"Endogenous Comparative Advantage"

Web Appendices

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This documents contains algebraic derivations that for the sake of brevity have been omitted from our paper "Endogenous Comparative Advantage". To make the derivations self-contained some equations have been duplicated from the main paper.

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B Derivation of Autarky Equilibria in Section 4.1

Here we provide the details of the equilibrium characterization under autarky in the parametrized version of the model discussed in Section 4.1.

B.1 Equilibria of Type A

This type of equilibrium has all b workers are allocated to the simple-sector and g workers are allocated to the high tech sector. The associated outputs are thus

$$\begin{aligned} x_1 &= c = \pi\eta \\ x_2 &= s = \pi(1-\eta) + (1-\pi)\,\eta, \end{aligned} \tag{B1}$$

and for the goods market clearing conditions to hold with individual demands given in Page 12 in the main paper it follows that

$$\frac{x_1}{x_2} = \frac{\alpha}{(1-\alpha)}\frac{1}{p_1}, \tag{B2}$$

so by combining (B1) and (B2) we get that the equilibrium price in this type of equilibrium must satisfy the expression for the price in the main paper:

$$p_1 = \frac{\alpha}{1-\alpha} \cdot \frac{\pi(1-\eta) + (1-\pi)\,\eta}{\pi\eta}. \tag{B3}$$

and for firms to earn zero profit the wages must be:

$$\begin{aligned} w_b &= 1 \\ w_g &= p_1 \underbrace{\frac{\pi\eta}{\pi\eta + (1-\pi)(1-\eta)}}_{\mu(g,\pi)}. \end{aligned} \tag{B4}$$

Hence all prices and quantities are determined under the assumption that the hypothesized allocation of workers constitutes an equilibrium. What remains to be checked is that there are no incentives to reallocate any workers given these prices and wages. For there to be no incentives to reallocate workers with signal g to sector 2 it must be that

$$\begin{aligned} p_1 \frac{\pi\eta}{\pi\eta + (1-\pi)(1-\eta)} &\underset{/(\text{B3})/}{=} \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta) + (1-\pi)\,\eta}{\pi\eta}\frac{\pi\eta}{\pi\eta + (1-\pi)(1-\eta)} \\ = \quad & \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta) + (1-\pi)\,\eta}{\pi\eta + (1-\pi)(1-\eta)} \geq 1 \Leftrightarrow \alpha\left(\pi(1-\eta) + (1-\pi)\,\eta\right) \geq (1-\alpha)\left(\pi\eta + (1-\pi)(1-\eta)\right) \\ \Leftrightarrow \quad & \alpha\left(\pi(1-\eta) + (1-\pi)\,\eta + \pi\eta + (1-\pi)(1-\eta)\right) = \alpha \geq \left(\pi\eta + (1-\pi)(1-\eta)\right) \\ \Leftrightarrow \quad & \pi\,(2\eta - 1) \leq \alpha + \eta - 1 \Leftrightarrow \pi \leq \frac{\alpha + \eta - 1}{2\eta - 1} \text{ (which can only hold if } \alpha \geq 1-\eta) \end{aligned} \tag{B5}$$

Finally, for there to be no incentives to reallocate workers with signal b to sector 1 it must be that

$$
\begin{aligned}
w_b &= 1 \geq p_1 \underbrace{\frac{\pi(1-\eta)}{\pi(1-\eta)+(1-\pi)\eta}}_{P(b,\pi)} = \\
&= \frac{\alpha}{1-\alpha} \frac{\pi(1-\eta)+(1-\pi)\eta}{\pi\eta} \frac{\pi(1-\eta)}{\pi(1-\eta)+(1-\pi)\eta} = \\
&= \frac{\alpha}{1-\alpha} \frac{1-\eta}{\eta} \Leftrightarrow \alpha \leq \eta,
\end{aligned}
\tag{B6}
$$

which completes the derivation summarized on page 4.1 in the main text.

