)$ which is less than the wage of g-signal workers employed in Sector 1, $p(\pi)\mu(g, \pi^j)$.

[20] This is necessary to have strictly positive incentives to invest in both countries

equilibrium conditions depends on the cost distribution, but examples can be constructed to this end.[21]

VI Stability, Welfare, and the Irrelevance of Size

Stability

A symmetric equilibrium replicating autarky always exists in the trade regime. However, this equilibrium can be unstable when the economy is open for trade.[22]

Consider a parameterization where π^A is a stable autarky equilibrium.[23] It follows that $\boldsymbol{\pi} = \left(\pi^A, \pi^A\right)$ is an equilibrium when the countries are allowed to trade.

We analyze the effects of small deviations from the symmetric equilibrium. Consider the change in relative price first. When $\pi^h = \pi^f = \pi$ and assuming again $\eta = 2/3$ and $\alpha = 1/2$, we are in the region where $\eta \geq \alpha$. The autarky equilibrium must be of type A. One can derive that when the equilibrium is of type A in both countries, the price is equal to $p(\pi^h, \pi^f) = (4 - \pi^h - \pi^f)/2(\pi^h + \pi^f)$,[24] therefore $p\left(\pi, \pi\right) = \left(2 - \pi\right)/2\pi$, which is consistent with (**??**). Differentiating these expression gives: $\frac{d}{d\pi} p(\pi, \pi) = \frac{-1}{(\pi)^2}$ (relevant under autarky), and $\frac{\partial}{\partial \pi^f} p(\pi^h, \pi^f) = \frac{-2}{\left(\pi^h + \pi^f\right)^2}$ (relevant with trade). Evaluating each expression at (π^A, π^A) we have:

$$\frac{d}{d\pi} p(\pi, \pi)\bigg|_{\pi=\pi^A} - \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f}\bigg|_{\pi^h=\pi^f=\pi^A} = \frac{-1}{\left(\pi^A\right)^2} - \frac{-2}{4\left(\pi^A\right)^2} = \frac{-1}{2\left(\pi^A\right)^2} < 0\ . \tag{25}$$

An increase in investments thus has a larger negative impact on the price in autarky, as intuition suggests. Autarky is equivalent to the trade regime with the added restriction that

[21] If one is willing to let the parameters of G be free, note for the sake of constructing a trade equilibrium that there is an infinite number of probability distributions satisfying the three restrictions on their domain that are needed for (π^h, π^f) to hold as a trade equilibrium together with π^A as an autarky equilibrium: $G(B^h(\pi^h, \pi^f)) = \pi^h$, $G(B^f(\pi^h, \pi^f)) = \pi^f$, and $G(B(\pi^A, \pi^A)) = \pi^A$.

[22] Because the model lacks real time, "stability" is a somewhat ad hoc criterion that corresponds to the adjustment dynamic where $\pi^j_{t+1} = G(B^j(\pi^j_t, \pi^k_t))$, $j, k = h, f$, $j \neq k$ (or the natural continuous analogue). Embedding the model in an OLG framework one obtains such dynamic system if one assumes that employers can not differentiate between workers of different cohorts.

[23] For example, when $\underline{c} < 0$, we know there is a unique autarky equilibrium, which must be stable since $G(B(\pi))$ must intersect the 45^o line from above.

[24] See the online appendix for the detailed derivation.

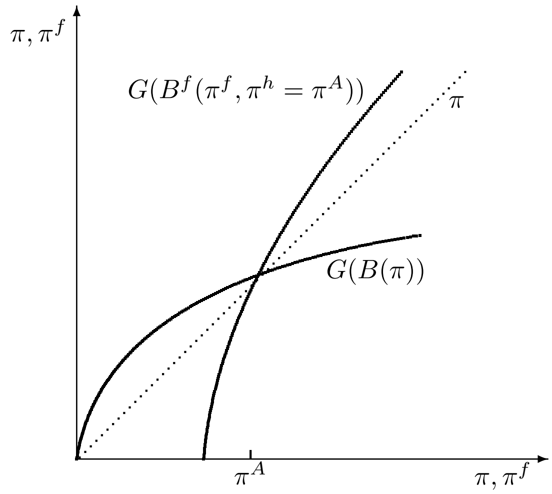

Figure 8: Best responses under trade and autarky, at the autarky equilibrium

$\pi^h = \pi^f = \pi$. We compare the effect of a change in investment on incentives to invest (**??**) between the regimes. In the autarky case, we restrict the two arguments of B^f to be equal, while they are unrestricted in the open economy case. With $\alpha = 1/2$ and $\eta = 2/3$, (**??**) further simplifies to obtain (using p^A as shorthand for $p(\pi^A, \pi^A)$) :

