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Towards a unified framework for high and low frequency return volatility modeling*

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This paper provides a selective summary of recent work that has documented the usefulness of high-frequency, intraday return series in exploring issues related to the more commonly studied daily or lower-frequency returns. We show that careful modeling of intraday data helps resolve puzzles and shed light on controversies in the extant volatility literature that are difficult to address with daily data. Among other things, we provide evidence on the interaction between market microstructure features in the data and the prevalence of strong volatility persistence, the source of significant day-of-the-week effect in daily returns, the apparent poor forecast performance of daily volatility models, and the origin of long-memory characteristics in daily return volatility series.

Key Words and Phrases: high-frequency data, ARCH, stochastic volatility, intraday seasonal, long memory.

1 Introduction

Following the introduction of the Autoregressive Conditional Heteroskedastic (ARCH) process in ENGLE (1982) and its extension into Generalized ARCH, or GARCH, in BOLLERSLEV (1986), research into time series modeling of financial market volatility has grown tremendously. This is documented in the surveys of, e.g., BOLLERSLEV, CHOU and KRONER (1992), BOLLERSLEV, ENGLE and NELSON (1994), GHYSELS, HARVEY and RENAULT (1996) and SHEPHARD (1996). The voluminous literature is inspired by the remarkable degree of robustness of the findings across asset classes, institutional settings and return frequencies. The standard ARCH or

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stochastic volatility models appear to fit the dynamic features of daily, weekly, monthly and quarterly return series from equity, commodity, bond, and currency markets about equally well in spite of the widely differing underlying claims and distinct institutional arrangements that surround trading in these securities. Moreover, the measures of volatility persistence are fairly similar across asset classes and adhere quite well to theoretical predictions regarding the short-run rate of decay in the estimated persistence measures as the return interval lengthens. In short, standard volatility models provide a natural first approximation to the statistical properties of univariate financial return series. Given the huge volume of literature and the plethora of applications to issues such as asset and derivatives pricing, portfolio allocation and risk management, it is hard not to deem this branch of the literature an unqualified success.

Nonetheless, several recent empirical findings have raised questions about the general usefulness of standard volatility models. First, with the advent of tick-by-tick data, volatility modeling has been pursued at still higher return frequencies, and it has become evident that the models break down at the intraday level. They no longer obey theoretical prescriptions regarding the degree of volatility persistence and, more generally, the parameter estimates defy the aggregation results linking the models at different return frequencies. In addition, there are numerous regularities in the intraday data that are entirely absent in the standard volatility specifications, such as pronounced, and largely deterministic, intraday volatility patterns and short-lived bursts in volatility associated with news releases. These features raise questions about the internal consistency of the volatility models, and how they relate to more specific market microstructure features. Furthermore, it questions the applicability of the models in an area of enormous practical relevance: the real-time environment of trading professionals such as money managers, portfolio insurers, derivatives traders and central bankers. Second, there is a large literature studying the forecast performance of volatility models at daily and weekly frequencies, both in-sample and out-of-sample. One typical approach is to measure the amount of variation in squared returns that is explained (predicted) by the corresponding one-step-ahead volatility forecasts. Mechanically, this is accomplished by regressing the realized squared returns on the associated forecasts, and then gauging the explanatory power via the regression R^2. Such studies report very low R^2 measures, typically in the order of 1–9%. Does this imply that a practitioner should pay little attention to volatility forecasts, since they only capture a small fraction of the ex-post return variability? A related set of findings is that alternative volatility models are hard to distinguish in terms of their forecast performance and, in particular, there is little stability in the models that perform best, either across assets or subsamples. The discrepancy between the highly significant in-sample parameter estimates on the one hand, and the apparent poor forecast performance and lack of robustness in terms of identifying the best performing model on the other has generated a certain amount of skepticism regarding the practical use of these models. Third, there is accumulating evidence that the standard models do not adequately capture the volatility persistence of financial return series in the very long run. Specifically, the implied geometric rate of decay in return variability appears to be too rapid relative to the long-run hyperbolic rate that has been documented in recent studies. While the ARCH and stochastic volatility models readily may be extended to encompass fractionally integrated processes that accommodate such long-memory features, there is a powerful alternative explanation for the findings, namely that there are infrequent structural breaks in the volatility generating process. In fact, the hypothesis of structural breaks in the mean dynamics has spurred a large literature in macroeconomics, focusing on whether unit roots (or very high persistence) is an inherent characteristic of the data or induced by rather infrequent structural shifts.

To investigate these issues, we rely on both theory and empirical work, but the unifying theme is that an understanding of the high-frequency features holds the key to improved inference, also at the lower frequencies. In so doing, we largely dispel the above reservations regarding standard volatility models. First, it is correct that other features are very important at the intraday level, including predictable intradaily activity patterns which induce corresponding volatility patterns across the trading day, and a systematic impact of news that generate bursts of volatility quite distinct from the longer-lasting effects captured by standard models. Nonetheless, we also document the presence of a persistent, slowly-changing volatility factor in intraday returns. This is the one factor that systematically ‘‘survives’’ aggregation to the daily level, and thus the one which is readily identified from lower-frequency returns. In contrast, the intraday pattern is largely annihilated by aggregation to the daily level, while the volatility impact of news announcements are muted. Such findings suggest how markets with diverse microstructures may produce similar volatility dynamics in lower frequency returns, although they may display very different characteristics at the higher frequencies. In essence, the level of liquidity, discreteness, intraday activity pattern and importance of news releases may differ, but the lower frequency behavior is still governed by the systematic and persistent volatility component which may be fairly similar across the asset classes and institutions.

Second, we show that the claims of poor volatility forecasting performance derive from a failure to contemplatle the workings of standard volatility models. By construction, these models are geared towards the systematic tendencies for returns to be more or less variable, i.e., their conditional variance. If, as is the case, daily returns possess a large idiosyncratic and unpredictable component, then it follows that observation-by-observation realized daily return variability must be predicted with very low precision. Logically, this is an implication of standard volatility models, and thus hardly an argument to use against them. In fact, we document that the real-world forecast performance with regard to squared returns is almost as good as one would expect from theory. Perhaps this is small consolation, as it leaves the question of usefulness unanswered: Do standard volatility models actually provide good forecasts? The answer is yes! The economically relevant concept of volatility is the latent conditional variance that drives return variability. By using high-frequency returns, we are able to measure this unobserved process at the daily level with a much higher degree of precision than is possible from daily returns. This is not only true in theory, but also confirmed in practice. What emerges is a striking resemblance between the theoretical forecast performance of volatility models and the ability of these models to predict the relevant volatility proxy at the daily level. Rather than concluding that the forecast performance is poor, we deem it remarkably good, given that any such model must suffer from some degree of misspecification. As such, the volatility forecasts are clearly highly relevant for a wide range of applications.

Third, an investigation of long memory in return volatility may seem to require a very long span of returns, and this raises the associated problem of testing for structural breaks. Instead, we pursue an approach that is inspired by theory. Rather than fixing the observation frequency at, e.g., the daily level, we note that the degree of fractional integration, under reasonable assumptions, should be invariant to the return interval. Thus, high-frequency returns should display the identical degree of long memory, and there are plenty of intraday observations over shorter time spans, so standard nonparametric estimates of the degree of fractional integration are feasible. This allows for direct comparison of the long-memory features across return frequencies, and facilitates investigation of the structural stability of this characteristic over time. Our empirical results are telling. The estimated coefficient of fractional integration is very similar across the high-frequency returns, and it is consistent with estimates obtained from much longer daily return samples. This supports the hypothesis that long memory is inherent to the return generating process, rather than an artifact of infrequent structural breaks. In addition, we provide an explicit model of information flow that is consistent with these features, thus documenting that there are potential rational economic forces behind the findings. A deeper comprehension of the sources of long-run dependencies in return volatility is potentially relevant for a wide range of issues. On the basis of our findings, we conjecture that high-frequency returns are going to be vital for further progress in this area.

The remainder of this article is structured as follows. Section 2 describes the data series underlying the empirical evidence. Section 3 illustrates the failure of the standard GARCH model at the intraday frequencies and links these findings to the pronounced volatility patterns in the high-frequency returns. Section 4 presents an explicit intraday volatility model that accommodates the intraday pattern, and then shows how it is possible to successfully recover the standard ARCH component from intraday returns. Additional market microstructure issues and their impact on the day-of-the-week effect are also discussed. Section 5 combines these findings with the recently developed aggregation theory for stochastic volatility diffusions to provide novel evidence regarding the forecasting performance of volatility models at the daily level. Section 6 demonstrates that high-frequency returns shed light on the issue of long memory in volatility. Finally, section 7 concludes.

2 Data and notation

To keep the exposition focused and manageable, all empirical work is based on data from a single market, the Deutschemark-U.S. dollar (DM-$) spot foreign exchange interbank market. This is not problematic, as qualitatively similar results have been documented for returns computed from future contracts on the Standard and Poors 500 equity index as well as the yen-dollar exchange rate. Our daily DM-\$ series covers March 14, 1979–September 29, 1993, excluding weekends and holidays, for a total of 3649 observations, i.e., $r_t = r_{(1),t}$; $t = 1, \ldots, 3649$. The notation, $r_{(m),t}$, denotes returns obtained by sampling m times per day ($m = 1/k, k \geq 1$) from the daily series, so five-day (weekly) returns are termed $r_{(1/5),t}$, while r_t, as indicated, is shorthand for the daily return realized on day t. The intraday returns are computed from all DM-\$ quotes appearing on the interbank Reuters network over the final year of our daily sample, i.e., October 1, 1992, through September 30, 1993. Since we focus on the relation between intraday and daily volatility, and not tick-by-tick data per se, the quotes are converted to continuously-compounded returns. First, an equidistant (log-) exchange rate series is defined by the linearly interpolated logarithmic midpoint of the recorded bid and ask quotes. Next, the corresponding returns are obtained via the (log-) differenced series. For further details on the construction of the return series, see Andersen and Bollerslev (1997a). Unfortunately, the properties of the return series at the very highest frequencies are heavily influenced by the quoting intensity, the discreteness of prices, and the tendency of foreign exchange dealers to position their quotes with a view towards inventory control. Only at the five-minute frequency does the spurious impact of these idiosyncratic microstructure features largely vanish. Further, in order to avoid confounding the evidence by the decidedly slower trading patterns over weekends, all returns from Friday 21:00 Greenwich Mean Time (GMT) through Sunday 21:00 GMT were excluded (the market effectively closes in the weekends). Thus, our sample covers 288 five-minute intervals over 260 days, for a total of 74,880 five-minute return observations: i.e., $r_{t,n} = r_{(288),t,n}$; $n = 1, \ldots, N$; $t = 1, \ldots, 260$, where $N = 288$ is the number of daily intervals. In analogy to the interdaily notation, $r_{(k),t,n}$ refers to the return over the nth interval on day t, (t, n), obtained by sampling the returns k times per day, $1 \leq k \leq 288$; $n = 1, \ldots, k$; and $r_{t,n}$ is shorthand for five-minute returns.