B.2 Equilibria of Type B

In this type of equilibrium, a fraction $\gamma \in (0,1)$ of g workers are in sector 1 and all b workers and the remaining g workers are allocated sector 2. In this case firms in sector 2 must be indifferent between which type of worker to hire, so $w_g = w_b = 1$. Firms in sector 1 must make zero profits, that is

$$
w_g = 1 = p_1 \frac{\pi\eta}{\eta\pi+(1-\eta)(1-\pi)} \Leftrightarrow p_1 = \frac{\eta\pi+(1-\eta)(1-\pi)}{\eta\pi}, \tag{B7}
$$

so the equilibrium price of good 1 is determined from this indifference condition. The associated outputs are

$$
\begin{aligned}
x_1 &= \gamma\eta\pi \\
x_2 &= (1-\gamma)(\eta\pi+(1-\eta)(1-\pi)) + (1-\eta)\pi + \eta(1-\pi),
\end{aligned}
\tag{B8}
$$

and by substituting (B8) into (B2) we find that goods market clearing requires that there is some $\gamma \in (0,1)$ such that

$$
\begin{aligned}
p_1 &= \frac{\eta\pi+(1-\eta)(1-\pi)}{\eta\pi} \\
&= \frac{\alpha}{1-\alpha} \frac{(1-\gamma)(\eta\pi+(1-\eta)(1-\pi)) + (1-\eta)\pi + \eta(1-\pi)}{\gamma\eta\pi} \Leftrightarrow \\
(1-\alpha)\gamma(\eta\pi+(1-\eta)(1-\pi)) &= \alpha((1-\gamma)(\eta\pi+(1-\eta)(1-\pi)) + (1-\eta)\pi + \eta(1-\pi)) \Leftrightarrow \\
\gamma(\eta\pi+(1-\eta)(1-\pi)) &= \alpha \Leftrightarrow \gamma = \frac{\alpha}{(\eta\pi+(1-\eta)(1-\pi))}
\end{aligned}
\tag{B9}
$$

Since workers with signal g are equally valuable in sector 1 and sector 2 the condition that there are no incentives to reallocate workers with signal b to sector 1 is automatically satisfied. Hence it only remains to check that γ derived in (B9) is a number on $(0,1)$. Obviously $\gamma > 0$, so we need to check that

$$
\begin{aligned}
\gamma &= \frac{\alpha}{(\eta\pi + (1-\eta)(1-\pi))} < 1 \Leftrightarrow \alpha < (\eta\pi + (1-\eta)(1-\pi)) \\
&\Leftrightarrow \alpha + \eta - 1 < \pi(2\eta - 1) \Leftrightarrow \pi > \frac{\alpha + \eta - 1}{2\eta - 1}.
\end{aligned}
\tag{B10}
$$

For $\alpha < \eta - 1$ this is satisfied for all $\pi \in [0,1]$, for $\alpha > \eta$ this is never satisfied, while for $1-\eta \leq \alpha \leq \eta$ this is a condition that says that for π above a threshold (given by $\frac{\alpha+\eta-1}{2\eta-1}$) the equilibrium is of this type.

B.3 Equilibria of Type C

Now, a fraction β of b workers and all g workers are allocated to the high tech sector, and the remaining b workers are allocated to the simple sector. This implies that the equilibrium wages must be

$$
\begin{aligned}
w_g &= p_1 \frac{\pi\eta}{\pi\eta + (1-\pi)(1-\eta)} \\
w_b &= p_1 \frac{\pi(1-\eta)}{\pi(1-\eta) + (1-\pi)\eta} = 1,
\end{aligned}
\tag{B11}
$$

and the second equation nails down the candidate equilibrium price as

$$
p_1 = \frac{\pi(1-\eta) + (1-\pi)\eta}{\pi(1-\eta)}. \tag{B12}
$$

The corresponding outputs are

$$
\begin{aligned}
x_1 &= \pi\eta + \pi(1-\eta)\beta \\
x_2 &= (\pi(1-\eta) + (1-\pi)\eta)(1-\beta)
\end{aligned}
\tag{B13}
$$