$$\left.\frac{dB^j(\pi, \pi)}{d\pi}\right|_{\pi=\pi^A} = \underbrace{\frac{\sqrt{p^A}}{6} \left.\frac{d\mu(g, \pi)}{d\pi}\right|_{\pi=\pi^A}}_{\text{“information effect”}} + \underbrace{\frac{1}{12\sqrt{p^A}} \left(\mu\left(g, \pi^A\right) + \frac{1}{p^A}\right) \left.\frac{dp(\pi, \pi)}{d\pi}\right|_{\pi=\pi^A}}_{\text{“price effect”}} \tag{26}$$

$$\left.\frac{\partial B^f\left(\pi^h, \pi^f\right)}{\partial \pi^f}\right|_{\substack{\pi^h=\pi^A \\ \pi^f=\pi^A}} = \underbrace{\frac{\sqrt{p^A}}{6} \left.\frac{d\mu\left(g, \pi^f\right)}{d\pi^f}\right|_{\pi^f=\pi^A}}_{\text{“information effect”}} + \underbrace{\frac{1}{12\sqrt{p^A}} \left(\mu\left(g, \pi^A\right) + \frac{1}{p^A}\right) \left.\frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f}\right|_{\substack{\pi^h=\pi^A \\ \pi^f=\pi^A}}}_{\text{“price effect”}}, \tag{27}$$

The effect on incentives is decomposed as a positive “information effect” and a negative “price effect”. The information effect in (**??**) is the same as in (**??**), but, by (**??**), the price effect is stronger in autarky, so the slope of $B^f\left(\pi^f, \pi^h = \pi^A\right)$ exceeds the slope of the autarky benefits of investment $B(\pi)$, when evaluating both functions at π^A (see Figure **??**). Hence, it is possible that $G(B^f\left(\pi^f, \pi^h = \pi^A\right))$ intersects the 45^o line from below at $\pi^f = \pi^A$ even if $G(B(\pi))$ intersects from above. Since the curve $G(B^f\left(\pi^f, \pi^h = \pi^A\right))$ intersecting the 45^o line from below is a *sufficient* condition for local instability this shows that the autarky equilibrium may be destabilized by opening up for trade.[25]

[25]Examples are easy to find. When c is uniformly distributed on $[0, 2]$, the unique (non-trivial) autarky

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}, c \sim U[-.02, .18]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = .1$	$\pi^f = .548$	$\pi = .269$
Per Capita Production	$y_1^h = 0, y_2^h = 1$	$y_1^f = .463,\ y_2^f = .226$	$y_1 = .179,\ y_2 = .577$
Per Capita Consumption	$x_1^h = .189\ x_2^h = .5$	$x_1^f = .274\ x_2^f = .726$	$x_1 = y_1\ x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = .090$	$B(\pi, \pi) = .034$
Gross expected utility	.307	.446	.321
Expected utility net of inv. cost	.308	.427	.319
Expected utility if invest	$.307 - c$	$.487 - c$	$.346 - c$
Expected utility if don't invest	.307	.397	.313
Wages	$w_g^h = 1,\ w_b^h = 1$	$w_g^f = 1.875,\ w_b^f = 1$	$w_g = 1.364,\ w_b = 1$
Expected Wage	1	1.452	1.154
Prices	$p_1 = 2.648$		$p_1 = 3.216$

Table 1: Trade and autarky equilibria in Example 1

Next, we illustrate some welfare properties of the equilibria with trade.

Example 1: specialization may be beneficial only to the rich country

Table ?? displays a parameterization where all country f citizens are better off in the asymmetric trade equilibrium than in the unique autarky equilibrium, and where all country h citizens are worse off in the asymmetric trade equilibrium than under autarky.[26]

Notice that the total world output of both goods is higher in the asymmetric equilibrium (see the second row of the table). While prohibitive trade barriers would make country h better off, it is also true that there exists transfer payments from f to h that can make both countries better off relative to the autarky equilibrium. Hence there are some productive gains from specialization despite the countries being fundamentally identical.

equilibrium is $\pi = .0067$. The equilibrium where $\pi^f = \pi^h = 0.067$ is unstable under trade, while an asymmetric equilibrium with $\pi^f = .0283$, $\pi^h = 0$ is stable.

