A.A 2021/22

EDAA Project

ESTIMATION AND DATA ANALYSIS WITH APPLICATIONS
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Summary

1) Introduction: Problem Statement

Estimation of the positioning of a mobile robot based on information of relative distance and / or orientation from one or more landmarks with different acquisition frequencies. 'Earth' surface environment.
State vector definition:

$$\boldsymbol{X} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

Where x and y are the Cartesian coordinates and θ is the angle formed between the direction of the robot and the x axis.
The motion of the robot can be described in a sufficiently accurate way by the time-continuous dynamic model:

$$\begin{cases} \dot{x} = v\cos(\theta) \\ \dot{y} = v\sin(\theta) \\ \dot{\theta} = \omega \end{cases}$$

Where v and ω are respectively tangential and angular speeds impressed on the robot.
In the absence of specific information, the presence of process noise modeled as a white stochastic process of known variance σ_q^2 was assumed.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \omega \end{bmatrix} + \begin{bmatrix} \varepsilon_x(t) \\ \varepsilon_y(t) \\ \varepsilon_\theta(t) \end{bmatrix}$$

2) Solution

The number of landmarks is parametric; it was faced the problem of how to position these landmarks to achieve the optimal formation in terms of maximizing the accuracy of the robot estimates.
In the article[1] the maximization of the logarithm of the FIM determinant is used as optimality criteria and exploiting the results obtained the maximum FIM determinant is obtained with the network of N landmarks regularly distributed around the target position.

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[1] Moreno-Salinas D, Pascoal AM, Aranda J. Optimal sensor placement for multiple target positioning with range-only measurements in two-dimensional scenarios. Sensors (Basel). 2013 Aug 16;13(8):10674-710. doi: 10.3390/s130810674. PMID: 23959235; PMCID: PMC3812623.

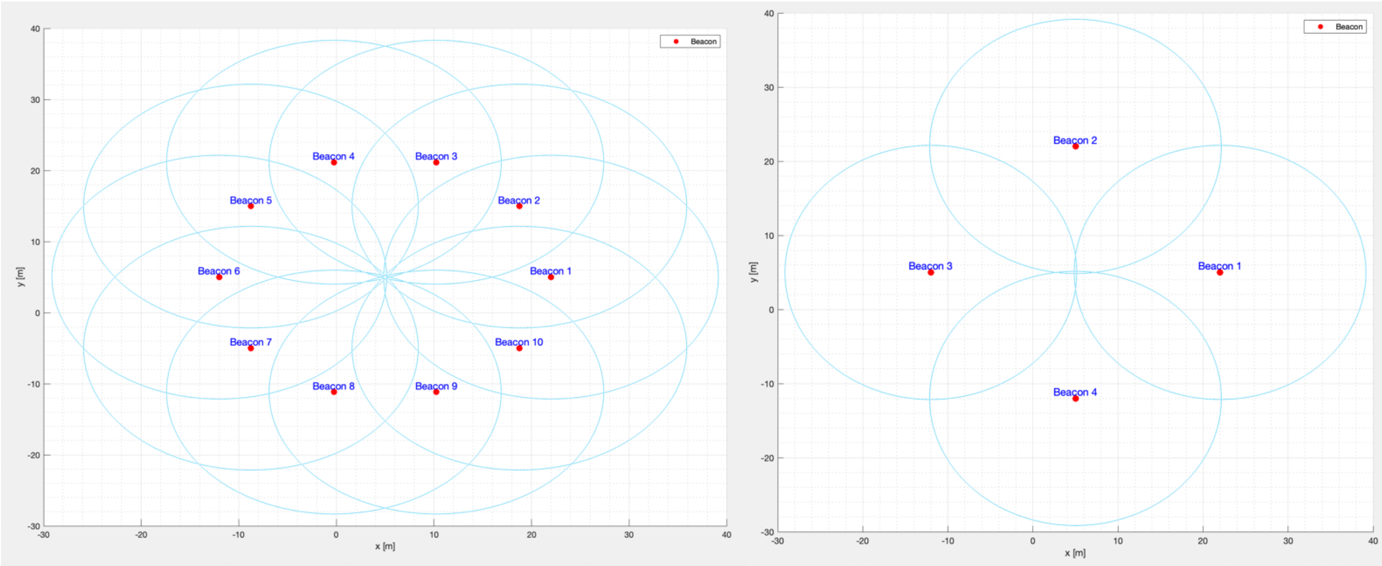

Figure 1Beacon positioning example for N=10 and N=4 distributed around a circumference of radius = 17 m and center C(5,5)

It can be seen from the circular range around each landmark that it has a maximum reachable distance, some numerical details can be seen in the next paragraph.

The model which describes robot trajectory is non-linear, so it is proceeded with the implementation of the Extended Kalman filter (EKF).