It follows directly from (B11) that $w_g > 1$, so there are no incentives to reallocate workers. The one condition that remains to be checked is that there exists $\beta \in (0,1)$ such that the goods market clears.. Substituting from (B13) into (B2) and using (B12) we get that this requires that

$$
\begin{aligned}
p_1 &= \frac{\pi(1-\eta) + (1-\pi)\eta}{\pi(1-\eta)} = \frac{\alpha}{1-\alpha} \frac{(\pi(1-\eta) + (1-\pi)\eta)(1-\beta)}{\pi\eta + \pi(1-\eta)\beta} \Leftrightarrow \\
\frac{1}{(1-\eta)} &= \frac{\alpha(1-\beta)}{(1-\alpha)(\eta + (1-\eta)\beta)} \Leftrightarrow (1-\alpha)(\eta + (1-\eta)\beta) = \alpha(1-\beta)(1-\eta) \Leftrightarrow \\
\beta(1-\eta) &= \alpha(1-\eta) - (1-\alpha)\eta \Leftrightarrow \beta = \frac{\alpha - \eta}{1-\eta}.
\end{aligned}
\tag{B14}
$$

Hence, this type of equilibrium exists if and only if $\alpha > \eta$.

C Derivation of Benefits from Investment Under Autarky (Model in Section 4.2)

Here we provide the derivations of the expression (22) in the main document.

In terms of p_1, w_g and w_b the gross incentives to invest are (expression (17) in main text)

$$E\left(v\left(w,p\right)|\text{inv}\right)-E\left(v\left(w,p\right)|\text{ don't inv}\right)=\frac{(2\eta-1)(w_g-w_b)}{(p_1)^{\alpha}}\alpha^{\alpha}(1-\alpha)^{1-\alpha} \tag{C1}$$

and since we can solve for p_1, w_g and w_b in terms of π and the parameters of the model we can derive a closed form expression for the gross benefits of investment as a function of π.

C.1 When $\alpha \leq \eta$

If $\alpha \leq \eta$ we showed that equilibria must be of type A or type B. When $\pi > \frac{\alpha+\eta-1}{2\eta-1}$ we then showed that the equilibrium must be of type B, implying that $w_g = w_b = 1$, so the benefits to invest are zero in this case. Hence

$$B\left(\pi\right)=0 \text{ for all } \pi>\frac{\alpha+\eta-1}{2\eta-1} \tag{C2}$$

Hence, it remains to derive $B(\pi)$ for $\pi \leq \frac{\alpha+\eta-1}{2\eta-1}$ in which case the equilibrium is of type A and w_g and w_b are given by the expressions in (B4) and p_1 by (B3). From (B4) we have that

$$\begin{aligned}
w_g-w_b &= p_1\frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)}-1=\frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\,\eta}{\pi\eta}\frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)}-1 \qquad \text{(C3)}\\
&= \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\,\eta}{\pi\eta+(1-\pi)(1-\eta)}-1\\
&= \frac{1}{1-\alpha}\frac{\alpha\left(\pi(1-\eta)+(1-\pi)\,\eta\right)-(1-\alpha)\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}\\
&= \frac{1}{1-\alpha}\frac{\alpha\left(\pi(1-\eta)+(1-\pi)\,\eta+\pi\eta+(1-\pi)\,(1-\eta)\right)\right)-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}\\
&= \frac{1}{1-\alpha}\frac{\alpha-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}
\end{aligned}$$

and from (B3) we have that

$$\frac{1}{p_1^{\alpha}}=\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\,\eta}\right)^{\alpha}, \tag{C4}$$

so substituting into (C1) we get

$$\begin{aligned}
B\left(\pi\right) &= (2\eta-1)\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\,\eta}\right)^{\alpha}\left(\frac{1}{1-\alpha}\frac{\alpha-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}\right)\alpha^{\alpha}(1-\alpha)^{1-\alpha} \qquad \text{(C5)}\\
&= (2\eta-1)\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\,\eta}\right)^{\alpha}\left(\frac{\alpha-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}\right),
\end{aligned}$$

so combing (C5) with (C2) (and observing that the right hand side of (C5) is negative if $\pi > \frac{\alpha+\eta-1}{2\eta-1}$) we get (22) in the main text.