[26] Although some agents change their investment behavior in the comparison across equilibria, this does not complicate Pareto comparisons. The crucial fact is that (in the example) both qualified and unqualified workers gain (lose) in country f (h). All workers in the rich country have the option to invest as in the autarky equilibrium, so revealed preferences imply that all workers gain. Similarly, in the poor country all workers have the option to invest as in the trade equilibrium when in autarky, so again, by revealed preferences, all workers are better off in autarky.

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}, c \sim U[.04, .24]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = 0$	$\pi^f = .353$	$\pi = 0$
Production	$y_1^h = 0,\ y_2^h = 1$	$y_1^f = .284,\ y_2^f = .323$	$y_1 = 0,\ y_2 = 1$
Consumption	$x_1^h = .107,\ x_2^h = .5$	$x_1^f = .177,\ x_2^f = .823$	$x_1 = y_1\ x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = .111$	$B(\pi, \pi) = 0$
Gross average utility	.232	.381	0
Avg. utility net of inv. cost	.232	.355	0
Expected utility if invest	$.232 - c$	$.452 - c$	n/a
Expected utility if don't invest	.232	.342	0
Wages	$w_g^h = 1,\ w_b^h = 1$	$w_g^f = 2.433,\ w_b^h = 1$	$w_g = n/a,\ w_b = 1$
Expected Wage	1	1.647	1
Prices	$p_1 = 4.660$		p_1 =n/a

Table 2: Trade and autarky equilibria in Example 2

Example 2: specialization may make both countries better off

In this example trade makes both countries better off. For maximal simplicity we rig this example so that the "free rider problem" in human capital investments is so severe that the unique equilibrium under autarky is the trivial equilibrium. However, with trade, the existence of the other country means that, for any investment π^f in country f, the price of good 1 is higher than without trade if there is no human capital investment in country h. Hence, trade allows a new market to emerge that would not operate without trade.

Table **??** summarizes one example where the market for good 1 only operates with international trade. There are multiple trade equilibria and the numbers in the table refer to the one with the largest fraction of investors in the country producing good 1.[27]

Consumers are happier when consuming both goods than when consuming only one good. Because a new market opens up, trade is beneficial for both countries.

Pareto Improving Inequality

The example presented above is extreme, but specialization through trade may more generally be viewed as an imperfect "solution" to the informational problem.[28] In the example,

[27] $(\pi^h, \pi^f) = (0, 0.0157)$, is also an equilibrium, but unlike the equilibrium in Table **??** it is unstable.
[28] For a detailed elaboration on this point in the context of discrimination, see **?**.

there is no way for a market to open unless the rewards for getting into the market are large enough. These rewards are bigger if only one country enters the market: the same "kick" from the local informational externality is generated at a smaller negative price effect. Specialization reduces the problem of under investment in human capital.

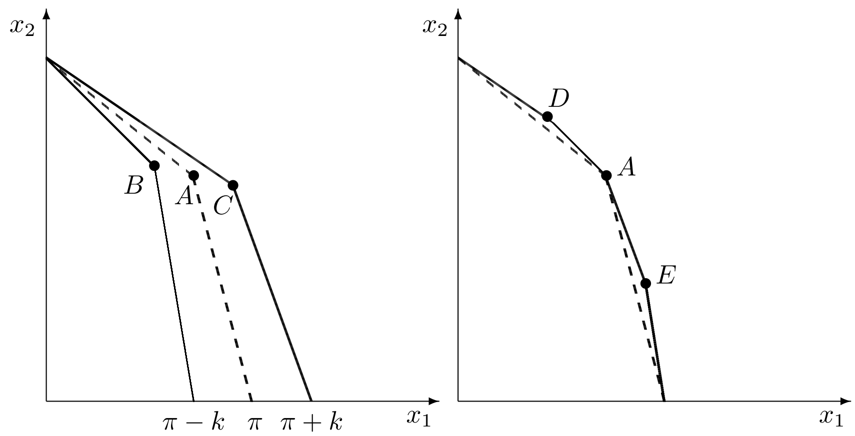

Figure 9: Specialization expands the world production possibilities

Even in less extreme cases, both countries may gain from specializing: it is *always* true that the production possibilities set expands when moving from a situation where both countries invest at the same rate to an asymmetric investment profile for a constant total quantity of investors in the world. Figure **??** assumes countries of equal size. In the left panel, the dashed line represents the frontier when both countries invest at π, whereas the continous lines (with kinks at B and C) illustrate the frontiers in each country at an asymmetric investment profile, but with the same aggregate investment.