Euler Discretization stopping at 1^{st} order

$$\dot{x} \approx \frac{k(KT + T) - x(KT)}{T} = \frac{x(k+1) - x(k)}{T}$$

The discretized nonlinear model is called $f_k\big(X(k), u(k), \varepsilon(k)\big)$ where $u(k)$ are the inputs to the system at step k; for simplicity of notation: $f_k \triangleq f_k\big(X(k), u(k), \varepsilon(k)\big)$.

$$f_k = \begin{cases} x(k+1) = x(k) + Tv(k)\cos(\theta) + \varepsilon_x(k) \\ y(k+1) = y(k) + Tv(k)\sin(\theta) + \varepsilon_y(k) \\ \theta(k+1) = \theta(k) + T\omega(k) + \varepsilon_\theta(k) \end{cases}$$

$$\varepsilon \sim N(0, Q(k))$$

where $Q(k)$ is a positive definite diagonal matrix with the elements of the diagonals equal to the variances.

For each landmark can be obtained two measurements:

1. Distance from the vehicle
2. Angle between vehicle and beacon

According to the problem statement, in the developed algorithm these measures can be available in AND or in OR:

1. Distance measures AND orientation measures
2. Only distance measures

3. Only orientation measures

In the following we will model case 1. where both measures are available.
The whole matrix of the measures Z is constructed by juxtaposing all the measures of type 1 to those of type 2.

$$Z(k) = \begin{bmatrix} \sqrt{\left(x_i - x(k)\right)^2 + \left(y_i - y(k)\right)^2} \\ \dots \\ arctan\left(\frac{y_i - y(k)}{x_i - x(k)}\right) - \theta(k) \\ \dots \end{bmatrix}$$

$$\text{for } i = 1, \dots, n$$

Where n is the total number of landmarks and (x_i, y_i) is the position in the Cartesian plane of the i-th landmark.
In parallel to the true values of the measurements, we will have the measurements Z_n, affected by noise, detected by the sensors:

$$Z_n(k) = Z(k) + \alpha\sqrt{R}$$

Where:

- α is a vector of $2N$ elements normally distributed between 0 and 1;
- N is the number of beacons;
- R is the covariance matrix associated with the measurements zero mean white noise $w(k)$; (i.e. $E[w(k)] = 0$ and $E[w(k) \cdot w(j)^T] = 0 \;\forall\, j, k$)
- $\sqrt{R}$ is the element-wise square root of R.

Proceeding with Jacobians calculus according to EKF algorithm in the book[2]:

$$F(k) = \left.\frac{df_k}{dx}\right|_{\substack{x=\hat{x}(k|k) \\ v=0}} = \left.\begin{bmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} & \frac{df_1}{d\theta} \\ \frac{df_2}{dx} & \frac{df_2}{dy} & \frac{df_2}{d\theta} \\ \frac{df_3}{dx} & \frac{df_3}{dy} & \frac{df_3}{d\theta} \end{bmatrix}\right|_{\substack{x=\hat{x}(k|k) \\ v=0}} = \begin{bmatrix} 1 & 0 & -Tv(k)\sin\left(\theta(k)\right) \\ 0 & 1 & Tv(k)\cos(\theta(k)) \\ 0 & 0 & 1 \end{bmatrix}_{\substack{x=\hat{x}(k|k) \\ v=0}}$$

$$L(k) = \left.\frac{df_k}{d\varepsilon}\right|_{\substack{x=\hat{x}(k|k) \\ v=0}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_{3x3}$$

System of equations of measures $h_k \triangleq h_k(X(k), w(k))$ is defined.

[2] Yaakov Bar-Shalom, Thiagalingam Kirubarajan, and X.-Rong Li. 2002. Estimation with Applications to Tracking and Navigation. John Wiley & Sons, Inc., USA.

$$h_k = \begin{cases} \sqrt{\left(x_i - x(k)\right)^2 + \left(y_i - y(k)\right)^2} + w_d(k) \\ \cdots \\ arctan\left(\dfrac{y_i - y(k)}{x_i - x(k)}\right) - \theta(k) + w_\theta(k) \\ \cdots \end{cases}$$
$$\text{for } i = 1, \dots, n$$

where:

- $w_d(k) \triangleq$ distance measurement error at step k;
- $w_\theta(k) \triangleq$ orientation measurement error at step k;

$$H(k) = \left.\frac{dh_k}{dx}\right|_{\substack{x=\hat{x}(k+1|k)\\ w=0}} = \left[\begin{matrix} \frac{-(x_i - x(k+1))}{\sqrt{(x_i-x(k+1))^2+(y_i-y(k+1))^2}} & \frac{-(y_i - y(k+1))}{\sqrt{(x_i-x(k+1))^2+(y_i-y(k+1))^2}} & 0 \\ \vdots & \vdots & \vdots \\ \frac{-(y(k+1)-y_i)}{(x(k+1)-x_i)^2+(y(k+1)-y_i)^2} & \frac{(x(k+1)-x_i)}{(x(k+1)-x_i)^2+(y(k+1)-y_i)^2} & -1 \\ \vdots & \vdots & \vdots \end{matrix}\right]_{\substack{x=\hat{x}(k+1|k)\\ w=0}}$$

There are i equations like the first row and i equations like the third-one (one for each landmark). If the beacons are not reachable, the respective lines of H will have null components.
Both in the case of unavailable measurements and in the case of OR measurements, the developed algorithm will adapt the matrix size of H to a square matrix $l \times l$ where l is the number of measurements valid at the $k-th$ iteration (also holds for the matrix R the same reasoning).