C.2 When $\alpha > \eta$

In this case the equilibrium is of type C and from (B11) we get that

$$\begin{aligned} w_g - w_b &= \frac{\pi(1-\eta)+(1-\pi)\eta}{\pi(1-\eta)} \frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 \\ &= \frac{\pi(1-\eta)+(1-\pi)\,\eta}{1-\eta} \frac{\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 \\ &= \frac{\pi+\eta-2\pi\eta}{1-(\pi+\eta-2\pi\eta)} \frac{\eta}{1-\eta} - 1, \end{aligned} \tag{C6}$$

and (B12) implies that

$$\frac{1}{p_1^{\alpha}} = \left(\frac{\pi\,(1-\eta)}{\pi(1-\eta)+(1-\pi)\,\eta} \right)^{\alpha}. \tag{C7}$$

Substituting (C6) and (C7) into (C1) we obtain

$$B\,(\pi) = (2\eta-1) \left(\frac{\pi\,(1-\eta)}{\pi(1-\eta)+(1-\pi)\,\eta} \right)^{\alpha} \left(\frac{\pi+\eta-2\pi\eta}{1-(\pi+\eta-2\pi\eta)} \frac{\eta}{1-\eta} - 1 \right) \alpha^{\alpha}(1-\alpha)^{1-\alpha}$$

D Characterization of trade equilibria

In this section we derive the equilibrium characterization and the ranges for different forms of equilibria for the model with international trade

With two countries, the number of potential forms of continuation equilibria now swells to 9: in each country the allocation of workers may be like in any of the three types of autarky equilibria (however, mixing in both countries is a knife-edge possibility). To reduce the number of cases we therefore set $\eta = 2/3$, $\alpha = 1/2$, and $\lambda^h = \lambda^f = 1/2$ in the analysis that follows. With these parameter values the continuation equilibrium can be of three different forms. If countries are labeled so that $\pi^h \leq \pi^f$ the possibilities are:

Type A^T Equilibria (according to signals in both countries). This is the obvious analogue to equilibria of Type A in the autarky model. If investments in both countries are close to the value when this occurs in autarky, the continuation equilibrium is of this form.

Type	A^T	B^T	C^T
$p\left(\pi^h,\pi^f\right)$	$\frac{4-\pi^f-\pi^h}{2(\pi^f+\pi^h)}$	$\frac{1+\pi^h}{2\pi^h}$	$\frac{2-\pi^f}{\pi^f}$
$w_g^h\left(\pi^h,\pi^f\right)$	$p\left(\pi^h,\pi^f\right)\frac{2\pi^h}{1+\pi^h}$	1	1
$w_b^h\left(\pi^h,\pi^f\right)$	1	1	1
$w_g^f\left(\pi^h,\pi^f\right)$	$p\left(\pi^h,\pi^f\right)\frac{2\pi^f}{1+\pi^f}$	$p\left(\pi^h,\pi^f\right)\frac{2\pi^f}{1+\pi^f}$	$p\left(\pi^h,\pi^f\right)\frac{2\pi^f}{1+\pi^f}$
$w_b^h\left(\pi^h,\pi^f\right)$	1	1	1
Exists when	$\pi^h \leq \pi^f \leq \frac{\pi^h(3-2\pi^h)}{1+2\pi^h}$	$\frac{\pi^h(3-2\pi^h)}{1+2\pi^h} \leq \pi^f \leq \frac{4\pi^h}{1+3\pi^h}$	$\pi^f \geq \frac{4\pi^h}{1+3\pi^h}$

Table 1: Continuation equilibria under international trade

Type B^T Equilibria (according to signals in f, mixing of good signals in h). In analogy with Type B equilibria in autarky, the equilibrium price is then determined from an indifference condition in the allocation of workers with signal g in country h.

Type C^T Equilibria (mixing of bad signals in f, all in low skill sector in h). This is just like a Type C equilibria in autarky, with some exogenous extra output of the low-tech good.