On the right panel the continuous line (with kinks at D, A and E) is the world production possibilities frontier under the same asymmetric investment profile assumed in the left panel. The dashed line reproduces the dashed line from the left panel. The total number of investors in the world is unchanged, but the world production possibilities set is larger when countries specialize. To understand, note that the efficient way of increasing x_1 starting from the vertical intercept is to first use good-signal workers from the country with higher investment, so initially the slope of the world production possibilities set must be the same as the set to the left with kink at C. The graph is drawn for the case where it is better to use high-signal workers from the low investment country than low signal workers from the high investment country in Sector 1, but the result is fully general.

The Irrelevance of Size

Changes of the relative size of the countries will in general affect the *asymmetric* equilibria due to price effects. The nature of such changes depends on the parameterization.

To illustrate that size does not confer special advantage as it does in agglomeration models, we construct examples showing that scale effects may go either way. One way is to look at the extreme case where λ^h is near zero (see Appendix **??** for details on the computation). In this case the foreign (big) country operates as in autarky. In the (small) home country instead, price effects are absent, because world price is only determined by investment in the the foreign country. The examples are computed by setting $\lambda^h = 1$

Figure **??** panel (A) was computed using $\alpha = 1/2$, $\eta = .97$, $\underline{c} = -0.005$, $\bar{c} = .095$ to illustrate one case where only the big country can be rich. There is a unique symmetric equilibrium at $\pi^A = 0.48$. At the autarky equilibrium, incentives to invest in human capital are decreasing in π in the large country. In the small country instead, additional investment does not have adverse price effects, and incentives B^O increase with π, but not at a fast enough rate: the best response for the small country $G(B^O(\pi, p = p^A))$ intersects the 45^0 line only below π^A. Therefore there are two asymmetric equilibria where the big country invests at $\pi^h = \pi^A = 0.48$ and the other at $\pi^f = 0.05$ or 0.29, both less than π^A.

In the example of Figure **??** panel (B) instead, the small country can be either richer or poorer than the big country. The figure was computed with the parameters as in Numerical Example 1 (except for country sizes). Workers' investment in the small country is more responsive than in the previous example at the autarky equilibrium, where the best response intersects the 45^0 line from below. Both $\left(\pi^h, \pi^f\right) = (.27, 0.1)$ and $\left(\pi^h, \pi^f\right) = (.27, 0.59)$ are equilibria. Note that the responsiveness of the best-response function to higher investment, which determines the location of the fixed points for the small country above and below π^A also depends on the shape of cost of investment distribution, therefore, by changing the

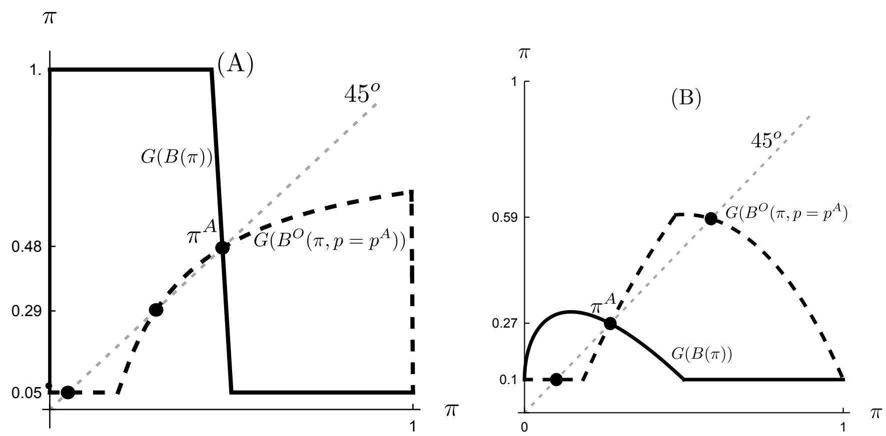

Figure 10: Equilibrium fixed point maps: large (solid line) and small (dashed line) country

shape of the cost distribution one can easily construct examples where *differences* between countries are large or small regardless of country size.

