

Comparison of integration methods in VEGAS+

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Classic VEGAS evaluates integrals through two strategies:

- **adaptive importance sampling**: estimation of the integral using a grid which is improved after each iteration.
- **stratified sampling**: integral is expressed as the sum of the contribution from different hypercubes in which the domain of integration is divided.

VEGAS+ adds a second adaptive strategy the **adaptive stratified sampling**: the number of integrand samples used in each hypercubes can vary from hypercube to hypercube while in classic VEGAS this number is fixed.

We benchmark the performance of these different methods of VEGAS+ to see whether VEGAS+ or the adaptive stratified sampling can perform better than the importance sampling already implemented in vegasflow.

Dimensional comparison

Comparison for Gaussian Function

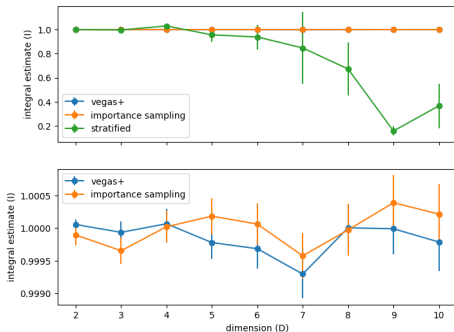


Figure: Comparison of Gaussian integral estimate I from 50 iterations with 12000 samples after a warmup of 5 iterations with 1000 samples.

Comparison for Gaussian Function

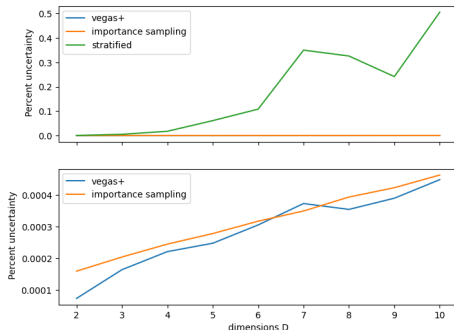


Figure: Comparison of percent uncertainty of Gaussian integral from 50 iterations with 12000 samples after a warmup of 5 iterations with 1000 samples.

Dimensional comparison

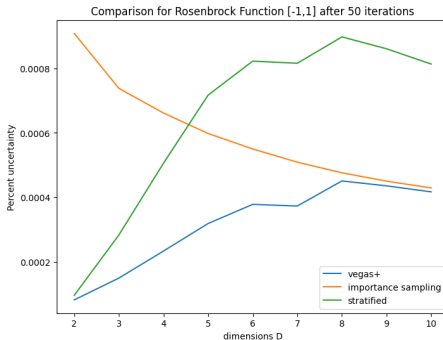


Figure: Comparison of percent uncertainty of Rosenbrock integral from 50 iterations with 12000 samples after a warmup of 5 iterations with 1000 samples.

Gaussian samples comparison

Comparison for Gauss Function dim = 4

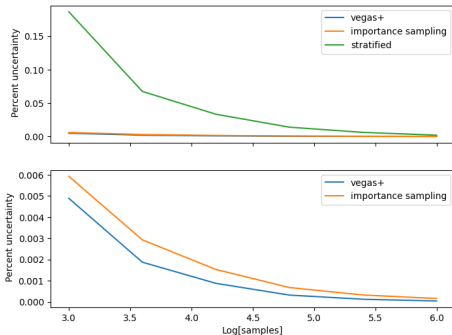


Figure: Comparison of percent uncertainty 4-d-Gaussian integral from 10 iterations with different samples per iteration after a warmup of 5 iterations with 1000 samples.

Comparison for Gauss Function dim = 8

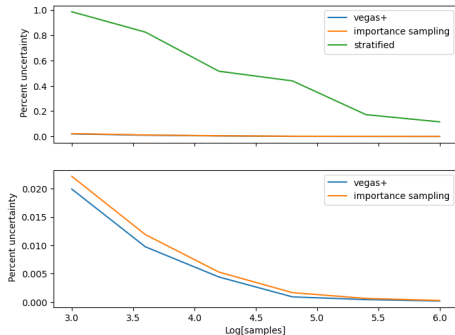


Figure: Comparison of percent uncertainty of 8-d-Gaussian integral from 10 iterations with different samples per iteration after a warmup of 5 iterations with 1000 samples.