The characterization of the relevant continuation equilibria is summarized in Table 1. It is understood that $\pi^h \leq \pi^f$, so Table 1 does provide a unique continuation equilibrium for any possible $(\pi) \neq (0,0)$ by reversing the roles of the countries when necessary. Figure 1 shows the regions of investment behavior where each type of equilibrium occurs.

D.1 Conditions that must hold in any equilibrium.

For the same reason as under autarky, market clearing and optimality on the behalf of consumers imply that (recall that $\alpha = 1/2$)

$$p_1 = \frac{x_2}{x_1} \tag{D1}$$

In addition, there will always be some workers producing the low tech good, so we know immediately that $w_b^h = 1$ in any equilibrium since we have labeled the countries so that the workers that have the lowest probability of being productive in the high tech sector are the workers with bad signals in country h.

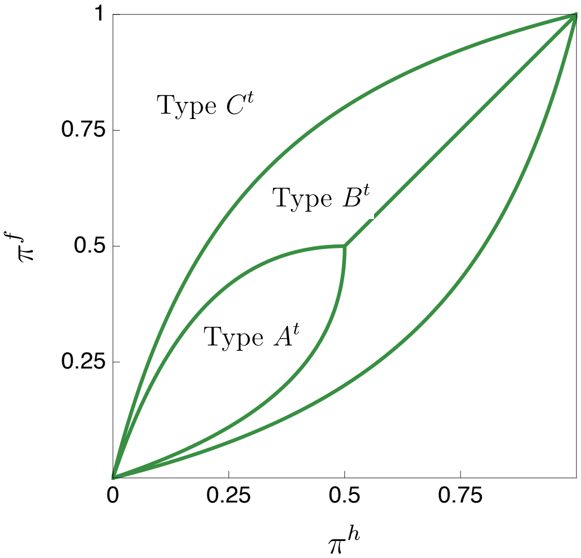

Figure 1: Types of asymmetric equilibria, with $\eta = 2/3, \alpha = 1/2$

D.2 Equilibria of type $\mathbf{A}^t$

In both countries g workers are employed in sector 1 and b workers in sector 2 ("according to signals" in both countries). For this to be an equilibrium we have that

$$\begin{aligned} w_b^f &= w_b^h = 1 \\ w_g^f &= p_1 \frac{\eta \pi^f}{\eta \pi^f + (1-\eta)\pi^f} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^f}{1+\pi^f} \\ w_g^h &= p_1 \frac{\eta \pi^h}{\eta \pi^h + (1-\eta)\pi^h} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^h}{1+\pi^h} \end{aligned} \tag{D2}$$

and outputs are given by

$$x_1 = \frac{2}{3}\left(\pi^f + \pi^h\right) \tag{D3}$$

$$\begin{aligned} x_2 &= \frac{1}{3}\pi^f + \frac{2}{3}\left(1-\pi^f\right) + \frac{1}{3}\pi^h + \frac{2}{3}\left(1-\pi^h\right) \\ &= \frac{2-\pi^f+2-\pi^h}{3} \end{aligned} \tag{D4}$$

so to satisfy (D1) p_1 must solve

$$p_1 = \frac{4-\pi^f-\pi^h}{2\left(\pi^f+\pi^h\right)} \tag{D5}$$

Since $\pi^h \leq \pi^f$ by labeling of the countries the two relevant conditions to check are that there are no incentives to use workers with signal b in the high tech sector in country f and that there are no incentives to use workers with signal g in the low tech sector in country h. The first of these restrictions implies that

$$p_1 \frac{\pi^f}{2-\pi^f} \leq 1 \tag{D6}$$

and the second implies that

$$\begin{aligned}
p_1 \frac{2\pi^h}{1+\pi^h} &= \frac{\left(4-\pi^f-\pi^h\right) 2\pi^h}{2\left(\pi^f+\pi^h\right)\left(1+\pi^h\right)} \geq 1 \Leftrightarrow \\
\left(4-\pi^f-\pi^h\right)\pi^h &\geq \left(\pi^f+\pi^h\right)\left(1+\pi^h\right) \Leftrightarrow \\
\pi^h\left(4-\pi^h-\left(1+\pi^h\right)\right) &= \pi^h\left(3-2\pi^h\right) \geq \pi^f\left(1+2\pi^h\right) \Leftrightarrow \\
\pi^f &\leq \frac{\pi^h\left(3-2\pi^h\right)}{\left(1+2\pi^h\right)}.
\end{aligned} \tag{D7}$$

