Lliurament Introducció a l'Astrofísica 2025:


Understanding Analemmas

In this exercise we want to understand the notion of an "analemma" starting from first principles.

1. Provide a short explanation of the term "analemma". You may search in the literature and the web. You are explicitly allowed to use artificial intelligence to help you throughout this entire task (being aware that AI sometimes provides you wrong information, the so-called "hallucinations", so please always double-check relevant parts, particularly formulae and code).

2. Determine the reason of the apparent movement of the Sun's position up-and-down on the analemma and how it relates to the Sun's declination.

3. Determine the reason of the apparent East-West movement of the Sun's position on the analemma and how it relates to the Hour Angle of the Sun.

4. To calculate analemmas we need to understand Keplerian motion of the Earth around the Sun, given the slight eccentricity $\varepsilon \approx 0.0167$ of the Earth's orbit. Kepler's third law tells us that the Earth's angular velocity varies along a year. The difference between its actual position, the so-called "**true anomaly**" (expressed in angle between the Sun-Earth at its reference at perihelion and its actual position) and the hypothetical position it would have if its orbit was circular, the so-called "**mean anomaly**", is called the

"**equation of center**". Can you find an exact **expression for the mean anomaly as a function of true anomaly** from Kepler's equation?

You will notice that this function cannot be easily inverted. Can you find an approximate expression for **true anomaly as a function of mean anomaly**?

5. From the above said, we now define the "**equation of time**", that is, the difference between "**apparent solar time**" and the "**mean solar time**". This equation is described by the superposition of two sine waves. Can you find out why there are two sine waves involved and why one has twice the angular velocity of the other? How is the equation of time related to sidereal time? Can you find a good approximation for the equation of time, valid for the year 2025?
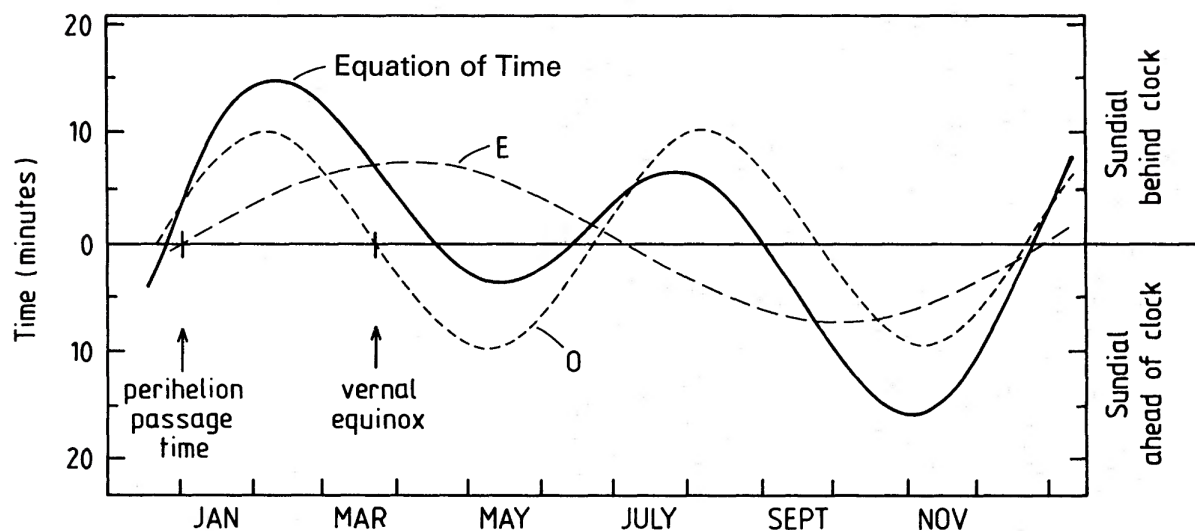
6. With the above, can you provide a short python script that plots the analemma of the Sun during the year 2025, as viewed from Bellaterra? When is the analemma upright and when is it inclined? Has our university drawn correct analemmas?
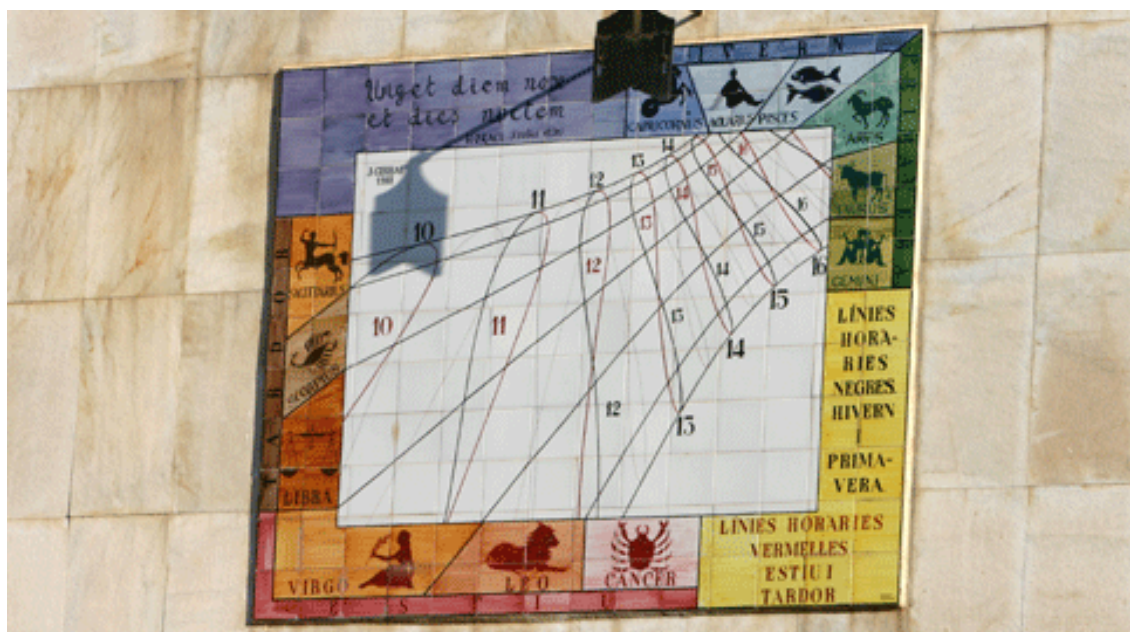
7. The *Stellarium* software allows us to view the analemma from any location on Earth. Could you cross-check the findings from your python code with Stellarium?

8. How do sundials (solar clocks) use the analemma to predict the correct time?

9. Finally, we would like to understand the influence of an orbit's eccentricity and the obliquity of a planet's rotation axis on the analemma: to do so, can you plot the analemma as seen from the following planets:
   - Mercury
   - Mars
   - Jupiter
   - Saturn

   Check out *Stellarium*'s feature to locate yourself on another planet and watch the analemma from there!

   And finally, the Moon's (lunar) analemma? What are the respective eccentricities and obliquities of these orbits?

A few references are provided in the "Bibliography" section of this task.

A good page with phantastic illustrations can be found at https://analemma.com/intro.html . Beware though that the mathematical details are insufficient on that page. Another page in Spanish is the one of the AARS: https://relojesdesol.info/node/748 .