Rosenbrock samples comparison

Comparison for Rosen Function dim = 4

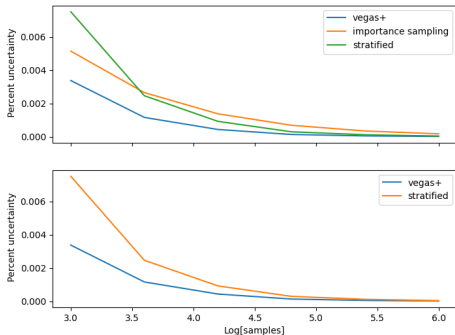


Figure: Comparison of percent uncertainty of 4-d-Rosenbrock integral from 10 iterations with different samples per iteration after a warmup of 5 iterations with 1000 samples.

Comparison for Rosen Function dim = 8

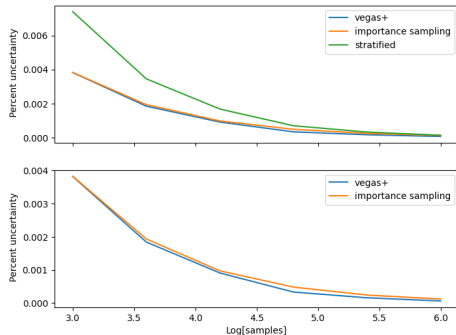


Figure: Comparison of percent uncertainty of 8-d-Rosenbrock integral from 10 iterations with different samples per iteration after a warmup of 5 iterations with 1000 samples.

Comparison for Physical integrand dim = 3

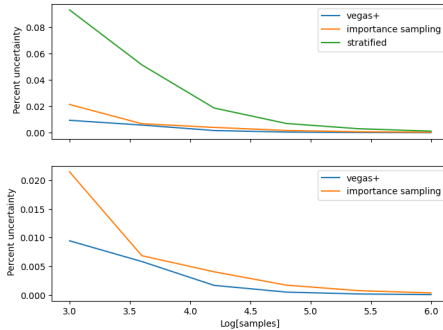


Figure: Comparison of percent uncertainty of 3-d physical integral from 10 iterations with different samples per iteration after a warmup of 5 iterations with 1000 samples.

Gauss 4-d performance comparison

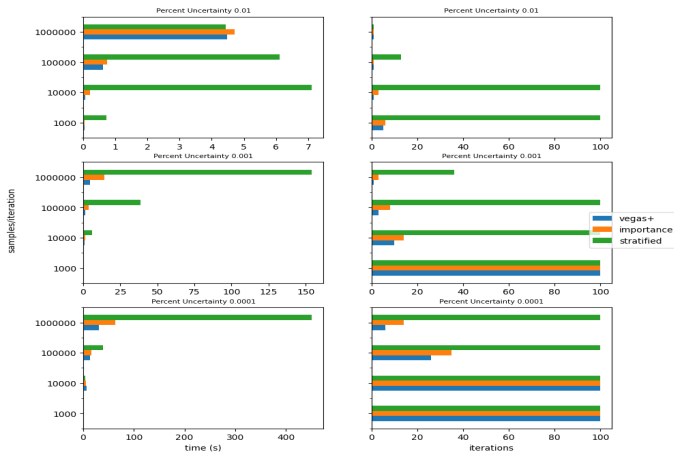


Figure: Comparison of performances between different integration methods at fixed percent uncertainty (10^{-2} , 10^{-3} or 10^{-4}) and at fixed samples per iteration ($10^3, 10^4, 10^5$ or 10^6) with 100 maximum iterations.