Finally, to see that (D6) is redundant we note that $\pi^f \geq \pi^h$ implies that

$$\begin{aligned}
p_1 &= \frac{4-\pi^f-\pi^h}{2\left(\pi^f+\pi^h\right)} \leq \frac{4-2\pi^f}{4\pi^f} = \frac{2-\pi^f}{2\pi^f} \Rightarrow \\
p_1 \frac{\pi^f}{2-\pi^f} &\leq \frac{2-\pi^f}{2\pi^f}\frac{\pi^f}{2-\pi^f} = \frac{1}{2} < 1,
\end{aligned} \tag{D8}$$

so (D6) is implies by the other equilibrium conditions.

D.3 Equilibria of type $\mathbf{B}^t$

Now, a fraction γ of g workers in country h and all $g$workers in country f are employed in sector 1 and a fraction $1-\gamma$ of g workers and all b workers in country h and all b workers in country f are employed in sector 2 ("mixing the good" in h and "according to signals" in f). Such an equilibrium must satisfy

$$\begin{aligned}
w_b^f &= w_b^h = 1 \\
w_g^f &= p_1 \frac{\eta\pi^f}{\eta\pi^f+(1-\eta)\pi^f} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^f}{1+\pi^f} \\
1 = w_g^h &= p_1 \frac{\eta\pi^h}{\eta\pi^h+(1-\eta)\pi^h} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^h}{1+\pi^h}
\end{aligned} \tag{D9}$$

and the associated outputs are

$$\begin{aligned}
x_1 &= \frac{2}{3}\pi^f + \gamma\frac{2}{3}\pi^h = \frac{2\left(\pi^f+\gamma\pi^h\right)}{3} \\
x_2 &= \underbrace{\frac{1}{3}\pi^f + \frac{2}{3}\left(1-\pi^f\right)}_{\text{bad signals in } f} + \underbrace{\frac{1}{3}\pi^h + \frac{2}{3}\left(1-\pi^h\right)}_{\text{bad signals in } h} + (1-\gamma)\underbrace{\left(\frac{2}{3}\pi^h + \frac{1}{3}\left(1-\pi^h\right)\right)}_{\text{good signals in } h} \\
&= \frac{2-\pi^f+2-\pi^h+(1-\gamma)\left(1+\pi^h\right)}{3}.
\end{aligned} \tag{D10}$$

Hence, to also satisfy market clearing (D1) $\gamma \in [0,1]$ must satisfy

$$\begin{aligned} p_1 &= \frac{1+\pi^h}{2\pi^h} = \frac{x_2}{x_1} = \frac{2-\pi^f+2-\pi^h+(1-\gamma)\left(1+\pi^h\right)}{2(\pi^f+\gamma\pi^h)} \Leftrightarrow \quad \text{(D11)} \\ (1+\pi^h)(\pi^f+\gamma\pi^h) &= \pi^h\left(2-\pi^f+2-\pi^h+(1-\gamma)\left(1+\pi^h\right)\right) \Leftrightarrow \\ \pi^f\left(1+\pi^h\right)+\gamma\left(\pi^h\left(1+\pi^h\right)\right) &= \pi^h\left(5-\pi^f\right)-\gamma\pi^h(1+\pi^h) \\ \gamma\left(2(1+\pi^h)\pi^h\right) &= \pi^h(5-\pi^f)-\left(1+\pi^h\right)\pi^f = 5\pi^h-\left(1+2\pi^h\right)\pi^f \Leftrightarrow \\ \gamma &= \frac{5\pi^h-\pi^f\left(1+2\pi^h\right)}{2(1+\pi^h)\pi^h} \end{aligned}$$

