Chapter 1

GCV

1.1 GCV-I-derivative

$\frac{dGCV}{d\lambda}(\lambda) = \frac{d(s\frac{SS_{res}(\lambda)}{dor^2(\lambda)})}{d\lambda} = s(\frac{dSS_{res}}{d\lambda}\frac{1}{dor^2} + SS_{res}\frac{d(dor^{-2})}{d\lambda})$

$[1] = \frac{dSS_{res}}{d\lambda} = \frac{d(<\hat{\boldsymbol{\varepsilon}},\hat{\boldsymbol{\varepsilon}}>)}{d\lambda} = 2 < \frac{d\hat{\boldsymbol{\varepsilon}}}{d\lambda}, \hat{\boldsymbol{\varepsilon}} >= 2 < \frac{d((\boldsymbol{Q}-\boldsymbol{Q}\boldsymbol{S})\boldsymbol{z})}{d\lambda}, \hat{\boldsymbol{\varepsilon}} >= -2 < \boldsymbol{Q}\frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} > \overset{\boldsymbol{Q}\in Sym}{=}$
$-2 < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \boldsymbol{Q}\hat{\boldsymbol{\varepsilon}} > \overset{\boldsymbol{Q}^2=\boldsymbol{Q}}{=} -2 < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >$

$[2] = \frac{d(dor^{-2})}{d\lambda} = -2\frac{1}{dor^3}\frac{d(dor)}{d\lambda} = -2\frac{1}{dor^3}\frac{d(s-q-tr(\boldsymbol{S}))}{d\lambda} = 2\frac{1}{dor^3}tr(\frac{d\boldsymbol{S}}{d\lambda})$

$\implies \frac{dGCV}{d\lambda}(\lambda) = s(- < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} > \frac{2}{dor^2} + SS_{res}\frac{2}{dor^3}tr(\frac{d\boldsymbol{S}}{d\lambda})) = \frac{2s}{dor^2}(\hat{\sigma}^2 tr(\frac{d\boldsymbol{S}}{d\lambda}) - < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >)$

1.2 GCV-II-derivative

$\frac{d^2GCV}{d\lambda^2}(\lambda) = 2s\frac{d(dor^{-2}(\hat{\sigma}^2 tr(\frac{d\boldsymbol{S}}{d\lambda}) - <\frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z},\hat{\boldsymbol{\varepsilon}}>))}{d\lambda}$

$[1] = \frac{d(dor^{-2})}{d\lambda}(\hat{\sigma}^2 tr(\frac{d\boldsymbol{S}}{d\lambda}) - < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >) = 2\frac{1}{dor^3}tr(\frac{d\boldsymbol{S}}{d\lambda})(\hat{\sigma}^2 tr(\frac{d\boldsymbol{S}}{d\lambda}) - < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >)$

$[2] = \frac{1}{dor^2}\frac{d(\hat{\sigma}^2)}{d\lambda}tr(\frac{d\boldsymbol{S}}{d\lambda}) = \frac{1}{dor^2}\frac{\frac{d(SS_{res})}{d\lambda}dor - SS_{res}\frac{d(dor)}{d\lambda}}{dor^2}tr(\frac{d\boldsymbol{S}}{d\lambda}) = \frac{1}{dor^2}\frac{-2<\frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z},\hat{\boldsymbol{\varepsilon}}>dor + SS_{res}tr(\frac{d\boldsymbol{S}}{d\lambda})}{dor^2}tr(\frac{d\boldsymbol{S}}{d\lambda}) =$
$\frac{1}{dor^3}(-2 < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} > + \hat{\sigma}^2 tr(\frac{d\boldsymbol{S}}{d\lambda}))tr(\frac{d\boldsymbol{S}}{d\lambda})$

$[3] = \frac{1}{dor^2}\hat{\sigma}^2 tr(\frac{d^2\boldsymbol{S}}{d\lambda^2})$

$[4] = -\frac{1}{dor^2} < \frac{d^2\boldsymbol{S}}{d\lambda^2}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >$

$[5] = -\frac{1}{dor^2} < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \frac{d\hat{\boldsymbol{\varepsilon}}}{d\lambda} >= \frac{1}{dor^2} < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \boldsymbol{Q}\frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z} >$

$\implies \frac{d^2GCV}{d\lambda^2}(\lambda) = \frac{2s}{dor^2}(\frac{1}{dor}[3\hat{\sigma}^2 tr(\frac{d\boldsymbol{S}}{d\lambda}) - 4 < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >]tr(\frac{d\boldsymbol{S}}{d\lambda}) + \hat{\sigma}^2 tr(\frac{d^2\boldsymbol{S}}{d\lambda^2}) + < \frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z}, \boldsymbol{Q}\frac{d\boldsymbol{S}}{d\lambda}\boldsymbol{z} >$
$- < \frac{d^2\boldsymbol{S}}{d\lambda^2}\boldsymbol{z}, \hat{\boldsymbol{\varepsilon}} >)$