Chapter 1

1.1 Multi-dimensional integration

1.1.1 A first example of MonteCarlo integration

MonteCarlo methods are a powerful tool which can provide the answer to a problem by simply running a simulation on the system studied. Montecarlo bla bla bla

In the case of multi-dimensional integration we are interested in performing the following integral I:

$$I = \int_V d^n\mathbf{x} f(\mathbf{x})$$

where V is the domain of the integration and f is a function of n variables $\mathbf{x} = (x_1, x_2, ..., x_n)$.

When perfoming the integral I the simulation comes down to a sampling of the integrand function. First we need to generate a set of random points $\mathbf{x}_i$ which belong to the integration domain V. The simplest way to performe the sampling is to pick random points uniformly distributed in the volume V.

An estimate of the integral using N random points can be given as:

$$I \approx I_{\text{MC}} = V \frac{1}{N} \sum_{\mathbf{x}_i \in V} f(\mathbf{x}_i)$$

I_{MC} is a random number, whose value depends on the sampled points, whose mean is given by the exact value of the integral I and the variance is given by:

$$\sigma_I^2 = \frac{1}{N} \big[V \int_V d^n\mathbf{x} f^2(\mathbf{x}) - I^2 \big]$$

This variance is asymptotically related to the variance of the random value I_{MC}, therefore we can estimate the value of σ_I^2 in the limit of large N as:

$$\sigma_I^2 \approx \sigma_{\text{MC}}^2 = \frac{1}{N-1} \Big[\frac{V^2}{N} \sum_{\mathbf{x}_i \in V} f^2(\mathbf{x}_i) - I_{MC}^2 \Big]$$

In both cases we can observe that the standard deviation decreases as the sample increased as $N^{-\frac{1}{2}}$.

- improvement with respect to quadrature formula
- problem of the boundaries solved
- problem of reducing the error -¿ importance sampling and stratified sampling