3 Intraday volatility patterns and volatility dynamics

Standard volatility models such as the popular GARCH(1,1) specification generally provide a good first approximation to the statistical features of financial return series at the daily, weekly or monthly horizons. Our DM-dollar series is no exception. To formally introduce the model, we let $\sigma^2_{(m),t}$ denote the conditional variance of $r_{(m),t}$ based on information through time $t - 1/m$. With a sampling frequency of m observations per day, the GARCH(1,1) model for $r_{(m),t}$ is given by the system,

$$r_{(m),t} = \sigma_{(m),t} \cdot z_{(m),t} \tag{1}$$

and

$$\sigma^2_{(m),t} = \psi_{(m)} + \alpha_{(m)} \cdot (\sigma_{(m),t-1/m} \cdot z_{(m),t-1/m})^2 + \beta_{(m)} \cdot \sigma^2_{(m),t-1/m} \tag{2}$$

where $\psi_{(m)} > 0$, $\alpha_{(m)} \geq 0$, $\beta_{(m)} \geq 0$, and $z_{(m),t}$ is i.i.d. with mean zero and variance one. It is straightforward to allow for a time-varying conditional mean, but mean variation at the high-frequency level is trivial in comparison with that of the second moments, and it has no impact on the estimated volatility dynamics. Since we focus on volatility, we avoid the notational burden associated with the presence of a non-zero mean term, although some of our empirical results are obtained from models that include simple ARMA-specifications for the expected return.

Consistent estimation and robust inference of the system (1)–(2) may be obtained by quasi-maximum likelihood (QML), see Bollerslev and Wooldridge (1992). This is important since specific distributional assumptions regarding the return innovation at a given frequency—so-called strong GARCH—invalidate the (strong) GARCH model at any other return horizon. Hence, to assess estimates across return frequencies within a coherent framework, we must relax the GARCH concept. Drost and Nijman (1993) provide the useful notion of weak GARCH. Heuristically, if GARCH(1,1) applies at one frequency, then equations (1)–(2) remain valid at all lower frequencies, except that the volatility term, $\sigma^2_{(m),t}$, no longer represents the conditional variance, but rather the linear projection of $r^2_{(m),t}$ on the Hilbert space spanned by $\{1, r_{(m),t-j/m}, r^2_{(m),t-j/m}\}_{j\geq 1}$. Within this framework, the interdaily volatility persistence parameters are tied to the ones at the daily level via the simple relation, $\alpha_{(m)} + \beta_{(m)} = (\alpha_{(1)} + \beta_{(1)})^{1/m}$. This implies, for example, that the imputed half-life of a shock to the volatility process should be constant across frequencies. Table 1 reports the estimated persistence parameters across frequencies between the daily and biweekly horizon along with the imputed half-lives. The relatively smooth decay in the persistence measure, $\alpha_{(m)} + \beta_{(m)}$, as the horizon lengthens and the stability of the half-lives suggest that the basic model performs well.

It seems natural to extend the above findings to the intraday setting. However, a reading of the literature reveals that the evidence in support of the GARCH specification in this context is, at best, mixed. Again, our DM-dollar series is representative. Table 2 records the estimated volatility persistence parameters for our five-minute return sample at various intraday frequencies. Initially, as the return horizon is lowered from the daily to the four-hour level, the persistence measure continues to increase. However, as the sampling frequency is elevated to the hourly level the volatility persistence measure collapses, only to resume its gradual increase as the frequency is heightened further towards the five-minute level. Moreover, the latter development is accompanied by relatively large estimated values of $\alpha_{(k)}$. This is inconsistent with the temporal aggregation results of Drost and Nijman (1993) and the results on continuous record asymptotics of GARCH(1,1) in Nelson (1990, 1992). Although $\alpha_{(k)} + \beta_{(k)}$ should converge to unity, as $k \to 0$, this should occur as $\alpha_{(k)} \to 0$ and $\beta_{(k)} \to 1$. In contrast, the estimated α-coefficients at the highest intraday frequencies exceed the corresponding estimates at the interdaily level. These findings are consistent with the extant literature. Engle, Ito and Lin (1990) and Hamao, Masulis and Ng (1990) primarily rely on returns over six hours or longer, and they report estimates of volatility persistence that are consistent with those from daily data.

Table 1. Persistence of MA(1)-GARCH(1,1) Models for Daily DM-dollar Exchange Rates

$$r_{(m),t} \equiv \Sigma_{\tau=t-1/m,\ldots,t} r_\tau = \mu_{(m)} + \theta_{(m)}\varepsilon_{(m),t-1/m} + \varepsilon_{(m),t} \qquad t = 1, 2, \ldots, [mT]$$
$$\sigma^2_{(m),t} = \omega_{(m)} + \alpha_{(m)}\varepsilon^2_{(m),t-1/m} + \beta_{(m)}\sigma^2_{(m),t-1/m}$$

$1/m$	$[Mt]$	$\alpha_{(m)}$	$\beta_{(m)}$	$\alpha_{(m)} + \beta_{(m)}$	Half Life
1	3649	0.105	0.873	0.978	31.2
		(0.015)	(0.015)		
2	1824	0.150	0.784	0.934	20.6
		(0.024)	(0.026)		
3	1216	0.106	0.813	0.919	24.8
		(0.021)	(0.037)		
4	912	0.167	0.713	0.879	21.6
		(0.036)	(0.042)		
5	729	0.182	0.611	0.794	15.0
		(0.049)	(0.081)		
6	608	0.191	0.646	0.838	23.5
		(0.049)	(0.060)		
7	521	0.129	0.674	0.803	22.2
		(0.049)	(0.071)		
8	456	0.170	0.563	0.733	17.8
		(0.051)	(0.176)		
9	405	0.133	0.641	0.774	24.3
		(0.067)	(0.230)		
10	364	0.174	0.434	0.607	13.9
		(0.079)	(0.186)		

The returns are based on 3649 daily quotes for the Deutschemark-U.S. dollar spot exchange rate from March 14, 1979, through September 29, 1993. Weekend and holiday quotes have been excluded. The length of the return interval equals $1/m$ days, for a total of $[Mt]$ observations, where [.] denotes the integer value. The half life of a shock to the conditional variance is calculated as $-\log(2)/\log(\alpha_{(m)} + \beta_{(m)})$, and is subsequently converted to trading days.

In contrast, Baillie and Bollerslev (1991) and Foster and Viswanathan (1995), on using hourly and half-hourly returns, find much lower volatility persistence. Moreover, Bollerslev and Domowitz (1993) report five-minute GARCH(1,1) estimates for $\alpha_{(k)} + \beta_{(k)}$ close to one but, as in Table 2, $\hat{\alpha}_{(k)}$ seems too large. Finally, Locke and Sayers (1993) find that one-minute returns display little volatility persistence, while Goodhart et al. (1993) detect very strong persistence in quote-by-quote data, but also note that the persistence declines markedly once information events are taken into account, thus suggesting that news arrivals may overwhelm the conditional heteroskedasticity at the extreme high frequencies.

These findings may appear puzzling. Why do the ARCH effects drop off precipitously at certain intraday frequencies, only to strengthen again at the very highest frequencies, albeit in a manner that is inconsistent with the observed effects at the interdaily horizons? Does this signal a fundamental breakdown in the ARCH process at the intraday level? Andersen and Bollerslev (1997a) instead point to the impact of a pronounced intraday return volatility pattern. Such regular intraday

Table 2. Persistence of MA(1)-GARCH(1,1) Models for Intraday DM-dollar Exchange Rates

$$r_{(k),t,n} \equiv \Sigma_{i=(n-1)N/k+1,\ldots,nN/k} r_{t,i} = \mu_{(k)} + \theta_{(k)}\varepsilon_{(k),t,n-1} + \varepsilon_{(k),t,n}$$
$$\sigma^2_{(k),t,n} = \psi_{(k)} + \alpha_{(k)}\varepsilon^2_{(k),t,n-1} + \beta_{(k)}\sigma^2_{(k),t,n-1} \qquad t = 1, 2, \ldots, 260;\ n = 1, \ldots, k$$

k	Tk	$\alpha_{(k)}$	$\beta_{(k)}$	$\alpha_{(k)} + \beta_{(k)}$	Half Life
288	74,880	0.193	0.822	1.015	∞
		(0.011)	(0.009)		
144	37,440	0.229	0.774	1.003	∞
		(0.012)	(0.008)		
96	24,960	0.273	0.708	0.981	533
		(0.018)	(0.014)		
72	18,720	0.287	0.677	0.964	375
		(0.019)	(0.016)		
48	12,480	0.322	0.579	0.901	200
		(0.035)	(0.033)		
36	9360	0.286	0.581	0.868	195
		(0.028)	(0.037)		
32	8320	0.306	0.521	0.828	165
		(0.035)	(0.042)		
24	6240	0.311	0.395	0.706	119
		(0.047)	(0.069)		
18	4680	0.261	0.456	0.718	167
		(0.039)	(0.074)		
16	4160	0.270	0.246	0.516	94
		(0.061)	(0.124)		
12	3120	0.018	0.969	0.988	6771
		(0.015)	(0.026)		
9	2340	0.016	0.975	0.991	12,159
		(0.008)	(0.013)		
8	2080	0.011	0.978	0.989	11,311
		(0.004)	(0.005)		
6	1560	0.011	0.979	0.990	17,084
		(0.004)	(0.005)		
4	1040	0.007	0.980	0.987	19,585
		(0.005)	(0.004)		
3	780	0.014	0.969	0.983	19,637
		(0.008)	(0.007)		
2	520	0.010	0.960	0.970	16,329
		(0.010)	(0.007)		

The returns are based on 288 interpolated five-minute logarithmic average bid–ask quotes for the Deutschemark-U.S. dollar spot exchange rate from October 1, 1992 through September 29, 1993. Quotes from Friday 21:00 Greenwich Mean Time (GMT) through Sunday 21:00 GMT have been excluded, resulting in a total of 74,880 return observations. The length of the different intraday return sampling intervals equal $5 \cdot k$ minutes. The model estimates are based on T/k non-overlapping return observations. The $\alpha_{(k)}$ and $\beta_{(k)}$ columns give the Gaussian quasi-maximum likelihood estimates for the GARCH(1,1) parameters. Robust standard errors are reported in parentheses. The half life of a shock to the conditional variance at sampling frequency k is calculated as $-\log(2)/\log(\alpha_{(k)} + \beta_{(k)})$, and converted into minutes.