$$M(k) = \left.\frac{dh_k}{dw}\right|_{\substack{x=\hat{x}(k+1|k)\\ v=0}} = I_{2lx2l}$$

3) Algorithm details

3.1) Max reachable distance of the landmark

It was considered the speed of flight of the acoustic waves in the air, equal to 343 m/s and based on the sampling time Tc of the signals sent by the robot, it was possible to see up to what maximum distance the landmark could have the measurement available; in the negative case the signal does not arrive at its destination in time and the measurement is lost.
Some dimensions of the simulation context:

- Sampling time $T_c = 0.15\ s$
- The N landmarks they are equally distributed along a circumference with a radius of about 17 meters and center C (5,5) (Obviously, these parameters must be adapted in such a way as to place the entire area that the robot can travel within the circle). So, the area restricted to the robot is approximately 900 m^2.
- Maximum distance that can be reached from a landmark $d_{max} = \frac{v_{sound} \cdot T_c}{2} = \frac{343\frac{m}{s} \cdot 0.15\ s}{2} = 25.73\ m$. (which is greater than the radius, but smaller than the diameter; it was done because if the robot moves fairly evenly throughout the circle, about three-quarters of the measurements will be achievable and a quarter not.)

d_{max} has a multiplicative term equal to 1/2 because by hypothesis the devices are not synchronized and therefore the round trip time is considered as the time necessary for communication.

3.2) Kalman filter calibration

In the simulation developed:

- $Q = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_\theta^2 \end{bmatrix}$

  Where:
  - $\sigma_x = 0.001\ m$
  - $\sigma_y = 0.001\ m$
  - $\sigma_\theta = \left(0.1 \frac{\pi}{180}\right)^\circ$

- $R = \begin{bmatrix} \sigma_d^2 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \sigma_{\theta'}^2 & 0 \\ 0 & 0 & 0 & \ddots \end{bmatrix}$

  assuming that all the landmark sensors have the same sensitivity to each other, and that the matrix R is a diagonal block matrix and that it has a number of lines equal to twice the number of landmarks (lines equally distributed for distance and orientation measurements) and:
  - $\sigma_d = 0.3\ m$
  - $\sigma_{\theta'} = \left(1 \cdot \frac{\pi}{180}\right)^\circ$

It is undisputed that Q << R, so the model will perform better than the measurements.

3.3) Management of incoming signals

A toy example is illustrated to show how the arrival of the marks was managed.
Suppose 4 beacons and a sampling time $T = 0.15\ s$. This is the startup situation of the remaining time available $t_a = T$ of a landmark to receive a signal (means the maximum time that the signal can take to reach its destination):

Start of Iteration 0 ($t = 0\ s$):