Finally, when the unique autarky equilibrium is at $\pi^A = 0$, if the large country is large enough only the small country can be richer. For example, if Example 2 above is extended to allow for different country sizes, the country must fully specialize in the low-tech industry if its size exceeds a critical value. Reducing the size of the country from 1/2 on the other hand only improves incentives. Hence, there are circumstances where *the only asymmetric equilibrium* is that the small country becomes rich.

Taken together, these examples show that there may be scale effects in favor of either the larger or the smaller economy, and that sometimes the equilibrium selection matters. However, these are not really "country-scale-effects". Instead, we prefer to think of them as scale effects that depend on relative size of the North to the South. To understand, suppose that there are n countries indexed by $j \in \{1, ..., n\}$, of size λ^j. Consider an equilibrium in this model where the set of countries is partitioned into the sets P and R and where $\pi^j = \pi^p$ for all $j \in P$ and $\pi^j = \pi^r$ for all $j \in R$. Finally let $\lambda^p = \sum_{j \in P} \lambda^j$ and $\lambda^r = \sum_{j \in R} \lambda^j$. This is an equilibrium if and only if (π^p, π^r) is an equilibrium in the two-country model with countries of sizes (λ^p, λ^r). There may of course be other equilibria as well, but at least for this form of equilibrium the size of the *individual country* is irrelevant and the relevant scale effect can be interpreted in our preferred manner.

A "development miracle" can be interpreted as a country that re-coordinates from being part of the developing world to being part of the developed world. The model cannot explain how such a re-coordination is achieved, but, if the economy is small, the effects on the rest of the world are negligible. In contrast, a simultaneous re-coordination of a significant fraction of the "South" leads to large relative price changes so that it is not worth the while as long as there is no change in the "North". Obviously, the model is too stylized for policy recommendations, but this nevertheless suggests that it may be misguided to use a few small successful countries as a model for all developing countries.

VII Concluding Remarks

We show that endogenous comparative advantages are possible between identical countries in an essentially neoclassical model. Specialization and trade arise due to an informational externality: workers are better informed than firms about their abilities.

Two-country model equilibria can be reinterpreted as n-country model equilibria where countries cluster in two groups in terms of level of development. Equilibria of the n-country model are neutral with respect to individual country sizes, so the model is consistent with a world with no particular relationship between size and development.

A natural extension is to introduce physical capital into the production technology. This would be interesting for analyzing the role of foreign capital and capital flight from poor countries. As this paper focuses on the effects asymmetric information about skills we have chosen to ignore physical capital. However, if capital and human capital were complementary in production, the effects analyzed in this paper would be reinforced.

To understand, suppose initially that capital cannot flow between countries. Except for a capital market equilibrium condition the model is more or less the same as the one without capital. Consider an asymmetric equilibrium under the assumption that initial capital endowments are identical. As capital is more useful in the high-tech industry the return on capital is higher in the rich country, so, with free capital mobility, the rich country must have a higher per capita level of capital. Notice that the movement of capital from the poor to the rich country affects incentives to invest positively in the rich country and negatively in the poor country, strengthening the incentives to specialize.[29]

Because this is a static model, we do not analyze incentives to migrate. Workers with good signals in poor countries may find it advantageous to migrate where their skills receive better rewards. However, such incentives are mitigated if employers recognize the workers' country of origin. When a foreign employer forms beliefs about a home country worker's productivity, she may take into account the worker's nationality, therefore the expected productivity computed by foreign and home country employers is the same. Then, incentives to acquire human capital are defined by citizenship, not residence.[30]

A Appendix: Proof of proposition ??.