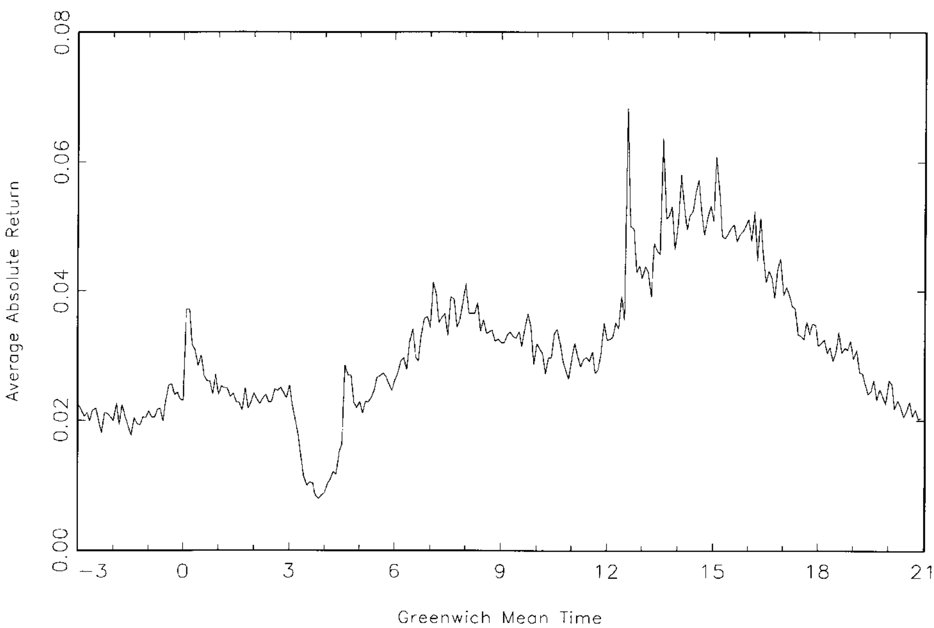

Fig. 1. Intraday Volatility Pattern. The figure displays the average absolute DM–dollar spot exchange rate returns for all five-minute intervals across the 24-hour trading day over the October 1, 1992, through September 30, 1993, sample period. The first interval corresponds to 20:55–21:00 GMT and the last represents 20:50–20:55 GMT. Returns over the weekend are excluded, resulting in a total of 260 trading days and 74,880 separate five-minute returns.

volatility features have been documented in equity markets by, e.g., WOOD, MCINISH and ORD (1985) and HARRIS (1986), and in foreign exchange markets by MÜLLER et al. (1990) and BAILLIE and BOLLERSLEV (1991). Figure 1 displays the average absolute five-minute returns over the 24-hour cycle, computed from our one-year DM-dollar sample. The trading day is defined from 21:00 (denoted by −3:00 in the figure) Greenwich Mean Time (GMT) to 21:00 GMT the following day. Inspection shows that the intraday volatility pattern largely reflects the ebb and flow of trading activity across the globe. The trading day starts in the Pacific zone, where banks located in New Zealand and Australia account for the majority of trading. At exactly 0:00 GMT, or 9:00 local Tokyo time, the Japanese market opens, and trading in Singapore and Hong Kong also picks up. This presumably accounts for the sudden jump in return volatility at this time. The drop between 3:00 and 4:30 GMT coincides with the regulated lunch period in the Tokyo market. Following the Tokyo lunch, the European markets begin trading, thus inducing a secular increase in return volatility. The subsequent drop is associated with the closing of the Asian segments of the market. Finally, volatility rises sharply as trading in North America opens up, producing an overlap in activity between the two primary centers for DM-dollar trading, London and New York. The subsequent decline reflects the closing of first the European and next the American trading segments.

Is it reasonable to expect this systematic volatility pattern to interfere with ARCH estimates at the daily level? The potential effect may be gauged from the magnitude of the intraday effects. From Figure 1, the average return volatility at the five-minute level fluctuates predictably by a factor of about 250%, even upon excluding the exceptional drop-off during the Tokyo lunch period. In contrast, a discrete jump in the GARCH(1,1) volatility factor of 25% from one day to the next is unusually large. Thus, it is evident that the systematic changes in return volatility within the day, associated strictly with the fluctuations in trading activity, may easily overwhelm the day-to-day movements in volatility due to ARCH effects at the daily frequencies. If this is correct, the typical correlation pattern in absolute and squared returns will be distorted at the intraday return horizons. This is indeed what happens, as can be seen in Figure 2. As lag-length increases, a gradual decline in the average correlations is visible, but it is largely obscured by a pronounced periodic, distorted u-shape pattern. The latter virtually seems to be superimposed on top of the gradually declining correlogram that one would expect from a high-frequency GARCH model. Moreover, the period of the u-shaped pattern is exactly 288 intervals, or one day. This suggests that the aggregation to daily returns serves to annihilate the impact of the intraday volatility pattern. Consequently, it is natural to conjecture that it is necessary to model jointly both the intraday pattern and the ARCH effects in order to recover

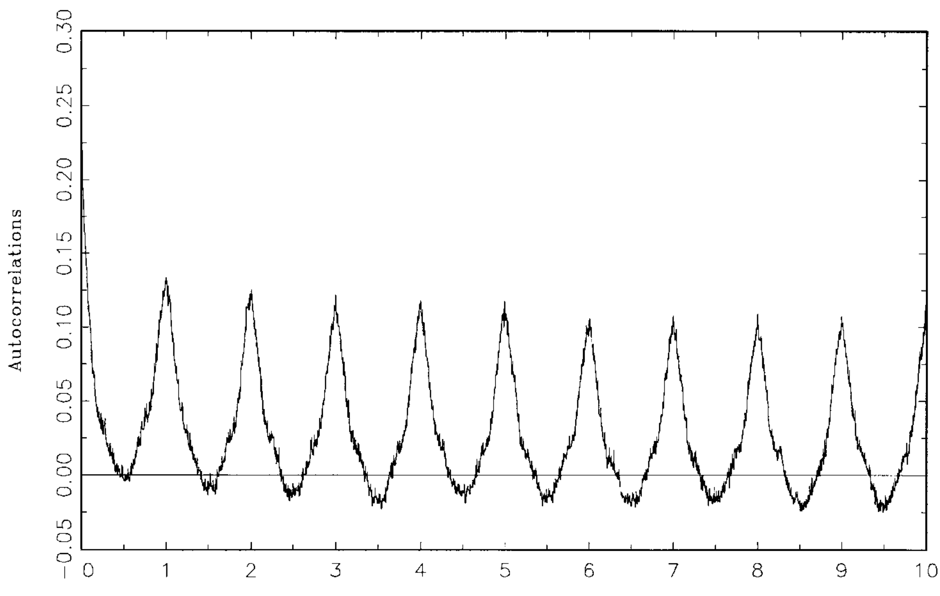

Fig. 2. Absolute Return Correlogram. The figure depicts the autocorrelations for demeaned five-minute absolute DM–dollar returns, calculated from observations over the October 1, 1992, through September 29, 1993, sample period. Returns over weekends, the Tokyo lunch period, 3:00–4:45 GMT, the main holidays and a few breakdowns in the data transmission are not included, resulting in a total of 69,420 five-minute returns.

meaningful estimates of the standard ARCH effects at the high-frequency level. We turn to this task in the following section.

4 A component model of the intraday return volatility process

This section provides a model for intraday return volatility that accounts for both the standard ARCH dynamics and the pronounced intraday volatility pattern. Moreover, specific features, e.g., short-lived volatility bursts associated with the release of macroeconomic news, are readily accommodated.

A natural extension of the return (innovation) equation (1) to a high-frequency setting, accommodating intraday volatility patterns and other systematic non-ARCH effects, takes the form

$$r_{(k),t,n} = \sigma_{(k),t,n} \cdot s_{(k),t,n} \cdot z_{(k),t,n} \tag{3}$$

where $\sigma_{(k),t,n}$ denotes a standard time-series (ARCH) based estimate of the conditional variance of $r_{(k),t,n}$ based on information through time $(t, n-1)$, $s_{(k),t,n}$ signifies the predictable component of the intraday volatility pattern as well as any additional systematic factors impacting the expected return volatility over the next interval of time, and $z_{(k),t,n}$ represents an i.i.d. error term with unit variance. Thus, the model generalizes the standard specification by including an additional multiplicative factor in the volatility process beyond the usual ARCH component.

A useful decomposition is achieved by squaring equation (3), taking logs, and rearranging,

$$2 \log | r_{(k,t,n} | - \log \sigma^2_{(k),t,n} = c + 2 \log s_{(k),t,n} + u_{(k),t,n} \tag{4}$$

where $c = E[\log z^2_{(k),t,n}]$, and $u_{(k),t,n} = \log z^2_{(k),t,n} - E[\log z^2_{(k),t,n}]$. Equation (4) may be interpreted as a regression relating the systematic deviation between the intraday squared returns and the associated ARCH-based volatility forecasts to a sole explanatory variable, the intraday volatility component, $s_{(k),t,n}$. However, in order to operationalize this "regression" interpretation of equation (4), we require an observable representation, or proxy, for the intraday ARCH-volatility component and the return innovation on the left-hand side as well as a specific parameterization of $s_{(k),t,n}$ in terms of measurable variables on the right-hand side. Theory has little to say regarding the specific shape of the intraday pattern, so we utilize a semi-nonparametric representation of this feature, essentially allowing the data to determine the shape of the pattern. A parsimonious specification, allowing for a smooth intraday pattern and the presence of dummy variables to capture specific discontinuities associated with market openings or news arrivals, is given by the Fourier flexible form (FFF) introduced by GALLANT (1981). Furthermore, we let the return innovation be given as the demeaned (by the sample mean) five-minute return, say $\hat{r}_{t,n}$, and $\sigma_{(k),t,n} = \hat{\sigma}_{(1),t}/N^{1/2}$, where $\hat{\sigma}_{(1),t}$ denotes the one-day-ahead GARCH(1,1) volatility forecast implied by parameter estimates obtained over the long daily sample that predates our annual five-minute sample, along with the daily return innovations observed through day $t-1$. The normalization factor $N^{1/2}$ simply converts the daily volatility factor into the high-frequency equivalent, assuming that the ARCH-volatility component is constant over the trading day. In summary, we obtain the following practical regression

$$x_{t,n} \equiv 2 \log |\hat{r}_{t,n}| - \log \hat{\sigma}^2_{(1),t} + \log N = \hat{c} + f(\theta, t, n) + \hat{u}_{(k),t,n} \tag{5}$$

where $c = E[\log z^2_{(k),t,n}] + E[\log \sigma^2_{(1),t} - \log \hat{\sigma}^2_{(1),t}]$, the error process $\{\hat{u}_{(1),t,n}\}$ is stationary, and the Fourier flexible form regressor is given by

$$f(\theta, t, n) = \mu_0 + \sum_{d=1}^{D} \lambda_d I_d(t, n) + \sum_{p=1}^{P} (\delta_{c,p} \cos 2\pi pn/N + \delta_{s,p} \sin 2\pi pn/N) \tag{6}$$