In order for $\gamma \leq 1$ it must be that

$$\begin{aligned} \frac{5\pi^h-\pi^f\left(1+2\pi^h\right)}{2(1+\pi^h)\pi^h} &\leq 1 \Leftrightarrow 5\pi^h-\pi^f(1+2\pi^h) \leq 2(1+\pi^h)\pi^h \Leftrightarrow \quad \text{(D12)} \\ \pi^f(1+2\pi^h) &\geq 5\pi^h-2(1+\pi^h)\pi^h = \pi^h(3-2\pi^h) \\ \pi^f &\geq \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \end{aligned}$$

In order for $\gamma \geq 0$ it must be that $\pi^f \leq \frac{5\pi^h}{1+2\pi^h}$, but this condition turns out to be redundant. The final condition for equilibrium is that there are no incentives to use workers with bad signals in f in the high tech sector, which implies that

$$p_1 \underbrace{\frac{\pi^f}{2-\pi^f}}_{P(b,\pi^f)} = \frac{1+\pi^h}{2\pi^h}\frac{\pi^f}{2-\pi^f} \leq 1 \Leftrightarrow \frac{2-\pi^f}{\pi^f}\frac{2\pi^h}{(1+\pi^h)} \geq 1 \quad \text{(D13)}$$

but this is just reversing the inequality in (D19), so this holds whenever $\pi^f \leq \frac{4\pi^h}{1+3\pi^h}$ and since

$$\frac{5\pi^h}{1+2\pi^h} - \frac{4\pi^h}{1+3\pi^h} > 0 \quad \text{(D14)}$$

we conclude that the condition for $\gamma \geq 0$ is automatically satisfied whenever there are no incentives to use bad signals in the high tech sector in f, so the relevant range for this equilibrium is when $\frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \leq \pi^f \leq \frac{4\pi^h}{1+3\pi^h}$.

D.4 Equilibria of type $\mathbf{C}^t$

In this case, all workers in country h are employed in sector 2, all $g$workers and a fraction β of b workers in country f are employed in sector 1. Let w_g^j and w_b^j denote the wages for high and low signal workers in country $j = h, f$. Given that all workers are in sector 2 in country h and that all

g workers are in sector 1 in country f it must be that

$$\begin{aligned} w_g^h &= w_b^h = 1 && \text{(D15)} \\ w_g^f &= p_1 \frac{\eta \pi^f}{\eta \pi^f + (1-\eta)\pi^f} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^f}{1+\pi^f} \\ 1 = w_b^f &= p_1 \frac{(1-\eta)\,\pi^f}{(1-\eta)\,\pi^f + \eta(1-\pi^f)} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{\pi^f}{2-\pi^f} \end{aligned}$$

These are simply zero profit conditions conditional on the hypothetical allocation of workers. Since all workers in h and a fraction $(1-\beta)$ of the $\frac{1}{3}\pi^f + \frac{2}{3}(1-\pi^f)$ workers in f with bad signals are in the low tech sector, so the output in that sector is

$$x_2 = 1 + (1-\beta)\left(\frac{1}{3}\pi^f + \frac{2}{3}(1-\pi^f)\right) = 1 + \frac{(1-\beta)\,(2-\pi^f)}{3}. \tag{D16}$$

The output in the high tech sector is

$$x_1 = \frac{2}{3}\pi^f + \beta\frac{1}{3}\pi^f = \frac{(2+\beta)\,\pi^f}{3} \tag{D17}$$

The price is now already determined by the last condition in (D15) and given this price, the consumers must rationally be willing to consume the quantities in (D16) and (D17), which by use of (D1) implies that $\beta \in [0,1]$ must satisfy

$$\begin{aligned} p_1 &= \frac{2-\pi^f}{\pi^f} = \frac{1+\frac{(1-\beta)(2-\pi^f)}{3}}{\frac{(2+\beta)\pi^f}{3}} = \frac{3+(1-\beta)\,(2-\pi^f)}{(2+\beta)\,\pi^f} \Leftrightarrow && \text{(D18)} \\ \left(2-\pi^f\right)(2+\beta) &= \left(3+(1-\beta)\,(2-\pi^f)\right) \Leftrightarrow \\ \left(2-\pi^f\right)((2+\beta)-(1-\beta)) &= 3 \Leftrightarrow \left(2-\pi^f\right)(1+2\beta) = 3 \Leftrightarrow \\ 2\beta\left(2-\pi^f\right) &= 3-\left(2-\pi^f\right) = 1+\pi^f \Leftrightarrow \\ \beta &= \frac{1+\pi^f}{2\,(2-\pi^f)} \end{aligned}$$