(Part 1) Consider an arbitrary equilibrium. Let $x^* = (x_1^*, x_2^*)$ denote the world production, where $x_i^* = \lambda^h x_i^{h*} + \lambda^f x_i^{f*}$ and x_i^{j*} denotes the production of good i in country j in equilibrium. Also let $l_i^{j*}(\theta)$ denote the corresponding input of workers with signal g in economy j and sector i. By profit maximization $p_i^* x_i^{j*} - \sum_{\theta\in g,b} w_\theta^{j*} l_i^{j*}(\theta) \geq p_i^* x_i^{j\prime} - \sum_{\theta\in g,b} w_\theta^{j*} l_i^{j\prime}(\theta)$ for any alternative plan $(x_i^{j\prime}, l_i^{j\prime}(\cdot))$. Adding over the two sectors and imposing the market clearing conditions on the labor market we conclude that $\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*}(\eta\pi^j + (1-\eta)(1-\pi^j)) - w_b^{j*}((1-\eta)\pi^j + \eta(1-\pi^j)) \geq \sum_{i=1,2} p_i^* x_i^{j\prime} - w_g^{j*}\left(l_1^j(g) + l_2^j(g)\right) - w_b^{j*}\left(l_1^j(b) + l_2^j(b)\right)$ for all possible alternative production plans (feasible as well as non-feasible in the aggregate). Now for any feasible alternative allocation $l_1^j(g) + l_2^j(g) \leq \eta\pi^j + (1-\eta)(1-\pi^j)$ and $l_1^j(b) + l_2^j(b) \leq (1-\eta)\pi^j + \eta(1-\pi^j)$, implying that $\sum_{i=1,2} p_i^* x_i^{j*} \geq \sum_{i=1,2} p_i^* x_i^{j\prime}$ for any feasible alternative $(x_1^{j\prime}, x_2^{j\prime})$. Since this must hold in each country we conclude that $p^* x^* \geq p^* x'$ for any alternative feasible world production vector $x' = (x_1', x_2')$. Moreover, in order for

[29] Details are available on request from the authors.

[30] However, employers may also condition their beliefs on migration status, which is as easily recognizable as citizenship. This raises the possibility that migrants acquire a reputation for higher investment than their fellow citizens who did not migrate, and that this belief is confirmed in equilibrium.

(x_1^{j*}, x_2^{j*}) to be profit maximizing it must be that $\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*} \left(\eta\pi^j + (1-\eta)(1-\pi^j)\right) +$ $w_b^{j*} \left((1-\eta)\pi^j + \eta(1-\pi^j)\right) = 0$. Finally, since u is homothetic, from standard arguments we have that if $(x_1^{j*}(w), x_2^{j*}(w))$ maximizes utility of a worker with income w, then $(\frac{w'}{w} x_1^{j*}(w), \frac{w'}{w} x_2^{j*}(w))$ maximizes utility of a worker with income w'. Consider the program

$$\max_{x_1, x_2} u(x_1, x_2) \tag{A1}$$

$$\text{s.t} \qquad p_1^* x_1 + p_2^* x_2 \leq p_1^* x_1^* + p_2^* x_2^* = p_1^* \sum_{j=h,f} \lambda^j x_1^{j*} + p_2^* \sum_{j=h,f} \lambda^j x_2^{j*},$$

where the star-superscript refers to equilibrium variables. The aggregate consumption bundle of any equilibrium must solve (**??**) because the problem gets the relative consumptions of x_1 and x_2 right and $p_1^* x_1^* + p_2^* x_2^*$ is the aggregate world income. We conclude that if x^* is an equilibrium world consumption plan it must solve (**??**). Since the set $X^W(\pi^h, \pi^f)$ is contained in the "budget set" of the representative consumer and $x^* \in X^W(\pi^h, \pi^f)$ it follows that x^* must be a solution to (**??**).