The sinusoids in (6) accommodate any smooth variation in the intraday pattern while the (zero-one) indicators, $I_d(t, n)$, allow for discontinuities in response to predetermined events that occur in the interval indexed by (t, n), such as macroeconomic announcements or the opening of trade in a relevant market segment. The strength of this effect is governed by the associated coefficient, λ_d, whereas the overall shape of the intraday pattern is captured by the δ-coefficients. By construction, the trigonometric terms have, as desired, a strict period of one trading day. A polynomial term may be added to the FFF expansion, but was deemed insignificant in the empirical work, and thus omitted here. Although the semi-nonparametric FFF representation is purely deterministic, our estimation procedure readily incorporates a stationary error process, allowing the FFF-functional to only reflect the systematic or expected, average effects.

The indicator variables may, as suggested, be used to capture predictable volatility shocks associated with specific events. A prominent example is the release of scheduled macroeconomic announcements in the U.S. at either 8:30 or 10:00 Eastern Standard Time (EST). Figure 3 displays the average absolute five-minute returns on days that were known in advance to contain the release of major U.S. macroeconomic statistics. The pronounced volatility spikes (or price jumps) at the exact moment of the news release, plus the subsequent heightened price variability for a period of about one or two hours, suggests that a unique "announcement effect"—distinct from ARCH effects and the intraday pattern—is present in the volatility process. Although the impact of these events will be estimated via their average effect (across several announcements) on the volatility pattern, it is obvious that individual announcements affect the price and volatility path differently depending on the size of the surprise component in the released figures relative to prior consensus forecasts and current market conditions. However, there is a large element of idiosyncratic noise associated with each event, and strong heteroskedasticity and serial correlation in the error terms of equation (5) are to be anticipated in the wake of a news release. Other candidate phenomena for predictable volatility effects in the foreign exchange interbank market

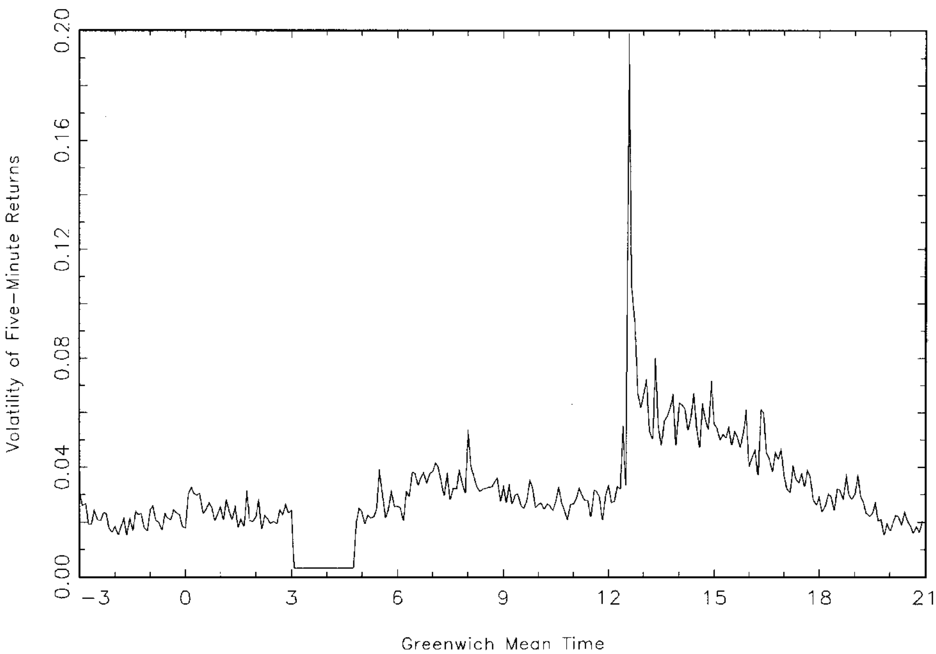

Fig. 3. Announcement Day Volatility (Summer Time). The figure plots the average absolute five-minute DM-dollar return starting with the 20:55–21:00 GMT interval and ending at 20:50–20:55 GMT for days with regularly scheduled U.S. macroeconomic announcements during the U.S. Summer Time regime over the October 1, 1992, through September 29, 1993, sample period, except that the Tokyo lunch period, 3:00–4:45 GMT, is assigned an artificially low return. These days each contain at least one release, at 8:30 Eastern Standard Time (12:30 GMT), of one of the following U.S. macroeconomic announcements: the employment report, the merchandise trade deficit, the producer price index, the advance durable goods report, estimates or revisions to the gross domestic product, retail sales, housing starts, leading indicators, and new jobless claims.

are regional holidays, regulated market openings (as exist in Japan), potential day-of-the-week effects, and weekend effects that may impact the market conditions on Monday morning and Friday afternoon. All of these are fundamentally distinct from the ARCH effects and the intraday pattern, and they are predictable as a function of calendar time, and thus anticipated by market participants, justifying their separate treatment.

Although the regression (5) may appear complicated, it is not hard to verify that the right hand side coefficients are estimated consistently by a simple OLS-regression, if the innovations to the ARCH-volatility process are independent of any stochastic error terms associated with the intraday effects and other included calendar-driven features, thus precluding correlation between regressors and error terms. In fact, we may even allow for misspecification of the interdaily volatility process, $\hat{\sigma}^2_{(1),t}$, under the weak requirement that the daily volatility estimation error constitutes a strictly stationary process which may simply be subsumed in the general error term, $\hat{u}_{(k),t,n}$. From this perspective, the main reason to include the time-varying daily volatility factor, $\hat{\sigma}^2_{(1),t}$, is to alleviate the serial correlation of the regression error, which should enhance the efficiency of the estimates in the final step. In the current application, we find that the intraday pattern is sufficiently dominant at the high-frequency level that the inference regarding the FFF-coefficients is virtually unaffected if we use the sample average $\bar{\sigma}$ instead of $\hat{\sigma}_{(1),t}$ in the regression (5). Finally, note that the log-transform, beyond affording tractability, reduces the impact of outliers on inference. The average five-minute price change is very small, so occasional jumps associated with five-minute returns of a quarter per cent or more represent large outliers that may distort the regression. Our procedure effectively eliminates such concerns.

Practical implementation of the FFF procedure requires a good deal of judgment and experimentation with the functional forms and lag lengths involved in the specification of $f(\theta, t, n)$. Here, we simply rely on the representation of Andersen and Bollerslev (1998a). First, the Tokyo lunch period is excluded from the estimation because of an exceptionally low quoting activity which renders reliable inference impossible. This is effectively a "weekend period", where trading occurs mostly in response to truly dramatic developments, resulting in a sample dominated by zero returns (no quoting) and a few large outliers. Second, the FFF-expansion includes only four sets of sinusoids, as additional terms provided insignificant improvements. Third, major announcements load onto a predetermined average volatility jump and a polynomial decay pattern that starts with the announcement and lasts for one hour (or two hours in the case of the main announcement, the Employment Report). Both U.S. and German announcements were identified, but only a few German ones are significant and these are not released according to a strict timetable as in the U.S. In the first round, we distinguish the U.S. Employment Report, announcements following the biweekly Bundesbank Meetings on German monetary policy, and Category I (U.S. GDP, trade balance, and durable goods) and Category II (U.S. PPI, retail sales, housing starts, leading indicators, jobless claims and factory orders, and German M3 figure) announcements. Fourth, a market opening dummy followed by a linear decay over one-half hour was used for the start of Japanese trading at 9:00 local time. Fifth, a regional holiday dummy captures the slowdowns that are less than true market closures, while more serious holiday periods (e.g., Christmas and New Year Day, U.S. Thanksgiving period) are dummied out in a fashion similar to the weekends and the Tokyo lunch. Sixth, Monday morning and Friday afternoons were allowed to be slower than the rest of the week due to a potential weekend spill-over effect. This impact is captured by two (polynomial regression) coefficients. Seventh, a number of significant failures in data transmission were identified and treated as market closures. Eight, specific day-of-the-week dummies were allowed. Ninth, the imposition of Summer Time in North America and Europe is accommodated by a one-hour parallel shift in the intraday pattern, and a "Summer slowdown" coefficient that reflects the increased gap between the North American and the Pacific trading regions during this period.