Gauss 4-d performance comparison

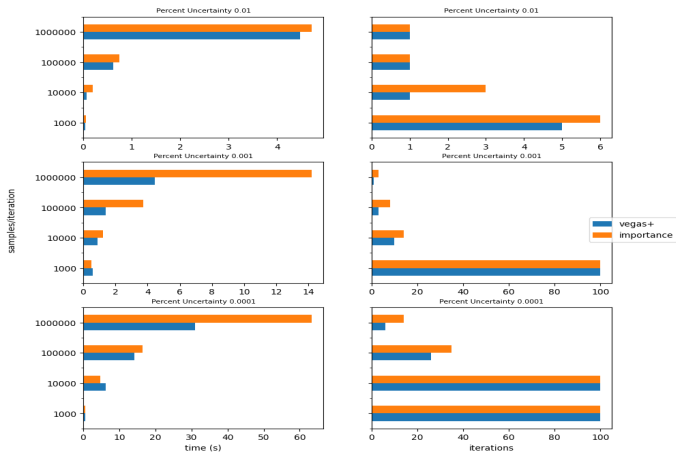


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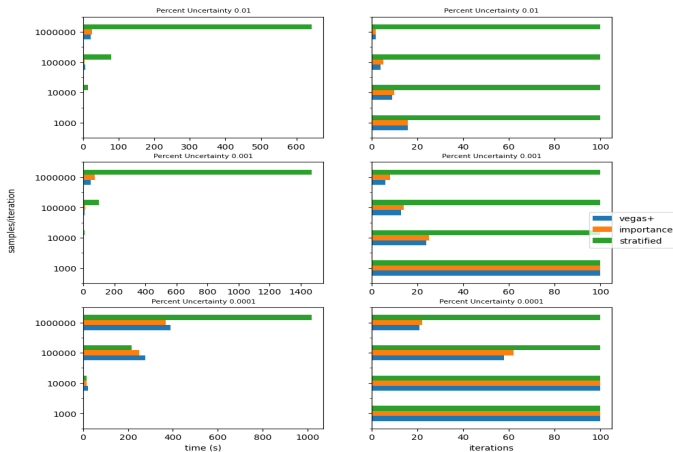


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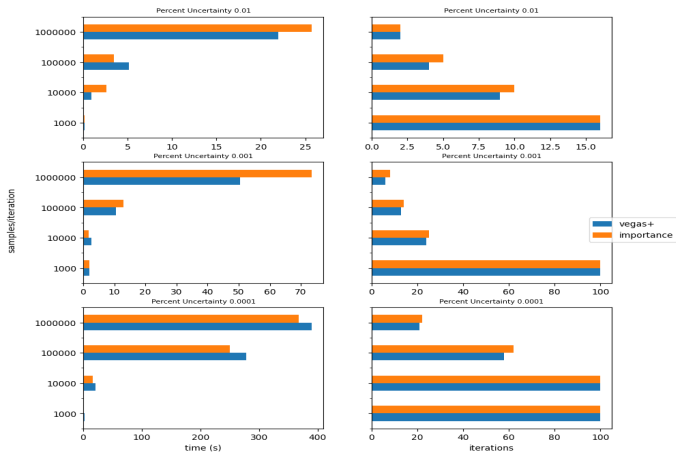


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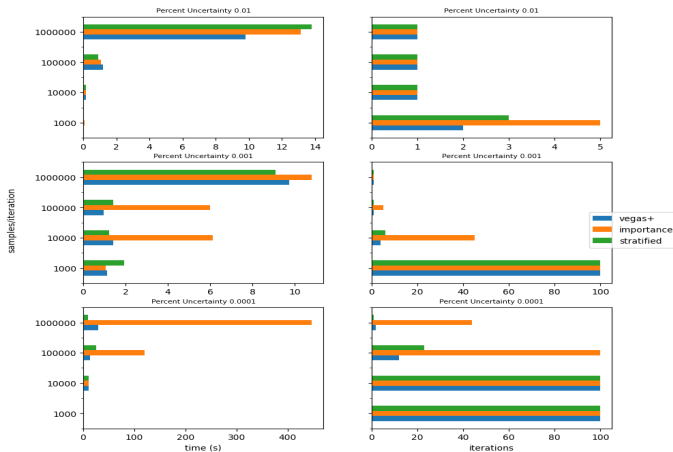


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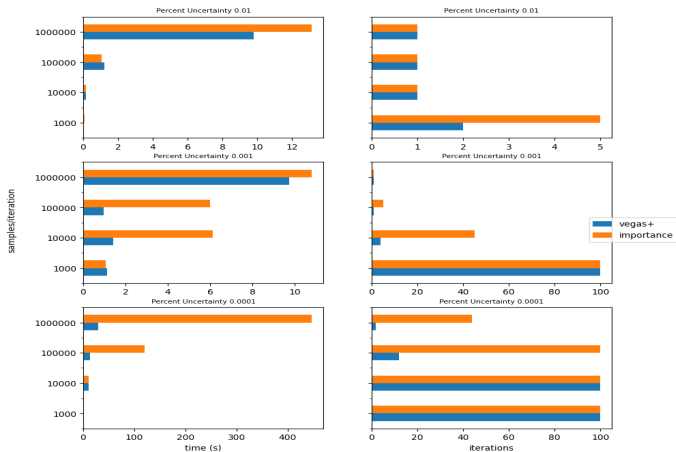