Since $\frac{1+\pi^f}{2(2-\pi^f)}$ is always positive, strictly increasing and equal to 1 when $\pi^f = 1$ we conclude that there is always some $\beta \in [0,1]$ such that the conditions in (D15) and (D1) are satisfied. The only condition that remains to be checked is that there are no incentives to employ workers with good signals in h in sector 1, that is that

$$\begin{aligned} p_1 \underbrace{\frac{2\pi^h}{2\pi^h+(1-\pi^h)}}_{P(g,\pi^h)} &= \frac{2-\pi^f}{\pi^f}\frac{2\pi^h}{(1+\pi^h)} \le w_g^f = 1 \Leftrightarrow && \text{(D19)} \\ \left(2-\pi^f\right)2\pi^h &\le \pi^f\left(1+\pi^h\right) \Leftrightarrow \frac{4\pi^h}{1+3\pi^h} \le \pi^f, \end{aligned}$$

which is satisfied in the region marked A^t in Figure 1).

E Stability Analysis in section 6.1

We show here that $\partial G(B^f(\pi))/\partial \pi^f > 1$ is a sufficient condition for instability. Consider the Jacobian of the difference equation system evaluated at the autarky equilibrium $\pi = (\pi^A, \pi^A)$:

$$\begin{bmatrix} G'\left(B\left(\pi\right)\right) \frac{\partial(B^f(\pi))}{\partial \pi^f}\Big|_{\pi=(\pi^A,\pi^A)} & G'\left(B\left(\pi\right)\right) \frac{\partial(B^f(\pi))}{\partial \pi^h}\Big|_{\pi=(\pi^A,\pi^A)} \\ G'\left(B\left(\pi\right)\right) \frac{\partial(B^h(\pi))}{\partial \pi^f}\Big|_{\pi=(\pi^A,\pi^A)} & G'\left(B\left(\pi\right)\right) \frac{\partial(B^h(\pi))}{\partial \pi^h}\Big|_{\pi=(\pi^A,\pi^A)} \end{bmatrix}. \tag{E20}$$

At the autarky equilibrium both the "cross derivatives" and the "own derivatives" are identical, so (dropping the common factor $G'\left(B\left(\pi\right)\right)$ the characteristic polynomial can be written as:

$$\begin{aligned} & \left(\frac{\partial(B^f(\pi))}{\partial \pi^f}\bigg|_{\pi=(\pi^A,\pi^A)} - \lambda \right)^2 - \left(\frac{\partial(B^f(\pi))}{\partial \pi^h}\bigg|_{\pi=(\pi^A,\pi^A)} \right)^2 \\ = \quad & \lambda^2 + 2\lambda \frac{\partial(B^f(\pi))}{\partial \pi^f}\bigg|_{\pi=(\pi^A,\pi^A)} + \left(\frac{\partial(B^f(\pi))}{\partial \pi^f}\bigg|_{\pi=(\pi^A,\pi^A)} \right)^2 - \left(\frac{\partial(B^f(\pi))}{\partial \pi^h}\bigg|_{\pi=(\pi^A,\pi^A)} \right)^2 \end{aligned} \tag{E21}$$

with roots:

$$\lambda_{1,2} = \frac{\partial(B^f(\pi))}{\partial \pi^f}\bigg|_{\pi=(\pi^A,\pi^A)} \pm \frac{\partial(B^f(\pi))}{\partial \pi^h}\bigg|_{\pi=(\pi^A,\pi^A)} \tag{E22}$$

The system is unstable if at least one of the roots is greater than one, therefore a sufficient condition is that the own derivative is greater than one.