Table 3 displays the parameter estimates and associated robust *t*-statistics. All main features except for the day-of-the-week effects are significant. Interestingly, the latter

Table 3. Flexible Fourier Form Regressions

Parameter	Full System	Day-of-Week Effect Excluded	Day-of-Week, Daily Volatility Excluded
$\hat{c}$	−1.77	−1.76	−1.85
	[−32.8]	[−69.2]	[−56.3]
	(−79.4)	(−155.6)	(−162.0)
$\delta_{c,1}$	−0.12	−0.13	−0.13
	[−4.41]	[−4.58]	[−4.78]
	(−8.27)	(−8.62)	(−8.77)
$\delta_{c,2}$	−0.13	−0.13	−0.13
	[−4.93]	[−5.09]	[−5.10]
	(−8.16)	(−8.30)	(−8.26)
$\delta_{c,3}$	−0.28	−0.28	−0.29
	[−11.8]	[−12.0]	[−11.4]
	(−18.4)	(−18.5)	(−18.5)
$\delta_{c,4}$	0.14	0.14	0.14
	[8.10]	[8.01]	[8.10]
	(10.6)	(10.6)	(10.6)
$\delta_{s,1}$	−0.62	−0.62	−0.62
	[−24.1]	[−23.8]	[−23.4]
	(−38.6)	(−38.5)	(−38.4)
$\delta_{s,2}$	−0.21	−0.21	−0.21
	[−10.4]	[−10.2]	[−10.2]
	(−14.3)	(−14.1)	(−14.0)
$\delta_{s,3}$	0.17	0.18	0.17
	[8.64]	[8.76]	[8.55]
	(11.9)	(12.1)	(11.8)
$\delta_{s,4}$	−0.01	−0.01	−0.01
	[−0.68]	[−0.46]	[−0.68]
	(−0.91)	(−0.62)	(−0.94)
Summer Slowdown	−1.14	−1.15	−1.08
	[−5.91]	[−5.95]	[−5.06]
	(−10.6)	(−10.7)	(−9.93)
Tokyo Opening	0.59	0.58	0.59
	[8.96]	[8.91]	[9.06]
	(9.81)	(9.76)	(9.76)
Holiday	−0.698	−0.712	−0.703
	[−5.76]	[−6.28]	[−6.87]
	(−13.86)	(−15.25)	(−14.93)
Employment Report	1.755	1.746	1.739
	[10.38]	[10.47]	[8.82]
	(11.11)	(11.09)	(10.95)
Category I Announcement	0.997	0.991	0.992
	[7.23]	[7.28]	[7.48]
	(8.35)	(8.33)	(8.26)
Category II Announcement	0.627	0.620	0.619
	[6.71]	[7.03]	[6.89]
	(8.64)	(8.65)	(8.56)
Bundesbank Meeting	1.465	1.457	1.492
	[6.20]	[6.20]	[6.43]
	(10.01)	(9.99)	(10.13)

Table 3 continued over page

Table 3. Continued

Parameter	Full System	Day-of-Week Effect Excluded	Day-of-Week, Daily Volatility Excluded
Monday Early	−0.301	−0.368	−0.529
	[−0.26]	[−0.32]	[−0.45]
	(−0.36)	(−0.44)	(−0.63)
	0.001	0.069	0.208
	[0.001]	[0.047]	[0.14]
	(0.001)	(0.065)	(0.19)
Friday Late	−0.609	−0.412	−0.437
	[−2.04]	[−1.43]	[−1.39]
	(−3.36)	(−2.37)	(−2.49)
	0.068	0.011	0.017
	[0.49]	[0.08]	[0.12]
	(0.88)	(0.14)	(0.22)
Tuesday	−0.065		
	[−0.95]	—	—
	(−2.17)		
Wednesday	0.006		
	[0.08]	—	—
	(0.19)		
Thursday	0.050		
	[0.74]	—	—
	(1.65)		
Friday	0.096		
	[1.31]	—	—
	(2.99)		

The table reports the parameter estimates, with robust standard errors in square brackets and regular OLS standard errors in parentheses, for the regression of logarithmic squared demeaned five-minute DM-\$ returns on deterministic regressors capturing calendar and announcement effects. The returns are calculated from interpolated five-minute logarithmic average bid–ask quotes for the DM-\$ spot exchange rate over the October 1, 1992 through September 29, 1993 sample period. Quotes from Friday 21:00 GMT through Sunday 21:00 GMT are excluded, resulting in a total of 74,880 return observations. The robust standard errors reflect a Newey and West (1987) type correction incorporating 289 lags. The regression equation takes the form

$$x_{t,n} \equiv 2 \cdot \log[\,|\,r_{t,n} - \bar{r}\,|\,] - \log \hat{\sigma}^2_{(1),t} = \hat{c} + f(\theta;\, t, n) + \hat{u}_{t,n}$$

where $r_{t,n}$ denotes the five-minute returns for interval n on day t, $\bar{r}$ the sample mean of the five-minute returns, $\hat{\sigma}^2_{(1),t}$ is an a priori estimate of the overall daily level of the five-minute return standard deviation, $\hat{u}_{t,n}$ is a mean zero error term, and $f(\theta;\, t, n)$ represents the deterministic calendar and announcement regressors. The volatility estimates, $\hat{\sigma}_{t,n}$, for interval n on day t, are obtained from an estimated MA(1)-GARCH(1,1) model fit to a longer daily sample of DM-\$ spot exchange rates from March 14, 1979 through September 29, 1993. Denoting the daily return standard deviation estimate by $\hat{\sigma}_{(1),t}$, the daily volatility factor is captured by $\hat{\sigma}_{t,n} \equiv N^{-1/2} \cdot \hat{\sigma}_{(1),t}$. The "Daily Volatility Excluded" column indicates that $N^{-1/2} \cdot \bar{\sigma}$ is used in place of $\hat{\sigma}_{t,n}$, where $\bar{\sigma}$ denotes the sample mean of $\hat{\sigma}_{(1),t}$. The $f(\theta;\, t, n)$ function is given by

$$f(\theta;\, t, n) = \mu_0 + \sum_{d=1}^{D} \lambda_d \cdot I_d(t, n) + \sum_{p=1}^{4} \left(\delta_{c,p} \cdot \cos \frac{p2\pi}{N} n + \delta_{s,p} \cdot \sin \frac{p2\pi}{N} n \right)$$

During the U.S. Summer Time, the sinusoids are translated leftward by one hour and an additional restricted second order polynomial allows for a volatility slowdown between 19:00 and 24:00 GMT. The $I_d(t, n)$ regressors indicate either regular dummy variables (in the case of holidays or weekdays) or a prespecified volatility response pattern associated with a calendar related characteristic or an announcement. A separate linear volatility decay is allowed for the Tokyo open, 00:00–00:35 GMT. Similarly, a restricted second order polynomial adapts to the volatility slowdown around the weekends, i.e., early Monday morning, 21:00–22:30 GMT, and late Friday, 17:00–21:00 GMT (U.S. Winter Time) or 16:00–21:00 GMT (U.S. Summer Time). Finally, the volatility decay pattern following announcements are restricted to last one hour (13 intervals), except for the Employment Report pattern which lasts two hours (25 intervals). All of the response patterns are approximated by a third order polynomial restricted to reach zero at the end of the response horizon. The announcement coefficients measure the extent to which the absolute returns load onto this pattern following the announcement. Category I comprises U.S. announcements on GDP, the trade balance, and durable goods, while Category II covers U.S. releases of PPI, retail sales, housing starts, leading indicators, jobless claims, and factory orders, and the German M3 figures.

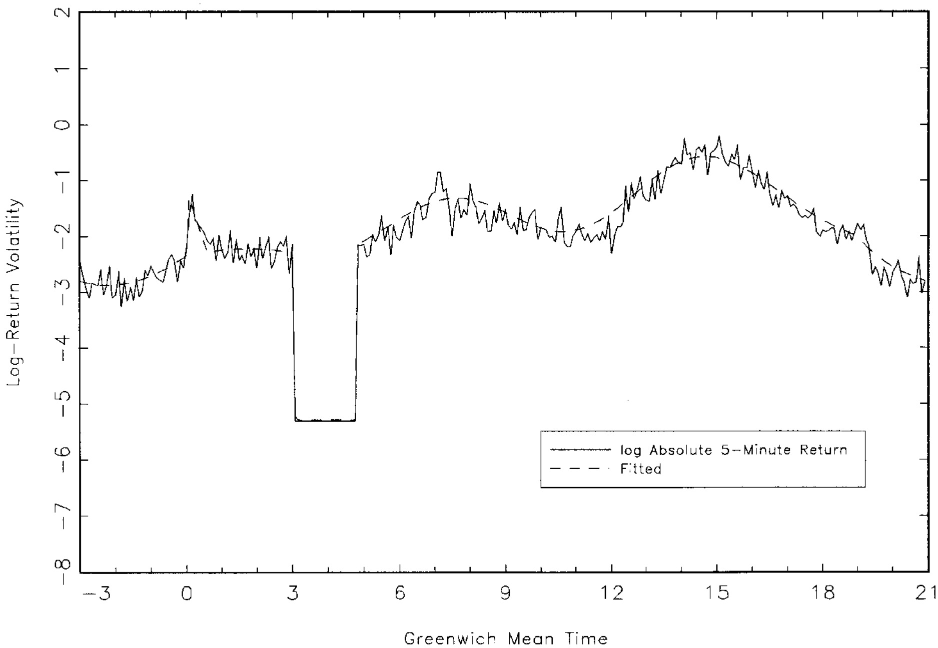

Fig. 4. Intraday Log-Return Volatility Fit (Summer Time). The figure graphs the Fourier flexible form fit to the average logarithmic squared, normalized and demeaned five-minute DM-dollar returns across the 24-hour weekday trading cycle plotted against the corresponding average sample values. The returns cover the U.S. Summer Time regime over the October 1, 1992, through September 29, 1993, sample period. Returns over the weekends are excluded. The GMT axis starts with the 20:55–21:00 GMT interval and ends at 20:50–20:55 GMT. The Tokyo lunch period, 3:00–4:45 GMT, is artificially assigned low returns, so this part is not fitted. The Summer Time average is based on 145 weekdays.

have been annihilated by the specific accounting for announcements which tend to be released on particular weekdays. This suggests that the significant day-of-the-week dummies reported in the extant literature may reflect a failure to incorporate scheduled news releases in the analysis. Since the day-of-the-week effect is largely void of economic justification, the following discussion relies on the estimates in column two, which exclude these dummies. The FFF-coefficients and the Tokyo opening effect define the estimated intraday volatility pattern. Figures 4 and 5 display these patterns in the log-volatility and the absolute return volatility dimension respectively. Included in the displays are the corresponding average (FFF-scaled logarithmic) absolute returns over all intervals during the U.S. Summer time period not affected by announcements, market closures, or regional holiday effects. Figure 4 indicates that the fit is excellent. The corresponding estimate in the absolute return dimension fares almost as well in spite of the more jagged appearance of the averaged absolute returns due to the higher degree of sensitivity to outliers.