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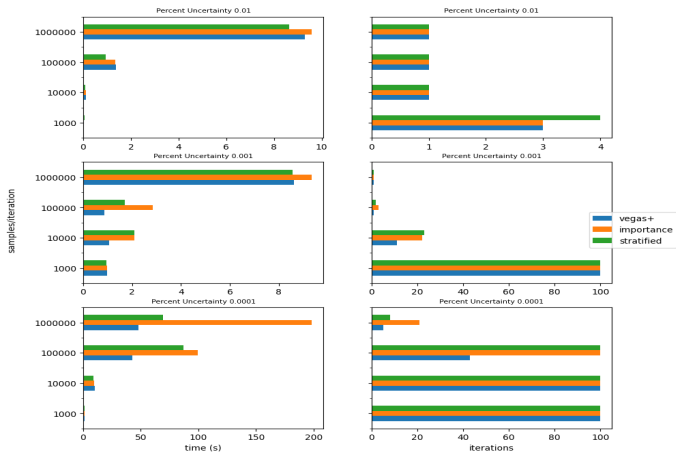


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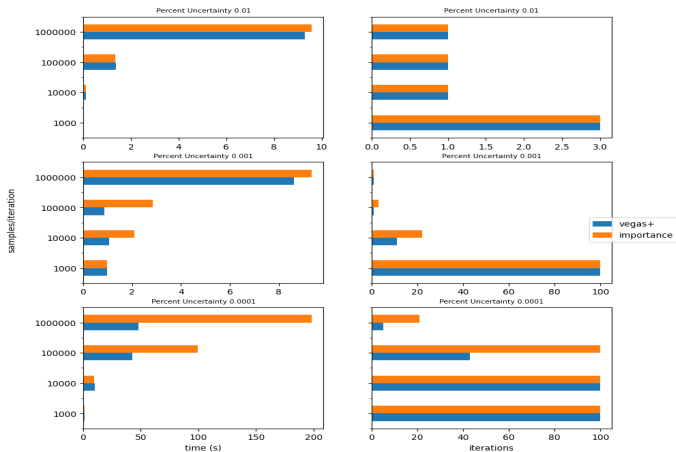


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Physical Integrand performance comparison

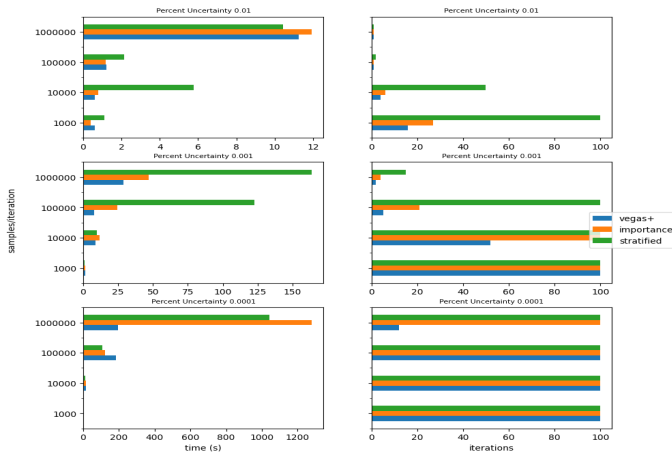


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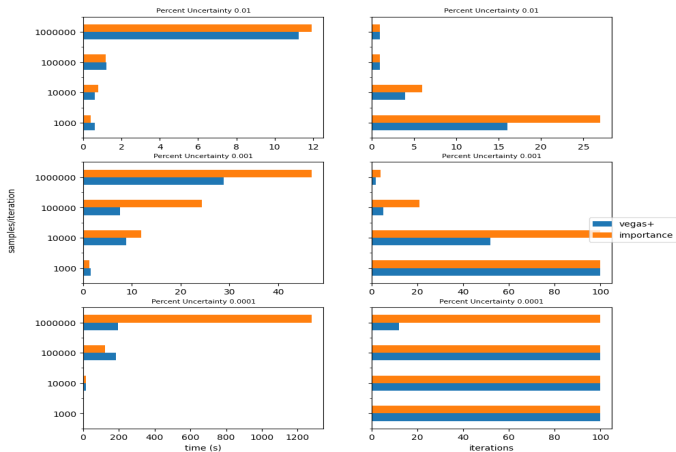


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