	Beacon 1	Beacon 2	Beacon 3	Beacon 4
$t_a\ [s]$	0.15	0.15	0.15	0.15

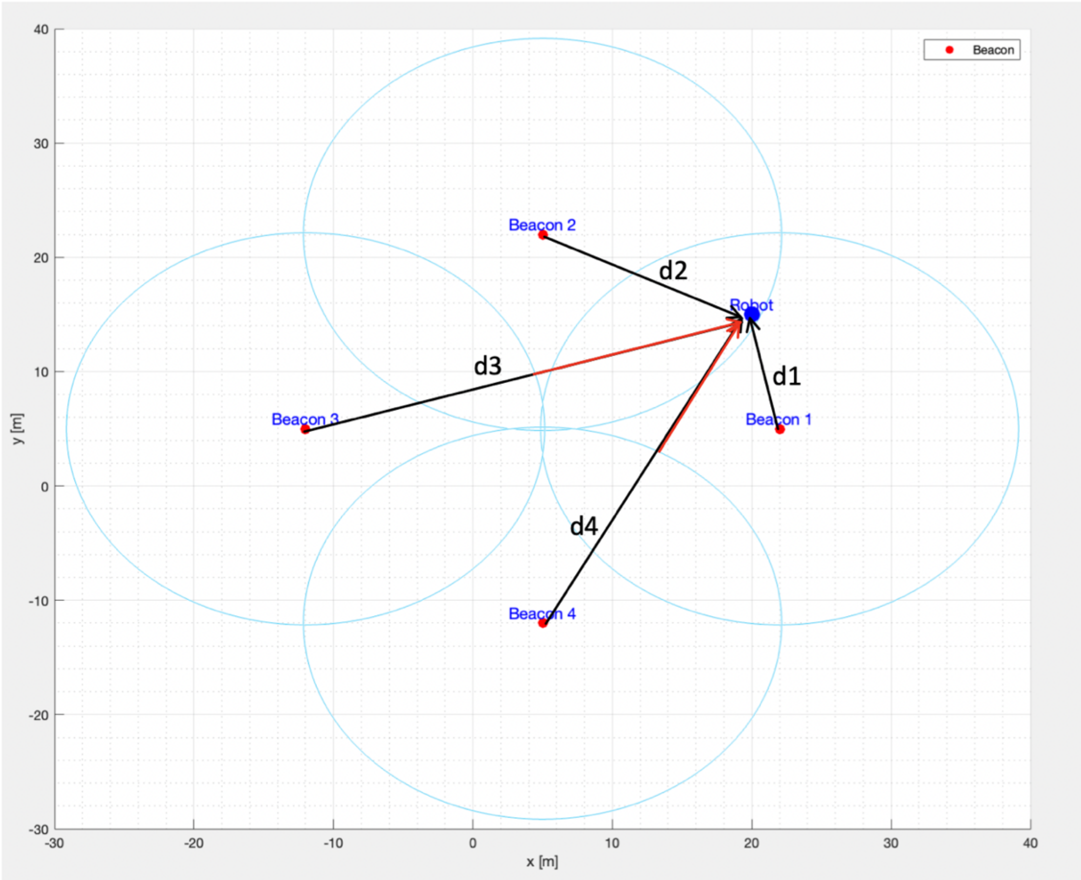

Figure 2 Beacon distances example

In this situation, the distance d1 certainly allows the beacon to receive the signal within 0.15 s, the beacon 2 has a distance d2 perfectly at the limit, while the distances d3 and d4 are greater than their maximum reachability range, so they will lead to a reception time certainly greater than 0.15 s.
The time of flight of the acoustic wave is defined by t_f and likely values are assigned for the 4 beacons:

Iteration 0 ($t = 0\ s$):

	Beacon 1	Beacon 2	Beacon 3	Beacon 4
$t_f\ [s]$	0.07	0.15	0.31	0.22

Note that in this implementation is allowed only a reception for iteration.

Beacons 1 and 2 will have the measures correctly available, on the contrary, beacons 3 and 4 the communication will not be successful, and the measure will be lost.

4) Results and Conclusion

4.1) Circular Path simulation

By way of example, the simulation takes place for $100\ s$ with the robot traveling a circular path at a tangential speed of $0.75\ \frac{m}{s}$ and an angular speed equal to $0.06\ \frac{rad}{s}$.