It is straightforward to extract explicit estimates of the immediate and cumulative expected impact of scheduled announcements, regional holidays and the Japanese

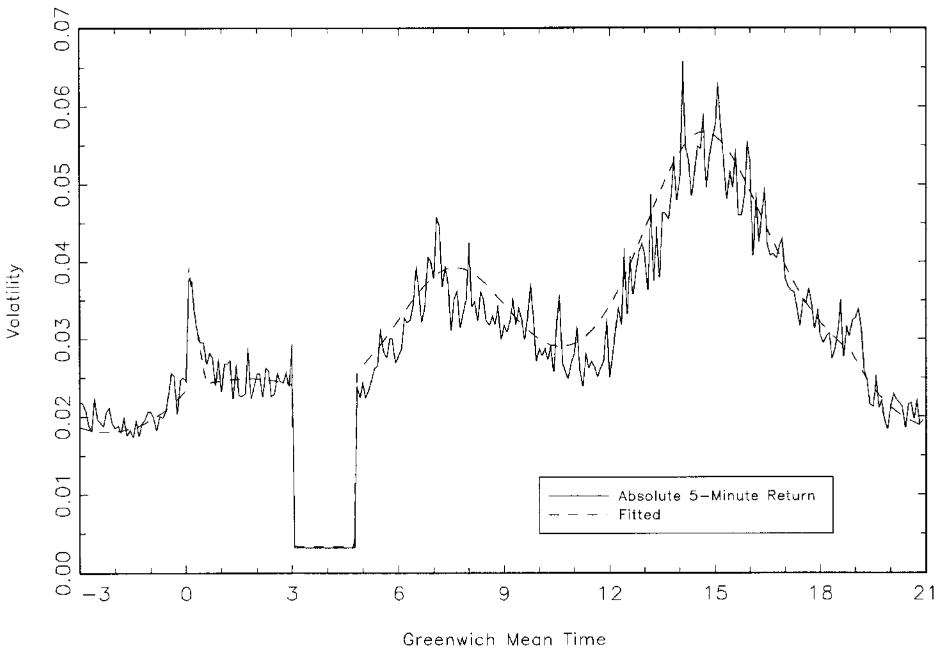

Fig. 5. Intraday Absolute Return Fit (Summer Time). The figure graphs the Fourier flexible form fit to the average absolute five-minute DM-dollar returns across the 24-hour weekday trading cycle plotted against the corresponding average sample values. The returns cover the U.s. Summer Time regime over the October 1, 1992, through September 29, 1993, sample period. Returns over the weekends are excluded. The GMT axis starts with the 20:55–21:00 GMT interval and ends at 20:50–20:55 GMT. The Tokyo lunch period, 3:00–4:45 GMT, is artificially assigned low returns, so this part is not fitted. The Summer Time average is based on 145 weekdays.

market opening for return volatility. In effect, this measures the economic significance of the various events in this particular metric. Since the magnitude of such effects is likely to be specific to the market under study, we refer the interested reader to Andersen and Bollerslev (1998a) for an example of this style of analysis. Of more general interest is the ability to recover the ARCH dynamics in the residual return innovations, once the systematic impact of all other factors have been accounted for. Formally, this is done via analysis of the *filtered returns*, $\hat{s}_{(k),t,n}^{-1}\hat{r}_{(k),t,n}$, which—absent estimation error—should have been stripped of the systematic impact of the intraday volatility pattern and other predictable non-ARCH components. As an illustration, contrast the raw absolute return correlogram in Figure 2 to that of the filtered absolute returns in Figure 6. The filtering has evidently annihilated almost the entire periodic component of the raw absolute return correlogram. This speaks to the success of our rather elaborate intraday volatility modeling procedure. Interestingly, a somewhat unexpected feature materializes: rather than decaying at a geometric rate, the correlogram appears to follow a much slower hyperbolic rate of decay. This suggests that the high-frequency data may be useful for identifying long-memory features in the data. We explore this topic more formally in section 6.

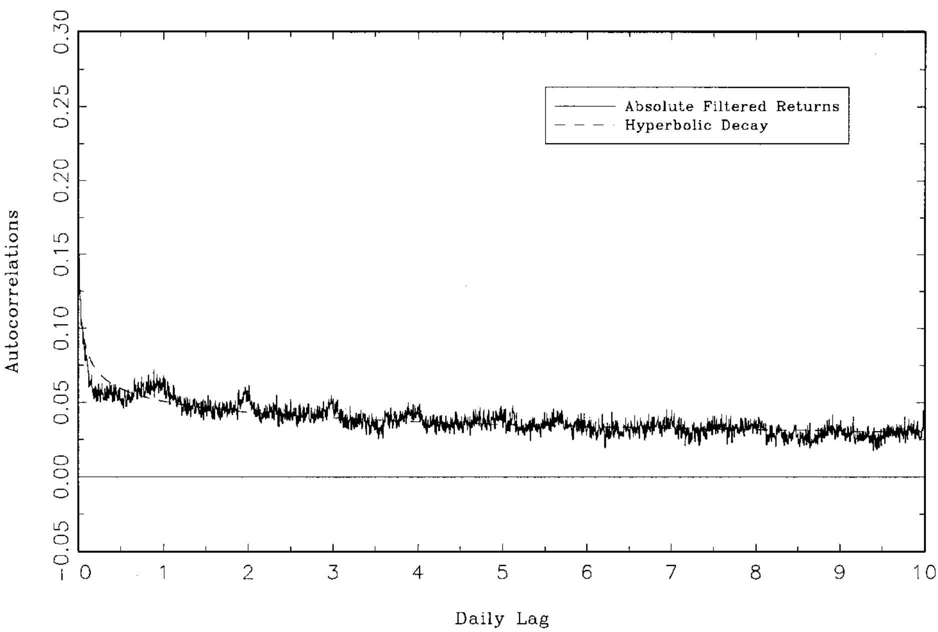

Fig. 6. Absolute Filtered Return Correlogram. The figures depicts the autocorrelations for filtered five-minute absolute DM–dollar returns, calculated from observations over the October 1, 1992, through September 29, 1993, sample period. Returns over weekends, the Tokyo lunch period, 3:00–4:45 GMT, the main holidays and a few breakdowns in the data transmission are not included, resulting in a total of 69,420 five-minute returns. The filtered returns further incorporate a standardization of the raw demeaned absolute returns by the estimated volatility impact of calendar, minor holiday and announcement effects.

5 Assessing volatility forecasts

The previous sections have shown that entirely new features of the return generating process become visible, once we move from daily to intraday returns. Nonetheless, it is not evident that we have learned anything new about the properties of daily returns, beyond perhaps the spurious nature of the day-of-the-week effect. Below, we show that the high-frequency returns indeed can help answer questions about volatility that are hard to address from daily returns alone. Specifically, this section explains why standard volatility models may provide poor forecasts of subsequent squared returns, and uses intraday returns to provide important corroborating evidence. This finally leads to the suggestion that intraday returns may be used for improved measurement of ex-post daily volatility.

Consider the popular ex-post squared return-volatility regression

$$r^2_{(m),t} = a_{(m)} + b_{(m)} \cdot \sigma^2_{(m),t} + u_{(m),t} \tag{7}$$

in which the explanatory power of the volatility forecast, $\sigma^2_{(m),t}$, is often gauged by the regression R^2, i.e., the sample variation in squared returns "explained" by the corresponding forecasts. Admittedly, there are a number of reasons to question the usefulness of this particular measure of predictive ability. In particular, it ignores predictable variation in the uncertainty associated with the forecasts. However, it has been widely used as a gauge to the practical importance of volatility forecasting.

We explore the regression (7) at the daily level, $m = 1$. First, a GARCH(1,1) model is estimated for the five-year daily sample prior to October 1, 1992. This relatively short sample alleviates concerns that a structural break in the volatility process may bias the forecasts. The resulting specification is, with robust standard errors provided below the parameter estimates,

$$\sigma^2_{(1),t} = 0.022_{(0.009)} + 0.068_{(0.016)} \cdot (\sigma_{(1),t-1} \cdot z_{(1),t-1})^2 + 0.898_{(0.023)} \cdot \sigma^2_{(1),t-1} \tag{8}$$

Out-of-sample forecasts, $\hat{\sigma}^2_{(1),t}$, for the following year are obtained from this model and the daily returns through $t-1$. Running the regression (7) based on the parameter values in (8) yields an R^2 of 0.047. This is consistent with the extant literature, which reports R^2 measures in the range 0.001 to 0.106, see, e.g., Akgiray (1989), Boudoukh, Richardson and Whitelaw (1997), Brailsford and Faff (1996), Canina and Figlewski (1993), Cumby, Figlewski and Hasbrouck (1993), Day and Lewis (1992), Dimson and Marsh (1990), Frennberg and Hansson (1995), Figlewski (1997), Heynen and Kat (1994), Jorion (1995, 1996), Pagan and Schwert (1990), Schwert (1989, 1990), Schwert and Seguin (1990) and West and Cho (1995). Most of the authors obviously recognize the inherent weaknesses of this measure, and several report alternative measures of predictive accuracy as well. However, they do not provide an explicit discussion of the expected value of the R^2 statistic.

Two questions come to mind. One, is the low degree of explanatory power disappointing? Second, does it indicate that such volatility forecasts are of limited practical value? The first question is readily answered. The low R^2 values should not be seen as surprising, but rather as a natural manifestation of the fact that the idiosyncratic variation, or noise, in financial returns is large. This implies that even an ideal estimate of return volatility will have low explanatory power vis-à-vis the corresponding future squared returns. For example, suppose returns are generated by the GARCH(1,1) model (1)–(2). Further assume, as is implicit in regression (7), that the fourth moment of $r_{(m),t}$ is finite. The population $R^2_{(m)}$ is then given by (Andersen and Bollerslev, 1997b):

$$R^2_{(m)} = \mathrm{Var}_{(m)}(\sigma^2_{(m),t}) \cdot \mathrm{Var}_{(m)}(r^2_{(m),t})^{-1} = \alpha^2_{(m)} \cdot (1 - \beta^2_{(m)} - 2 \cdot \alpha_{(m)} \cdot \beta_{(m)})^{-1} \tag{9}$$

One may verify that this R^2 is bounded from above by $\kappa^{-1}_{(m)}$, the inverse of the kurtosis for the return innovation, $z_{(m),t}$. Thus, for conditionally normal innovations, the population R^2 cannot exceed $\frac{1}{3}$, and for the empirically relevant case of conditionally fat-tailed innovations the upper bound is lower still. Furthermore, for realistic parameter values for $\alpha_{(m)}$ and $\beta_{(m)}$, the population $R^2_{(m)}$ is significantly below the upper bound. In other words, low R^2s are not an anomaly, but rather a direct implication of standard volatility models. Specifically, for the point estimates in equation (8), the implied population $R^2_{(1)}$ is 0.064. While this is slightly higher than the actual measure derived from our DM-dollar regression, it is consistent with the sample values reported in the literature.

The results illustrate that poor predictive power with respect to future squared returns is an inherent feature of standard volatility models. Thus, while such findings cannot be used as evidence against the models, they may appear to indicate some degree of practical irrelevance. If a perfectly specified model only explains a small fraction of the variability in squared returns, why then pay attention to the forecasts? The answer is that the notion of volatility, which is relevant for ex-ante financial decision making and asset pricing, is not accurately measured by the ex-post daily squared returns. The argument is most conveniently cast in the usual continuous-time setting of finance. In addition, this framework allows for an explicit adoption of a stochastic volatility diffusion for asset returns that generates weak-form GARCH(1,1) behavior at any discrete sampling frequency, thus accommodating the findings of ARCH-features at arbitrary return horizons.