(Part 2) Let x^* solve (**??**) and let $V = \left\{x \in R_+^2 | u(x) > u(x^*)\right\}$. Quasi-concavity implies that V is a convex set. The set $X^W(\pi^h, \pi^f)$ is also convex (see Page **??**). Moreover, $V \cap X^W(\pi^h, \pi^f) = \emptyset$, so the separating hyperplane theorem (Theorem 11.3. in **?**) implies that there exists some p^* such that $p^* x \geq p^* x^*$ for all $x \in V$ and $p^* x \leq p^* x^*$ for every $x \in X^W(\pi^h, \pi^f)$. Let the wages be given by $w_g^{j*} = \max\{p_1^* \mu(g, \pi^j), p_2^*\}$ and $w_b^{j*} = \max\{p_1^* \mu(b, \pi^j), p_2^*\}$, and let the allocation of workers be as in the planning solution. Observe in particular that if $p_1^* \mu(\theta, \pi^j) > p_2^*$, then no worker with signal θ is employed in Sector 2 in the allocation that produces x^*. This is most easily seen in the differentiable case, where the optimality condition to (**??**) implies that $\frac{\partial u(x^*)}{\partial x_1^*} / \frac{\partial u(x^*)}{\partial x_2^*} = \frac{p_1^*}{p_2^*} > \frac{1}{\mu(g, \pi^j)}$. But, $\frac{1}{\mu(\theta, \pi^j)}$ is the cost of producing an extra unit of good 1 by giving up some country j workers with good signal currently in production of good 2, so we conclude that as if representative consumer would be better off if some of these workers would be switched to the production of good 1, contradicting optimality of x^* if $p_1^* \mu(\theta, \pi^j) > p_2^*$ and some of the j workers are assigned to Sector 2. A symmetric argument holds for when the inequality is reversed. Hence, if $l_1^{*j}(\theta) > 0$, then $p_1^* \mu(\theta, \pi^j) = \max\{p_1^* \mu(\theta, \pi^j), p_2^*\} = w_\theta^{j*}$, implying that the profit from hiring any quantity workers with signal θ is zero in Sector 1, whereas if $l_1^{*j}(\theta) = 0$, then $p_1^* \mu(\theta, \pi^j) \leq \max\{p_1^* \mu(\theta, \pi^j), p_2^*\} = w_\theta^{j*}$, so no gain can be earned from hiring a positive quantity. The argument for Sector 2 is symmetric, which leads us to conclude that the outputs and (implicit) allocation of workers in the solution to (**??**) are consistent with profit maximizing behavior given the prices and wages constructed. ■

B Appendix: Equilibria when One Country is a Small Open Economy

Equilibria can be calculated by solving two separate (different) one-dimensional fixed point problems. Consider the incentives to invest in a country with fraction of investors π under the "small open economy" assumption that the price (of good 1) is fixed at p. Equilibrium wages in the small open economy are determined to generate zero profits: $w_g^O = \max\{p\mu(g, \pi), 1\}$ and $w_b^O = \max\{p\mu(b, \pi), 1\}$. The incentive to invest in the small open economy, denoted $B^O(\pi; p)$, is thus (using (**??**)),

$$B^O(\pi; p) = \frac{(2\eta - 1)\alpha^\alpha (1-\alpha)^{1-\alpha}}{p^\alpha} \max\{\max\{p\mu(g, \pi), 1\} - \max\{p\mu(b, \pi), 1\}, 0\}. \tag{B2}$$

If p^A is well defined (i.e., whenever the autarky equilibrium is non-trivial), then

$$\begin{aligned} B^h(\pi^h, \pi^A) &\to B^O(\pi^h, p = p^A) \text{ for all } \pi^h \in [0,1] \text{ as } \lambda^h \to 0 \qquad \text{(B3)} \\ B^f(\pi^h, \pi^A) &\to B(\pi^f) \text{ for all } \pi^f \in [0,1] \text{ as } \lambda^h \to 0, \end{aligned}$$

Assume parameters imply a unique autarky equilibrium, and call it π^A. Let p^A denote the associated autarky price. If $\pi = \pi^A$, then $B^O(\pi^A; p = p^A) = B(\pi^A)$, and π^A solves

$$\pi = G(B^O(\pi; p = p^A)). \tag{B4}$$

While both (**??**) and the autarky fixed point equation have π^A as a common solution, incentives diverge for other values of π since in autarky the price changes as π changes whereas there are no such price effects in (**??**). Equation (**??**) will therefore in many cases have solutions different from π^A. Now, if π^O solves (**??**) and if $\frac{d}{d\pi}\big|_{\pi=\pi^A} [\pi - G(B(\pi))] \neq 0$ and

$\frac{d}{d\pi}\Big|_{\pi=\pi^O}\left[\pi - G\left(B^O\left(\pi; p = p^A\right)\right)\right] \neq 0$, then, for λ^h small enough, there exists an equilibrium $\left(\pi^{h*}, \pi^{f*}\right)$ in the trade model near $\left(\pi^O, \pi^A\right)$.[31]

[31]The slope condition for the autarky equilibrium is satisfied under the conditions that guarantee the equilibrium uniqueness. Its role is that if the equilibrium was at a tangency with the 45^0 line, the slightest effect from abroad could eliminate the equilibrium.