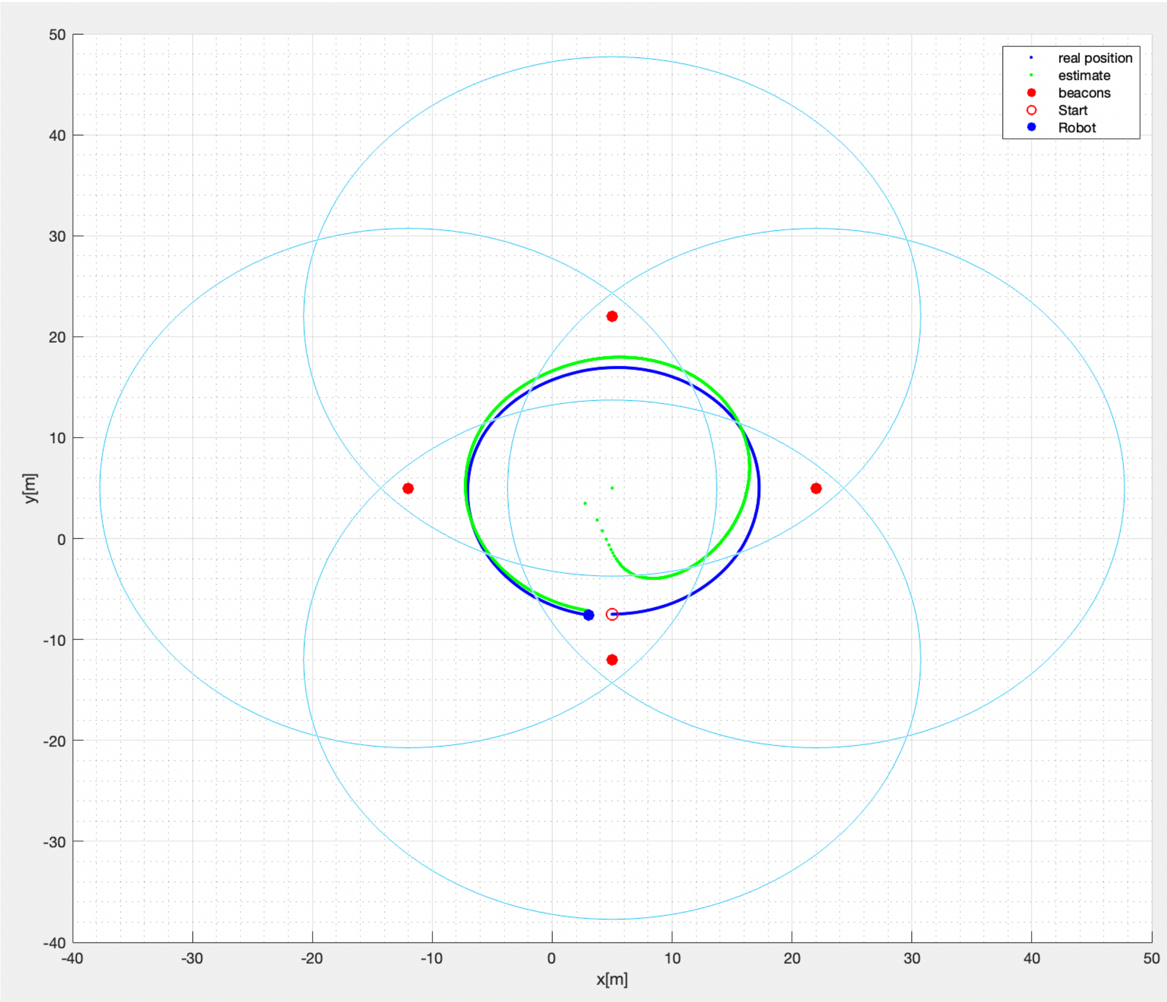

Figure 3 Circular path simulation

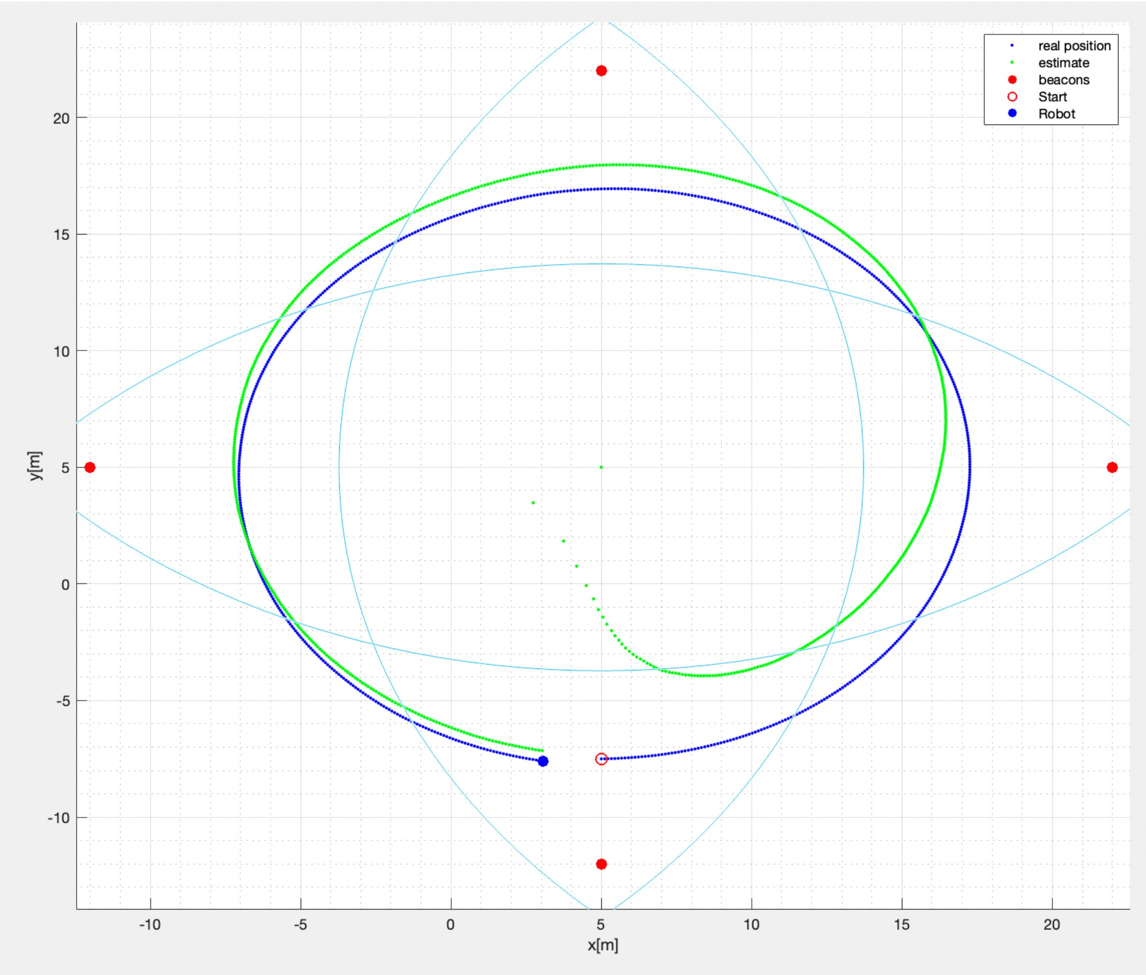

Figure 4 Detail of Figure 3

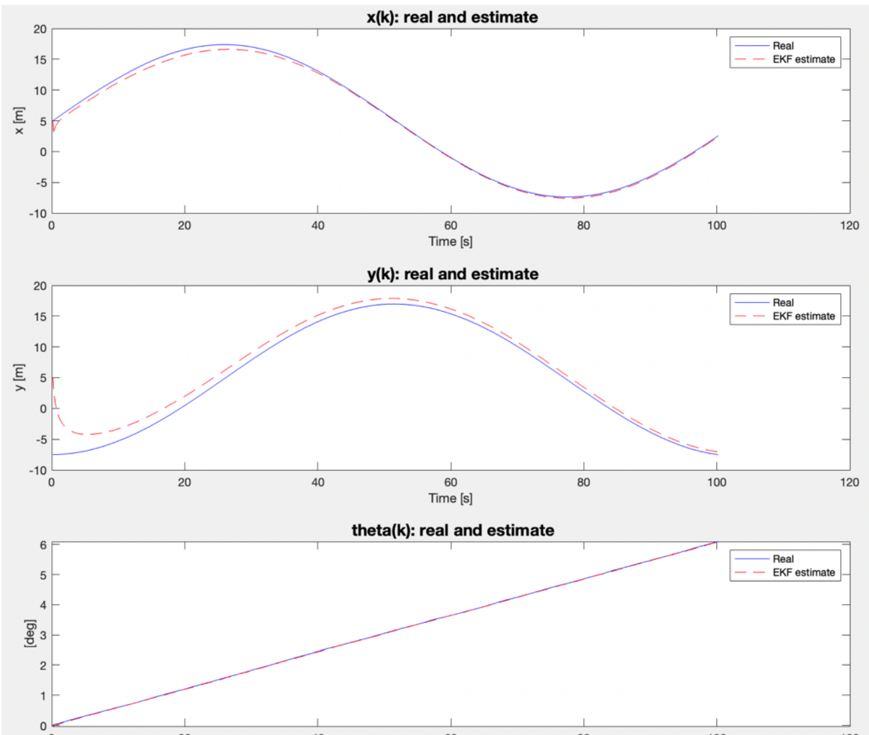

Figure 5 Comparison of the state vector between model and estimate