Specifically, let the instantaneous returns, or change in log-exchange rates, $\mathrm{d}p_t$, be generated by the continuous-time martingale:

$$\mathrm{d}p_t = \sigma_t \cdot \mathrm{d}W_{p,t} \tag{10}$$

where $W_{p,t}$ denotes a standard Wiener process, and σ_t follows a separate diffusion. Predictability in the mean could easily be incorporated into the analysis, but the assumption of serially uncorrelated mean-zero returns greatly simplifies the notation. This assumption is also consistent with the empirical evidence for the DM-$ exchange rate analyzed throughout. It follows from (9) by Ito's lemma that the conditional variance for the one-day return, $r_{t+1} \equiv p_{t+1} - p_t$, is given by

$$E_t(r^2_{t+1}) = E_t\left(\int_0^1 \sigma^2_{t+\tau}\,\mathrm{d}\tau\right) = \int_0^1 E_t(\sigma^2_{t+\tau})\,\mathrm{d}\tau \tag{11}$$

This suggests that the daily return volatility is linked directly to the time series behavior of the quantity $\int_0^1 \sigma^2_{t+\tau}\,\mathrm{d}\tau$. The latter represents a random latent daily volatility factor, i.e., the average realization of the stochastic return volatility process over the trading day. It provides the natural generalization of the discrete-time daily volatility factor, $\sigma^2_{(1),t}$, to a diffusion setting, and it has also been explicitly derived as the key input to the pricing of derivative securities under stochastic volatility; see, e.g., Hull and White (1987), Melino (1994), Scott (1987) and Wiggins (1987).

It is evident that the performance of volatility models ideally should be assessed against the quantity that they seek to forecast, i.e., $\int_0^1 \sigma^2_{t+\tau}\,\mathrm{d}\tau$. Since this is an unobserved random variable, this is infeasible. In equation (7), the volatility forecast is instead matched up with the observed daily squared return which, from equation (11), provides an unbiased estimator of the latent volatility factor. The problem is, as emphasized in the context of equation (9), that the daily squared return is a very noise estimator of actual volatility. As such, the lack of predictive power may reflect the measurement error inherent in this proxy for the volatility factor rather than the performance of the volatility forecasts. From this perspective, a natural approach is to seek improved measurements of the latent volatility factor. In theory, this is feasible when returns are observed at high frequencies. Specifically, if the discretely sampled returns are serially uncorrelated and the sample path for σ_t is continuous, the theory of quadratic variation implies (see, e.g., KARATZAS and SHREVE, 1988) that,

$$p \lim_{m\to\infty}\left(\int_0^1 \sigma^2_{t+\tau}\, d\tau - \sum_{j=1}^{m} r^2_{(m),t,j}\right) = 0 \tag{12}$$

Hence, the daily volatility factor is, in principle, observable from the sample path realization of the returns process. In reality, due to discontinuities in the price process and a plethora of market microstructure effects, we do not obtain a continuous reading from a diffusion, so the limiting result will not apply literally. Nonetheless, it suggests that using the cumulative sum of squared five-minute returns, $\Sigma_{j=1,\ldots,288} r^2_{(288),t,j}$, rather than the usual daily squared return, may greatly improve ex-post volatility measurement, in turn resulting in more meaningful volatility forecast evaluations.

Inspired by equation (12), we construct a cumulative five-minute squared return measure for each trading day in our one-year high-frequency sample. This provides an alternative *intraday cumulative squared return* measure of ex-post daily volatility. Upon inserting this measure as the dependent variable in the volatility forecast regression (7), while retaining the identical daily GARCH(1,1) forecasts as the regressor, we obtain an R^2 of 0.479. Thus, simply by replacing the original measure of daily realized volatility by a seemingly more accurate high-frequency based alternative, the explanatory power of the identical volatility forecasts increases from less than 5% to about 48%. This suggests that standard volatility models do provide forecasts that are strongly correlated with an empirically quantifiable measure of return variability, even at the one-day-ahead horizon. This phenomenon is readily observed in Figures 7 and 8, which relate the forecasts to scaled versions of the alternative ex-post return volatility measures. Clearly, the extreme in- and outliers in the squared return series reflect the presence of a large idiosyncratic return innovation. As predicted, the cumulative five-minute squared return series is much less erratic, and the improved coherence between the GARCH forecasts and the ex-post volatility proxies is striking.

Although the strong improvement in the R^2 is encouraging, it still leaves the basic question unanswered. If cumulative squared intraday returns provide a reliable indicator of return variability, should we then expect an even higher correlation between forecasts and ex-post volatility measures? In other words, is an R^2 of 48% satisfactory? This question cannot be answered without a concrete specification of the

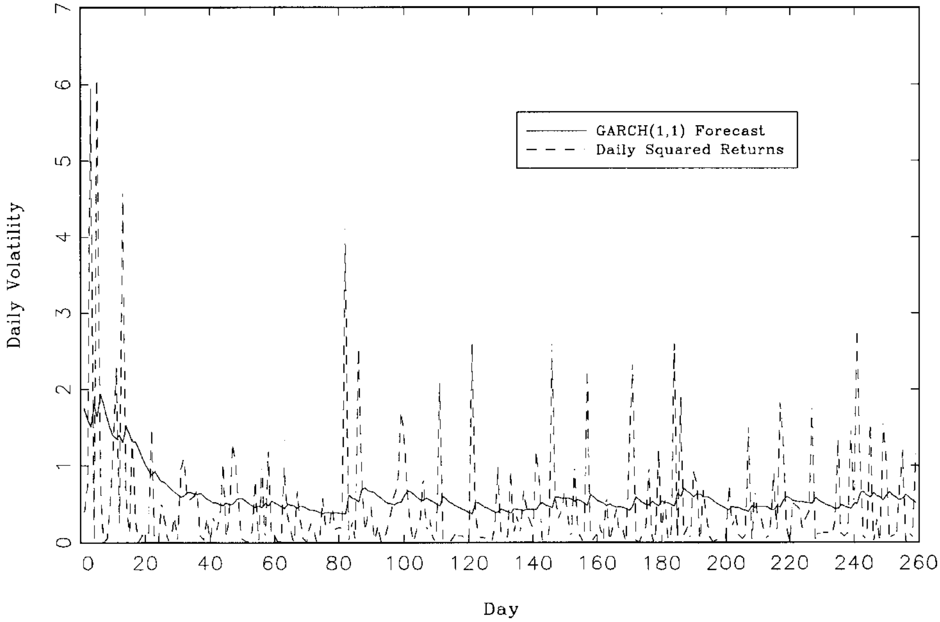

Fig. 7. Daily Squared Returns and GARCH Forecasts. The figure plots the conditional variance estimates for the daily weekdays DM–dollar returns over October 1, 1992, through September 29, 1993, obtained from a MA(1)-GARCH(1,1) model, along with the corresponding realized daily squared returns. Both series have been normalized to average unity over the one-year sample.

volatility process. A natural benchmark is given by the diffusion limit of the GARCH(1,1), as developed in NELSON (1990). It takes the form

$$d\sigma_t^2 = \theta(\omega - \sigma_t^2) \cdot dt + (2\lambda\theta)^{1/2}\sigma_t^2 \cdot dW_{\sigma,t} \tag{13}$$

where $\omega > 0$, $\theta > 0$, $0 < \lambda < 1$, and the Wiener processes, $W_{p,t}$ and $W_{\sigma,t}$, are independent. DROST and WERKER (1996) show that this diffusion is consistent with weak-form GARCH(1,1) at any discrete-time sampling frequency. Thus, if the returns process is governed by the system (10) and (13), one would expect GARCH(1,1) to perform well, and the population R^2 within this setting to provide a natural upper bound for the explanatory power of the GARCH(1,1) volatility forecasts. There is no analytic expression for the population R^2 in this case, but we may obtain the value by simulation. First, we use the direct mapping between the daily (weak-form) GARCH parameters in equation (2) and the corresponding coefficients in the stochastic volatility diffusion (13), as detailed in DROST and WERKER (1996). Concretely, our daily GARCH estimates imply that $(\omega, \theta, \lambda) = (0.636, 0.035, 0.296)$. Second, from this characterization of the returns process we generate long simulated samples, compute ex-post volatility measures such as daily squared returns and intraday (five-minute) cumulative squared returns, and construct the corresponding daily GARCH forecasts. The resulting R^2 from the volatility regression (7) can be made arbitrarily accurate by increasing the simulation size and exploiting variance

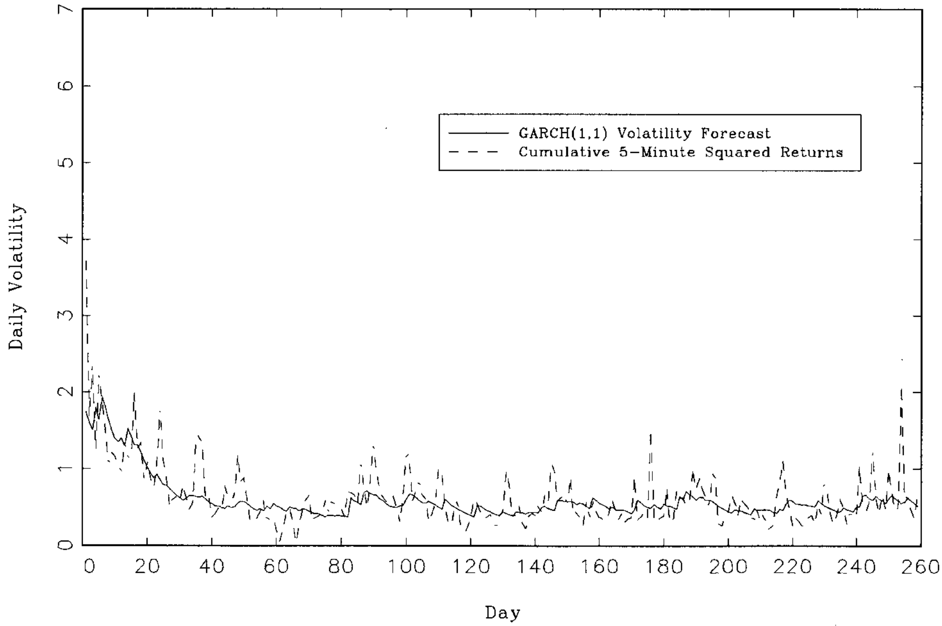

Fig. 8. Cumulative Intraday Squared Returns and GARCH Forecasts. The figure plots the conditional variance estimates for the daily weekday DM–dollar returns from October 1, 1992, through September 29, 1993, obtained from a MA(1)-GARCH(1,1) model, along with the corresponding realized cumulative squared five-minute returns. Both series have been normalized to average unity on a daily basis over the one-year sample.

reduction techniques. In so doing, we find that the GARCH(1,1) forecasts generate a population R^2 of 0.063, when daily squared returns are used as the dependent ex-post volatility measure, and an R^2 of 0.483 when the five-minute cumulative squared returns are used. These results should be compared with the R^2s of 0.047 and 0.479 obtained from the actual DM-dollar data. The close correspondence between the theoretical measures and their empirical counterparts suggest that, at least along this dimension, the plain-vanilla GARCH(1,1) performs about as well as one could ever hope for!