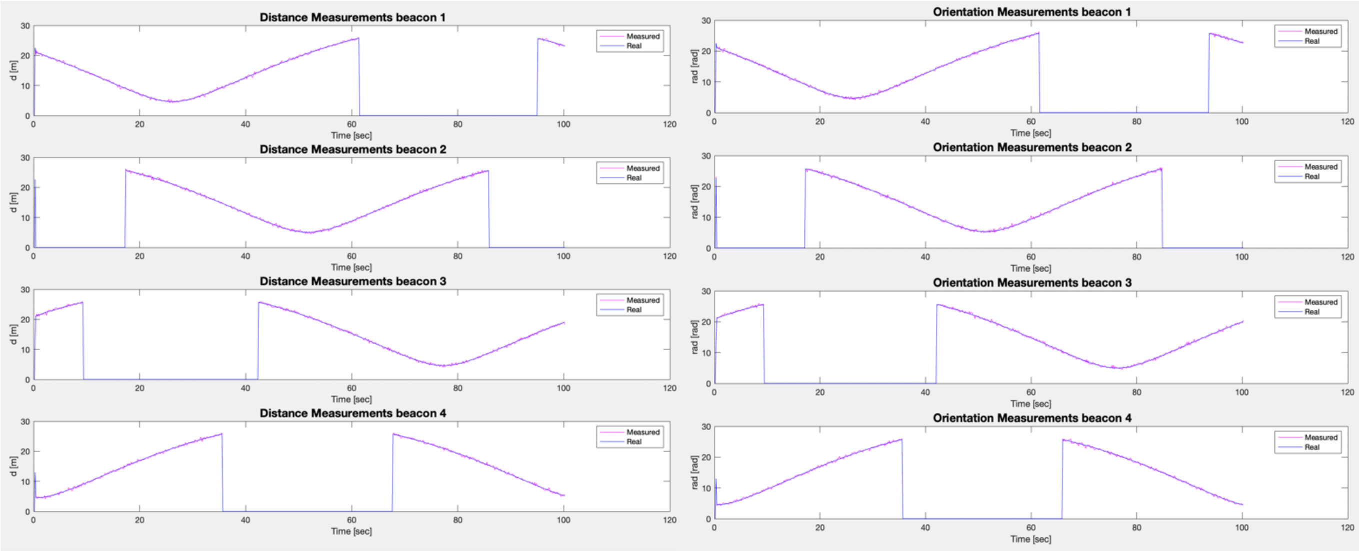

Figure 6 Validity of the measures

4.2) Covariance matrices

4.2.1) Case distance measures AND orientation measures

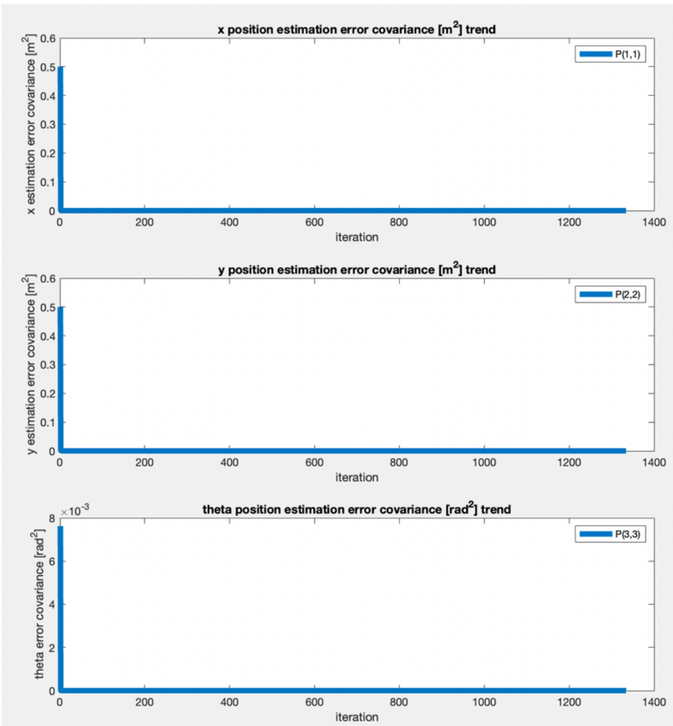

Figure 7 Covariance Matrices in case of distance measures AND orientation measures

4.2.2) Case only distance measures

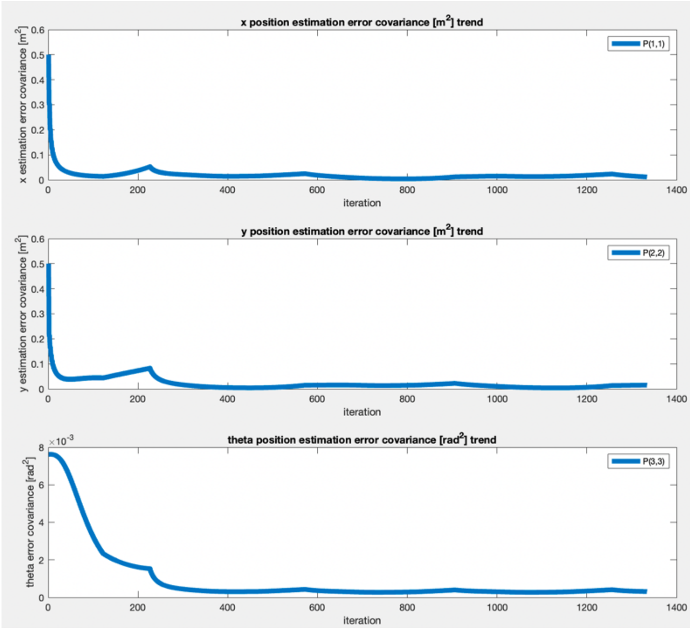

Figure 8 Covariance Matrices in case of only distance measures

4.2.3) Case only orientation measures

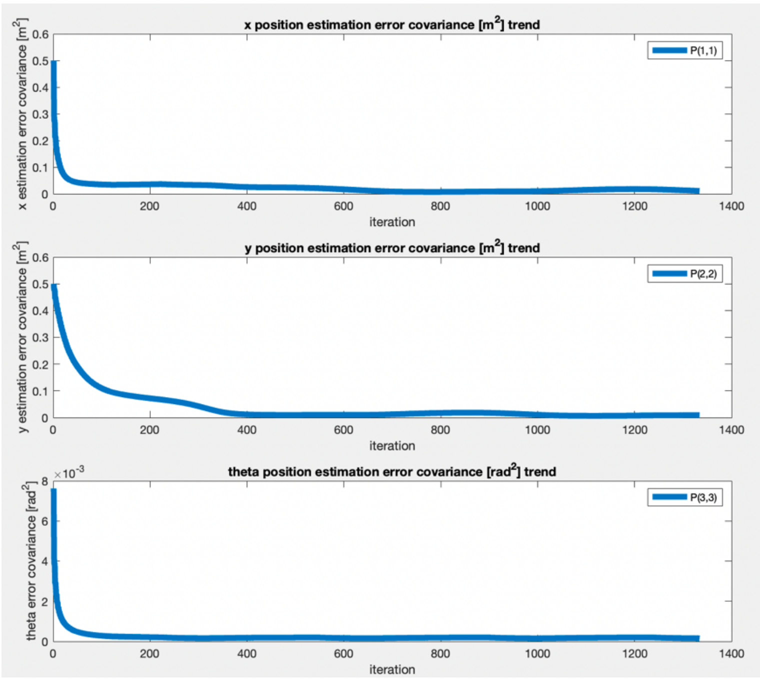

Figure 9 Covariance matrices Case only orientation measures

4.3) Estimation error

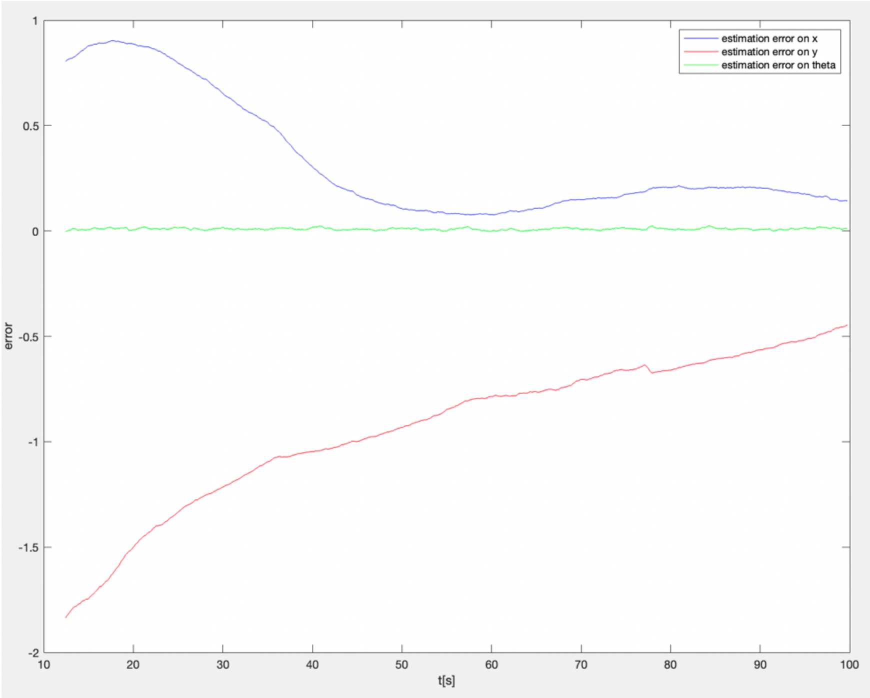

Figure 10 Estimation error

To evaluate the estimation error on the 3 variables of the state vector, the execution time of the algorithm was increased up to 1000 s.

It can be seen graphically from figure 11 and numerically from the following Monte Carlo simulation that $E[\tilde{X}(k|k)] = E\left[X(k|k) - \hat{X}(k|k)\right] = 0$ because Kalman filter is an unbiased estimator.

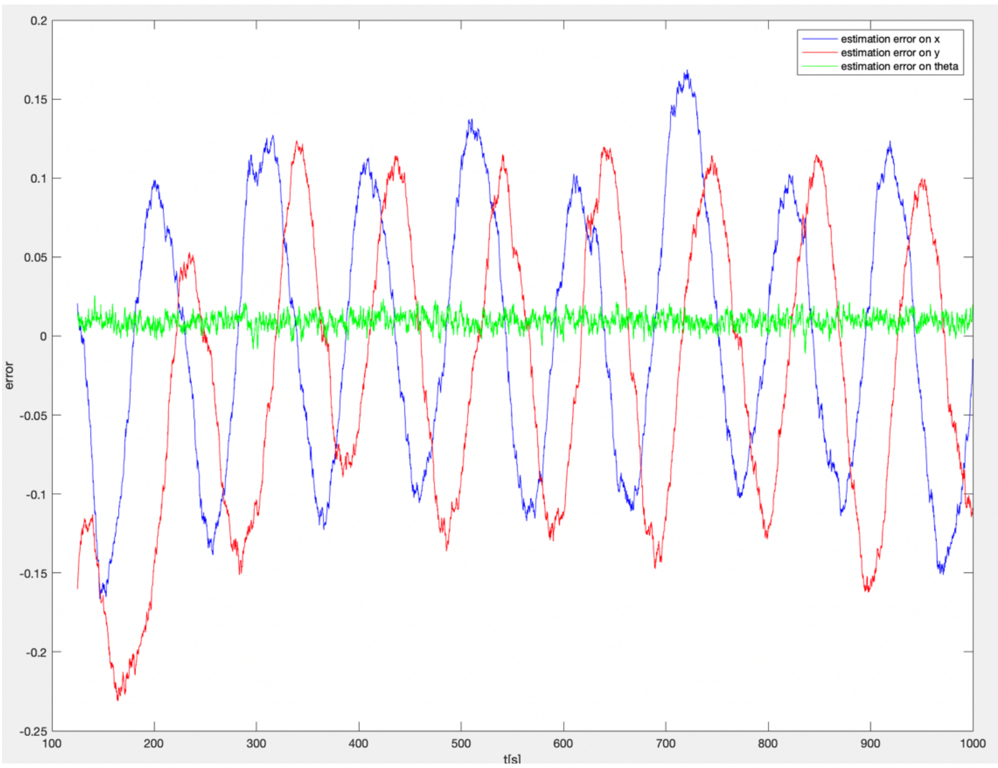

Figure 11 Estimation error on 1000 s

4.3.1) Monte Carlo Simulation

The EKF simulation was repeated for N = 1000 times; at each iteration, a value (for each of the 3 components) of the estimation error was sampled and the "temporal" position of the selected value is a uniformly distributed random variable that assumes values between 1 and the number of iterations of the EKF algorithm.
Downstream, the sample mean of the 3 components of the estimation error was calculated.
By way of example, the results of a simulation are reported:

$$\overline{\tilde{X}(1)} = 0.0346\ m$$
$$\overline{\tilde{X}(2)} = \ -0{,}126\ m$$
$$\overline{\tilde{X}(2)}\ = 0{,}028\ rad$$

Bibliography

[1] Moreno-Salinas D, Pascoal AM, Aranda J. Optimal sensor placement for multiple target positioning with range-only measurements in two-dimensional scenarios. Sensors (Basel). 2013 Aug 16;13(8):10674-710. doi: 10.3390/s130810674. PMID: 23959235; PMCID: PMC3812623.

[2] Yaakov Bar-Shalom, Thiagalingam Kirubarajan, and X.-Rong Li. 2002. Estimation with Applications to Tracking and Navigation. John Wiley & Sons, Inc., USA.

[3] http://www.diag.uniroma1.it/~oriolo/amr/slides/Localization3_Slides.pdf