The simulation approach can be exploited for other purposes. Andersen and Bollerslev (1998b) provide a detailed analysis of the measurement error that is incurred by using the intraday cumulative squared returns at various sampling frequencies as proxies for the actual latent volatility process. The results solidify the above findings. For instance, it is shown, in the context of the GARCH(1,1) example, that the ex-post volatility measure based on five-minute returns is very close to the ideal limiting continuous-record case indicated in equation (12). As a result, the limiting population R^2 associated with daily GARCH(1,1) forecasts and exact measurements of volatility from a continuous price path is 0.495, just slightly above the 0.483 established earlier for the case where ex-post volatility is measured by five-minute return observations. This further supports the hypothesis that the cumulative squared five-minute returns provide a useful empirical proxy for the latent volatility factor at the daily level.

6 Long-memory in return volatility

It has long been recognized that return volatility in financial markets is highly persistent, and consistent with the findings for the daily DM-dollar series in section 3 above, many studies have argued that the volatility process appear to be integrated in variance in the sense of the IGARCH model of ENGLE and BOLLERSLEV (1986). However, these findings have recently been challenged by a number of authors including BAILLIE et al. (1996), DACOROGNA et al. (1993), DING et al. (1993), and HARVEY (1994), who argue that the long-run dependencies in the volatility are better characterized by a slowly mean-reverting fractionally integrated process. Whereas the standard covariance stationary ARCH type models imply a fast geometric rate of decay or infinite persistence, as in the IGARCH model, volatility shocks in the fractionally integrated models dissipate at a slow hyperbolic rate of decay. As such, the models are said to exhibit long memory. These issues have implications for a range of issues, including the proper approach to statistical inference in financial markets and practical long-run financial planning and asset pricing, as exemplified by the option pricing results in BOLLERSLEV and MIKKELSEN (1996, 1997).

Although the statistical evidence in favor of long memory appear quite strong, there are plausible alternative hypotheses that must be addressed. In particular, it may be the case that the volatility process is subject to occasional structural breaks. From the corresponding literature on long-run dependencies in the conditional mean, such infrequent changes in the data generating process are known to induce spurious indications of strong persistence, or apparent unit roots, in the data. A similar issue may plague the estimation of volatility dynamics. However, in contrast to the mean dynamics, the analysis of high-frequency data holds the promise of better distinguishing long-memory volatility processes from mixtures of standard volatility models with unobserved transitions or break-points. In particular, as noted in ANDERSEN and BOLLERSLEV (1997b), in the case of long memory dependencies, the autocorrelations of the absolute or squared returns from different sampling frequencies should eventually exhibit the identical long-run hyperbolic rate of decay, see also CHAMBERS (1998).

Following ANDERSEN and BOLLERSLEV (1997b), these ideas may also be rationalized from an economic perspective. Return volatility is often modeled as driven by the random arrival of new information within the so-called mixture-of-distribution characterization of the price process, e.g., CLARK (1973), TAUCHEN and PITTS (1983), HARRIS (1987), ROSS (1989) and ANDERSEN (1996). If the volatility is seen as impacted by numerous different information arrival processes with distinct persistence characteristics, the mixture of these arrival processes may induce long memory in the volatility process. Since the resulting degree of fractional integration should be invariant across sampling frequencies, high-frequency data provide the opportunity to estimate this parameter over relatively short calendar time spans involving a large number of observations. The intertemporal stability of the estimated degree of fractional integration provides a further test of this hypothesis.

To illustrate, consider again the five-minute DM-dollar return series. Let $\rho(|r|, j)$ denote the jth order autocorrelation coefficient for the absolute returns process. A fractionally integrated, or long memory, volatility process then implies that the correlogram dies out at a hyperbolic rate, i.e.

$$\rho(|r|, j) \sim j^{2d-1}, \quad \text{for } 0 < d < \tfrac{1}{2} \text{ and } j \to \infty \tag{14}$$

In contrast, standard short memory models imply a geometric decay in the autocorrelogram, or $\rho(|r|, j) \sim k^j$ for $j \to \infty$, where $k > 0$. However, we have already observed that the correlogram for the absolute filtered returns appears to decay at a hyperbolic rate, consistent with equation (14), and indicated by the dotted line in the Figure 6, corresponding to $d = 0.387$. This approach for estimating d is obviously heuristic, but this value for d turns out to be suggestive of the findings obtained by more formal means below.

First, recall that the process y_t is defined to be integrated of order d, or $I(d)$, if $(1 - L)^d y_t$ is stationary and ergodic with a positive and bounded spectrum across all frequencies, where L denotes the lag operator, and the fractional differencing operator is defined by the binomial expansion, $(1 - L)^d \equiv \Sigma_{j=0,\ldots,\infty} [\Gamma(j-d)\Gamma(-d)/\Gamma(j+1)]L^j$, with $\Gamma(\cdot)$ denoting the gamma function. Now, let $f_{|r|}(\omega)$ denote the spectrum for the absolute returns process. If the process is fractionally integrated of order d, or $I(d)$, $0 < d < \frac{1}{2}$, the spectrum at the lowest frequencies, or $\omega \approx 0$, is approximately given by $f_{|r|}(\omega) \approx c \cdot |\omega|^{-2 \cdot d}$, where c denotes a normalizing constant. Thus, on taking logarithms,

$$\ln(f_{|r|}(\omega)) \approx \ln(c) - 2 \cdot d \cdot \ln(\omega) \tag{15}$$

An approximate log–linear relationship for low values of ω is evident in Figure 9, which displays the estimated spectrum for the five-minute DM-\$ absolute return series. Note also the spikes at $\omega = 2 \cdot \pi/288 \approx 0.0218$ and the seasonal harmonics corresponding to the daily frequency.

The relationship in equation (15) forms the basis for several alternative semiparametric frequency-domain estimation procedures. For illustration, we apply two such estimators for d. The first is the Geweke and Porter-Hudak (1983) procedure which estimates d directly from the sample periodogram regression implied by equation (15) over suitably low-frequency ordinates. The second is the procedure proposed by Robinson (1994) based on a ratio of the averaged periodogram over appropriate ranges of the low-frequency ordinates. For the five-minute DM-dollar absolute return series these two estimates are 0.321 and 0.385, respectively. In order to check the aforementioned invariance, we applied the identical semiparametric estimation procedures to the entire set of return frequencies listed in Table 2. The results, reported in Andersen and Bollerslev (1997b), show that the estimates for d

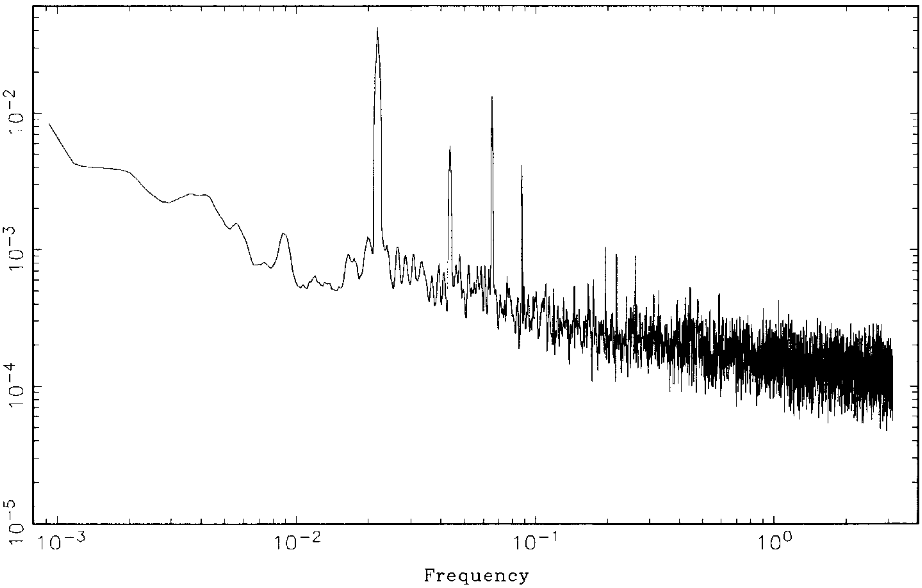

Fig. 9. Spectrum of Five-Minute Absolute Returns. The figure graphs the spectrum for the absolute five-minute DM-dollar returns on weekdays over the October 1, 1992, through September 29, 1993, sample period, with a total of 74,880 return observations. The spectrum is estimated as the smoothed sample periodogram using a triangular kernel with bandwidth of ten.

are remarkably stable across the different absolute return horizons. Furthermore, the results are fully consistent with the suggestive evidence from the time domain autocorrelogram, once the complex high-frequency dynamics have been explicitly accounted for. However, the frequency domain estimates are obtained without making any assumptions about the short-run dynamics. Finally, it is also noteworthy that these estimates for d are directly in line with the findings from the extant literature that relies on interdaily returns over much longer time spans. For instance, on applying the same two semiparametric estimation procedures to the long daily DM-dollar series, ANDERSEN and BOLLERSLEV (1997b) obtain estimates of 0.344 and 0.301, respectively.

In conclusion, the striking stability of the estimates for the degree of fractional integration in the absolute return process across varying return horizons and widely different time spans clearly suggest that the long memory feature is an inherent characteristic of the return generating process. Moreover, it is obvious that the high-frequency returns permit a much more thorough and extensive investigation of such features than would be possible from interdaily data alone.

7 Conclusion

We have argued that high-frequency intradaily returns contain a great deal of information about financial market volatility beyond that available from daily or lower frequency observations. Also, this additional information is not simply restricted to intradaily phenomena. The high-frequency returns may be used to construct improved ex-post interdaily return volatility measurements, enhance our ability to separate short-run and long-run volatility components, increase our understanding of day-of-the-week and news announcement effects, and even allow for improved estimates of the apparent long-memory features in return volatility. Only future research will reveal the full extent of these and other related new insights.

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